

Falling Natural Rates, Rising Housing Volatility and the Optimal Inflation Target

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Abstract

The decline in natural interest rates in advanced economies over the past decades has been accompanied by a significant increase in the volatility of housing prices. We show that the monetary policy implications of these macroeconomic trends depend - in the presence of a lower-bound constraint on nominal rates - on the source of increased housing price volatility. If housing price expectations are rational, increased housing price volatility reflects more volatile housing demand shocks. The Ramsey optimal inflation target then increases only minimally as average natural rates fall. Instead, if housing price volatility is partly due to speculative housing price beliefs, as suggested by survey data, then lower natural rates endogenously trigger larger fluctuations in belief-driven housing price fluctuations. This causes the volatility of the natural rate to increase and exacerbates the lower-bound problem. As a result, the Ramsey optimal inflation target rises much more strongly as the natural rate falls.

JEL Codes: E31, E44

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1 Introduction

The persistent fall in natural interest rates and long-term growth rates represent some of the most troubling macroeconomic trends in advanced economies over the past decades (Holston et al. (2017), Del Negro et al. (2017) and Fujiwara et al. (2016)).¹ These trends have received considerable attention in policy circles and the academic literature alike, as weak economic growth and a persistent decline in the natural rate pushed many central banks for extended

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¹The natural rate of interest is the real interest rate that is consistent with a stable inflation rate.

periods towards the Effective Lower Bound (ELB), i.e., to a situation where nominal interest rates are zero or slightly negative.

This paper documents that the fall in long-term growth rates and natural interest rates in advanced economies has been associated with two additional adverse macroeconomic trends. First, the volatility of housing prices has increased considerably over recent decades. This trend is concerning because increased housing price volatility has the potential to significantly increase macroeconomic instability, as happened during the 2007-09 global recession. Second, estimates of the natural rate of interest suggest that the *volatility* of the natural rate of interest has increased at the same time as its average level fell.

Both of these developments are troubling from the perspective of a central bank confronted with falling levels of the natural rate of interest. Given the downward pressure that falling natural rates exert on nominal rates and given the inability to lower nominal rates below the ELB, central banks may become increasingly unable to stabilize housing markets or - more generally - unable to implement the natural rate of interest via appropriate changes in nominal interest rates.

While central banks in advanced economies have responded to this situation by deploying a variety of large quantitative easing programs, they have nevertheless persistently and significantly undershot their inflation targets.² This suggests that the ELB constraint on nominal interest rates represents an important impediment to the conduct of monetary policy and that the newly developed quantitative easing toolkit is at most an imperfect substitute for standard interest rate policy.

The goal of this paper is to present a parsimonious structural economic model that allows to jointly discuss these macroeconomic trends and to determine their implications for monetary policy design. Particular emphasis is given to the question how these trends affect the inflation target that the central bank should optimally pursue.³

To achieve these goals, we consider a sticky price model featuring a lower-bound constraint for nominal interest rates and a housing sector. The model is consistent with lower growth rates triggering lower natural rates, which in turn increase the volatility in housing markets. Yet, the monetary policy implications of increased housing market volatility depends crucially on the economic drivers responsible for the increase in housing price volatility.

If housing price volatility is driven purely by structural economic shocks, whose fluctuations may have increased over time, then the optimal inflation target should increase only mildly as trend growth rates (and natural rates) fall. Instead, if housing price fluctuations are partly driven by speculative housing price beliefs, as suggested by household survey data, then average inflation must rise considerably more as trend growth rates (and natural rates) fall.

This result emerges because an increase in housing price volatility that is driven by increased fluctuations in subjective beliefs also increases the volatility of the natural rate. The latter is in line with the data and exacerbates the lower-bound problem for monetary policy. Monetary policy optimally responds to this situation by promising higher inflation

²This holds particularly true for the larger and more closed currency areas, e.g., the Euro Area and the United States, where the exchange rate is less relevant for domestic inflation dynamics.

³The inflation target is defined as the average inflation rate emerging under Ramsey optimal monetary policy.

rates.⁴

To analyze monetary policy in the presence of speculative private sector expectations, we consider a setting in which households and firms are *internally rational*, i.e., maximize utility (or profits) conditional on their subjective beliefs about variables outside of their control (Adam and Marcet (2011)). Internal rationality encompasses fully rational expectations as a special case, when subjective beliefs coincide with the objective outcomes generated by the model.

Our analysis considers both a setting with rational housing price expectations and a setting with subjective housing beliefs in which household extrapolate (to some extent) past housing capital gains into the future, following Adam et al. (2016).⁵ To make the deviations from rational expectations as parsimonious as possible, we assume throughout the paper that expectations about variables other than housing prices (and associated rental rates) are rational at all times.

The subjective belief setup replicates important features of households' forecast errors about future housing prices that we document using the Michigan household survey. In particular, the survey expectations display pro-cyclical capital gain expectations, while actual capital gains are counter-cyclical. Moreover, expectations are adjusted sluggishly, as past forecasts revisions predict future forecast errors in the same direction. When considering the whole dynamics of housing-price forecasts, we document that housing-price expectations initially underreact, followed by a delayed overshooting.

Besides matching features of the forecast error structure, extrapolation of past capital gains generates housing prices that are characterized by momentum and mean reversion: housing prices display occasional and long-lived boom-bust patterns in line with the empirically observed behavior of housing prices (Glaeser and Nathanson (2017)). Importantly, these boom-bust episodes emerge with higher likelihood when the average natural interest rate is low. This is the case because housing prices become more sensitive to subjective housing price expectations when real interest rates are low.

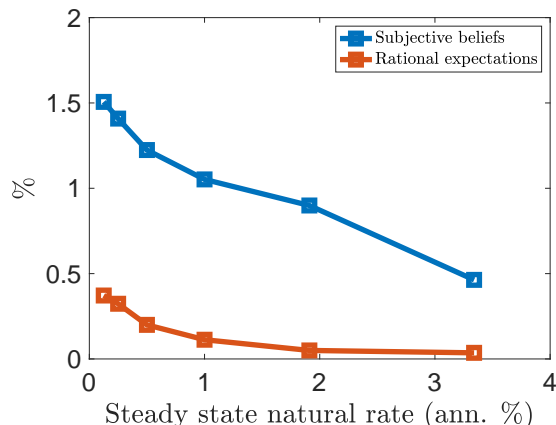
The fact that belief-driven housing price fluctuations become more prevalent with low natural rates, implies that the natural rate of interest becomes more volatile. This is so because agents optimally condition their subjective consumption and investment plans on their subjective housing price beliefs. The real interest rate that is consistent with the *objectively* efficient consumption allocation thus depends on agents' subjective housing price beliefs, whose volatility has increased. The subjective belief model can thus generate falling average natural rates, rising housing price volatility and rising natural rate volatility solely from a downward trend in the trend growth rate of the economy.

We calibrate the model such that it matches the observed standard deviation of the price-to-rent ratio and the natural interest rate in the United States before 1990. When moving from a relatively high average value for the natural rate, as observed before the 1990s, to

⁴The presence of subjective housing beliefs also makes it optimal for monetary policy to lean against shocks that affect housing demand, unlike in a setting with rational housing expectations.

⁵Experimental evidence on housing price expectations (Armona et al. (2019)) supports the notion that housing price expectations are extrapolative. Soo (2018) constructs a housing sentiment index based on media coverage of housing news, documenting the extrapolative nature of house price expectations. Extrapolative capital gain and return expectations in stock markets have been documented by Greenwood and Shleifer (2014) and Adam et al. (2017).

