

# **Heterogeneous Expectations and the Business Cycle at the Effective Lower Bound**

**EDMM Conference, CNB**

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Bank of Canada

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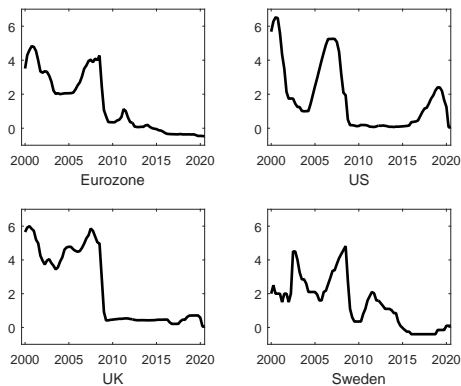


Figure: Interest rates over 2000-2020.

- ▶ **Main Goal:** studying the interaction between expectations and the business cycle over the ELB period.
- ▶ What are the implications of heterogeneous and de-anchored expectations on post-GFC dynamics?
  - ▶ Follow-up on Özden and Wouters (2020).

- ▶ Monetary policy and regime switching
- ▶ Rational Expectations and Adaptive Learning in Regime Switching Models
- ▶ Heterogeneous Expectations
- ▶ Estimations and Counterfactuals

# ELB in a Regime-switching Framework

- ▶ Standard 3-equation New Keynesian model (Model details)

- ▶ Monetary policy:

$$r_t = \max\{r^*, \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \varepsilon_{r,t}^T\},$$

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- ▶ which can be approximated as a 2-regime process:

$$\begin{cases} r_t(s_t = T) = \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \varepsilon_{r,t}^T, \\ r_t(s_t = Z) = \varepsilon_{r,t}^Z. \end{cases}$$

- ▶ The second regime is modeled as a *Stochastic Lower Bound*.

► Transition matrix:  $Q = \begin{bmatrix} q_t^T & 1 - q_t^T \\ 1 - q_t^Z & q_t^Z \end{bmatrix}$ ,

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► Regime switching in the shadow rate:

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► Heterogeneity in expectations: rational agents and adaptive learners.

# Rational Expectations in Regime-switching Models

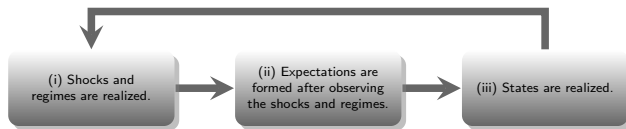


Figure: Intra-period timeline in a regime-switching Rational Expectations system.

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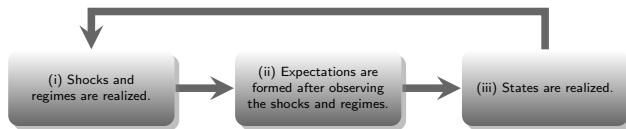


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## ► Two assumptions within the ELB context:

- Agents know the consequences of the ELB (no transitory period).
- Agents know the regime probabilities: at all times, they hold realistic beliefs about the duration of ELB.

# Rational Expectations in Regime-switching Models

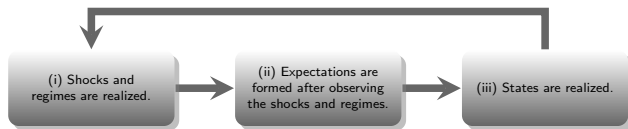


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## ► Two assumptions within the ELB context:

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## ► Taylor rule regime is determinate, ELB is indeterminate.

- **Long-run Taylor principle:** the model is stable as long as the *expected exit probability from the ELB*  $(1 - q^Z)$  is large enough.

# Estimated ELB Durations in SW Model, Özden & Wouters (2020)

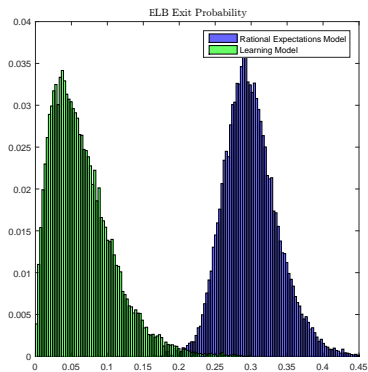


Figure: Estimated ELB exit probabilities in RE and the best-fitting learning model.

