Output Gap Estimation and Monetary Policy with Imperfect Knowledge

Pei Kuang   Kaushik Mitra   Li Tang

University of Birmingham

Presentation at Czech National Bank conference "Expectations in Dynamic Macro Models"

August 31, 2021

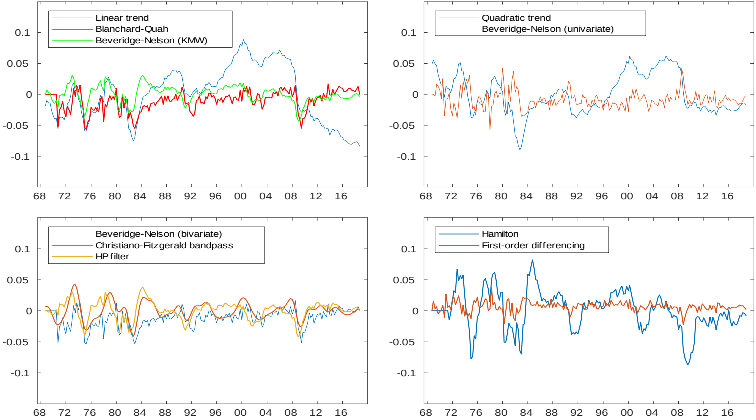

Figure: Output gap (OG) estimates for the US economy

- Econometricians offer central banks lots of OG estimation (or detrending) methods.
- Some methods intrinsically produce large and volatile OG estimates (e.g., polynominal detrending, Hamilton filter); others produce smaller estimates (e.g., bandpass filter, BN-KMW filter).

- Does the choice of OG estimation methods matter for welfare?
- Are some methods better than other methods (in terms of welfare)?
- Optimal monetary policy, given a method adopted by the policymaker?

Welfare implications of detrending (OG estimation)

- Under imperfect knowledge, detrending interacts with monetary policy decisions and macro outcomes, which has important welfare implications.
    - OG estimates from the chosen method(s) –> policy decisions –> macroeconomic outcomes –> OG estimates.
- The choice of a different detrending (OG estimation) method –> different interest rate decisions and macro outcomes.
- This paper: bridge the econometrics literature on macroeconomic detrending (or OG estimation) and analysis of monetary policy and welfare.
    - Develop a NK model in which the policymaker learns about the OG using a detrending method.

This paper

1. Characterize (numerically) the condition under which the policymaker can avoid very large welfare losses due to learning instability, for 11 OG estimation methods & several monetary policies
2. For each OG estimation method, compute optimized coefficients for Taylor-type interest rules (IT or AIT rules) and associated losses
3. Analyze stability and optimal policies under alternative trend assumptions & robust optimal policy against uncertainty about the underlying trend
4. Evaluate the reliability of detrending methods (or revision properties of OG estimates), with a crucial difference from existing literature (e.g., Orphanides and van Norden, 2002)

Model

- These issues can be analyzed, for any country, using any preferred model, for any (OG estimation) method of interest.
- This paper: a three-equation NK model with learning for the US economy, plus
  - The Okun's law with a coefficient of 2 (unemployment rates are needed for two detrending methods, i.e., Blanchard-Quah method, bivariate BN filter)

Dynamic IS curve and Phillips curve

- Dynamic IS curve

$$x_t = 0.95 x^e_{t+1} + 0.02 x_{t-1} - \underset{(0.104)}{0.296} \left(i^e_t - \pi^e_{t+1} \right) + e_{x,t} \quad (1)$$

$$e_{x,t} = \underset{(0.068)}{0.39} \, e_{x,t-1} + \nu_{x,t}, \qquad \sigma_x = 0.58\% \quad (2)$$

- Phillips curve

$$\pi_t = 0.99 \pi^e_{t+1} + \underset{(0.011)}{0.027} x_t + e_{\pi,t} \quad (3)$$

$$e_{\pi,t} = \underset{(0.069)}{0.36} \, e_{\pi,t-1} + \nu_{\pi,t}, \ \sigma_\pi = 0.41\%. \quad (4)$$

Estimation following Orphanides and Williams (2007 JME) with SPF forecasts. Note: $t-1$ dating & sample period: 1971 Q1 - 2019 Q4

Trend productivity and output process

- Trend productivity process (a_t^p) and potential output (y_t^p) process

$$\Delta a_t^p = g + e_{a,t} \tag{5}$$
$$\Delta y_t^p = \Delta a_t^p \tag{6}$$

  - $g = 2.98\%$ per annum & $\sigma_{e_{a,t}} = 0.5\%$.