Figure 1: Average Inflation under Optimal Monetary Policy



Notes: The figure reports the optimal inflation target (y-axis) for different average levels of the natural rate (x-axis) in the presence of a zero lower bound constraint. The red line depicts the optimal target for the case with rational housing price beliefs and the blue line the one with subjective housing price beliefs. In the absence of a lower bound constraint, i.e., if quantitative easing measures are a perfect substitute for nominal rate reductions below the lower bound constraint, the optimal inflation target is zero, independently of the average level of the natural rate.

the lower average level observed thereafter, the rational expectations model generates an increase in the volatility of the price-to-rent ratio that is much smaller than in the data and no additional volatility for the natural rate.⁶ As a result, even when the average natural rate falls to an annual level of 0.125%, the average inflation rate under optimal monetary policy increases by only 0.4%. Under rational expectations and optimal monetary policy, lower natural rates thus do not justify significant increases in the inflation target. This is illustrated in figure 1 which reports the increase in the optimal inflation target (y-axis) due to an ELB constraint on nominal rates as a function of the average natural rate (x-axis).

In contrast, the model with subjective housing price beliefs matches the bulk of the observed increase in housing volatility following the observed decline in natural rates after 1990, without having to resort to increased volatility of housing demand shocks. This is the case because lower natural rates amplify belief-driven fluctuations in housing prices and thereby magnify momentum effects in prices. The more volatile housing beliefs contribute to a considerable increase in the volatility of the natural rate, close to the one estimated for the U.S. after 1990. In response, average inflation under Ramsey optimal monetary policy increases substantially more compared to the setting with rational expectations. For an average natural rate of 0.125% per year, optimal monetary policy gives rise to an average inflation rate of 1.5%, see figure 1.

Additionally to the higher inflation target under subjective beliefs, we find that it is optimal for the policymaker to "lean against the wind". After a housing demand shock, the model with subjective beliefs triggers additional increases in housing prices due to the extrapolative expectations. These inefficient increases in housing prices distort the optimal

⁶Under rational expectations, housing-price-to-rent ratios are solely driven by housing preference shocks, which themselves do not affect the natural rate.

consumption allocation and fuel a housing investment boom. To counteract this, the monetary authority increases the nominal interest rate. Under rational expectations, on the other hand, the policymaker does not react to housing shocks as these are fully efficient.

The rest of the paper is structured as follows. Section 2 discusses the related literature and section 3 presents the empirical facts about trend growth rates, natural rates and housing prices. Section 4 presents the economic model, which allows for subjective beliefs, a lower bound constraint and a housing sector and section 5 derives the nonlinear optimal monetary policy problem. The two alternative drivers of house price fluctuations —housing preference shocks and subjective housing price beliefs—are introduced in Section 6. Section 7 presents a quadratic approximation to the monetary policy problem, which allows obtaining important analytic insights into the structure of the policy problem. We calibrate the model in section 8 and present our main quantitative results in section 9. Section 10 concludes.

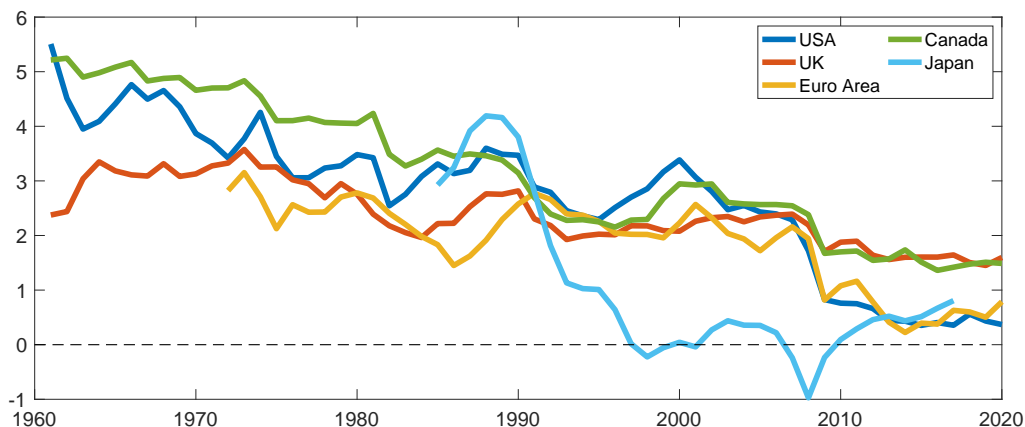
2 Related Literature

Eggertsson et al. (2019) and Le Bihan et al. (2019) also study the implications of lower natural rates for monetary policy. Consistent with our findings, they show that a substantial increase in the inflation target is a promising monetary instrument to deal with the effective lower bound (ELB) problem. We add to their work by studying Ramsey optimal policy, by considering a model featuring a housing sector, and by allowing for the presence of subjective beliefs.

Earlier work discussed Ramsey optimal monetary policy in the presence of an ELB constraint, but abstracted from housing markets and the presence of subjective beliefs. A seminal early contribution is due to Eggertsson and Woodford (2003), who show how commitment to future inflation can help ameliorating the effects of not being able to lower nominal rates further at the ELB. The one-time nature of the shock in their analysis precluded, however, a discussion of the effects of the ELB on average inflation. Adam and Billi (2006) and Coibion et al. (2012) discuss Ramsey optimal monetary policy under rational expectations in a setting where repeated shocks cause the ELB to be occasionally binding. They find that ELB episodes tend to be short and infrequent under optimal policy, so that average inflation increases by a few basis points only, compared to a setting that ignores the existence of the ELB constraint. The present paper shows that this conclusion is altered in a model with housing and in the presence of subjective housing price expectations.

As we show, the implications of asset price booms and busts for monetary policy generally depend on the source of these fluctuations. Iacoviello (2005) introduces housing markets into a monetary business cycle model and shows that output and inflation volatility are almost independent of whether or not the monetary authority responds to housing prices. House price fluctuations in Iacoviello (2005), however, are driven by structural shocks and thus efficient. In the present setting, which allows for belief-driven fluctuations in housing prices, it becomes optimal for monetary policy to “lean against” housing price fluctuations. Adam and Woodford (2020) and Caines and Winkler (2020) also find that leaning against housing price increases can be optimal in the presence of distorted beliefs. We add to this literature by taking into account the ELB constraint on nominal interest rates, which has important implications for the optimal inflation target.

Figure 2: The evolution of Natural Rates



Notes: This figure shows the evolution of the natural rate of interest for different advanced economies. The natural rate for USA, Canada, and the Euro area is taken from Holston et al. (2017). The natural rate for Japan is taken from Fujiwara et al. (2016).

More generally, Kaplan et al. (2020) argue that subjective beliefs about future house prices were the most important driver of housing prices in the U.S. economy during the boom and bust phase around the Great Recession. Adam et al. (2012) find that subjective housing price dynamics can explain the dynamics of housing booms and current accounts in the G7 economies up until the Great Recession. Adelino et al. (2017) document that rising home prices and increased housing price expectations were the main contributor to the observed increase in U.S. mortgage debt before the Great Recession.⁷ Section 6.3 in the present paper shows that the housing price expectations of U.S. households, as collected in the Michigan survey, are in fact inconsistent with the RE hypothesis. Taken together these contributions render credibility to the notion that subjective housing price beliefs play an important role for housing price dynamics. As a result it becomes important to consider the monetary policy implications of such fluctuations in the presence of an ELB constraint.

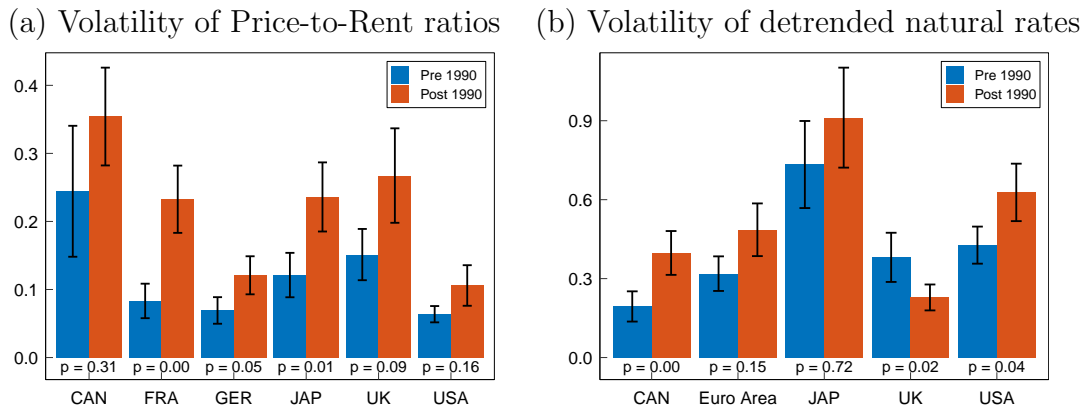
Clearly, the present paper abstracts from a number of economic features that can generate additional effects on the optimal inflation target, e.g., cash-distortions as analyzed in Khan et al. (2003), nominal wage rigidity considered in Benigno and Ricci (2011), or the relative price trends recently studied in Adam and Weber (2019, 2020).