- ▶ Average expected duration in RE model: 3.34 quarters.
- ▶ Average expected duration in adaptive learning models: 26.04 quarters.

# Adaptive Learning in Regime-switching Models

$$\begin{cases} r_t(s_t = T) = \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_y y_t) + \varepsilon_{r,t}^T, \\ r_t(s_t = Z) = \varepsilon_{r,t}^Z. \end{cases}$$

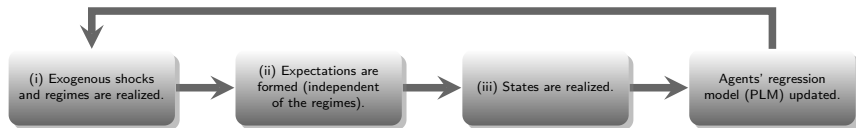


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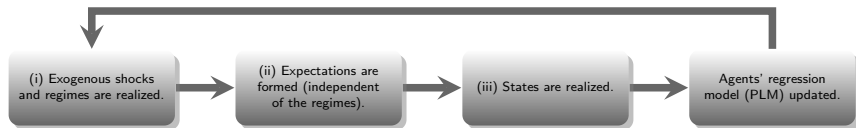


Figure: Intra-period timeline in an adaptive learning system with regime switching.

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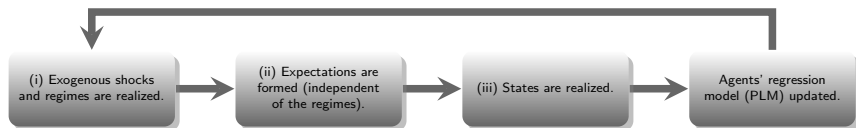


Figure: Intra-period timeline in an adaptive learning system with regime switching.

- ▶ Regime transitions are unobserved and unknown (beliefs are period specific).
- ▶ Consequences of the ELB are ex-ante unknown, and learned over time.
- ▶ **Taylor rule regime is E-stable, while ELB is E-unstable.**
- ▶ Potential de-anchoring of expectations and deflationary spirals with long ELB periods.
- ▶ E-stability illustration in the NK Model.

# Counterfactuals: Effects of Learning on the Probability of leaving ELB

	REE	REE-MS	Learning
(Log-) Bayes Factor	1	21.54	35.87

Table: Bayes Factors for some estimated RE and learning models on U.S. data, 1966:I-2016:IV.

- Monte Carlo simulations with different gain values for the learning models:

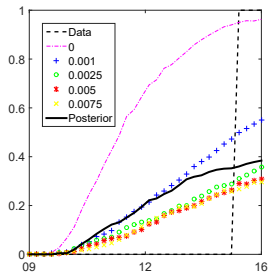


Figure: Distributions of the ELB duration in a learning model after the GFC with different .

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- Shadow rate is ignored in their forecasts (**De-stabilizing force**)
- The fractions are determined following a standard Brock & Hommes (1999) heuristic switching:

$$n_t^{RE} = \frac{\exp(-\chi FE_t^{RE})}{\exp(-\chi FE_t^{RE}) + \exp(-\chi FE_t^L)}, \quad n_t^L = \frac{\exp(-\chi FE_t^L)}{\exp(-\chi FE_t^{RE}) + \exp(-\chi FE_t^L)},$$

- The expectational switching is modeled as a **regime-switching** process.