Policymaker: learning, policy decisions and loss function

- Commits to a rule with a zero low bound (ZLB) on nominal interest rate

$$i_t = \max\{\theta_r i_{t-1} + \theta_\pi \pi_{t-1} + \theta_x \widetilde{x}_{t-1} + e_{i,t}, -(\pi^* + r^*)\}$$

  - $\pi^* = 2\%$ & $r^* = 4\%$ per annum.

- Applies an OG estimation method to a rolling data sample of 120 periods (30 years)

$$y_t = y_t^{tr} + \widetilde{x}_t \tag{7}$$

  - y_t^{tr} : trend component; $\widetilde{x}_t$: cyclical component.

- Loss function

$$L = Var(\pi) + \lambda_x Var(x)$$

  - $\lambda_x = 0.003$ (Giannoni and Woodford, 2005).

Households' expectations and learning

- Simple forecasting model

$$\begin{aligned} \pi_t &= \gamma^{\pi} + \epsilon_t^{\pi} && (8) \\ i_t &= \gamma^{i} + \epsilon_t^{i} && (9) \\ \Delta y_t &= \gamma^{\Delta y} + \epsilon_t^{\Delta y} && (10) \end{aligned}$$

- Belief updating

$$\begin{aligned} \gamma_t^{\pi} &= \gamma_{t-1}^{\pi} + 0.008(\pi_t - \gamma_{t-1}^{\pi}) && (11) \\ \gamma_t^{i} &= \gamma_{t-1}^{i} + 0.004(i_t - \gamma_{t-1}^{i}) && (12) \\ \gamma_t^{\Delta y} &= \gamma_{t-1}^{\Delta y} + 0.003(\Delta y_t - \gamma_{t-1}^{\Delta y}) && (13) \end{aligned}$$

Gain parameters calibrated by minimizing model-implied forecasts and corresponding SPF forecasts.

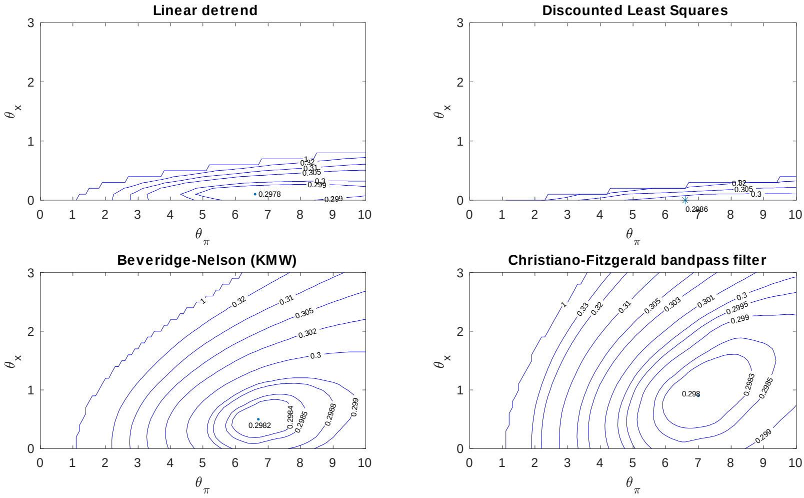

Figure 1: Iso-loss contour (no inertia)

Size of stability region: Bandpass (79.8%) & BN-KMW (70.6%) >> Linear (16.3%) & DLS (1.7%). (Note: losses are calculated based on 40,000 simulation periods & no projection facility.)

