MACROECONOMIC CREDIT RISK MODEL[135]

1. INTRODUCTION

As mentioned in this Report, credit risk – despite having declined – remains the main risk to financial stability in the Czech Republic. At the level of individual debtors, credit risk depends primarily on the individual characteristics of those debtors (financial results, ability to repay, age, income, etc.), while at the systemic level, credit risk can be expected to develop cyclically depending on the evolution of key macroeconomic indicators.

This article focuses on the macroeconomic default rate model in the Czech economy. The aim is to produce a model allowing us to estimate the expected proportion of bad loans in the total loan portfolio of banks in response to the evolution of key macroeconomic indicators. The proportion of bad loans is one of the inputs to the stress testing model developed by the CNB.[136] It has so far been regarded as a constant parameter estimated from extreme historical events. The new approach enables modelling of the impacts of various macroeconomic shocks on loan portfolio quality and subsequently, in combination with the stress-testing system, on the capital of the entire banking system. Such shocks may be set either expertly on the basis of historical experience or constructed in the form of alternative scenarios linked to the CNB's main macroeconomic forecasting model.

The article is structured in the following way. Section 2 gives a brief summary of the possible theoretical approaches to credit risk modelling and also notes the approaches to this issue applied by other central banks. Section 3 discusses the time series used to estimate the model. Section 4 describes the results of the model applied to the Czech economy. The next section focuses on the application of the outputs of the model for stress testing. The conclusion sums up the results achieved and discusses other possible areas of development of credit risk modelling in the Czech Republic. The article also includes a technical annex which presents a description of the theoretical assumptions and the derivation of the econometric model used.

2. CREDIT RISK MODELS

2.1. Basic approaches to credit risk modelling

There are two main classes of models in credit risk modelling. The first type, which aims to estimate the risk profiles of individual debtors and is applied mainly in the everyday work of commercial banks, can be described as an individual credit risk model. Even within these models banks may include macroeconomic indicators among the explanatory variables in order to avoid the problem of pro-cyclical credit risk assessment.[137] Outputs from individual credit risk models can be used for calculating banks' capital requirements under the Internal Ratings Based approach (IRB) of the New Basel Capital Accord (NBCA),[138] which will be binding as from 2007.

This article makes use of the other type of credit models based on macroeconomic credit risk modelling. These models aim to estimate changes in credit risk at the aggregate level; they are therefore used for evaluating systemic risk or for evaluating financial stability in the economy.

Three main approaches are used within macroeconomic credit risk models. These approaches are based methodologically on models of individual risks. The first, traditional and frequently used view is based on finding an empirical relationship between a dependent variable representing loan portfolio quality and key macroeconomic indicators.[139] The transmission channel between macroeconomic indicators and the credit risk indicator is relatively difficult to trace. The second approach works with more advanced models based on structural models of individual risk, which are grounded on a microeconomic explanation of the creation of credit risk.[140] The third method for modelling credit risk is to apply "reduced models", which use data on the market prices of corporate bonds and shares as inputs. The advantage of this third type of model is the use of information hidden in such prices. However, these models are of little use for credit risk analysis in the Czech Republic, given its underdeveloped capital market.

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135 Petr Jakubík, CNB.

136 The stress testing methodology is described in detail in Čihák and Heřmánek (2005).

137 That is, the problem where the credit risk of a single entity would be assessed in positive terms during a period of economic growth and in negative terms during a period of economic slowdown. Credit risk models which would fail to address the issue of pro-cyclicality might result in a further strengthening of the economic downturn.

138 See Gordy (2003) and Finger (2001); a single factor model was used to calibrate risk weights for the purposes of BASEL II (probability of default, correlation of debtors' assets for individual risk classes). Applications of the model to the German economy can be found, for example, in Rösch (2003) or Hamerle, Liebig and Scheule (2004).

139 Empirical models of credit risk are discussed, for example by Bunn, Cunningham and Drehmann (2005), Deutsche Bundesbank (2005), Babouček and Jančar (2005) and Virolainen (2004).