## Heterogeneous Expectations II

- ▶ Combining heterogeneous expectations and monetary policy results in a 4-regime setup:

Monetary policy \ Expectations	Adaptive Learning	Anchored at RE
(Active) Taylor rule	Regime 1	Regime 2
ELB	Regime 3 (de-stabilizing)	Regime 4 (stabilizing)

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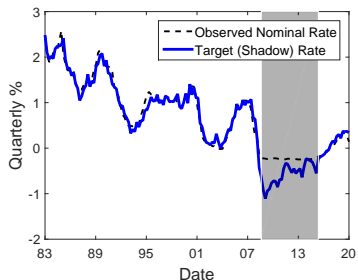
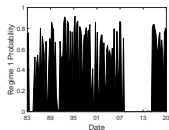


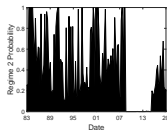
Figure: Shadow rate and the nominal interest rate in the 4-regime learning model.

- ▶ Estimated as an endogenous regime-switching model, based on the methods of Özden and Wouters (2020).
- ▶ Illustration of the filter

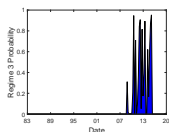
# Estimation Results: Regime Probabilities



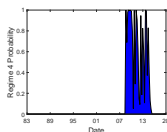
(a) Regime 1: Taylor Rule + Learning.



(b) Regime 2: Taylor Rule + RE.



(c) Regime 3: ELB + Learning.



(d) Regime 4: ELB + RE.

Figure: Estimated regime probabilities in the heterogeneous expectations model.

- ▶ pre-GFC (Regimes 1 and 2): nearly equal weights on anchored (RE) and de-anchored (learning) expectations.
- ▶ ELB period over 2008-2015 (Regimes 3 and 4): anchored expectations receive twice as much.

Some parameter estimates.

# Estimation Results: some parameters in adaptive learners' PLM

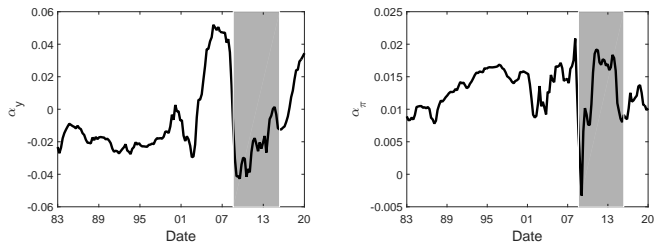
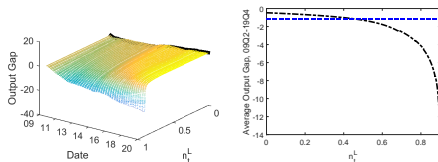


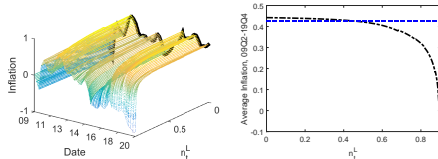
Figure: Intercept parameters in adaptive learners' PLM.

- The crisis period emerges as a *wave of pessimism* on adaptive learners.

# Counterfactual Experiments: what happens if there are more adaptive learners?



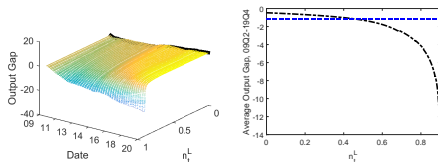
(a) Output gap



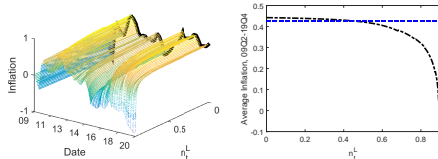
(b) Inflation

Figure: Period-specific time paths (left) and averages (right) for inflation and output gap: 2009Q4 to 2019Q4.

# Counterfactual Experiments: what happens if there are more adaptive learners?



(a) Output gap



(b) Inflation

Figure: Period-specific time paths (left) and averages (right) for inflation and output gap: 2009Q4 to 2019Q4.

- ▶ Taylor rule probabilities over the counterfactual period
- ▶ Counterfactuals with fractions and gains
- ▶ Pandemic period

- ▶ Parsimonious heterogeneous expectations model to assess the degree of de-anchoring and learning over the ELB.
- ▶ More learning (larger fraction of adaptive learners, or higher gains) are associated with adverse outcomes.
- ▶ Limiting the de-anchoring of expectations is important to avoid longer ELB durations and deflationary spirals.