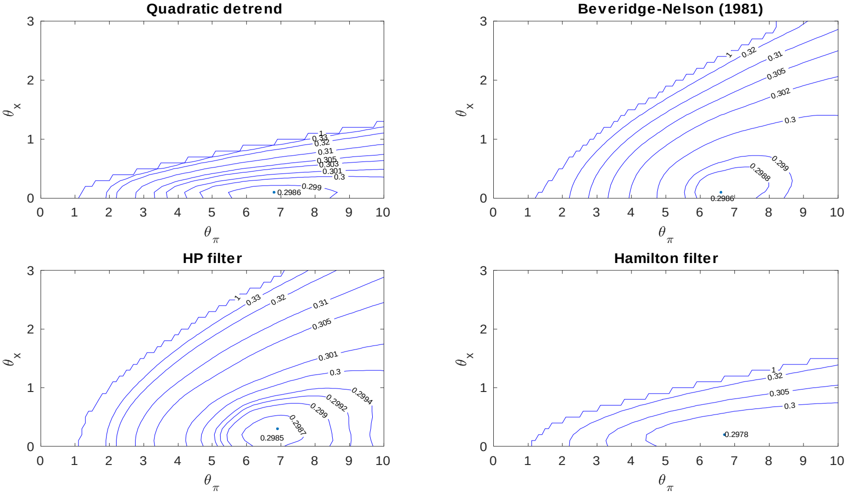

Figure 2: Iso-loss contour (no inertia)

Size of stability region: Quadratic (23.2%), BN (56.4%), HP (66.4%), Hamilton (28.7%). E.g., HP filter > Hamilton filter

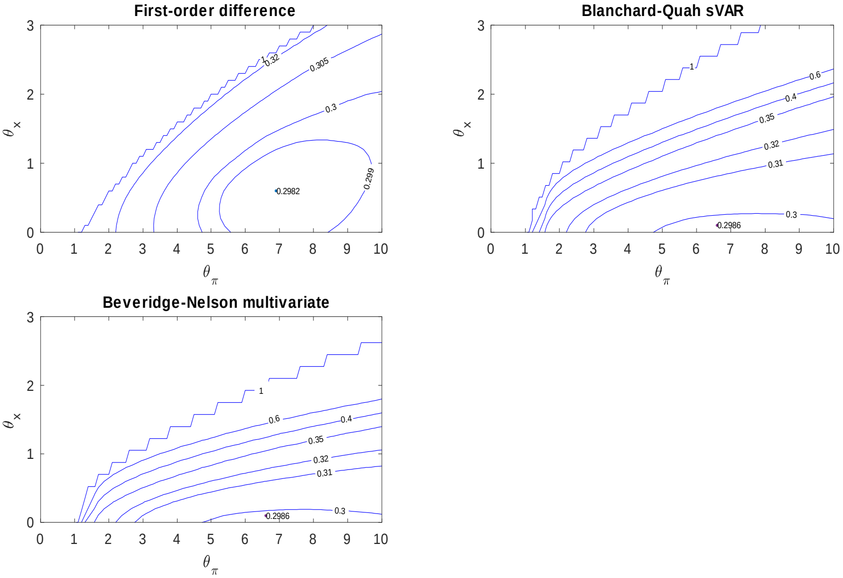

Figure 3: Iso-loss contour (no inertia)

Stability results

- Ranking according to the size of stability regions
  - {bandpass filter, BN-KMW filter} > {HP filter, BQ method, first-order differencing} > {multivariate BN filter, BN (1981) filter} > {Hamilton filter} > {linear detrending, quadratic detrending} > {DLS detrending}
- The detrending methods which intrinsically produce larger and more volatile OG estimates (e.g., polynominal detrending, Hamilton filter) –> more volatile beliefs –> more prone to learning instability and very large welfare losses.
  - For the detrending methods which intrinsically produce smaller OG estimates (e.g., BN-KMW filter, bandpass filter) –> less volatile beliefs –> less prone to learning instability.

