

# Inflation Persistence, Noisy Information and the Phillips Curve

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## This project

1. Document a fall in inflation persistence and volatility since the mid 1980s
2. Show that the NK model cannot explain the fall in persistence
3. Document a change in information frictions in the mid 1980s
4. Build a noisy information framework
5. Implications on the (lack of) flattening in the Phillips curve

# Inflation dynamics have changed since the mid 1980s

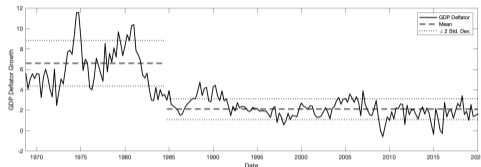


Figure: Time series of inflation, with subsample mean and standard deviation.

Table 1: Summary statistics over time.

	1968:Q4–2020:Q1	1968:Q4–1984:Q4	1985:Q1–2020:Q1
Mean	3.362	6.160	2.117
Volatility	2.400	2.234	1.016
First-Order Autocorrelation	0.880	0.754	0.505

Structural Break

Autocorrelation function

Rolling Sample Regression

Time-varying parameter regression

Unit root test

GARCH

## Inflation dynamics in NK models

# Inflation dynamics in NK models

- ▶ Understand the change in inflation dynamics via structural framework
- ▶ Determinants of inflation persistence and volatility
- ▶ Study 3 causal explanations: changes in...
  - ▶ Structural shocks (no change on persistence, fall in volatility)
    - Benchmark
    - Technology & cost-push shocks
  - ▶ Monetary stance (no change on persistence, fall in volatility)
    - Taylor rule
    - Indeterminacy
    - Discretion
    - Commitment
  - ▶ Intrinsic persistence (small change in persistence, fall in volatility)
    - Price indexation
    - Trend inflation
- ▶ NK framework (extended in several dimensions) cannot explain the change in persistence
- ▶ Propose a noisy information environment

## Noisy Information

# Motivation for Noisy Information

- ▶ Federal Reserve disclosure policy over time
  - ▶ Before 1967: Fed policy decisions announced **once a year** in the Annual Report
  - ▶ **1967**: release the directive in the Policy Report (PR) **90 days after** the decision
  - ▶ **1976**: enlarged the PR and reduced delay to **45 days**
  - ▶ **1977-1993**: objectives, 'tilt', ranking of policy factors, minutes
  - ▶ **1994**: **immediate** release of PR if action
  - ▶ **1999**: **immediate** release of tilt
  - ▶ **2000**: **immediate** announcement after each meeting
- ▶ Quantify this increase in information using the Coibion & Gorodnickenko (AER 2015) regression design

# Empirical Evidence on Dispersed Information

- ▶ Expectation data from the Survey of Professional Forecasters
  - ▶ Forecasters are asked to report their nowcast, forecast of next quarter, ..., up until a year
  - ▶ Quarterly, 1968:IV-2020:I
  - ▶ Alleviates the concern that firms could figure out Fed actions by hiring market watchers
- ▶ Measure of belief formation frictions: [Coibion & Gorodnickenko \(AER 2015\)](#) underrevision
  - ▶ Define  $\pi_{t+3,t} = \frac{\text{Deflator}_{t+3} - \text{Deflator}_{t-1}}{\text{Deflator}_{t-1}}$
  - ▶ Denote individual  $i$ 's forecast made in period  $t$  of annual inflation as  $\mathbb{E}_{it}\pi_{t+3,t}$
  - ▶ Denote average forecast as  $\overline{\mathbb{E}}_t\pi_{t+3,t} = \frac{1}{N_t} \sum_{i=1, \dots, N_t} \mathbb{E}_{it}\pi_{t+3,t}$
- ▶ Structural break version of CG

$$\pi_{t+3,t} - \overline{\mathbb{E}}_t\pi_{t+3,t} = \alpha + (\beta + \beta_{*,\mathbb{1}_{\{t \geq t^*\}}}) (\mathbb{E}_t\pi_{t+3,t} - \mathbb{E}_{t-1}\pi_{t+3,t}) + u_t$$



Table: Regression table

	(1) CG Regression	(2) 1968:IV-1984:IV	(3) 1985:I-2020:I	(4) Structural Break
Revision	1.230*** (0.250)	1.414*** (0.283)	0.169 (0.193)	1.501*** (0.317)
Revision $\times \mathbb{1}_{\{t \geq t^*\}}$				-1.111*** (0.379)
Constant	-0.0875 (0.0696)	0.271 (0.185)	-0.317*** (0.0478)	-0.135* (0.0690)
Observations	197	58	139	197

HAC robust standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Rolling Sample

Time-varying parameter

Livingston

# Noisy Information NK model

## ► Assumptions:

1. Households and the Central bank have FIRE
2. Firms have RE but cannot observe the state of the economy
  - Monetary policy shock  $v_t$  is the only aggregate state variable
  - Each firm  $j$  observes noisy signal  $x_{jt}$  on the CB action  $v_t$ ,

$$x_{jt} = v_t + u_{jt}, \quad \text{with } u_{jt} \sim \mathcal{N}(0, \sigma_u^2)$$

## ► Model equations:

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t \tilde{y}_{t+1}$$

$$\pi_{jt} = \kappa \theta \mathbb{E}_{jt} \tilde{y}_t + (1 - \theta) \mathbb{E}_{jt} \pi_t + \beta \theta \mathbb{E}_{jt} \pi_{j,t+1}, \quad \pi_t = \int \pi_{jt} dj$$

$$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t, \quad v_t = \rho_v v_{t-1} + \varepsilon_t^v, \quad \varepsilon_t^v \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Derivation PC

Optimal Expectations

Parameters

## ► Model the increase in information disclosure as a decrease in $\sigma_u$ (structural break)

# Inflation dynamics in the NINK model

## Proposition 1 (Full Proposition)

*In the dispersed information framework, equilibrium inflation dynamics are given by*

$$\pi_t = \vartheta \pi_{t-1} - \psi_\pi \chi(\vartheta) v_t$$

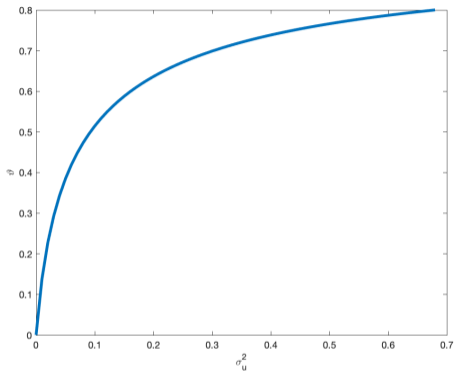
*where  $\vartheta(\sigma_u, \phi_\pi) \in (0, \rho_v)$  governs information frictions, and in the limit of no info frictions ( $\sigma_u \rightarrow 0$ ),  $\vartheta \rightarrow 0$  and  $\chi \rightarrow 1$*