The second approach was chosen to develop a model for the purposes of stress testing in the CNB. The aim was to estimate the potential future development of bad loans in banks' portfolios in response to changes in the macroeconomic environment. The approach chosen by the CNB is based on a Merton-type model[141], which models a debtor's default as an event occurring if the return on his assets falls below a certain threshold. An assumption was made for the model that the threshold depends on macroeconomic variables. The threshold is most probably lower in recessions and higher in booms. This model furthermore assumes that the value of the return depends, in addition to observed factors, on unobservable (latent) factors which can be explained microeconomically and which are assumed to have particular distributions.[142]

2.2 Credit risk models in other central banks

Most central banks employ some form of sensitivity analysis or stress testing, but only a few of them use a macroeconomic credit model. Where central banks do use such macroeconomic credit models, they are mostly empirical-type models, as, for example, in the case of the United Kingdom, Germany, Belgium and Finland. The Bank of England uses an empirical model[143] which estimates the bankruptcy rate of non-financial corporations and the default rate in the mortgage and credit card portfolios. The outputs generated in this manner are then entered into credit loss estimation models as explanatory variables. The default rates are estimated from real GDP, the real interest rate, unemployment, the corporate debt ratio and other aggregate indicators. Finland uses a macroeconomic model based on logistic regression[144] which explains the default rate relationship for individual sectors of the economy using macroeconomic indicators.[145] This model regards real GDP, nominal interest rates and the debt ratios of the individual sectors investigated as the explanatory variables. The default rate is modelled using the bankruptcy rate of companies in the total number of companies for the given sector of the economy. The Hungarian central bank is also preparing a credit model which uses the number of bankruptcies of companies for individual sectors of the economy, based on the approach employed by the Finnish central bank. Germany used a regression model estimated on a panel of German banks. The dependent variable here is a logistic transformation of the proportion of provisions in the credit portfolio. This model works with the change in the risk-free interest rate, GDP growth and loan portfolio growth as the macroeconomic indicators in the role of explanatory variables.[146] The Belgian central bank uses a model based on logistic regression estimating the aggregate default rate of the corporate sector. The output gap, nominal long-term interest rates and the lagged rate of aggregate corporate default are used as the explanatory variables. Generally speaking, the development of macroeconomic credit risk models has become an important area of interest of central banks as institutions pursuing financial stability. However, the topic associated with these models is undergoing very rapid development and there is no overall consensus on which model is the best.

3. DATA USED

Quarterly data for the Czech economy have been used for all calculations. The model is based on time series of bad loans and selected macroeconomic indicators.

3.1. Bad loans

The (dependent) credit risk variable or default variable estimated in the model can be defined in several ways. A default event is commonly defined as payment delinquency. A 12-month default probability is usually employed in credit risk assessments. This is defined for a given moment as the probability of a default event occurring in a 12-month period following that given moment, provided that the given person did not default in the period immediately preceding the given moment. This definition thus corresponds to new default events in the economy.

140 Structural models are addressed, for example, by Jakubík (2006).

141 Merton-type models are based on option pricing models, which estimate the value of a company as a price of a put option. This idea was discussed for the first time in Merton (1974).

142 Only one unobservable factor was considered for the purposes of the estimate, so the model is referred to as one-factor. A detailed and more technical description of the approach used can be found in the Annex.

143 A description of the macroeconomic credit risk model used by the Bank of England can be found in Bunn, Cunningham and Drehmann (2005).