3 Natural Rates and Housing Prices in Advanced Economies

The secular decline in natural rates. Natural real interest rates have displayed a steady downward trend in the G7 economies over the past decades. Figure 2 illustrates these trends

⁷Further evidence on the important role of expectations as drivers of house prices is documented, e.g., in Case and Shiller (1988), Shiller (2007), Piazzesi and Schneider (2009), Case et al. (2012), and Ben-David et al. (2019).

Figure 3: The volatility of detrended Natural Rates and Price-to-Rent Ratios



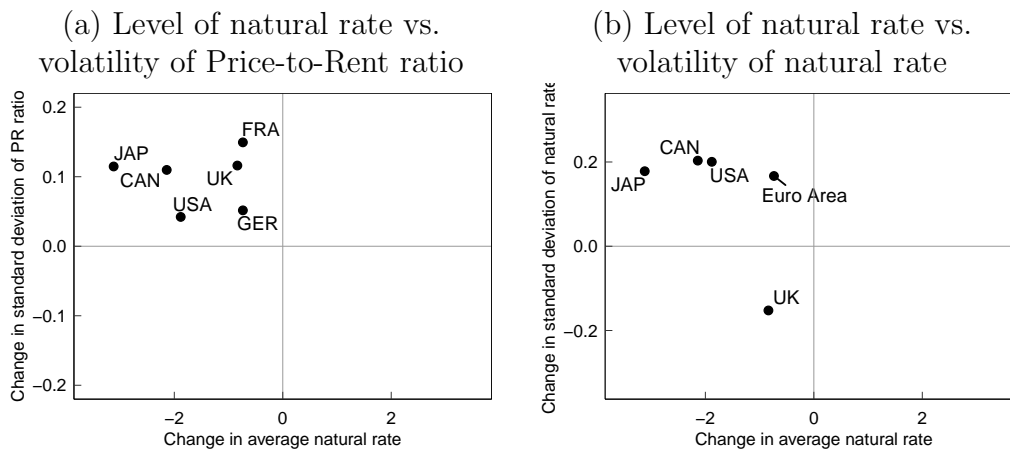
Notes: Panel (a) shows the standard deviation of the price-to-rent ratio for different advanced economies. Panel (b) shows the standard deviation of the natural rate (linearly detrended). The blue bars show the estimates for the pre-1990 period, and the red bars for the post-1990 period. The black whiskers denote 90%-confidence bands. The reported p -values are for the null hypothesis that the volatility has not changed from pre- to post-1990.

using the estimated natural rates of Holston et al. (2017) for USA, Canada and the Euro area, and from Fujiwara et al. (2016) for Japan.

The rising volatility of housing prices. A concurrent development is that the volatility of housing prices has increased markedly in the G7 economies over the period 1970-2019. It is generally difficult to estimate the volatility of housing prices in a precise manner because housing prices tend to display a high degree of persistence over time.⁸ Therefore, we compare the volatility of the Price-to-Rent ratio in the periods pre-1990 and post-1990. More specifically, we compute standard deviations in terms of percentage deviations from the sample mean of the considered period, so as to be consistent with the theoretical model developed later on. We take the Price-to-Rent ratios from the OECD database. Figure 3(a) reports the estimated standard deviation of the Price-to-Rent ratios for the two periods, together with 90%-confidence bands (black whiskers), and the p -value of the null-hypothesis that the standard deviations have not changed across the two subsamples. The point estimate of the standard deviation of the Price-to-Rent ratio has increased in all economies. This said, due to the large estimation uncertainty, not all increases in the point estimates are statistically significant at conventional significance levels. At the 10% confidence level, four of the six observations are statistically significant. Appendix B.1 shows the robustness of this finding. Figure 12(a) shows that the results are robust to considering the absolute standard deviation across the two subperiods and Figure 13(a) shows that they are robust to using different sample split points.

⁸In the Appendix we consider the autocorrelation of the annual Price-to-Rent ratios in the G7 economies. Table 5 reports the autoregressive coefficient of an estimated AR(1) process in the Price-to-Rent ratios. The table reports estimates for the whole sample, as well as for the first and last 30 years of the sample period. Price-to-Rent ratios are overall highly persistent and for some countries the point estimates are very close to unity, especially post-1990.

Figure 4: Cross-country correlation between falling natural rates and increasing volatilities



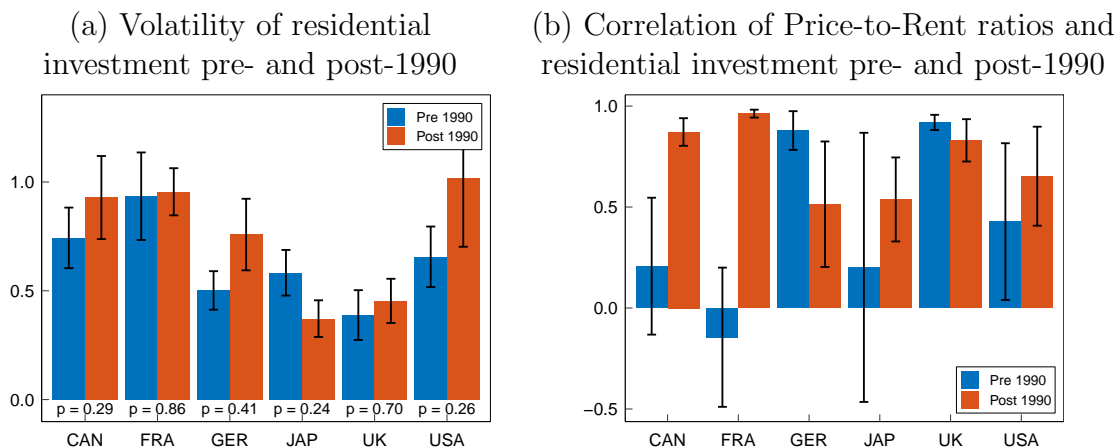
Notes: Panel (a) plots the pre-post-1990 changes in the average natural rates (the evolution of which are shown in Figure 2) against the changes in the Price-to-Rent ratio volatility (as shown in Figure 3(a)).

The increase in natural rate volatilities. While the decline in natural rate *levels* has received considerable attention in the literature, the evolution of the *volatility* of the natural rate over time has received virtually no attention. Figure 3(b) shows the standard deviation of the natural rate pre- and post-1990 after taking out a linear time trend. The figure reports the standard deviations for the two periods, together with 90% confidence bands (black whiskers), and the p -values for the null hypothesis that the standard deviation has not changed from pre- to post-1990. The standard deviations have increased over time in all currency areas, except for the UK. Yet, since estimation uncertainty is relatively large, the increase in the point estimates are statistically significant only for the United States and Canada. Appendix B.1 shows that the increase in the natural rate volatility is a robust finding as well. Figure 13(b) shows the robustness to using different sample split points and Figure 12(b) shows robustness to considering the standard deviation of the undetrended natural rate.