## Proposition 2

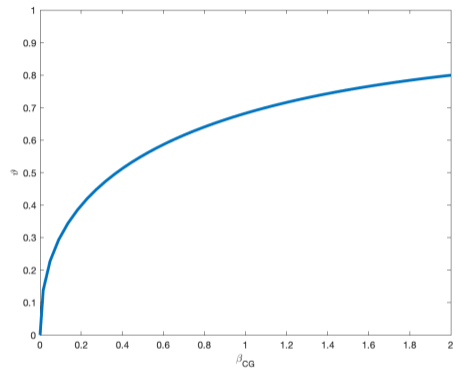
*The theoretical counterpart of the coefficient  $\beta_{CG}$  is given by*

$$\beta_{CG} = \frac{\lambda^2 \{ (1 - \lambda^2) \vartheta^2 (1 - \rho \vartheta) + [\rho(\vartheta - \lambda) - \vartheta \lambda (1 - \rho \lambda)] (1 - \vartheta \lambda) \}}{(\rho - \lambda)(\vartheta - \lambda)(1 - \vartheta \lambda)}$$

*where  $\lambda = \rho(1 - G) \in (0, 1)$*



(a) Inflation persistence increasing in noise



(b) Inflation persistence increasing in CG coefficient

## Persistence Result

- ▶ Calibration to match  $\beta_{CG}$ :

	Pre-1985	Post-1985
$\phi_\pi$	1	2
$\sigma_u$	$\beta_{CG} = 1.501$	$\beta_{CG} = 0.390$

- ▶ The model produces a fall in inflation persistence: from  $\vartheta_{pre} = 0.739$  to  $\vartheta_{post} = 0.444$
- ▶ Data:  $\rho_{\pi,pre} = 0.814$  and  $\rho_{\pi,post} = 0.491$

	(1) OLS	(2) Newey
$\pi_{t-1}$	0.814*** (0.0481)	0.814*** (0.0483)
$\pi_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$	-0.323*** (0.0807)	-0.323*** (0.0804)
Observations	207	207

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

“Inflation Disconnect” Puzzle and  
(lack of) flattening of Phillips Curve

## “Inflation Disconnect” Puzzle

- ▶ Fall in the sensitivity of inflation wrt other (real) variables (DelNegro et al. 2020)
- ▶ Most well-known inflation dynamics in the NKPC

$$\pi_t = \kappa \tilde{y}_t + \beta \mathbb{E}_t \pi_{t+1}$$

- ▶ Inflation *only* related to output *through* the slope  $\kappa$ 
  - ▶ Literature has extensively focused on showing that  $\kappa$  has flattened
  - ▶ Inflation independent of any other (real) variable
- ▶ Show the “inflation disconnect” puzzle without resorting to  $\kappa$
- ▶ Two tests
  - ▶ Agnostic stance in expectations
  - ▶ Using our NI framework disconnect occurs via expectation formation changes

# Test 1: Agnostic stance

- Recall the individual Phillips curve for firm  $j$

$$\pi_{jt} = \kappa\theta\mathbb{E}_{jt}\tilde{y}_t + (1 - \theta)\mathbb{E}_{jt}\pi_t + \beta\theta\mathbb{E}_{jt}\pi_{j,t+1}$$

- Iterating forward and aggregating, we can write

$$\pi_t = \kappa\theta \sum_{k=0}^{\infty} (\beta\theta)^k \bar{\mathbb{E}}_t^f \tilde{y}_{t+k} + (1 - \theta) \sum_{k=0}^{\infty} (\beta\theta)^k \bar{\mathbb{E}}_t^f \pi_{t+k}$$

- Inflation related to output via  $\kappa$  and *general* expectations
- Test structural break in  $\kappa$  *controlling for* non-standard expectations Livingston

Table: Regression table

	(1) NKPC	(2) Break Output	(3) Break
$\bar{\mathbb{E}}_t^f \tilde{y}_t$	0.108*** (0.0330)	0.0781* (0.0451)	0.0810* (0.0459)
$\bar{\mathbb{E}}_t^f \tilde{y}_{t+4}$	0.290*** (0.0707)	0.316*** (0.0793)	0.215** (0.0931)
$\bar{\mathbb{E}}_t^f \pi_t$	1.252*** (0.0906)	1.214*** (0.0963)	1.205*** (0.119)
$\bar{\mathbb{E}}_t^f \pi_{t+4}$	-0.327*** (0.100)	-0.308*** (0.104)	-0.256** (0.125)
$\bar{\mathbb{E}}_t^f \tilde{y}_t \times \mathbb{1}_{\{t \geq t^*\}}$		0.0775 (0.0596)	0.0885 (0.0593)
$\bar{\mathbb{E}}_t^f \tilde{y}_{t+4} \times \mathbb{1}_{\{t \geq t^*\}}$		-0.0846 (0.0858)	0.0990 (0.109)
$\bar{\mathbb{E}}_t^f \pi_t \times \mathbb{1}_{\{t \geq t^*\}}$			0.0305 (0.177)
$\bar{\mathbb{E}}_t^f \pi_{t+4} \times \mathbb{1}_{\{t \geq t^*\}}$			-0.262 (0.173)
Constant	-0.233*** (0.0581)	-0.210** (0.102)	-0.160 (0.0969)



## Test 2: NI Phillips Curve

- ▶ Anchoring, myopia and relevance of future output gaps

### Proposition 3 (Full)

The as if Phillips curve dynamics are described by

$$\pi_t = \omega_{\pi\pi}\pi_{t-1} + \kappa\tilde{y}_t + \delta_{\pi y}\mathbb{E}_t\tilde{y}_{t+1} + \delta_{\pi\pi}\beta\mathbb{E}_t\pi_{t+1}$$

*Red terms endogenous to  $\sigma_u$*

- ▶ Model implied dynamics in the pre-1985 period  $(\phi_\pi, \beta_{CG}) = (1, 1.50)$

$$\pi_t = 0.562\pi_{t-1} + 0.172\tilde{y}_t + 0.000\mathbb{E}_t\tilde{y}_{t+1} + 0.405\mathbb{E}_t\pi_{t+1}$$

- ▶ Model implied dynamics in the pre-1985 period  $(\phi_\pi, \beta_{CG}) = (2, 0.39)$

$$\pi_t = 0.399\pi_{t-1} + 0.172\tilde{y}_t - 0.114\mathbb{E}_t\tilde{y}_{t+1} + 0.633\mathbb{E}_t\pi_{t+1}$$

- ▶ Suppose an econometrician assumes  $\tilde{y}_t \sim \text{AR}(1)$ :

$$0.172\tilde{y}_t - 0.114\mathbb{E}_t\tilde{y}_{t+1} = (0.172 - 0.114\rho_y)\tilde{y}_t$$