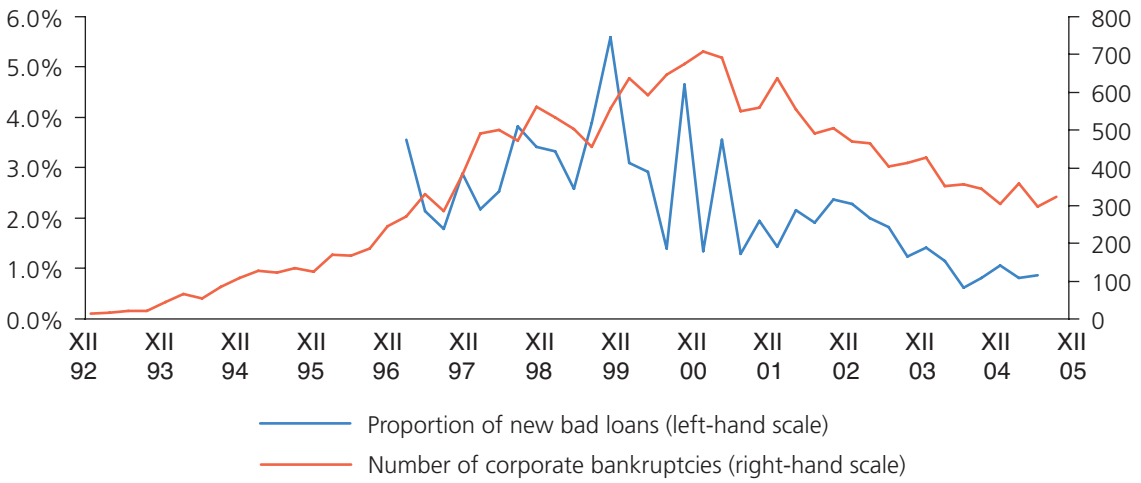
144 The regressive logistic model corresponds to a linear regression applied after logistic transformation of the dependent variable. The logistic transformation of the dependent variable *y* corresponds to $\ln \frac{y}{1-y}$. For credit models, this expression transforms the original value from the interval [0;1] to values from the entire real axis.

145 The macroeconomic credit risk model for the Finnish economy is described in Virolainen (2004).

146 The macroeconomic model of aggregate corporate default is discussed in National Bank of Belgium (2005).

In our model, the default rate was modelled by the proportion of new bad loans in the total volume of loans in the economy.[147] Quarterly time series of new bad loans were available from 1997 Q1 to 2005 Q3. They were, however, affected by one-off measures entailing reclassification of outstanding mortgage-backed loans in 1999–2001.[148] This period saw significant deviations in the calculated proportion of newly classified loans in the banking portfolio. However, this reclassification did not in fact change the true quality of these portfolios and can be seen as a way of making the indicator of the stock of classified loans more realistic.

Chart 1 – Corporate default rate
(quarterly data)



Source: **CNB, Ministry of Justice of the Czech Republic**

The special (dummy) variable used took a value of 1 for quarters when the monitored indicator saw significant deviations from the observed trend. The quarters include 1999 Q3, 1999 Q4, 2000 Q4 and 2002 Q2. In other cases, this variable takes the value of 0. The dummy variable so defined corresponds to the effect of changes in the approach to loan classification.

An alternative approach to approximating the default rate in the economy is to use time series of the number of adjudicated bankruptcies or compositions. This approach has been used, for example, to estimate the macroeconomic credit risk model of the Finnish economy.[149] For the Czech Republic, such data have been available since the start of the transformation. However, they have probably had a higher information content only since the late 1990s.[150] The quarterly development of the number of adjudicated bankruptcies in the Czech Republic is demonstrated in Chart 1. In practice there seems to be a lag between the filing of a petition for bankruptcy and the actual adjudication, and the default event in the loan portfolio usually precedes the adjudication of bankruptcy. The application of such time series for the Czech economy may also be limited by the frequent amendments made to the relevant legislation.[151] Given these facts, the time series of bankrupts in the end was not used to estimate the macroeconomic credit model for the Czech economy. Nevertheless, Chart 1 confirms the similar development of this time series and the share of growth in classified loans in the loan portfolio.

147 That is, loans which became "bad" in the given quarter. The moment of default means the time when the loan was classified as substandard or worse for the first time. Shifts within the "bad" loans category (for example, a further downgrading of the loan from doubtful to loss) will not affect the default rate according to this definition. This variable does not correspond to the proportion of total classified or non-performing loans, which are not an optimum measure of credit risk as they may include loans which were first classified a very long time ago and which remain in the loan portfolio, for example, for accounting purposes and are not related to the current economic situation.

148 CNB Provision of 17 September 1997 stipulating the principles for classifying loan receivables and for provisioning for these receivables, as amended.