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- ▶ **Future work:**
  - ▶ Extension with QE to disentangle the effects of different unconventional policies.
  - ▶ MP & Fiscal policy interactions over the ELB period.

## Appendix: Standard 3-equation New Keynesian Model

- ▶ IS Equation:

$$y_t = (1 - \iota_y)E_t y_{t+1} + \iota_y y_{t-1} - \frac{1}{\tau}(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t},$$

- ▶ New Keynesian Phillips Curve:

$$\pi_t = \beta((1 - \iota_p)E_t \pi_{t+1} + \iota_p \pi_{t-1}) + \kappa y_t + \varepsilon_{\pi,t},$$

- ▶ AR(1) Shocks:

$$\begin{cases} \varepsilon_{y,t} = \rho_y \varepsilon_{y,t-1} + \eta_{y,t}, \\ \varepsilon_{\pi,t} = \rho_\pi \varepsilon_{\pi,t-1} + \eta_{\pi,t}, \end{cases}$$

- ▶ Monetary policy with a Taylor rule, subject to the ELB:

$$r_t = \max\{r^*, \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_y y_t)\}$$

# Illustration E-stability Regions in the Standard New Keynesian Model

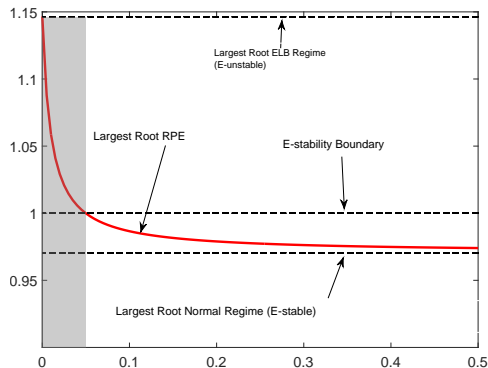


Figure: E-stability regions in the standard New Keynesian model as a function of the ELB exit probability  $1 - p^Z$ .

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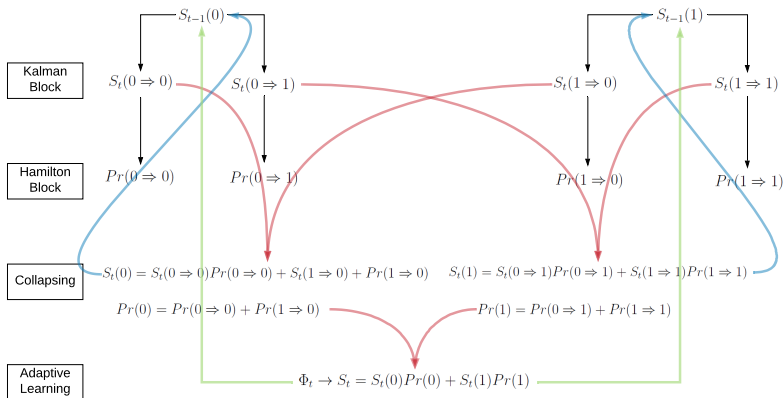
# Estimation Results

	RE (No switching)	RS-RE (2 regime)	Learning (2 regime)	Heterogeneous Exp. (4 regime)
Param.	Post. Mean	Post. Mean	Post. Mean	Post. Mean
$\kappa$ (NKPC Slope)	0.003	0.002	0.01	0.003
$\iota_y$ (Indexation)	0.274	0.243	0.113	0.322
$\iota_\pi$ (Indexation)	0.293	0.273	0.213	0.344
$\gamma$ (Gain)	-	-	0.007	0.009
(log)-likl	89.07	122.09	126.01	122.99

Table: Estimation results with U.S. data, 198Q1-2019Q4.

