DOES MY MODEL PREDICT A FORWARD GUIDANCE PUZZLE?

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OBJECTIVES

We provide sufficient conditions to identify when a forward guidance puzzle will occur and provide a novel resolution to the Forward Guidance Puzzle: sunspots.

1. Show that (I)terative (E)xpectation Stabiliy is a sufficient condition for ruling out the Forward Guidance Puzzle
2. IE-stability conditions are distinct from determinacy conditions, which illustrates that indetermancy does not imply the existence of the Forward Guidance Puzzle
3. Show that indeterminacy zero lower bound solutions resolve the Forward Guidance Puzzle

SIMPLE NK MODELS

Application to [3] BNK model:

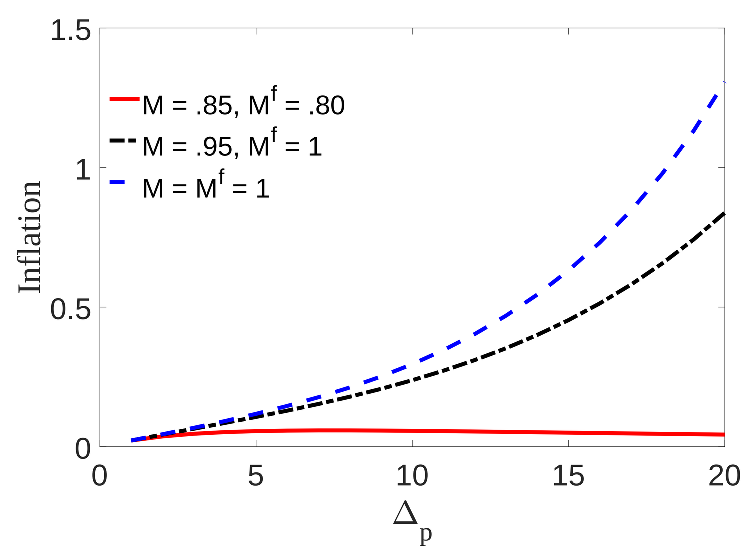
$$\begin{aligned} x_t &= M\mathbb{E}_t x_{t+1} - \sigma\left(i_t - \mathbb{E}_t \pi_{t+1}\right) \\ \pi_t &= M^f \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t \\ i_t &= \phi_\pi \pi_t, \end{aligned}$$

where M and M^f represent additional discounting of future expectations. Nests usual model when $M^f = M = 1$.

IE-stability condition:

$$\phi_\pi + \frac{(1-\beta M^f)(1-M)}{\kappa\sigma} > 1$$

The above is satisfied only if M^f and M are small enough...



Notes: The initial responses of inflation to an anticipated interest rate changes that take place Δ_p periods in the future for different values of discounting M and M^f. When $M = M^f = 1$, the standard model is obtained.

Proposition 3: *Consider the New Keynesian model and suppose* $\phi_{\pi,T^a} < \frac{\kappa + (\beta-1)\phi_{x,T^a}}{\kappa}$.

1. *All MSV FGA solutions of the model exhibit the Forward Guidance Puzzle.*
2. *There exist sunspot FGA solutions that do not exhibit the Forward Guidance Puzzle if and only if* $\phi_{\pi,T^*} < \frac{\kappa+(\beta-1)\phi_{x,T^*}}{\kappa}$

FORWARD GUIDANCE SOLUTIONS AND IE-STABILITY

The connection between IE-stability and the Forward Guidance Puzzle is obtained by studying RE models in the form considered by [1]:

$$\begin{aligned} y_t &= \Gamma(\theta) + A(\theta) y_{t-1} + B(\theta)\mathbb{E}_t y_{t+1} + D(\theta)\omega_t \\ \omega_t &= \rho(\theta)\omega_{t-1} + \varepsilon_t \end{aligned}$$

where y_t is a $n \times 1$ vector of endogenous variables with $m \leq n$ jump variables, ω_t is $l \times 1$ vector of exogenous variables, ε_t is a vector of exogenous white noise innovations, and θ is the vector parameters.