149 Macroeconomic models of the credit risk of the Finnish economy using the number of corporate bankruptcies can be found in Virolainen (2004) and Jakubík (2006).

150 The time series of bankruptcies shows that the number of bankruptcies at the start of the 1990s was very low, probably as a result of inadequate legislation.

151 Legislative aspects are discussed in detail in the following article *The Impact of Insolvency Law on Financial Stability* in the thematic part of this report.

3.2. Macroeconomic indicators considered

Various macroeconomic indicators are used as explanatory variables relating to the indicator of the default rate in the economy. Interest rates and gross domestic product are most commonly considered in this context.[152] Gross domestic product (GDP) is a basic indicator of the cyclical position of the economy. A decline or low growth in GDP affects credit risk, for example via negative effects on corporate earnings, wage growth, unemployment or prices of assets (such as real estate), which, in turn, leads to a deterioration in loan portfolio quality. A rise in interest rates affects the loan portfolio in a similar way, increasing the costs of corporate and household financing, decreasing the market value of assets, etc.

In the case of GDP, annual real GDP growth was applied. One-month and one-year PRIBOR interbank rates were considered as nominal interest rates. Real interest rates[153] were deflated ex post by the consumer price index. The real effective exchange rate and the nominal koruna-euro and koruna-dollar rates were also considered among the explanatory variables. They are important for credit risk given the nature of the Czech economy as a small open economy where the financial condition of the corporate sector in particular strongly depends on the exchange rate. The last indicator used was the level of indebtedness of the economy, measured by the ratio of client loans to GDP, which approximates the exposure of the financial sector to the rest of the private sector.

In selecting the set of macroeconomic indicators, the issue of the interpretability of the results obtained was also taken into account. Emphasis was put on obtaining the relationship between credit risk, represented by growth in bad loans in the banking portfolio, and the macroeconomic indicators which already enter the stress testing scenarios.[154] Another partial limitation on the selection of the variables was the effort to link this credit risk model to the results of the CNB's macroeconomic forecast.[155]

4. ESTIMATION OF THE MODEL

Taking into account the criteria for the selection of variables relating to the stress testing scenarios and the outputs of the CNB's macroeconomic forecast, we selected the statistically best model containing GDP, the nominal interest rate, inflation and the dummy variable for the purposes of a change in methodology with a subsequent one-off impact on reclassification of the loan portfolio. In the case of GDP, non-lagged annual real GDP growth was used. The statistically most significant interest rate was the nominal 1Y PRIBOR lagged by four quarters. In the case of inflation, the annual rate of growth of the average quarterly CPI lagged by two quarters was the most significant. The model was also tested without the dummy variable. This gave very similar results, although it slightly overestimated the default rate at the end of the period under review, showing that the chosen model has some degree of robustness.

Table 1 shows the results of the estimated model.[156] All the estimates were significant at least at the 5% confidence level. The default rate in the economy is negatively related to gross domestic product, hence higher GDP growth leads to lower credit risk. By contrast, the level of credit risk is positively related to interest rates, which is also consistent with economic intuition. Including inflation in the model reduces the effect of nominal

Tab. 1 – Model of the default rate in the economy

Description of variable corresponding to estimated coefficient	Denoted by	Estimate	Standard error	Pr>ltl
Constant (β_0)	c	-2.0731	0.1019	<0.0001
Gross domestic product (β_1)	hdp	-4.9947	1.9613	0.0162
Nominal interest rate (β_2)	R_{t-4}	2.7839	0.9076	0.0045
Inflation (β_3)	π_{t-2}	-2.4364	1.0994	0.0344
Dummy (β_4)	dum	0.3296	0.0663	<0.0001
Effect of latent factor (ρ)		0.0121	0.0032	0.0008

Source: CNB

152 For a discussion of the issue of explanatory macroeconomic indicators, see, for example, Virolainen (2004), Deutsche Bundesbank (2005), Rösch (2003) and Jakubík (2006).

153 An internal CNB calculation based on CPIs and continuous weights corresponding to the average previous annual trade turnover was used to calculate the real exchange rate.