The link between natural rates levels and volatilities. We take the observations presented thus far as tentative evidence that the documented fall in the level of the natural rate is associated with an increase in its volatility. To visualize this relationship, Figure 4(a) plots the change in the natural rates between pre-1990 and post-1990 against the changes in the volatility of the Price-to-Rent ratios. Across all countries, falling natural rates have been associated with an increased housing price volatility. Similarly, Figure 4(b) plots the changes in the natural rate levels against changes in the natural rate *volatilities* between the two periods. Across countries, except the UK, falling natural rates have been associated with increased natural rate volatilities.

Real effects of housing price fluctuations. To show that fluctuations in housing prices are relevant for the real economy, in particular, investment into residential housing, we

Figure 5: Increase in housing investment volatility, and high correlation between housing prices and housing investment



Notes: Panel (a) shows the standard deviation of residential investment. Panel (b) shows the correlation between the Price-to-Rent ratio and the residential investment-to-output ratio (RIY-ratio). The blue bars show the estimates for the pre-1990 period, and the red bars for the post-1990 period. The black whiskers denote the 90%-confidence bands. The p -values correspond to the test whether or not the values changed from pre- to post-1990.

present two additional exercises. Panel (a) in Figure 5 reports that the standard deviation of the residential investment to output ratio (RIY-ratio) in the pre- and post-1990 subsamples. It shows that, similar to the increased housing price volatilities, the standard deviation of the RIY-ratio has increased across all countries, except for Japan. Panel (b) in Figure 5 depicts the correlation between the PR-ratio and the RIY-ratio in the two sample periods. While the correlation in the first sample period is statistically insignificant for three of the considered countries, the correlation is very positive and statistically significant in the latter sample period in all countries. This suggests that, at least in the second half of the sample, there exists a strong association between housing prices and housing investment.

4 A Sticky Price Model with Housing

We build in our analysis on the sticky price model with housing and subjective beliefs developed in Adam and Woodford (2020). We augment it by considering a rental market for housing and by explicitly incorporating a zero lower bound constraint for nominal interest rates.⁹ We also allow for more general forms of belief distortions in the private sector, including subjective beliefs that imply that private decisionmakers may not know the equi-

⁹The existence of an occasionally binding ZLB constraint critically complicates the nature of the optimal policy problem because the consumption Euler equation is then an implementability constraint that needs to be taken into account when designing optimal monetary policy. When the ZLB is assumed to be never binding, one can ignore the consumption Euler equation when determining Ramsey optimal monetary policy, because interest rates can be determined ex-post to make the Euler equation hold.

librium mapping from fundamentals to market outcomes.¹⁰ Admitting these more general forms of belief distortions allows the model to generate realistic amounts of housing price volatility, without having to resort to the presence of large housing preference shocks, and it allows replicating important features of the structure of housing forecast errors, as observed in survey data.

Following Adam and Marcet (2011), we consider an economy populated by internally rational decisionmakers. Households (firms) maximize utility (profits), but entertain a potentially subjective probability measure \mathcal{P} , which assigns probabilities to all external variables, i.e., to all variables that agents take as given in their decision problem. These variables include fundamental shocks, as well as competitive market prices (wages, goods prices, housing prices and rents). The setup delivers rational expectations in the special case when \mathcal{P} is the rational probability measure.

The economy is made up of identical infinitely-lived households, each of which maximizes the following objective function¹¹

$$U \equiv E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \left[\tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj + \tilde{\omega}(D_t + D_t^R; \xi_t) \right], \quad (1)$$

subject to a sequence of flow budget constraints

$$C_t + B_t + (D_t - (1 - \delta)D_{t-1}) \frac{q_t^u}{\tilde{u}_C(C_t; \xi_t)} + k_t + R_t D_t^R = \tilde{d}(k_t; \xi_t) \frac{q_t^u}{\tilde{u}_C(C_t; \xi_t)} + \int_0^1 w_t(j) H_t(j) dj + \frac{B_{t-1}}{\Pi_t} (1 + i_{t-1}) + \frac{\Sigma_t}{P_t} + \frac{T_t}{P_t}, \quad (2)$$

where C_t is an aggregate consumption good, $H_t(j)$ is the quantity supplied of labor of type j and $w_t(j)$ the associated real wage, D_t the stock of owned houses, D_t^R the units of rented houses, $\delta \in [0, 1]$ the housing depreciation rate, q_t^u the real price of houses in marginal utility units, defined as

$$q_t^u \equiv q_t \tilde{u}_C(C; \xi_t),$$

where q_t is the real house price in units of consumption. The variable q_t^u provides a measure of whether housing is currently expensive or inexpensive, in units that are particularly relevant for determining housing demand. The variable k_t denotes investment in new houses and $\tilde{d}(k_t; \xi_t)$ the resulting production of new houses.¹² The variable $B_t \equiv \tilde{B}_t/P_t$ denotes the real value of nominal government bond holdings \tilde{B}_t , P_t the nominal price of consumption, $\Pi_t = P_t/P_{t-1}$ the inflation rate, i_t the nominal interest rate, R_t the real rental rate for housing units, and ξ_t is a vector of exogenous disturbances, which may induce random shifts in the functions \tilde{u} , \tilde{v} , $\tilde{\omega}$ and \tilde{d} . The variable T_t denotes nominal lump sum transfers (taxes

¹⁰The setup in Adam and Woodford (2020) assumed absolute continuity between subjective and objective beliefs over arbitrary finite horizons, which implies that agents know the equilibrium mapping from fundamentals to market outcomes. The present setting nests this setup as a special case.

¹¹It cannot be common knowledge to households that they are representative whenever \mathcal{P} deviates from the rational measure.

¹²We consolidate housing production into the household budget constraint. It would be equivalent to have instead a separate housing production section that is owned by households.

if negative) from the government and Σ_t nominal profits accruing to households from the ownership of firms.

Households discount future payoffs at the rate $\beta \in (0, 1)$. Since our model is formulated in terms of growth-detrended variables, the discount rate β jointly captures the time preference rate $\tilde{\beta} \in (0, 1)$ and the steady-state growth rate of marginal utility. Letting $g_c \geq 0$ denote the steady-state growth rate of consumption in non-detrended terms, we have

$$\beta \equiv \tilde{\beta} \frac{\tilde{u}_C(C(1+g_c); \underline{\xi})}{\tilde{u}_C(C; \underline{\xi})}, \quad (3)$$

where $\underline{\xi}$ denotes the steady state value of the disturbance ξ_t . When the growth rate g_c of the economy falls, then the discount rate β increases because marginal utility falls less fast. We can thus capture the fall in economic growth rates via a simple increase in the time discount rate β . In line with the empirical evidence, the increase in the discount rate β will cause safe interest rates to fall.

The aggregate consumption good is a Dixit-Stiglitz aggregate of each of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \quad (4)$$

with an elasticity of substitution $\eta > 1$. We further assume isoelastic functional forms

$$\begin{aligned} \tilde{u}(C_t; \xi_t) &\equiv \frac{C_t^{1-\tilde{\sigma}-1} \bar{C}_t^{\tilde{\sigma}-1}}{1-\tilde{\sigma}^{-1}}, \\ \tilde{v}(H_t(j); \xi_t) &\equiv \frac{\lambda}{1+\nu} (H_t(j))^{1+\nu} \bar{H}_t^{-\nu}, \\ \tilde{\omega}(D_t + D_t^R; \xi_t) &\equiv \xi_t^d (D_t + D_t^R), \\ \tilde{d}(k_t; \xi_t) &\equiv \frac{A_t^d}{\tilde{\alpha}} k_t^{\tilde{\alpha}}, \end{aligned} \quad (5)$$

where $\tilde{\sigma}, \nu > 0$, $\tilde{\alpha} \in (0, 1)$ and $\{\bar{C}_t, \bar{H}_t, \xi_t^d, A_t^d\}$ are bounded exogenous and positive disturbance processes which are among the exogenous disturbances included in the vector ξ_t .