Table: Regression table

	(1) Standard NKPC	(2) Break	(3) DI NKPC	(4) Break Output	(5) Break Inflation
$\tilde{y}_t$	-0.0261 (0.0236)	-0.114** (0.0452)	0.192** (0.0941)	0.273* (0.142)	0.265** (0.112)
$\pi_{t+1}$	0.991*** (0.0175)	0.996*** (0.0171)	0.677*** (0.0740)	0.646*** (0.0876)	0.499*** (0.104)
$\pi_{t-1}$			0.309*** (0.0743)	0.340*** (0.0873)	0.481*** (0.102)
$\tilde{y}_{t+1}$			-0.221** (0.104)	-0.307** (0.142)	-0.272** (0.120)
$\tilde{y}_t \times \mathbb{1}_{\{t \geq t^*\}}$		0.122** (0.0566)		-0.183 (0.198)	
$\tilde{y}_{t+1} \times \mathbb{1}_{\{t \geq t^*\}}$				0.191 (0.196)	
$\pi_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$					-0.366* (0.200)
$\pi_{t+1} \times \mathbb{1}_{\{t \geq t^*\}}$					0.406* (0.209)
Observations	203	203	203	203	203

HAC robust standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## Backup Slides

# Structural Break Back

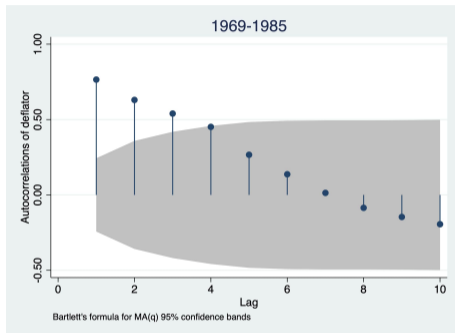
- ▶ Wald test positive about a structural break in 1985:Q1
- ▶ Regress

$$\pi_t = \alpha_\pi + \alpha_{\pi,*} \mathbb{1}_{\{t \geq t^*\}} + \rho_\pi \pi_{t-1} + \rho_{\pi,*} \pi_{t-1} \mathbb{1}_{\{t \geq t^*\}} + \varepsilon_t$$

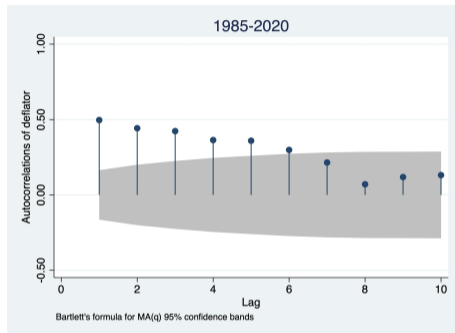
	(1)	(2)	(3)	(4)	(5)	(6)
	Deflator		CPI		PCE	
$\pi_{t-1}$	0.880*** (0.0466)	0.785*** (0.0755)	0.738*** (0.0628)	0.793*** (0.0827)	0.816*** (0.0461)	0.837*** (0.0672)
$\pi_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$		-0.287** (0.144)		-0.497*** (0.143)		-0.434*** (0.117)
Constant	0.400** (0.166)	1.320*** (0.471)	1.008*** (0.262)	1.396** (0.542)	0.618*** (0.182)	0.990** (0.431)
Constant $\times \mathbb{1}_{\{t \geq t^*\}}$		-0.263 (0.543)		0.370 (0.607)		0.283 (0.477)
Observations	206	206	206	206	206	206

Standard errors in parentheses

# Autocorrelation Function [Back](#)



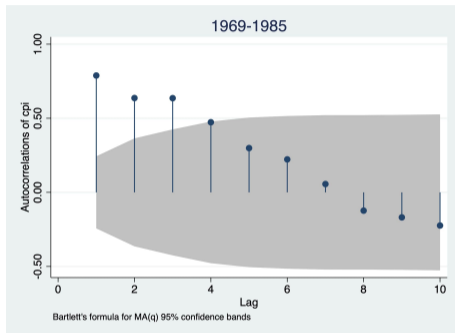
(a) GDP Deflator, 1969-1985



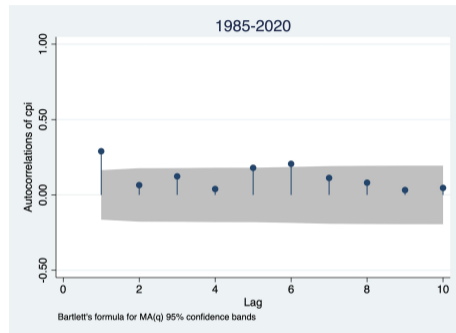
(b) GDP Deflator, 1985-2020

Figure: Autocorrelation function of GDP Deflator

# Autocorrelation Function [Back](#)



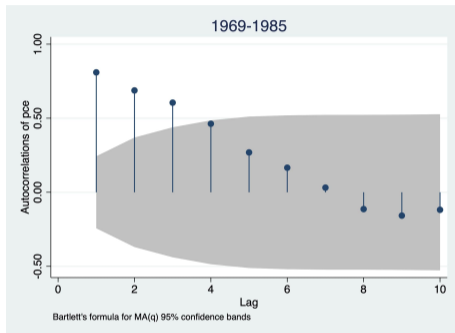
(a) CPI, 1969-1985



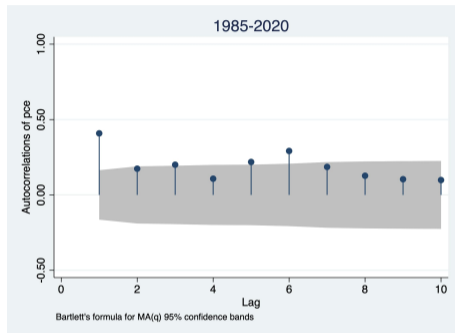
(b) CPI, 1985-2020

Figure: Autocorrelation function of CPI

# Autocorrelation Function [Back](#)



(a) PCE, 1969-1985

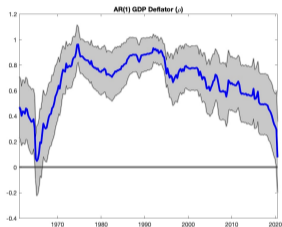


(b) PCE, 1985-2020

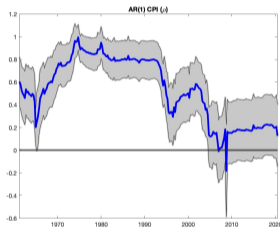
Figure: Autocorrelation function of PCE

# Rolling-Sample Regression Back

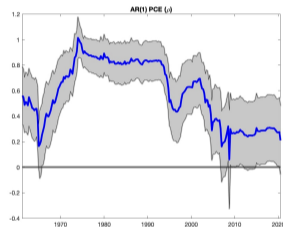
- ▶ Regress  $\pi_t = \rho_\pi \pi_{t-1} + \varepsilon_t$  using 14-year window samples



(a) GDP Deflator



(b) CPI



(c) PCE

Figure: First-order autocorrelation of GDP Deflator, CPI and PCE, rolling sample



# Unit Root Test Back

- ▶ Cross-sample unit root analysis
  - ▶ Augmented Dickie-Fuller
  - ▶ Phillips-Perron
- ▶ Null hypothesis (unit root) cannot be rejected in the pre-1985 sample
- ▶ Strong rejection of the null in the post-1985 sample