154 These indicators thus affect the resulting capital adequacy in the stress testing through two channels. The first acts directly via their effect on banks' balance sheets, while the other operates indirectly via the estimate of credit risk.

155 The results of the CNB's macroeconomic forecast are regularly discussed in the Inflation Reports published quarterly by the CNB.

156 See the Annex for the technical specification of the model used.

interest rates lagged by four quarters by real inflation lagged by two quarters. For this reason, the estimate of the coefficient representing inflation in the model is negative. The combination of nominal interest rates and inflation demonstrates that the credit default rate in the Czech economy depends on real interest rates rather than nominal rates, although the estimated coefficients are not exactly the same and have different lags. The statistical significance of the effect of the unobservable component shows that this factor is still necessary for explaining the dependent variable, despite the inclusion of macroeconomic indicators.[157] This result implies that the default rate in the economy is also affected by other factors than macroeconomic indicators.

The estimated form of the functional relationship for the development of the default rate in the economy is provided by equation (1).[158]

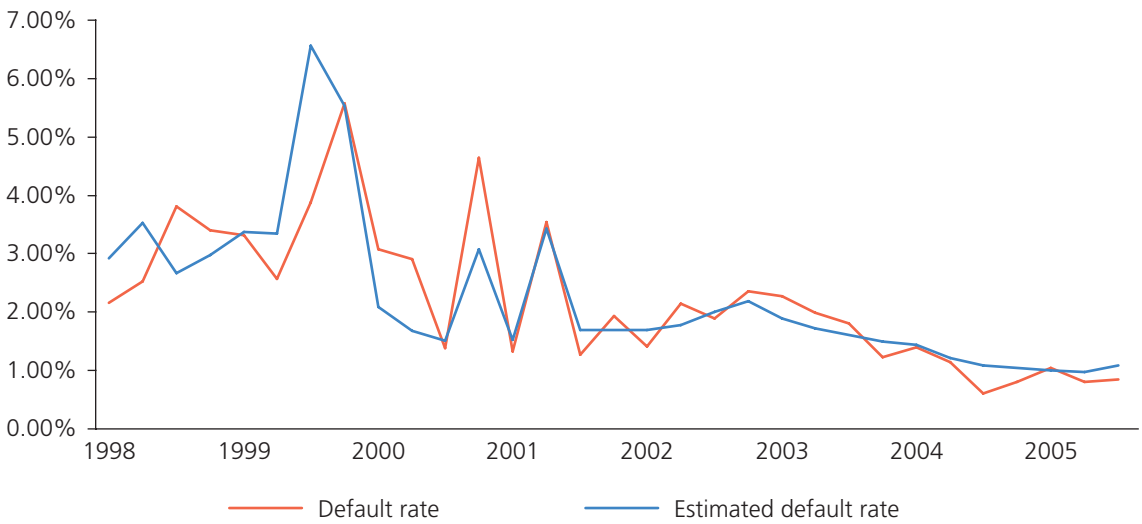
$$df_t = \phi(-2{,}0731 - 4{,}9947hdp_t + 2{,}7839R_{t-4} - 2{,}4364\pi_{t-2} + 0{,}3296dum_t) \quad (1)$$

The dummy variable will continue to take the value of zero for the credit risk estimates. This implies that relationship (1) can be simply rewritten in the form of (2) for the purposes of estimating the quarterly default rate.

$$df_t = \phi(-2{,}0731 - 4{,}9947hdp_t + 2{,}7839R_{t-4} - 2{,}4364\pi_{t-2}) \quad (2)$$

The coefficients from equations (1) and (2) cannot be simply interpreted as the commonly used elasticities of the impacts of the relevant macroeconomic factors on credit risk, as they are further recalculated using the cumulative distribution function of a normal distribution, hence their impact is not linear. A simple sensitivity analysis of the impacts of changes in the macroeconomic variables is given in section 5 of this article.