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# Filtering Illustration with 2-regimes



# Probabilities of the Taylor Rule Regime

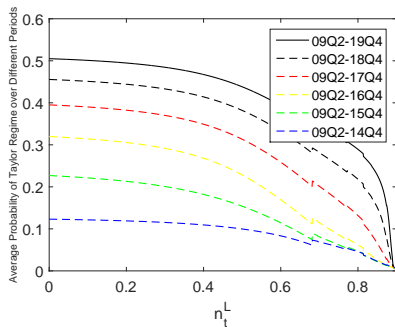
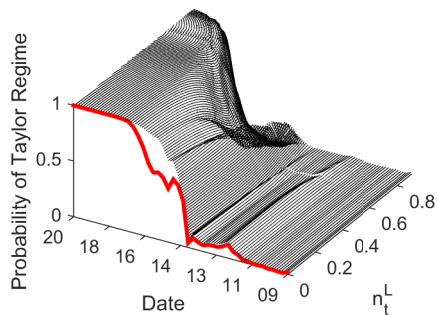
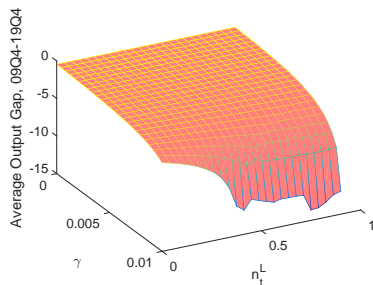


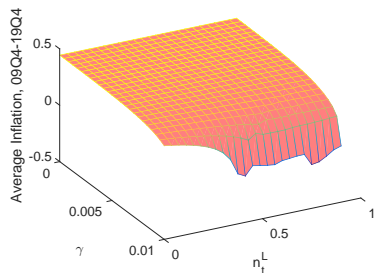
Figure: Probability of the Taylor Rule regime over different periods between 2009Q4 to 2019Q4.

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# Counterfactual Experiments: Different gains and fractions



(a) Output Gap



(b) Inflation

Figure: Averages for output gap and inflation between 2009Q4 to 2019Q4 as a function of the fraction of adaptive learners  $n_t^L$  and the constant gain  $\gamma$ .

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# Counterfactuals over 2020Q1-2024Q4: Probability of Returning to Taylor Rule Regime

	Degree of de-anchoring		
Date	Endogenous	fixed 10 %	fixed 90 %
20Q1	98.3%	98.3%	98.3%
20Q2	0.06%	0.06%	11.8%
21Q4	3.99%	04.3%	0.6%
22Q4	32.8%	33.7%	21.05%
23Q4	45.3%	44.6%	33.3%
24Q4	54.1%	52%	41.5

Table: Probabilities of returning back to the Taylor rule regime over 2020-2024 with different fractions of adaptive learners.

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## ► Studies on Survey Expectations:

- Branch, W.A. and Evans, G.W., 2006. A simple recursive forecasting model. *Economics Letters*, 91(2), pp.158-166.
- Ormeño, A. and Molnár, K., 2015. Using survey data of inflation expectations in the estimation of learning and rational expectations models. *Journal of Money, Credit and Banking*, 47(4), pp.673-699.

## ► Estimated Models:

- Slobodyan, S. and Wouters, R., 2012a. Learning in a medium-scale DSGE model with expectations based on small forecasting models. *American Economic Journal: Macroeconomics*, 4(2), pp.65-101.
- Slobodyan, S. and Wouters, R., 2012b. Learning in an estimated medium-scale DSGE model. *Journal of Economic Dynamics and control*, 36(1), pp.26-46.
- Milani, F., 2007. Expectations, learning and macroeconomic persistence. *Journal of Monetary Economics*, 54(7), pp.2065-2082.
- Milani, F., 2011. Expectation shocks and learning as drivers of the business cycle. *The Economic Journal*, 121(552), pp.379-401.
- Hommes, C., Mavromatis, K., Özden, T. and Zhu, M., 2019. Behavioral Learning Equilibria in the New Keynesian model. DNB Working paper, (654).

## ► Experiments:

- Adam, K., 2007. Experimental evidence on the persistence of output and inflation. *Economic Journal* 117, 603-636.
- Assenza, T., Heemeijer, P., Hommes, C. H., & Massaro, D., 2019. Managing self organization of expectations through monetary policy: a macro experiment. *Journal of Monetary Economics*.