Our specification includes two housing related disturbances, namely ξ_t^d which captures shocks to housing preferences and A_t^d shocks to the productivity in the construction of new houses. We impose linearity in the utility function (5) as this greatly facilitates the characterization of optimal policy, with rented and owned housing units being perfect substitutes. We could introduce a weight on rental units relative to housing units that would allow us to perfectly match the average price-to-rent ratio we observe in the data. But since this does not change any other results, we abstract from such a scaling parameter and assign equal weight to housing and renting in the utility.

The housing demand shock ξ_t^d evolves according to

$$\xi_t^d / \underline{\xi}^d = (\xi_{t-1}^d / \underline{\xi}^d)^{\rho_\xi} e^{\varepsilon_t^d}, \quad (6)$$

where $\underline{\xi}^d$ is the steady state value of the housing demand disturbance, ε_t^d an i.i.d. innovation satisfying $E[e^{\varepsilon_t^d}] = 1$, and $0 \leq \rho_\xi < 1$ captures the persistence of housing demand disturbances.

Each differentiated good is supplied by a single monopolistically competitive producer; there is a common technology for the production of all goods, in which (industry-specific) labor is the only variable input,

$$y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi}, \quad (7)$$

where A_t is an exogenously varying technology factor, and $\phi > 1$. The Dixit-Stiglitz preferences (4) imply that the quantity demanded of each individual good i will equal¹³

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta}, \quad (8)$$

where Y_t is the total demand for the composite good defined in (4), $p_t(i)$ is the (money) price of the individual good, and P_t is the price index,

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \quad (9)$$

corresponding to the minimum cost for which a unit of the composite good can be purchased in period t . Total demand is given by

$$Y_t = C_t + k_t + g_t Y_t, \quad (10)$$

where g_t is the share of the total amount of composite goods purchased by the government, treated here as an exogenous disturbance process.

4.1 Household Optimality Conditions

Internally rational households choose state-contingent sequences for the choice variables $\{C_t, H_t(j), D_t, D_t^R, k_t, B_t\}$ so as to maximize (1), subject to the budget constraints (2), taking as given their beliefs about the processes $\{P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t/P_t, T_t/P_t\}$, as determined by the (subjective) measure \mathcal{P} .

We shall be particularly interested in the policy implications generated by subjective housing price beliefs. To insure that an optimum exists in the presence of potentially distorted beliefs about the housing price q_t^u , we require housing choices to lie in some compact choice set $D_t \in [0, D^{\max}]$, where $D^{\max} < \infty$ is an arbitrarily large but finite upper bound on the quantity of housing the household can purchase. We choose D^{\max} large enough, such that it will never bind in equilibrium.

The first order conditions give rise to an optimal labor supply relation

$$w_t(j) = \frac{\tilde{v}_H(H_t(j); \xi_t)}{\tilde{u}_C(C_t; \xi_t)}, \quad (11)$$

¹³In addition to assuming that household utility depends only on the quantity obtained of C_t , we assume that the government also cares only about the quantity obtained of the composite good defined by (4), and that it seeks to obtain this good through a minimum-cost combination of purchases of individual goods.

a consumption Euler equation

$$\tilde{u}_C(C_t; \xi_t) = \beta E_t^{\mathcal{P}} \left[\tilde{u}_C(C_{t+1}; \xi_{t+1}) \frac{1 + i_t}{P_{t+1}/P_t} \right], \quad (12)$$

an equation characterizing optimal investment in new houses

$$k_t = \left(A_t^d q_t^u \frac{C_t^{\bar{\sigma}-1}}{\bar{C}_t^{\bar{\sigma}-1}} \right)^{\frac{1}{1-\bar{\alpha}}}, \quad (13)$$

an optimality condition for rental units

$$\xi_t^d = R_t \tilde{u}_C(C_t, \xi_t), \quad (14)$$

and a set of conditions determining the optimal housing demand D_t :

$$\begin{aligned} q_t^u &< \xi_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} q_{t+1}^u && \text{if } D_t = D^{\max} \\ q_t^u &= \xi_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} q_{t+1}^u && \text{if } D_t \in [0, D^{\max}] \\ q_t^u &> \xi_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} q_{t+1}^u && \text{if } D_t = 0. \end{aligned} \quad (15)$$

With rational expectations, the upper and lower holding bounds never bind.¹⁴ We are, however, interested in how the presence of belief distortions about future housing values affect equilibrium outcomes. With subjective housing price expectations, the holding bounds in equation (15) can potentially bind under the *subjectively* optimal plans. This explains why an internally rational household can hold subjective housing price expectations, even if it holds rational expectations about the preference shocks ξ_T^d in equation (15).¹⁵

Forward-iterating on equation (12), which holds with equality under all belief-specifications, delivers a present-value formulation of the consumption Euler equation

$$\tilde{u}_C(C_t; \xi_t) = \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left[\tilde{u}_C(C_T; \xi_T) \beta^T \prod_{k=0}^{T-t} \frac{1 + i_{t+k}}{P_{t+k+1}/P_{t+k}} \right], \quad (16)$$

which will be convenient to work with, especially under subjective belief specifications. Household choices must also satisfy the transversality constraint

$$\lim_{T \rightarrow \infty} \beta^T E_t^{\mathcal{P}} (\tilde{u}_C(C_T; \xi_T) B_T + D_T q_T^u) = 0. \quad (17)$$

Optimal household behavior under potentially distorted beliefs is jointly characterized by equations (11) and (13)-(17).

¹⁴The upper bound D^{\max} has been chosen sufficiently large for this to be true. The lower bound is never reached because the housing production function satisfies Inada conditions.

¹⁵See Adam and Marcet (2011) for a detailed discussion of this point.

4.2 Optimal Price Setting by Firms

The producers in each industry fix the prices of their goods in monetary units for a random interval of time, as in the model of staggered pricing introduced by Calvo (1983) and Yun (1996). Producers use the representative households' subjectively optimal consumption plans to discount profits and are assumed to know the product demand function (8). They need to formulate beliefs about the future price levels P_T , industry wages $w_T(j)$, aggregate demand Y_T , and productivity A_T .

Let $0 \leq \alpha < 1$ be the fraction of prices that remain unchanged in any period. A supplier i in industry j that changes its price in period t chooses its new price $p_t(i)$ to maximize

$$E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi(p_t(i), P_T, w_T(j), Y_T, A_T), \quad (18)$$

where E_t^P denotes the expectations of price setters conditional on time t information, which are identical to the expectations held by consumers. Firms discount random nominal income in period T using households' subjective stochastic discount factor $Q_{t,T}$, which is given by

$$Q_{t,T} = \beta^{T-t} \frac{\tilde{u}_C(C_T, \xi_T)}{\tilde{u}_C(C_t, \xi_t)} \frac{P_t}{P_T}.$$

The term α^{T-t} in equation (18) captures the probability that a price chosen in period t will not have been revised by period T , and the function $\Pi(p_t(i), \dots)$ indicates the nominal profits of the firm in period t , as discussed next.

Profits are equal to after-tax sales revenues net of the wage bill. Sales revenues are determined by the demand function (8), so that (nominal) after-tax revenue equals

$$(1 - \tau_t) p_t(i) Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta}.$$

Here τ_t is a proportional tax on sales revenues in period t , $\{\tau_t\}$ is treated as an exogenous disturbance process, taken as given by the monetary policymaker. We assume that τ_t fluctuates over a small interval around a non-zero steady state level $\bar{\tau}$. We allow for exogenous variations in the tax rate in order to include the possibility of "pure cost-push shocks" that affect the equilibrium pricing behavior while implying no change in the efficient allocation of resources.

The labor demand of firm i at a given industry-specific wage $w_t(j)$ can be written as

$$h_t(i) = \left(\frac{Y_t}{A_t} \right)^\phi p_t(i)^{-\eta\phi} P_t^{\eta\phi}, \quad (19)$$

which follows from (7) and (8). Using this, the nominal wage bill is given by

$$P_t w_t(j) h_t(i) = P_t w_t(j) \left(\frac{Y_t}{A_t} \right)^\phi p_t(i)^{-\eta\phi} P_t^{\eta\phi}.$$

Subtracting the nominal wage bill from the above expression for nominal after tax revenue, we obtain the function $\Pi(p_t(i), P_T, w_T(j), Y_T, A_T)$ used in (18).