Chart 2 – Estimated default rate model
(quarterly data)



Source: **CNB**

The ability to explain the quarterly default rate by means of the estimated model (1) is shown in Chart 2. The estimated model is a variant of the binary choice model[159], to which the standard approaches to measuring the statistical significance of an estimate cannot be applied. However, there are numerous less common indicators which can be applied and which suggest that the model has a good performance.[160]

157 The latent factor expressed the unobservable part of the macroeconomic risk in the model, which cannot be explained by macroeconomic indicators. See the Annex for a detailed explanation of this coefficient.

158 The symbol ϕ denotes the distribution function of the normal distribution. Although the coefficient ρ from Table 1 is not present in equation (1), it is necessary for the estimate of the model. See the technical annex to this article for a detailed discussion of its role in the estimate and interpretation.

159 Binary models generally consider situations with two possible realisations of a dependent variable (0,1). They consist in estimating the probability of one of the events occurring. A formal description of the model used is included in the technical annex to this article.

160 For example, pseudo-coefficients of determination: Estrella $R_E^2=0{,}97$, Cragg-Uhler $R_{CU1}^2=0{,}95$, $R_{CU2}^2=0{,}95$, Veall-Zimmermann $R_{VZ}^2=0{,}80$. See the Annex for definitions of the indicators.

5. USE OF THE MODEL IN STRESS TESTING

Using the estimated model, the impacts of macroeconomic shocks on the default rate of the banking portfolio can be tested at the level of the aggregate economy. The estimated model is based on quarterly time series, so the estimated default rate is also a quarterly figure, which needs to be annualised for the purposes of stress testing.[161]

In order to forecast the default rate, we have to set the inputs to the macroeconomic credit model, which will also be used as the stress testing parameters. They include non-lagged annual real growth, nominal annual interest rates lagged by four quarters and annual inflation lagged by two quarters relative to the forecast horizon. These values can be set either expertly or as a percentage deviation from the macroeconomic forecasts drawn up by the CNB or as outputs from the CNB's macroeconomic model under an assumption of significant, improbable, but not entirely impossible, negative macroeconomic shocks.

The following Table 2 gives the results of the macroeconomic credit model for different combinations of values for GDP growth, nominal interest rates and the inflation rate. These are merely illustrative examples of the sensitivity of the credit risk indicator for different combinations of the explanatory variables, and are not the actual values entering the stress testing. Table 2 shows that the sensitivity of credit risk for example to a change in GDP growth of 1 percentage point differs *ceteris paribus* depending on the rate of such growth. For higher GDP growth rates, the impacts of a decline in growth of 1 percentage point are lower than for lower growth rates. The underlying reason is that the chosen variant of the model or estimation of the model (6) uses a calculation based on the cumulative distribution function of a normal distribution. A similar conclusion applies to the other variables in the model.

Tab. 2 – Sensitivity analysis of the model (quarterly change in bad loans in response to the value of exogenous variables)[162]

		GDP growth					
CPI	R	-2%	-1%	0%	1%	2%	3%
	2%	2.6%	2.3%	2.1%	1.8%	1.6%	1.4%
	4%	3.0%	2.6%	2.4%	2.1%	1.8%	1.6%
1%	6%	3.4%	3.0%	2.7%	2.4%	2.1%	1.9%
	8%	3.8%	3.4%	3.0%	2.7%	2.4%	2.1%
	10%	4.3%	3.8%	3.4%	3.1%	2.7%	2.4%
	4%	2.8%	2.5%	2.2%	2.0%	1.7%	1.5%
2%	6%	3.2%	2.8%	2.5%	2.2%	2.0%	1.8%
	8%	3.6%	3.2%	2.9%	2.6%	2.3%	2.0%
	10%	4.1%	3.6%	3.3%	2.9%	2.6%	2.3%
	4%	2.6%	2.4%	2.1%	1.9%	1.6%	1.4%
3%	6%	3.0%	2.7%	2.4%	2.1%	1.9%	1.7%
	8%	3.4%	3.0%	2.7%	2.4%	2.2%	1.9%
	10%	3.9%	3.5%	3.1%	2.8%	2.5%	2.2%
	6%	2.8%	2.5%	2.3%	2.0%	1.8%	1.6%
4%	8%	3.2%	2.9%	2.6%	2.3%	2.0%	1.8%
	10%	3.7%	3.3%	2.9%	2.6%	2.3%	2.1%
	6%	2.7%	2.4%	2.1%	1.9%	1.7%	1.5%
5%	8%	3.1%	2.7%	2.4%	2.2%	1.9%	1.7%
	10%	3.5%	3.1%	2.8%	2.5%	2.2%	1.9%