<i>p</i> -values, null = series has unit root		
1969-2020		
Variable	ADF	Phillips-Perron
GDP Deflator	0.23	0.02
CPI	0.11	0.00
PCE	0.16	0.00
1969-1985		
Variable	ADF	Phillips-Perron
GDP Deflator	0.15	0.07
CPI	0.17	0.09
PCE	0.055	0.09
1985-2020		
Variable	ADF	Phillips-Perron
GDP Deflator	0.07	0.00
CPI	0.00	0.00
PCE	0.01	0.00

# Time-Varying Parameter Regression Back

- ▶ Consider the framework

$$\pi_t = \rho_t \pi_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

- ▶ Persistence coefficient follows a random walk

$$\rho_{t+1} = \rho_t + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma_u)$$

- ▶ Bayesian estimation, prior selection is standard following Nakajima (2011)

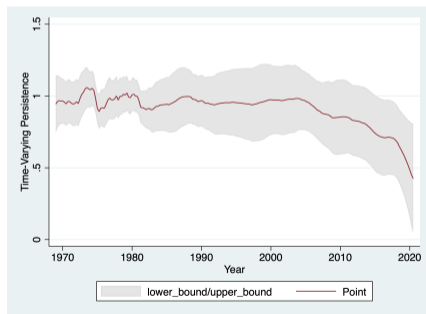


Figure: Persistence over time

- ▶ GMM regression on
 
$$i_t = (\phi_\pi + \phi_{\pi,*} \mathbb{1}_{\{t \geq t^*\}}) \pi_t + \phi_y y_t + \varepsilon_t$$
- ▶ Instruments: four lags of
  - ▶ effective Fed funds rate
  - ▶ GDP deflator
  - ▶ CBO output gap
  - ▶ commodity price inflation
  - ▶ real M2 growth
  - ▶ spread between long-term bond rate and 3-month Treasury Bill

Table: Regression table

	(1) 1969:IV-2020:I	(2) Break
$\pi_t$	1.154*** (0.112)	1.323*** (0.140)
$y_t$	0.353*** (0.121)	0.309** (0.128)
$\pi_t \times \mathbb{1}_{\{t \geq t^*\}}$		0.958*** (0.284)
Observations	204	204

Standard errors in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

# Benchmark

- ▶ Dynamic IS curve

$$\tilde{y}_t = -\frac{1}{\sigma} (i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t \tilde{y}_{t+1} \quad (1)$$

- ▶ NK Phillips curve

$$\pi_t = \kappa \tilde{y}_t + \beta \mathbb{E}_t \pi_{t+1} \quad (2)$$

- ▶ Monetary policy rule

$$i_t = \phi_\pi \pi_t + \phi_y y_t + v_t, \quad v_t = \rho_v v_{t-1} + \varepsilon_t^v, \quad \varepsilon_t^v \sim \mathcal{N}(0, \sigma_\varepsilon^2) \quad (3)$$

- ▶ Introducing (3) into (1), we can write (1)-(2) as a system of two first-order forward-looking stochastic equations
- ▶ Inflation dynamics are given by

$$\begin{aligned} \pi_t &= -\psi_\pi v_t \\ &= \rho_v \pi_{t-1} - \psi_\pi \varepsilon_t^v \end{aligned} \quad (4)$$

where  $\psi_\pi$  is decreasing in  $\phi_\pi$

## Measuring the Shock Process

- ▶ Problem:  $v_t$  is unobservable, but we have estimates on monetary policy shocks  $\varepsilon_t^v$  from Romer and Romer (2004), updated until 2007 by Wieland & Yang (2020)
- ▶ Solution: indirect estimation on  $\rho_v$
- ▶ Using the AR(1) property of the  $v_t$  shock process, we can write the Taylor rule as

$$i_t = \rho_v i_{t-1} + (\phi_\pi \pi_t + \phi_y y_t) - \rho_v (\phi_\pi \pi_{t-1} + \phi_y y_{t-1}) + \varepsilon_t^v \quad (5)$$

- ▶ An estimate of the first autoregressive coefficient identifies monetary policy persistence

## Persistence

$$i_t = \rho_v i_{t-1} + (\phi_\pi \pi_t + \phi_y y_t) - \rho_v (\phi_\pi \pi_{t-1} + \phi_y y_{t-1}) + \varepsilon_t^v \quad (6)$$

- ▶ As before, we rely on a structural break analysis but our results are consistent with alternative persistence measures
- ▶ First, we estimate

$$i_t = \alpha_i + \alpha_{i,*} \mathbb{1}_{\{t \geq t^*\}} + \rho_i i_{t-1} + \rho_{i,*} i_{t-1} \mathbb{1}_{\{t \geq t^*\}} + \gamma \mathbf{X}_{t,t-1} + u_t$$

using unrestricted GMM, where  $\mathbb{1}_{\{t \geq t^*\}}$  is an indicator variable equal to 1 if the period is within the post-1985 era

- ▶ However, notice that  $\rho_v$  also interacts with lagged inflation and output gap in (6)
- ▶ To account for this, we estimate a structural break in (6) using a restricted GMM

$$i_t = \alpha_i + \alpha_{i,*} \mathbb{1}_{\{t \geq t^*\}} + \rho_i i_{t-1} + \rho_{i,*} i_{t-1} \mathbb{1}_{\{t \geq t^*\}} + \gamma \mathbf{X}_{t,t-1} + u_t$$

	(1)	(2)	(3)	(4)
	Unrestricted GMM		Restricted GMM	
$i_{t-1}$	0.941*** (0.0184)	0.939*** (0.0448)	0.972*** (0.0119)	0.931*** (0.0365)
$i_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$		-0.00261 (0.0591)		-0.0537 (0.0632)
Constant	0.122 (0.118)	0.305 (0.473)	0.0770* (0.0467)	0.851** (0.373)
Constant $\times \mathbb{1}_{\{t \geq t^*\}}$		-0.123 (0.436)		-0.813 (0.559)
Observations	203	203	203	203

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

# Summary

- ▶ The full NK model cannot explain the fall in inflation persistence, since it is inherited from the monetary shock process which did not change over time
- ▶ It can rationalize the fall in inflation volatility through a contemporaneous fall in the elasticity of interest rates with respect to output and inflation

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## Technology and Cost-push Shocks

- ▶ Extend the basic framework to demand (technology) and supply (cost-push) shocks,  $a_t$  and  $u_t$

- ▶ Demand side:

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1}) - (1 - \rho_a)\psi_{ya}a_t + \mathbb{E}_t \tilde{y}_{t+1} \quad (7)$$

- ▶ Supply side:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t + u_t \quad (8)$$

- ▶  $a_t$  and  $u_t$  follow AR(1) processes with persistence  $\rho_a$  and  $\rho_u$
- ▶ Inflation dynamics follow