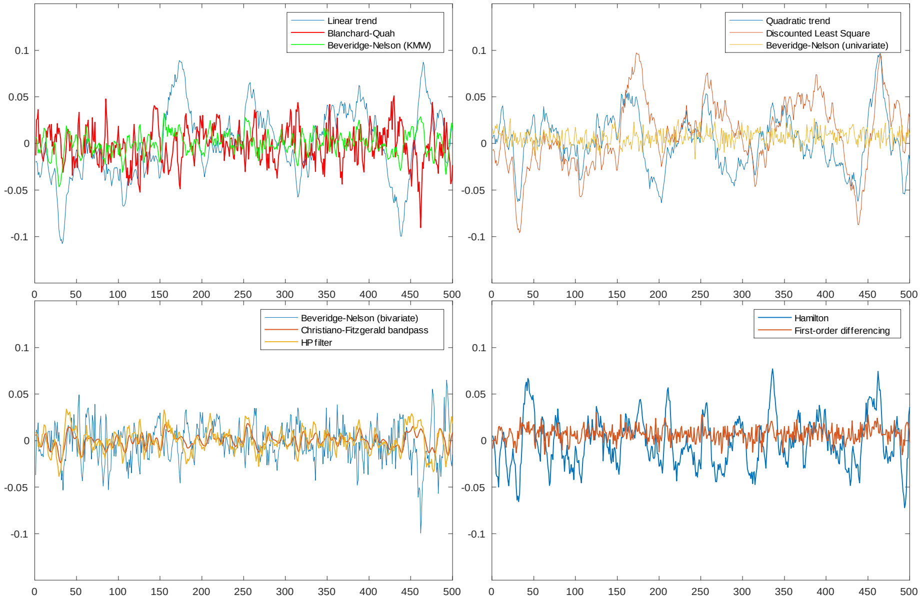

Figure: OG estimates from simulated model with original Taylor rule

Optimal policy and associated loss ($\theta_r = 0.6$)

Method	Coeff. θ_π	θ_x	SD σ_π	σ_x	Loss
Panel A: methods intrinsically producing smaller and less volatile OG estimates					
CF bandpass filter	2.9	0.5	0.475	4.899	0.298
BN-KMW filter	2.9	0.4	0.475	4.884	0.297
First-order differencing	3.1	0.6	0.477	4.840	0.297
BN filter	2.9	0.4	0.476	4.884	0.298
HP filter	2.9	0.3	0.475	4.919	0.298
Panel B: methods intrinsically producing larger and more volatile OG estimates					
Bivariate BN filter	2.8	0.0	0.475	4.943	0.299
Blanchard-Quah	2.8	0.0	0.475	4.943	0.299
Hamilton filter	3.1	0.2	0.475	4.876	0.297
Quadratic trend	2.8	0.0	0.475	4.943	0.299
Linear trend	3.1	0.1	0.475	4.886	0.298
DLS filter	2.8	0.0	0.475	4.943	0.299

1. Small difference in optimal loss across detrending methods.
2. Optimal θ_x in Panel B are small and smaller than optimal θ_x in Panel A.

A 4% inflation target (comparing with 2% in baseline)

- For all detrending methods, the chance of hitting ZLB $\downarrow$ $=>$ volatility of inflation and output gaps $\downarrow$ $=>$ volatility in beliefs $\downarrow$ $=>$ less prone to learning instability $=>$ the size of stability region $\uparrow$.
  - e.g., HP filter (66.4% to 79.3%), CF bandpass filter (79.8% to 85.8%). BN-KMW filter (70.6% to 86.9%), BQ filter (64.2% to 79.5%)
  - no change in the rank of the detrending methods (based on the size of the stability regions)
  - 10% – 20% $\downarrow$ in the chance of hitting ZLB & optimal welfare $\uparrow$.

Negative interest rates

- Similar to increasing inflation target.
- Implementation the model: removing ZLB (or imposing a negative lower bound, say -0.5% per annum).
- For all detrending methods, volatility of inflation and OG $\downarrow$ –> belief volatility $\downarrow$ –> the size of stability region $\uparrow$ & optimal welfare $\uparrow$

Average inflation targeting (AIT)

- AIT policy:

$$i_t = \max\left\{\theta_\pi \overline{\pi}^{AIT}_{t-1} + \theta_x \widehat{x}_{t-1} + e_{i,t}, -(\pi^* + r^*)\right\}$$

where $\overline{\pi}^{AIT}_t = (1-\omega)\,\overline{\pi}^{AIT}_{t-1} + \omega\pi_t$ & $\omega = 0.2$.