The results of the macroeconomic credit model are used in the current version of stress testing for estimating the proportion of bad loans in the portfolio, which is then entered in the stress testing as an input parameter. The credit model allows us to generate bad loans in the banking portfolio as a result of a shock in the form of a change in real GDP growth, nominal interest rates or inflation.

161 For the annualisation methods, see the preceding article *Summary of Results of Stress Tests in Banks* in this Financial Stability Report.

162 The sensitivity analysis uses non-lagged GDP growth, CPI inflation lagged by 2 quarters and nominal interest rates lagged by 4 quarters.

6. CONCLUSIONS

In order to develop a macroeconomic credit risk model for the Czech economy, we used a one-factor Merton-type model estimated for the aggregate economy. The model confirmed a very strong link between bank portfolio quality and the macroeconomic environment. The estimated macroeconomic credit risk model was incorporated into the existing version of stress testing, thus allowing us to find a link between credit portfolio quality and the macroeconomic environment. One of the possible improvements to the model would be to make it dynamic, which would make it possible to take into account the correlation of assets over time. The chosen type of model also allows it to be extended by including microeconomic data in the model or estimating the model based on sectoral data. The issue of estimating the probability of default in the aggregate credit portfolio is closely associated with a variable referred to as "loss given default". The current incorporation of the macroeconomic credit model into stress testing assumes the worst case scenario, i.e. a 100% loss. The modelling of the impact of macroeconomic shocks on the volume of bad loans in the portfolio could be made more precise in the future by estimating a model of loss given default as a function of the probability of default based on aggregate data.

ANNEX

The Macroeconomic Model Used – One-Factor Merton-type Model

The following equations describe the version of the latent factor model used, which appears in numerous papers.[163] The fundamental idea is based on the Merton model.[164] A random process with a standard normal distribution is assumed for the standardised logarithmic return on assets of a firm. The discrete normal logarithmic return satisfies the following equation for each firm in the economy.

$$R_{it} = \sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} \tag{3}$$

R denotes the logarithmic return on assets for each firm *i* at time *t*. *F* corresponds to the logarithmic return in the economy independent of firm *i* at time *t*, which is assumed to be a random variable with a standard normal distribution. This variable represents the part of the return which is not specific to the firm and can thus satisfy the general conditions for profitability of firms in the economy. *U* denotes the return specific to the firm, which is again assumed to be random with a standard normal distribution. The two random variables are also assumed to be serially independent. The coefficient ρ expresses the correlation between the returns on assets of any two debtors. Given these assumptions, the logarithmic return on assets of each firm also has a standard normal distribution. The model is based on the Merton approach, according to which a default event occurs if the return on a firm's assets falls below a certain threshold. The applied variant of the model also assumes that the value of this threshold changes depending on changes in the macroeconomic environment. The value is modelled as a linear combination of macroeconomic variables. Based on all these assumptions, the probability of default of the firm can be derived, with ϕ denoting the distribution function of the normal distribution and x_{jt} denoting the macroeconomic indicators included in the model (gross domestic product, nominal interest rate, inflation and dummy variable).

$$p_{it} = P(R_{it} < T) = P(\sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} < \beta_0 + \sum_{j=1}^{K} \beta_j x_{jt}) = \phi(\beta_0 + \sum_{j=1}^{K} \beta_j x_{jt}) \tag{4}$$

This enables us to derive the relationship for the conditional probability of default in response to the realisation of an unobservable factor (f_t denotes realisation of the unobservable factor F_t).[165]

$$p_i(f_t) = \phi\left(\frac{\beta_0 + \sum_{j=1}^{K} \beta_j x_{jt} - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right) \tag{5}$$

The same formal notation can be reached if we consider the effect of macroeconomic indicators within a factor which corresponds to the return independent of the firm, that is, a factor common to the entire portfolio of firms under consideration.