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t + \psi_{\pi u} u_t$$

- ▶ First-order autocorrelation coefficient  $\rho_1$  depends critically on the  $\rho_x$ 's

$$\rho_1 = \frac{\rho_v \frac{\psi_{\pi v}^2 \sigma_{\varepsilon v}^2}{1-\rho_v^2} + \rho_a \frac{\psi_{\pi a}^2 \sigma_{\varepsilon a}^2}{1-\rho_a^2} + \rho_u \frac{\psi_{\pi u}^2 \sigma_{\varepsilon u}^2}{1-\rho_u^2}}{\frac{\psi_{\pi v}^2 \sigma_{\varepsilon v}^2}{1-\rho_v^2} + \frac{\psi_{\pi a}^2 \sigma_{\varepsilon a}^2}{1-\rho_a^2} + \frac{\psi_{\pi u}^2 \sigma_{\varepsilon u}^2}{1-\rho_u^2}}$$

- ▶ We already documented no change in  $\rho_v$
- ▶ Find evidence on a structural break in  $\rho_a$  and  $\rho_u$

# Technology Shock

- ▶ Use three data series used in the literature
- ▶ Fernald (2014) estimates directly (log) technology  $a_t$
- ▶ Francis et al. (2014) and Justiniano et al. (2011) estimate the technology shock  $\varepsilon_t^a$ 
  - ▶ Indirect estimation of  $\rho_a$  using the natural real interest rate process
  - ▶ The natural real rate is given by  $r_t^n = \sigma\psi_{ya}(\rho_a - 1)a_t$ , which can be rewritten as

$$r_t^n = \rho_a r_{t-1}^n - \sigma\psi_{ya}(1 - \rho_a)\varepsilon_t^a \quad (9)$$

- ▶ We use the Federal Reserve estimate of the natural interest rate series, produced by Holston (2017), as our proxy for  $r_t^n$

	(1) Technology	(2) SB	(3) Natural rate	(4) SB	(5) Natural rate	(6) SB
(Log) $TFP_{t-1}$	0.998*** (0.00454)	0.990*** (0.00860)				
(Log) $TFP_{t-1}$ change		0.00323 (0.00339)				
Natural rate $_{t-1}$			0.951*** (0.0317)	0.945*** (0.0327)	0.963*** (0.0367)	0.957*** (0.0404)
Technology shock in Francis et al. (2014)			0.0511** (0.0234)	0.0514** (0.0237)		
Natural rate $_{t-1}$ change				-0.0106 (0.0129)		-0.00863 (0.0141)
Technology shock in Justiniano et al. (2011)					0.0191 (0.0278)	0.0195 (0.0280)
Constant	0.00360 (0.00327)	0.00743* (0.00445)	0.128 (0.0968)	0.162 (0.109)	0.0878 (0.114)	0.123 (0.140)
Observations	186	186	163	163	160	160

Robust standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

# Cost-Push Shock

- ▶ Nekarda & Ramey (2010) estimate the structural time-varying price-cost markup
- ▶ Two different measures of the cost-push shock
  - ▶ In the first, rely on a Cobb-Douglas production function in order to estimate the markup,
  - ▶ In the second, rely on a CES production function, estimating labor-augmented technology using long-run restrictions as in Gali (1999)

	(1) Cobb-Douglas	(2) SB	(3) CES	(4) SB
Markup <sub>t-1</sub>	0.945*** (0.0246)	0.938*** (0.0305)	0.963*** (0.0234)	0.947*** (0.0252)
Markup <sub>t-1</sub> change		0.00187 (0.00436)		0.00472 (0.00419)
Constant	0.0280** (0.0125)	0.0307** (0.0146)	0.0189 (0.0117)	0.0252** (0.0120)
Observations	195	195	195	195

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

# Optimal Monetary Policy under Discretion

- ▶ In the pre-1985 period, inflation dynamics follow

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t + \psi_{\pi u} u_t$$

- ▶ In the post-1985 period with optimal policy, the central bank minimizes the welfare losses experienced by a representative consumer,

$$\mathbb{E}_0 \sum_{k=0}^{\infty} \beta^k \left( \pi_t^2 + \frac{\kappa}{\epsilon} x_t^2 \right)$$

where  $x_t$  is the welfare-relevant output gap, subject to the Phillips curve

$$\pi_t = \kappa x_t + \xi_t,$$

where  $\xi_t \equiv \beta \mathbb{E}_t \pi_{t+1} + u_t$  is treated as a non-policy shock

- ▶ Inflation dynamics follow

$$\pi_t = \rho_u \pi_{t-1} + \psi_d \varepsilon_t^u$$

where  $\psi_d > 0$  depends on deep parameters and inflation persistence is inherited from the cost-push shock.

- ▶ Compared to the pre-1985 dynamics there is no significant change in inflation persistence:
  - ▶ in the pre-period, model persistence is around 0.95,
  - ▶ while in the post-period persistence is around 0.96, the estimated persistence of cost-push shocks throughout both periods.
- ▶ Therefore, such change in the policy stance would have generated an *increase* in inflation persistence, which rules out this explanation.

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# Optimal Monetary Policy under Commitment

- ▶ In the pre-1985 period, inflation dynamics follow

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t + \psi_{\pi u} u_t$$

- ▶ In the post-1985 period with optimal policy, the central bank minimizes the welfare losses experienced by a representative consumer,

$$\mathbb{E}_0 \sum_{k=0}^{\infty} \beta^k \left( \pi_t^2 + \frac{\kappa}{\epsilon} x_t^2 \right)$$

where  $x_t$  is the welfare-relevant output gap, subject to the Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t$$

- ▶ Inflation dynamics follow

$$\pi_t = \rho_c \pi_{t-1} + \psi_c \Delta u_t$$

where  $\psi_c > 0$  depends on deep parameters and  $\rho_c$  governs inflation persistence, which depends on deep parameters.



- ▶ Standard parameterization yields  $\rho_c = 0.31$ , excessive fall in inflation persistence
- ▶ Commitment implies an *as if* Taylor rule in which  $\phi_\pi$  rose from 1 to 6.5, inconsistent with our empirical evidence.
- ▶ Output gap as persistent as inflation!