- For all detrending methods, chance of hitting ZLB $\downarrow$ $->$ inflation volatility $\downarrow$ & belief volatility $\downarrow$ $->$ significant $\uparrow$ in the size of stability region
- No change in the rank of detrending methods (according to the size of stability region).
- Inflation volatility $\downarrow$ $->$ optimal welfare $\uparrow$

Dual mandate regime

- Yellen (2012): equal weights on inflation and unemployment gap volatility => $\lambda_x = 0.015$ (for quarterly data).
- Comparing with the baseline results (where $\lambda_x = 0.003$), for all detrending methods
  - No change in stability regions (or rank of detrending methods);
  - Optimal response to output gap $\theta_x \uparrow$ & optimal response to inflation $\theta_\pi \downarrow$;
  - Large $\downarrow$ in θ_π & small $\uparrow$ in θ_x −> interest rate volatility $\downarrow$ −> chance of hitting ZLB $\downarrow$.

Alternative trend assumptions

- Assumption 1 (baseline) a stochastic trend with only level shocks
- Assumption 2: linear and deterministic trend
- Assumption 3: a stochastic trend with both level and growth shocks to the trend
- The results are robust to the three assumptions, e.g., no change in the rank of detrending methods (according to the size of stability regions).
  - Assumption 3: the size of stability regions $\downarrow$ & optimal welfare $\downarrow$ (comparing with baseline)
- Robust (min-max) optimal policy against uncertainty about assumption 1 – 3 = optimal policy under assumption 3
  - Assumption 3 is always the worst-case scenario.

Revision properties of OG estimates

Table: Properties of revisions to output gap estimates

Method	Mean	SD	Max	Min	AR
Linear trend	2.70	3.36	10.24	-11.71	0.999
Quadratic trend	2.19	2.75	9.81	-10.15	0.997
HP filter	1.02	1.28	5.18	-5.68	0.974
Hamilton filter	0.56	0.72	4.54	-3.97	0.875
Christiano-Fitzgerald	0.63	0.80	3.11	-3.47	0.938
BN filter	1.33	1.51	6.53	-8.25	0.903
BN-KMW	0.16	0.20	0.70	-0.83	0.973
BN multivariate	1.49	2.41	12.78	-12.37	0.881
Blanchard-Quah	1.48	2.17	9.56	-11.18	0.905

Note: calculations based on the OG estimates in the baseline model with optimal policy and no inertia.

With considering that detrending affects DGP =>

Reliability ranking: {BN-KMW filter} > {Christiano-Fitzgerald bandpass filter, Hamilton filter} > {HP filter} > {BN filter, bivariate BN filter, BQ filter} > {linear, quadratic detrending} > {DLS detrending}.

Conclusions I

- Analyze stability and optimal policy for 11 detrending methods
  - Ranking based on the size of stability region
    - {bandpass filter, BN-KMW filter} $>$ {HP filter, BQ method, first-order differencing} $>$ {multivariate BN filter, BN (1981) filter} $>$ {Hamilton filter} $>$ {linear detrending, quadratic detrending} $>$ {DLS detrending}
  - The methods which intrinsically produce large and volatile OG estimates (e.g., Hamilton filter, polynomial detrending) are prone to learning instability and very large welfare losses. Optimal response to OG is little.
  - The methods which intrinscially produce smaller OG estimates (e.g., BN-KMW filter, bandpass filter) give the policymaker large manoeuvring space to avoid very large losses. Optimal response to OG is bigger.

Conclusions II

- For all detrending methods, increasing the inflation target, or allowing negative interest rates, or adopting average inflation targeting $=>$ the size of stability regions $\uparrow$ & optimal welfare$\uparrow$
- The findings still hold under three alternative trend assumptions. And robust optimal policy against uncertainty under three trend assumptions = optimal policy when the trend is stochastic and contains level and growth shocks.
- Evaluate reliability of the detrending methods, considering the interaction of detrending with policy decisions and macro outcomes
  - Ranking based on reliability: {BN-KMW filter (KMW, 2016)} $>$ {Christiano-Fitzgerald bandpass filter, Hamilton filter} $>$ {HP filter} $>$ {BN filter, bivariate BN filter, BQ filter} $>$ {linear, quadratic detrending} $>$ {DLS detrending}.