Each of the suppliers that revise their prices in period t chooses the same new price p_t^* , that maximizes (18). The first-order condition with respect to $p_t(i)$ is given by¹⁶

$$E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Pi_1(p_t(i), P_T, w_T(j), Y_T, A_T) = 0.$$

The equilibrium choice p_t^* , which is the same for each firm i in industry j , is the solution to this equation. Letting p_t^j denote the price charged by firms in industry j at time t , we have $p_t^j = p_t^*$ in periods in which industry j resets its prices and $p_t^j = p_{t-1}^j$ otherwise.

Under the assumed isoelastic functional forms, the optimal choice has a closed-form solution

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^P \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} (1-\tau_T) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}}. \quad (20)$$

The price index evolves according to a law of motion

$$P_t = [(1-\alpha) p_t^{*1-\eta} + \alpha P_{t-1}^{1-\eta}]^{\frac{1}{1-\eta}}, \quad (21)$$

as a consequence of (9). The equilibrium inflation in any period is characterized by

$$\left(\frac{P_t}{P_{t-1}}\right)^{\eta-1} = \frac{1 - (1-\alpha) \left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha}. \quad (22)$$

The welfare loss from price adjustment frictions can be captured by price dispersion, which is defined as

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t^j}{P_t}\right)^{-\eta(1+\omega)} dj \geq 1, \quad (23)$$

where

$$\omega \equiv \phi(1+\nu) - 1 > 0$$

is the elasticity of real marginal cost in an industry with respect to industry output.

Using equation (21) together with the fact that the relative prices of the industries that do not change their prices in period t remain the same, one can derive a law of motion for the price dispersion term Δ_t of the form

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}), \quad (24)$$

with

$$h(\Delta_t, P_t/P_{t-1}) \equiv \alpha \Delta_t \left(\frac{P_t}{P_{t-1}}\right)^{\eta(1+\omega)} + (1-\alpha) \left(\frac{1 - \alpha \left(\frac{P_t}{P_{t-1}}\right)^{\eta-1}}{1-\alpha}\right)^{\frac{\eta(1+\omega)}{\eta-1}}.$$

¹⁶Note that supplier i 's profits in (18) are a concave function of the quantity sold $y_t(i)$, since revenues are proportional to $y_t(i)^{\frac{\eta-1}{\eta}}$ and hence concave in $y_t(i)$, while costs are convex in $y_t(i)$. Moreover, since $y_t(i)$ is proportional to $p_t(i)^{-\eta}$, the profit function is also concave in $p_t(i)^{-\eta}$. The first-order condition for the optimal choice of the price $p_t(i)$ is the same as the one with respect to $p_t(i)^{-\eta}$; hence the first-order condition with respect to $p_t(i)$ is both necessary and sufficient for an optimum.

As is commonly done, we assume that the initial degree of price dispersion is small ($\Delta_{-1} \sim O(2)$).

Equations (20), (22), and (24) jointly define a short-run aggregate supply relation between inflation, output and house prices (via the aggregate demand equation (10) and (13)), given the current disturbances ξ_t , and expectations regarding future wages, prices, output, consumption and disturbances. Equation (24) describes the evolution of the costs of price dispersion over time.

For future reference, we remark that all firms together make total profits equal to

$$\frac{\Sigma_t}{P_t} = (1 - \tau_t)Y_t - w_t H_t, \quad (25)$$

where $w_t H_t = \int_0^1 w_t(j) H_t(j) dj$.

4.3 Government

The government consumes goods $g_t Y_t$, imposes a sales tax τ_t , issues nominal bonds $\tilde{B}_t \equiv P_t B_t$, and pays for lump sum transfers T_t to households. The government budget constraint is given by

$$B_t = B_{t-1} \frac{1 + i_{t-1}}{P_t/P_{t-1}} + \frac{T_t}{P_t} + (g_t - \tau_t) Y_t.$$

For simplicity, we assume that lump sum transfers (taxes if negative) are set such that they keep real government debt constant at some initial level B_{-1} . This implies that government transfers are given by

$$\frac{T_t}{P_t} = -(g_t - \tau_t)Y_t + B_{t-1} \left(1 - \frac{1 + i_{t-1}}{P_t/P_{t-1}}\right). \quad (26)$$

4.4 Market Clearing Conditions

Using (10) and (13), one can express the market clearing condition for the consumption/investment good as

$$Y_t = \frac{C_t + \Omega_t C_t^{\frac{\bar{\sigma}-1}{1-\bar{\alpha}}}}{1 - g_t}, \quad (27)$$

where

$$\Omega_t \equiv \left(A_t^d \bar{C}_t^{-\bar{\sigma}-1} q_t^u\right)^{\frac{1}{1-\bar{\alpha}}} > 0 \quad (28)$$

is a term that depends on exogenous shocks and belief distortions in the housing market only, see equation (15). The previous two equations implicitly define a function

$$C_t = C(Y_t, q_t^u, \xi_t), \quad (29)$$

which delivers the market clearing consumption level, for a given output level Y_t , given housing prices q_t^u and given exogenous disturbances ξ_t .

The market clearing condition for housing is

$$D_t = (1 - \delta)D_{t-1} + \tilde{d}(k_t; \xi_t), \quad (30)$$

and rental market clearing requires

$$D_t^R = 0. \quad (31)$$

Labor market clearing requires that the supply of labor of type j in (11) is equal to labor demand of industry j , which is given by (19), as all firms in the industry charge the same price. This delivers

$$w_t(j) = \frac{\tilde{v}_H(H_t(j); \xi_t)}{\tilde{u}_C(C_t; \xi_t)} = \frac{\lambda (H_t(j))^\nu \bar{H}_t^{-\nu}}{C_t^{-\bar{\sigma}-1} \bar{C}_t^{\bar{\sigma}-1}} = \lambda \frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\bar{\sigma}-1}} \left(\frac{Y_t}{A_t} \right)^{\nu\phi} C_t^{\bar{\sigma}-1} \left(\frac{p_t^j}{P_t} \right)^{-\nu\eta\phi}, \quad (32)$$

where $p_t^j = p_t^*$ in periods where industry j can adjust prices and $p_t^j = p_{t-1}^j$ otherwise.

4.5 Internally Rational Expectations Equilibrium

We are now able to define an *internally rational expectations equilibrium (IREE)*.

Definition 1 *An internally rational expectations equilibrium is a bounded stochastic process for $\{Y_t, C_t, k_t, D_t, \{w_t(j)\}, p_t^*, P_t, \Delta_t, q_t^u, i_t\}_{t=0}^\infty$ satisfying the aggregate supply equations (20), and (22), the law of motion for the evolution of price distortions (24), the household optimality conditions (13), (15), (16), and the market clearing conditions (27), (30) and (32), where the latter has to hold for all industries j .*

The equilibrium features ten variables (counting the continuum of wages as a single variable) that must satisfy nine conditions, leaving one degree of freedom to be determined by monetary policy.¹⁷ In the special case with rational expectations, $E_t^P[\cdot] = E_t[\cdot]$, the IREE is a rational expectations equilibrium (REE).

Given the equilibrium outcomes, the remaining model variables can be determined as follows. Equilibrium profits are given by equation (25) and equilibrium taxes by equation (26). Equilibrium labor supply $H_t(j)$ follows from equation (11) for each type of labor j . Equilibrium bond holdings satisfy $B_t = B_{-1}$ and equilibrium inflation is

$$\Pi_t \equiv P_t/P_{t-1}.$$

Equilibrium rental units follow from equation (31) and equilibrium rental prices from equation (14).