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# Price Indexation

- ▶ Changes in  $\phi_\pi$  and  $\phi_y$  have no effect on inflation persistence *unless* there is aggregate anchoring
- ▶ Generate aggregate anchoring through price indexation
- ▶ A restricted firm resets its price partially indexed to past inflation:  $p_{it} = p_{i,t-1} + \omega\pi_{t-1}$
- ▶ Otherwise standard
- ▶ Phillips curve modified to

$$\Delta_t = \kappa\tilde{y}_t + \beta\mathbb{E}_t\Delta_{t+1},$$

where  $\Delta_t := \pi_t - \omega\pi_{t-1}$

- ▶ Inflation dynamics given by

$$\pi_t = \rho_\omega\pi_{t-1} + \psi_\omega v_t$$

where  $\rho_\omega$  is decreasing in  $\phi_\pi$

## Trend Inflation

- ▶ Ascari & Sbordone (2014) and Stock & Watson (2007) document a fall in trend inflation from 4% in the pre-1985 period to 2% afterwards
- ▶ Log-linearize the model around a steady-state with positive trend inflation
- ▶ Creates intrinsic persistence through relative price dispersion, which is a backward-looking variable that has no first-order effects in the benchmark
- ▶ Demand side unaffected, Supply side (Phillips curve) now a system of three equations

$$\pi_t = \Xi_1 \psi_t + \Xi_2 y_t + \Xi_3 \mathbb{E}_t \psi_{t+1} + \Xi_4 \mathbb{E}_t \pi_{t+1}$$

$$\psi_t = \Gamma_1 s_t + \Gamma_2 y_t + \Gamma_3 \mathbb{E}_t \psi_{t+1} + \Gamma_4 \mathbb{E}_t \pi_{t+1}$$

$$s_t = \Lambda_1 \pi_t + \Lambda_2 s_{t-1}$$

- ▶  $\Lambda_2(\bar{\pi})$  increasing in  $\bar{\pi}$
- ▶ Inflation dynamics given by

$$\pi_t = \rho_{\bar{\pi}} \pi_{t-1} + \psi_{\bar{\pi}} v_t + \xi_t,$$

where  $\xi_t$  is an MA( $\infty$ ) process and  $\rho_{\bar{\pi}}$  is decreasing in  $\phi_{\pi}$  and  $\phi_y$ , and increasing in  $\bar{\pi}$

## Regression Design

- ▶ The previous regression design is motivated by the Kalman filter
- ▶ Suppose that an agent wants to forecast an unobserved fundamental  $v_t$ ,

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

where  $\varepsilon^v \sim \mathcal{N}(0, \sigma_\varepsilon^2)$

- ▶ Instead of observing the fundamental, agents observe a noisy signal

$$x_{it} = v_t + u_{it}$$

with  $u_i \sim \mathcal{N}(0, \sigma_u^2)$

- ▶ An agent optimal expectation (Kalman filter) takes the following form

$$\mathbb{E}_t v_t = (1 - G)\mathbb{E}_{t-1} v_t + Gx_{it}$$

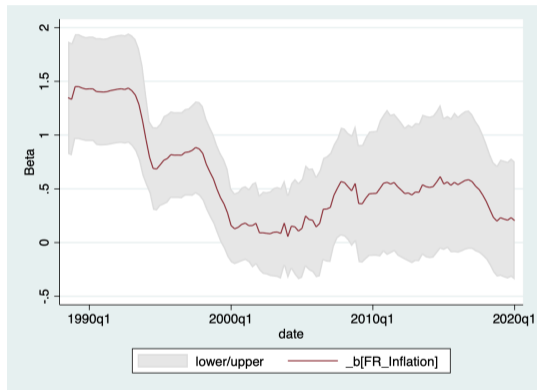
where  $G$  is the Kalman gain, the weight that agents (optimally) assign on new information  $x_{it}$  relative to the previous forecast, which depends on  $\sigma_\varepsilon^2$  and  $\sigma_u^2$

- ▶ One can show

$$v_{t+1} - \mathbb{E}_t v_{t+1} = \frac{1 - G}{G} (\mathbb{E}_t v_{t+1} - \mathbb{E}_{t-1} v_{t+1}) + u_t$$

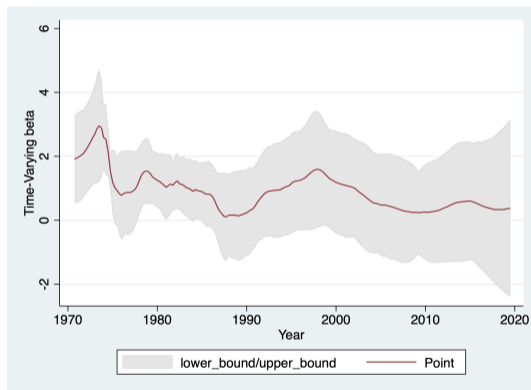
- ▶ Hence, an estimate of  $\beta$  pins down information frictions!

# Rolling Sample [Back](#)



# Time-Varying Parameter Regression [Back](#)

$$\pi_{t+3,t} - \mathbb{E}_t \pi_{t+3,t} = \beta_t (\mathbb{E}_t \pi_{t+3,t} - \mathbb{E}_{t-1} \pi_{t+3,t}) + u_t$$



- Survey conducted semiannually, estimate the following structural-break variant

$$\pi_{t+2,t} - \mathbb{E}_t \pi_{t+2,t} = \alpha_{CG} + (\beta_{CG} + \beta_{CG*} \mathbb{1}_{\{t \geq t^*\}}) (\mathbb{E}_t \pi_{t+2,t} - \mathbb{E}_{t-2} \pi_{t+2,t}) + u_t \quad (10)$$

Table: Regression table

	(1) CG Regression	(2) Structural Break
Revision	0.380* (0.202)	0.412** (0.204)
Revision $\times \mathbb{1}_{\{t \geq t^*\}}$		-0.880** (0.414)
Constant	-0.183* (0.102)	-0.105 (0.119)
Observations	146	146

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## Derivation Phillips curve

- ▶ Continuum of firms indexed by  $j \in \mathcal{I}_f = [0, 1]$
- ▶ Each firm is a monopolist producing a differentiated intermediate-good variety, producing output  $Y_{jt}$  and setting nominal price  $P_{jt}$  and making real profit  $D_{jt}$

- ▶ Production function

$$Y_{jt} = N_{jt}^{1-\alpha} \quad (11)$$

- ▶ Firm  $j$ 's program

$$P_{jt}^* = \arg \max_{P_{jt}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt} \left\{ \Lambda_{t,t+k} \frac{1}{P_{t+k}} [P_{jt} Y_{j,t+k|t} - C_{t+k}(Y_{j,t+k|t})] \right\}$$
$$\text{s.t. } Y_{j,t+k|t} = \left( \frac{P_{jt}}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$$

where

- ▶  $\Lambda_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma}$  is the stochastic discount factor,
- ▶  $C_t(\cdot)$  is the (nominal) cost function,
- ▶  $Y_{j,t+k|t}$  denotes output in period  $t+k$  for a firm  $j$  that last reset its price in period  $t$ .



► FOC

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt} \left[ \Lambda_{t,t+k} Y_{j,t+k|t} \frac{1}{P_{t+k}} (P_{jt}^* - \mathcal{M} \Psi_{j,t+k|t}) \right] = 0$$

where

- $\Psi_{j,t+k|t} \equiv C'_{t+k}(Y_{j,t+j|t})$  denotes the (nominal) marginal cost for firm  $j$ ,
- $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ .

- Log-linearizing around the zero inflation steady-state, we obtain the familiar price-setting rule

$$p_{jt}^* = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{jt} (\psi_{j,t+k|t} + \mu) \quad (12)$$

where

- $\psi_{j,t+k|t} = \log \Psi_{j,t+k|t}$
- $\mu = \log \mathcal{M}$ .