163 The latent factor model is found, for example, in Jakubík (2006), Rösch (2005), Céspedes and Martín (2002), Cipollini and Missaglia (2005) and Lucas and Klaassen (2003).

164 The model was formulated for the first time in Merton (1974).

165 The unobservable factor, or latent factor, is a random variable representing the return on assets of firms which is common to firms in the whole economic sector studied, in our case the whole aggregate economy. The realisation of this random variable cannot be observed, but one can make an assumption regarding its distribution. A normal distribution of this variable is considered here, although other forms of distribution, such as a logistic distribution, could also be used.

If we furthermore assume a homogenous portfolio of firms in the economy whose returns on assets correspond to process (3), the average default rate in the economy is then – based on the law of large numbers – equivalent to the probability of default of a firm. Given the assumption of homogeneity of firms in the economy, it is more appropriate to estimate the model on the basis of sectoral data. However, as the necessary data were not available, the model was only estimated on aggregate data for the whole economy. Therefore, some of the factors which might play a significant role in a particular sector may not be significant in the model estimated for the whole economy.

In order to estimate the model (4), a relationship with a conditional number of defaults of firms depending on the realisation of the random variable F representing the latent factor was used. The conditional number of defaults depending on the realisation of the random factor is a random variable which, under the given assumptions, has a binomial distribution, with the parameters of conditional probability $p(f_t)$ given by equation (5) and the number of firms N_t.

$$D(f_t) \approx Bi(N_t, p(f_t)) \tag{6}$$

The total number of firms and the number of firms in default in the economy were not available for individual periods. Aggregate data on growth in banks' bad loans were employed in the estimation of the model for individual quarters. To this end, the following line of reasoning was followed. Each koruna of a loan was considered an individual loan of a single client. In such case, therefore, the random variable *D* corresponds to the number of new bad koruna loans, or the growth in the volume of bad loans, while *N* stands for the total volume of loans granted. A default event is represented here by non-repayment of a loan of CZK 1. Under these assumptions, the volume of bad loans can be modelled by means of the relation (6).[166] The model was estimated by maximising a likelihood function containing a random latent factor, which was assumed to have a standard normal distribution.

A number of characteristics measuring the quality of the estimate can be applied to the above described model. One of the tests of model quality is a test of the hypothesis that all the coefficients β_j except the constant member are zero (H_0: $\beta_1 = \beta_2 = \ldots = \beta_K = 0$). This hypothesis can be tested by means of the likelihood ratio $\lambda = L_C / L_U$. The known result says that – 2lnλ is an asymptotic chi-distributed variable with K degrees of freedom.[167] The results of the test rejected the hypothesis at a significance level of less than 1%.

The observed criteria of the pseudo-coefficients of determination based on the likelihood function also bear out the good quality of the model. These coefficients should be in the interval [0;1], with results close to 1 attesting to very good model quality.

$$R_E^{\ 2} = 1 - \left(\frac{\ln L_U}{\ln L_C)} \right)^{-\frac{2}{n}\ln L_C} = 0{,}97$$ Estrella (1998)

$$R_{CU1}^{\ 2} = 1 - \left(\frac{L_C}{L_U} \right)^{\frac{2}{n}} = 0{,}95$$ Cragg-Uhler (1970)

$$R_{CU2}^{\ 2} = \frac{1 - \left(\frac{L_C}{L_U} \right)^{\frac{2}{n}}}{1 - L_C^{\ \frac{2}{n}}} = 0{,}95$$ Cragg-Uhler (1970)

$$R_{VZ} = \frac{2(\ln L_U - \ln L_C)}{2(\ln L_U - \ln L_C) + n} \frac{2\ln L_C - n}{2\ln L_C} = 0{,}80$$ Veall-Zimmermann (1992)

166 The assumption regarding koruna loans is somewhat simplified, as koruna loans are not in fact independent.

167 The known result of the distribution – 2lnλ is given, for example, in Rao (1973).

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