5 The Nonlinear Optimal Policy Problem

We shall consider Ramsey optimal policies in which the policymaker chooses the sequence of policy rates, prices and allocations to maximize household utility under rational expectations, subject to the constraint that prices and allocations constitute an Internally Rational Expectations Equilibrium. The policymaker thus maximizes utility under a probability measure

¹⁷The transversality condition (17) must also be satisfied in equilibrium, but is not imposed as an equilibrium condition, as it will hold for all belief specifications considered below.

that is different from the one entertained by households, whenever the latter hold distorted beliefs. Benigno and Paciello (2014) refer to such a policymaker as being 'paternalistic'.

The objective of the policymaker is to maximize household utility. Using equation (8) to express the relative quantities demanded of the differentiated goods each period as a function of their relative prices and the linear dependence of utility on the stock of assets, we can write the utility flow to the representative household in the form

$$u(Y_t, q_t^u; \xi_t) - v(Y_t; \xi_t)\Delta_t + \bar{\xi}_t \frac{A_t^d}{\bar{\alpha}} k_t^{\bar{\alpha}},$$

with

$$\begin{aligned} u(Y_t, q_t^u; \xi_t) &\equiv \tilde{u}(C(Y_t, q_t^u, \xi_t); \xi_t) \\ v(y_t^j; \xi_t) &\equiv \tilde{v}(f^{-1}(y_t^j/A_t); \xi_t), \end{aligned}$$

where Δ_t , defined in equation (23), captures the misallocations from price dispersion. The term

$$\bar{\xi}_t \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t} \beta^{T-t} \xi_T^d]$$

captures the present value contribution from new housing investment. We can use (13), and (29) to express k_t in terms of Y_t , q_t^u and exogenous shocks. Hence, we can express the policy maker's objective of maximizing (1) under rational expectations, as maximizing

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t),$$

where the flow utility is given by

$$\begin{aligned} U(Y_t, \Delta_t, q_t^u; \xi_t) &\equiv \frac{\bar{C}_t^{\bar{\sigma}-1} C(Y_t, q_t^u, \xi_t)^{1-\bar{\sigma}-1}}{1-\bar{\sigma}-1} \\ &\quad - \frac{\lambda}{1+\nu} \bar{H}_t^{-\nu} \left(\frac{Y_t}{A_t}\right)^{1+\omega} \Delta_t \\ &\quad + \frac{A_t^d \bar{\xi}_t^d}{\bar{\alpha}} \Omega(q_t^u, \xi_t)^{\bar{\alpha}} C(Y_t, q_t^u, \xi_t)^{\frac{\bar{\alpha}}{1-\bar{\alpha}} \bar{\sigma}-1}, \end{aligned} \quad (33)$$

which is a monotonically decreasing function of Δ given Y , q^u and ξ and where $\Omega(q^u, \xi)$ is the function defined in (28). The only endogenous variables that are thus relevant for evaluating the policymaker's objective function are Y_t , Δ_t and q_t^u . Note that the policymaker holds rational expectations about these variables and maximizes expected utility under the rational probability measure, while agents might hold subjective beliefs.

The non-linear optimal monetary policy problem is then given by

$$\max_{\{Y_t, q_t^u, p_t^*, w_t(j), P_t, \Delta_t, i_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t, q_t^u; \xi_t) \quad (34)$$

subject to

$$\left(\frac{p_t^*}{P_t}\right)^{1+\eta(\phi-1)} = \frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \frac{\eta}{\eta-1} \phi w_T(j) \left(\frac{Y_T}{A_T}\right)^{\phi} \left(\frac{P_T}{P_t}\right)^{\eta\phi+1}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} (1 - \tau_T) Y_T \left(\frac{P_T}{P_t}\right)^{\eta}} \quad (35)$$

$$w_t(j) = \lambda \frac{\bar{H}_t^{-\nu}}{\bar{C}_t^{\bar{\sigma}-1}} \left(\frac{Y_t}{A_t}\right)^{\phi\nu} C(Y_t, q_t^u, \xi_t)^{\bar{\sigma}-1} \left(\frac{p_t^*}{P_t}\right)^{-\eta\phi\nu} \quad (36)$$

$$(P_t/P_{t-1})^{\eta-1} = \frac{1 - (1 - \alpha) \left(\frac{p_t^*}{P_t}\right)^{1-\eta}}{\alpha} \quad (37)$$

$$\Delta_t = h(\Delta_{t-1}, P_t/P_{t-1}) \quad (38)$$

$$\tilde{u}_C(C(Y_t, q_t^u, \xi_t); \xi_t) = \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left[\tilde{u}_C(C_T; \xi_T) \beta^T \prod_{k=0}^{T-t} \frac{1 + i_{t+k}}{P_{t+k+1}/P_{t+k}} \right] \quad (39)$$

$$q_t^u = \xi_t^d + \beta(1 - \delta) E_t^{\mathcal{P}} q_{t+1}^u \quad (40)$$

where the initial price level P_{-1} and the initial price dispersion Δ_{-1} are given. Equation (36) insures that wages clear current labor markets. Similarly, by setting $C_t = C(Y_t, q_t^u, \xi_t)$ on the left-hand side of the consumption Euler equation (39), we impose market clearing for output goods in period t . Similarly, setting q_t^u equal to the value defined in (40) insures market clearing in the housing market.¹⁸ Firms' subjective expectations about future wages and households' subjectively optimal consumption plans for the future, however, will generally not be consistent with labor market or goods market clearing in the future, whenever beliefs deviate from rational ones.

To be able to analyze the policy problem (34) further, it will be necessary to be more specific about the beliefs \mathcal{P} entertained by households and firms about external variables entering their decision problem.

6 Housing Prices Dynamics: Alternative Views

We shall now consider two alternative belief settings that give rise to different drivers for housing price dynamics.¹⁹

The first setting we consider is entirely standard and assumes that agents hold rational expectations (RE) about all variables. Housing prices will then exclusively be driven by housing demand shocks. While this is a useful benchmark to consider, the assumption of rational housing price expectations is not supported by empirical evidence available from survey expectations about future housing prices, as we show in section 6.3 below. We therefore consider a second setting that allows for subjective housing price and rental rate

¹⁸This holds as long as D^{\max} is chosen sufficiently large, such that it never binds along the equilibrium path.

¹⁹Recall from our earlier discussion that firms must hold beliefs about future values of P_t , $w_t(j)$, Y_t and that households must hold beliefs about future values of $(P_t, w_t(j), q_t^u, R_t, i_t, \Sigma_t/P_t, T_t/P_t)$. Both actors must additionally hold beliefs about the fundamental shocks entering their decision problem. We always assume that these beliefs are rational.

beliefs. We maintain the assumption of rational expectations for all other variables to make a minimal deviation from the standard setting. The setting with subjective housing beliefs will align well with the properties of survey expectations, as we document in section 6.3. Housing prices will then depend on two economic forces: housing demand shocks and subjective optimism/pessimism about future housing prices.

6.1 Housing Prices under Rational Expectations

The following lemma summarizes the housing price dynamics under rational expectations (RE):²⁰

Lemma 1 *Under RE, the equilibrium housing price is*

$$q_t^{u,RE} = \bar{\xi}_t^d, \quad (41)$$

where

$$\bar{\xi}_t^d \equiv \sum_{T=t}^{\infty} E_t[(1-\delta)^{T-t} \beta^{T-t} \xi_T^d] \quad (42)$$

captures the present value contribution to household utility from new housing investment. The price-to-rent (PR) ratio is

$$PR_t^{RE} = \frac{q_t^{u,RE}}{\xi_t^d}. \quad (43)$$

To a first-order approximation, we have

$$\widehat{q}_t^{u,RE} = \frac{1 - \beta(1-\delta)}{1 - \beta(1-\delta)\rho_\xi} \widehat{\xi}_t^d, \quad (44)$$

$$\widehat{PR}_t^{RE} = \left(\frac{\beta(1-\delta)(\rho_\xi - 1)}{1 - \beta(1-\delta)\rho_\xi} \right) \widehat{\xi}_t^d \quad (45)$$

where hatted variables denote percent deviations from steady state.