- ▶ The (log) marginal cost for firm  $j$  at time  $t + k|t$  is

$$\begin{aligned}\psi_{j,t+k|t} &= w_{t+k} - mpn_{j,t+k|t} \\ &= w_{t+k} - [\log(1 - \alpha) - \alpha n_{j,t+k|t}]\end{aligned}$$

where

- ▶  $mpn_{j,t+k|t}$  denotes (log) marginal product of labor for a firm that last reset its price at time  $t$ ,
- ▶  $n_{j,t+k|t}$  denotes (log) employment in period  $t + k$  for a firm that last reset its price at time  $t$
- ▶ Let  $\psi_t \equiv \int_{\mathcal{I}_f} \psi_{jt}$  denote the (log) average marginal cost

$$\psi_t = w_t - [\log(1 - \alpha) - \alpha n_t]$$

- ▶ The following relation holds

$$\begin{aligned}\psi_{j,t+k|t} &= \psi_{t+k} + \alpha(n_{jt+k|t} - n_{t+k}) \\ &= \psi_{t+k} + \frac{\alpha}{1 - \alpha}(y_{jt+k|t} - y_{t+k}) \\ &= \psi_{t+k} - \frac{\alpha\epsilon}{1 - \alpha}(p_{jt}^* - p_{t+k})\end{aligned}\tag{13}$$

- ▶ Introducing (13) into (12), we can rewrite the firm price-setting condition as

$$p_{jt}^* = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{jt} (p_{t+k} - \Theta \hat{\mu}_{t+k})$$

where

- ▶  $\hat{\mu} = \mu_t - \mu$  is the deviation between the average and desired markups,
- ▶  $\mu_t = -(\psi_t - p_t)$ ,
- ▶  $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$
- ▶ Suppose that firms observe the aggregate prices up to period  $t - 1$ ,  $p^{t-1}$
- ▶ Then we can restate the above condition as

$$p_{jt}^* - p_{t-1} = -(1 - \beta\theta)\Theta \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{jt} \hat{\mu}_{t+k} + \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{jt} \pi_{t+k}$$

- ▶ Define the firm-specific inflation rate as  $\pi_{jt} = (1 - \theta)(p_{jt}^* - p_{t-1})$
- ▶ Then we can write the above expression as

$$\pi_{jt} = -(1 - \theta)(1 - \beta\theta)\Theta \mathbb{E}_{jt} \hat{\mu}_t + (1 - \theta)\mathbb{E}_{jt} \pi_t + \beta\theta \mathbb{E}_{jt} \pi_{j,t+1}$$

where  $\pi_t = \int_{\mathcal{I}_f} \pi_{jt} dj$ .

- ▶ Using the aggregate household's labor supply condition we can write

$$\hat{\mu}_t = - \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t$$

- ▶ Finally, we can write the individual Phillips curve as

$$\begin{aligned} \pi_{jt} &= (1 - \theta)(1 - \beta\theta)\Theta \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \mathbb{E}_{jt} \tilde{y}_t + (1 - \theta)\mathbb{E}_{jt} \pi_t + \beta\theta\mathbb{E}_{jt} \pi_{i,t+1} \\ &= \kappa\theta\mathbb{E}_{jt} \tilde{y}_t + (1 - \theta)\mathbb{E}_{jt} \pi_t + \beta\theta\mathbb{E}_{jt} \pi_{i,t+1} \end{aligned} \quad (14)$$

where  $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$ , and the aggregate Phillips curve can be written as

$$\pi_t = \kappa\theta \sum_{k=0}^{\infty} (\beta\theta)^k \bar{\mathbb{E}}_t^f \tilde{y}_{t+k} + (1 - \theta) \sum_{k=0}^{\infty} (\beta\theta)^k \bar{\mathbb{E}}_t^f \pi_{t+k} \quad (15)$$

## Parameter Values

Parameter	Description	Value	Source/Target
$\sigma$	IES	1	Galí (2015)
$\beta$	Discount factor	0.99	Galí (2015)
$\varphi$	Inv. Frisch Elas.	5	Galí (2015)
$1 - \alpha$	Labor share	0.75	Galí (2015)
$\theta$	Calvo lottery	0.75	Galí (2015)
$\epsilon$	Elas. Subs. between goods	9	Galí (2015)
$\rho_v$	Monetary shock process persistence	0.94	Estimated
$\phi_y$	Output coefficient Taylor rule	0.5	Estimated
$\phi_{\pi,pre}$	Inflation coefficient Taylor rule pre-1985	1	Estimated
$\phi_{\pi,post}$	Inflation coefficient Taylor rule post-1985	2	Estimated
$\sigma_{u,pre}^2$	Signal innovation variance pre-1985	0.445	$\beta_{CG,pre}$ in Estimated
$\sigma_{u,post}^2$	Signal innovation variance post-1985	0.095	$\beta_{CG,post}$ in Estimated

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## Optimal expectations

- ▶ Aggregate inflation persistence depends on individual expectation's anchoring
- ▶ Guessing (and verifying) dynamics for  $\tilde{y}_t$  and  $\pi_t$ , we can rewrite the firm problem as

$$\pi_{jt} = -\frac{1 + \rho_v \chi \left( \psi_\pi + \frac{\sigma \psi_y}{1 - \frac{\vartheta}{\rho_v}} \right)}{\sigma + \phi_y} \kappa \theta \mathbb{E}_{jt} v_t + \left[ 1 - \theta - \kappa \theta \frac{\phi_\pi - \vartheta}{\sigma(1 - \vartheta) + \phi_y} \right] \mathbb{E}_{jt} \pi_t + \beta \theta \mathbb{E}_{jt} \pi_{j,t+1}$$

### Proposition 4

*Firm  $i$ 's nowcast of the monetary policy shock process is*

$$\mathbb{E}_{it} v_t = \mathbb{E}_{i,t-1} v_t + G(x_{it} - \mathbb{E}_{i,t-1} v_t) \quad (16)$$

*where the Kalman gain is given by  $G(\rho, \sigma_\varepsilon, \sigma_u) = 1 - \frac{\lambda}{\rho}$ . Firm  $i$ 's expectations of current aggregate output and individual future inflation as*

$$\begin{aligned} \mathbb{E}_{it} \pi_t &= \mathbb{E}_{i,t-1} \pi_t + G_1(x_{it} - \mathbb{E}_{i,t-1} v_t) \\ \mathbb{E}_{it} \pi_{i,t+1} &= \mathbb{E}_{i,t-1} \pi_{i,t+1} + G_2(x_{it} - \mathbb{E}_{i,t-1} v_t) \end{aligned}$$

*where  $G_k(\beta, \sigma, \theta, \kappa, \phi_\pi, \phi_y, \rho, \sigma_\varepsilon, \sigma_u)$  for  $k = \{1, 2\}$ , satisfying  $G_1 < G_2 < G$ .*