Definition: Forward Guidance Announcement *A Forward Guidance Announcement (FGA) is a tuple $\{\theta_{T^a}, \theta_{T^*}\}$ such that $T^* - T^a = \Delta_p > 0$, where θ_{T^a} is the vector of structural parameters that governs the economy from the time of the announcement, T^a, until time $T^* - 1$. θ_{T^*} is the vector of structural parameters that governs the economy at time $t \geq T^*$.*

Definition: Impact *The contemporaneous impact of an FGA is defined as $|y_{ss} - \mathbb{E}[y_{T^a}]|$, where the $\mathbb{E}[y_{T^a}]$ is the unconditional expectation of the vector of endogenous variables at time $t = T^a$ and y_{ss} is the steady state of the model when $t < T^a$.*

Definition: Forward Guidance Puzzle *An FGA $\{\theta_{T^a}, \theta_{T^*}\}$ is said to exhibit The Forward Guidance Puzzle if its impact is unbounded as $\Delta_p \to \infty$.*

Forward Guidance Annoucement modeled as:

$$y_t = \begin{cases} \Gamma_a + A_a y_{t-1} + B_a \mathbb{E}_t y_{t+1} + D_a \omega_t & \text{if } t < T^* \\ \Gamma_* + A_* y_{t-1} + B_* \mathbb{E}_t y_{t+1} + D_* \omega_t & \text{if } t \geq T^* \end{cases}$$

where $B_* = B(\theta_{T^*})$ and $B_a = B(\theta_{T^a})$, ect...

FGA Solution solved as in [2] via backward recursion: Let $j = T^* - t$

$$\begin{aligned} \bar{a}_j &= \left(I - B_a \bar{b}_{j-1}\right)^{-1}\left(\Gamma_a + B_a \bar{a}_{j-1}\right) && (1) \\ \bar{b}_j &= \left(I - B_a \bar{b}_{j-1}\right)^{-1} A_a && (2) \\ \bar{c}_j &= \left(I - B_a \bar{b}_{j-1}\right)^{-1}\left(B_a \bar{c}_{j-1}\rho_a + D_a\right) && (3) \end{aligned}$$

where $\bar{a}_0 = \bar{a}(\theta_{T^*})$, $\bar{b}_0 = \bar{b}(\theta_{T^*})$, and $\bar{c}_0 = \bar{a}(\theta_{T^*})$ are the RE solutions in the terminal regime.

Equations 1, 2, and 3 are a T-map!