# Optimal expectations

- ▶ Exogenous variable forecast: expectations will only update by a factor  $G \in (0, 1)$ , a firm does not need to infer others' decision
  - ▶ agents only need to rely on their private information, since others' actions do not determine the forecasted variable
- ▶ Forecasts of endogenous variables depend on others' actions, giving rise to higher-order beliefs
  - ▶ degree of anchoring is higher at each belief order
  - ▶ larger anchoring in expectations of endogenous aggregates

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Table: Regression table

	(1) NKPC	(2) Break Output	(3) Break
$\bar{\mathbb{E}}_t^f \tilde{y}_{t+2,t}$	1.014*** (0.262)	1.402*** (0.438)	1.079** (0.418)
$\bar{\mathbb{E}}_t^f \tilde{y}_{t+4,t+2}$	-0.0717 (0.335)	-0.680 (0.553)	-0.354 (0.533)
$\bar{\mathbb{E}}_t^f \pi_{t+2,t}$	-0.0552 (0.0652)	-0.0352 (0.0602)	-0.264*** (0.0836)
$\bar{\mathbb{E}}_t^f \pi_{t+4,t+2}$	-0.0375 (0.151)	-0.123 (0.147)	0.237 (0.180)
$\bar{\mathbb{E}}_t^f \tilde{y}_{t+2,t} \times \mathbb{1}_{\{t \geq t^*\}}$		-0.892* (0.526)	-0.598 (0.509)
$\bar{\mathbb{E}}_t^f \tilde{y}_{t+4,t+2} \times \mathbb{1}_{\{t \geq t^*\}}$		0.882 (0.662)	0.555 (0.641)
$\bar{\mathbb{E}}_t^f \pi_{t+2,t} \times \mathbb{1}_{\{t \geq t^*\}}$			0.303*** (0.0955)
$\bar{\mathbb{E}}_t^f \pi_{t+4,t+2} \times \mathbb{1}_{\{t \geq t^*\}}$			-0.486** (0.191)
Constant	-0.115 (0.250)	0.388 (0.398)	0.479 (0.460)
Observations	99	99	99

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$



## Proposition 5 [Back](#)

In the dispersed information framework, equilibrium output gap and inflation dynamics are given by

$$\begin{aligned}\tilde{y}_t &= -\frac{\vartheta(\phi_\pi - \vartheta)}{\sigma(1 - \vartheta) + \phi_y} \pi_{t-1} - \psi_y \frac{\chi(\vartheta)}{1 - \vartheta/\rho} v_t \\ \pi_t &= \vartheta \pi_{t-1} - \psi_\pi \chi(\vartheta) v_t\end{aligned}$$

where

$$\chi(\vartheta) = \left( 1 - \frac{\frac{\kappa\sigma\vartheta(\phi_\pi - \vartheta)}{\sigma(1 - \vartheta) + \phi_y}}{(1 - \rho\beta)[\sigma(1 - \rho) + \phi_y] + \kappa(\phi_\pi - \rho) + \frac{\kappa\sigma\vartheta(\phi_\pi - \vartheta)}{\sigma(1 - \vartheta) + \phi_y}} \right) \left( 1 - \frac{\vartheta}{\rho} \right) \in (0, 1)$$

and  $\vartheta$  is a scalar that is given by the reciprocal of the largest root of the following cubic

$$\mathcal{P}(z) = (\beta\theta - z)(z - \rho^{-1})(z - \rho) - \frac{\sigma_\varepsilon^2}{\rho\sigma_u^2} z\theta \left[ \beta - z \left( 1 + \frac{\kappa(\phi_\pi - \vartheta)}{\sigma(1 - \vartheta) + \phi_y} \right) \right] \quad (17)$$

## Proposition 6 ( [Back](#) )

The ad-hoc hybrid dynamics (??) produces identical dynamics to the dispersed information model if

$$\begin{aligned} B - \varphi &= \omega_f \delta (AB + \rho B) \\ \omega_b &= (I_2 - \omega_f \delta A) A \end{aligned}$$

for certain matrices  $\omega_b$  and  $\omega_f$

$$\omega_b = \begin{bmatrix} \omega_{b,11} & \omega_{b,12} \\ \omega_{b,21} & \omega_{b,22} \end{bmatrix} \quad \text{and} \quad \omega_f = \begin{bmatrix} \omega_{f,11} & \omega_{f,12} \\ \omega_{f,21} & \omega_{f,22} \end{bmatrix}$$

where

$$A = \begin{bmatrix} 0 & -\frac{\vartheta(\phi_\pi - \vartheta)}{\sigma(1-\vartheta) + \phi_y} \\ 0 & \vartheta \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -\psi_y \\ -\psi_\pi \left(1 - \frac{\vartheta}{\rho}\right) \end{bmatrix} \chi(\vartheta)$$

## Corollary 1 ( [Back](#) )

The as if DIS and Phillips curve dynamics are described by

$$\begin{aligned}\tilde{y}_t &= \frac{\omega_{y\pi}}{\sigma} \pi_{t-1} - \frac{1}{\sigma} \mathbb{E}_t r_t + \frac{\delta_{yy}}{\sigma} \mathbb{E}_t \tilde{y}_{t+1} + \frac{\delta_{y\pi} - 1}{\sigma} \mathbb{E}_t \pi_{t+1} \\ \pi_t &= \omega_{\pi\pi} \pi_{t-1} + \kappa y_t + \delta_{\pi y} \mathbb{E}_t \tilde{y}_{t+1} + \delta_{\pi\pi} \beta \mathbb{E}_t \pi_{t+1}\end{aligned}$$

where

$$\omega_{y\pi} = (\sigma + \phi_y) \omega_{b,12} + \phi_\pi \omega_{b,22}$$

$$\omega_{\pi\pi} = \omega_{b,22} - \kappa \omega_{b,12}$$

$$\delta_{yy} = \frac{\sigma}{\sigma + \phi_y + \kappa \phi_\pi} [(\sigma + \phi_y)(\omega_{f,11} + \kappa \omega_{f,12}) + \phi_\pi(\omega_{f,21} + \kappa \omega_{f,22})]$$

$$\delta_{y\pi} = \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \{ (1 - \beta \phi_\pi)[(\sigma + \phi_y)\omega_{f,11} + \phi_\pi \omega_{f,21}] + (\kappa + \beta\sigma + \beta\phi_y)[(\sigma + \phi_y)\omega_{f,12} + \phi_\pi \omega_{f,22}] \}$$

$$\delta_{\pi y} = \frac{\sigma}{\sigma + \phi_y + \kappa \phi_\pi} [(\omega_{f,21} - \kappa \omega_{f,11}) + \kappa (\omega_{f,22} - \kappa \omega_{f,12})]$$

$$\delta_{\pi\pi} = \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \{ (1 - \beta \phi_\pi)(\omega_{f,21} - \kappa \omega_{f,11}) + [\kappa + \beta(\sigma + \phi_y)] (\omega_{f,22} - \kappa \omega_{f,12}) \}$$