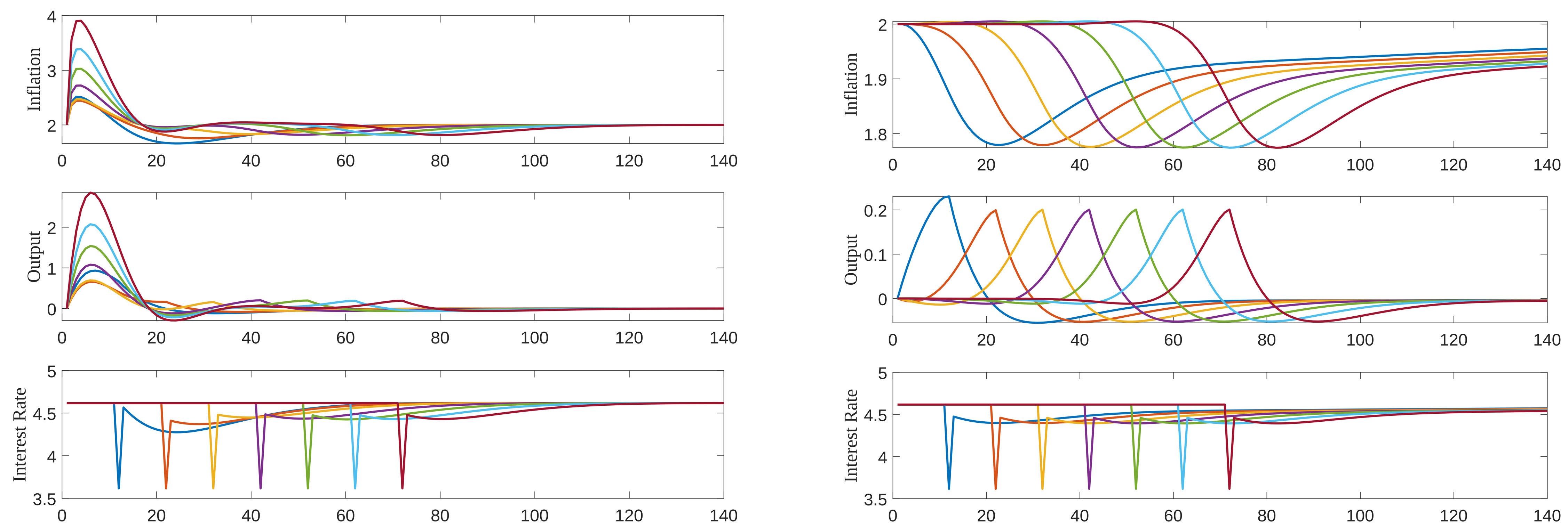
- If IE-unstable, then y_{T^a} diverges as policy is pushed farther out
- The Forward Guidance Puzzle is the result of an unstable T-map

Proposition 2 *The impact of a FGA $\{\theta_{T^a}, \theta_{T^*}\}$ is bounded as $\Delta_p \to \infty$ if*

1. *$\bar{\phi}(\theta_{T^a})$ exists*
2. *$\bar{\phi}(\theta_{T^a})$ is IE-stable*
3. *and $\phi_0(\theta_{T^*})$ is in the appropriate neighborhood of $\bar{\phi}(\theta_{T^a})$*

THE FORWARD GUIDANCE PUZZLE IN SMETS AND WOUTERS

We show how the method of [2] and [4] may be combined to construct sunspot FGA solutions. Simplifying the Taylor rule in the Smets and Wouters model, we can study IRFs to announced 100 basis point shocks under a fixed policy rate.



Notes: The plots shows the paths of output, inflation, and interest rates for anticipated 100 basis point monetary policy shocks with $\Delta_p = 10, \ldots, 70$.

ABSTRACT

We provide sufficient conditions for when rational expectations models predict bounded responses of endogenous variables to forward guidance announcements. The conditions coincide with a special case of the (E)xpectation-stability conditions that govern when agents can learn a Rational Expectations Equilibrium. The conditions are distinct from the determinacy conditions and are applicable in a wide variety of models, including Markov-switching models. Using the conditions, we show a novel resolution of the Forward Guidance Puzzle: sunspots. Under passive monetary policy, conditioning on a sunspot bounds the stimulus provided by forward guidance announcements in both the simple and medium-scale New Keynesian environments.

CONCLUSION

- We show that IE-stability predicts when MSV solutions exhibit well-behaved responses to forward announcements in a general class of structural RE models.
- We developed tools for recovering sunspot solutions to forward guidance announcements.
- We illustrate that IE-stability, and not indeterminacy, predicts when forward guidance announcements have reasonable effects by constructing IE-stable solutions of indeterminate models, including medium-scale models and regime-switching models.

REFERENCES

[1] George W Evans and Seppo Honkapohja. *Learning and expectations in macroeconomics*. Princeton University Press, 2001.

[2] Mariano Kulish and Adrian Pagan. Estimation and solution of models with expectations and structural changes. *Journal of Applied Econometrics*, 2017.

[3] Xavier Gabaix. A behavioral new keynesian model. *American Economic Review*, 110(8):2271–2327, 2020.

[4] Francesco Bianchi and Giovanni Nicolò. A generalized approach to indeterminacy in linear rational expectations models. *Quantitative Economics*, forth.