Proof. See Appendix C. ■

Equations (44) and (45) show that housing prices display large and persistent variations under rational expectations (RE), whenever housing demand shocks $\widehat{\xi}_t^d$ display large and persistent variations. In fact, housing demand shocks are a commonly used modelling approach for generating housing price fluctuations in the RE literature (Iacoviello (2005)). Equation (44) shows, however, that housing price volatility falls when the average natural rate falls: the coefficient pre-multiplying $\widehat{\xi}_t^d$ is positive but declines as the discount factor β increases.

The situation is different for the price-to-rent ratio: the pre-multiplying coefficient in equation (45) is negative and declines further as β increases, so that the volatility of the

²⁰Under the RE hypothesis, households hold rational expectations about the prices $\{P_t, w_t(j), q_t^u, R_t, i_t\}$, firms hold rational expectations about $\{P_t, w_t(j), Y_t\}$, and all actors hold rational expectations about the exogenous fundamentals. Household expectations about $\{\Sigma_t/P_t, T_t/P_t\}$ are given by equations (25) and (26), evaluated with rational output expectations and the optimal choices for $\{H_t, k_t, B_t\}$.

price-to-rent ratio increases as natural rates falls.²¹ Yet, this effect will be quantitatively small: since housing shocks need to be very persistent to replicate the persistent swings in the level of housing prices, the persistence parameter ρ_ξ must be close to one. In the limit case where housing shocks follow a random walk ($\rho_\xi = 1$), the volatility of the price-to-rent ratio is independent of the discount factor.

The latter feature explains why the increase in the price-to-rent ratio volatility predicted under RE, following a drop in the natural rate from pre-1990 levels to post-1990 levels, will be less than the empirically observed increase. Matching the increase in housing price volatility under RE thus requires assuming that the volatility of housing demand shocks $\widehat{\xi}_t^d$ increased over the period in which natural rates decreased.

6.2 Housing Prices with Subjective Housing Price Beliefs

Our second belief setting considers a single deviation from the rational expectations assumption, which allows for the presence of subjective capital gain expectations in housing markets and associated subjective beliefs for rental prices. Specifically, we shall use the subjective specification used in Adam et al. (2016), which implies that investors (weakly) extrapolate past capital gains. As we show in the next section, this setup allows replicating important features of the forecast error structure about future housing prices present in the Michigan survey.

To isolate the effects of speculative housing and rent price expectations, we continue to assume that households hold rational expectations about all other prices, i.e., about $\{P_t, w_t(j), i_t\}$. Likewise, firms hold rational expectations about $\{P_t, w_t(j), Y_t\}$ and all actors continue to hold rational expectations about the exogenous fundamentals. Beliefs about profits and lump sum taxes, $\{\Sigma_t/P_t, T_t/P_t\}$ continue to be determined by equations (25) and (26), evaluated with rational output expectations and the subjectively optimal choices for $\{H_t, k_t, B_t\}$.

The introduction of subjective housing price expectations is motivated by the observation that reconciling survey expectations about future return expectations with the actual behavior of future returns requires introducing some departure from the rational expectations hypothesis (Adam et al. (2017)). These departures could take the form of extrapolative expectations (Barberis et al. (2015), Adam and Merkel (2019)), learning about underlying trends in price growth (Adam et al. (2016)), or learning from life-time experience (Collin-Dufresne et al. (2016), Nagel and Xu (2019), Malmendier and Nagel (2011, 2016)).²² We show in Section 6.3 that our subjective belief model matches observed patterns in survey expectations about future housing prices.

²¹The different signs of the pre-multiplying coefficients are empirically problematic, as they imply that changes in housing prices and changes in the PD ratio should be (perfectly) negatively correlated. In the data, however, the correlation is positive and close to one.

²²Adam et al. (2020) show that this discrepancy between survey return expectations and actual return behavior cannot be explained away by assuming that survey respondents confound beliefs and preferences when answering the survey questions and report, for instance, risk-neutral or pessimistically tilted expectations. Armona et al. (2019) provide experimental evidence for extrapolative expectation formation about house prices. Soo (2018) documents the extrapolative nature of housing expectations based on a constructed housing sentiment index.

The subjective belief setup of Adam et al. (2016) stipulates that households perceive risk-adjusted price growth to follow

$$\frac{q_t^u}{q_{t-1}^u} = b_t + \varepsilon_t, \quad (46)$$

where b_t is an unobserved persistent growth component given by

$$b_t = b_{t-1} + \nu_t$$

and $\varepsilon_t \sim iiN(0, \sigma_\varepsilon^2)$ and $\nu_t \sim iiN(0, \sigma_\nu^2)$ are independent transitory shocks. Adam et al. (2016) show that Bayesian belief updating implies that subjective conditional expectations are given by:²³

$$E_t^{\mathcal{P}} \left(\frac{q_{t+1}^u}{q_t^u} \right) = \beta_t, \quad (47)$$

where β_t evolves according to

$$\beta_t = \min \left\{ \beta_{t-1} + \frac{1}{\alpha} \left(\frac{q_{t-1}^u}{q_{t-2}^u} - \beta_{t-1} \right), \beta^U \right\}, \quad (48)$$

with $1/\alpha$ denoting the (steady-state) Kalman gain, which depends on the subjectively perceived values for $(\sigma_\varepsilon^2, \sigma_\nu^2)$. The upper bound $\beta^U < (\beta(1-\delta))^{-1}$ insures that optimism is bounded from above, so as to keep subjectively expected utility finite.²⁴

Equation (48) implies that subjective optimism rises (falls), when agents observe risk-adjusted capital gains that exceed (fall short of) their prior expectations. Since realized housing prices q_t^u will depend positively on expected future housing price growth β_t , this setup can generate persistent housing booms (busts) from endogenous belief dynamics: as housing prices increase (decrease), they trigger rising (falling) optimism and therefore future housing price increases (decreases), consistent with the empirical observation that capital gains in housing markets display considerable auto-correlation over time.

Associated with the subjective housing price expectations are subjectively optimal consumption plans, as determined by the Euler equation (16). The subjectively optimal consumption plans determine, according to equation (14), the subjective rental price expectations that households must entertain together with their subjective housing price expectations. These subjective rental price expectations are required to make the household's subjective beliefs consistent with internal rationality.²⁵

To insure that the subjectively optimal consumption plans satisfy the transversality condition (17), we impose that equation (46) describes subjective housing price beliefs for an arbitrarily long but finite amount of time $t < \bar{T} < \infty$ and that households hold rational expectations in the long-run, i.e. for all periods $t \geq \bar{T}$. Agents thus perceive

$$q_t^u = \bar{\xi}_t^d \quad \text{for all } t \geq \bar{T}, \mathcal{P} \text{ almost surely,}$$

²³This assumes that agents entertain a normally distributed conjugate prior belief about the unobserved persistent component b_t .

²⁴The bounding function in (48) is a special case of the bounding function used in equation (27) in Adam et al. (2016), which is obtained by setting $\beta^L = \beta^U$.

²⁵Households' subjective beliefs are otherwise inconsistent with the household first-order conditions. This shows how internal rationality imposes restrictions on what rational agents can plausibly believe about external variables.

where $\bar{\xi}_t^d$ is the rational expectations housing price, as determined in equation (42). Appendix C.1 shows that this assumption is sufficient to insure that the transversality condition is satisfied.²⁶

The following lemma derives the equilibrium PR-ratio and housing price under the considered subjective belief specification.

Lemma 2 *With subjective housing price expectations, the equilibrium housing price for periods $t < \bar{T}$ is given by*

$$q_t^{u,P} = \frac{1}{1 - \beta(1 - \delta)\beta_t} \xi_t^d, \quad (49)$$

where β_t evolves according to (48). The percent deviation of housing prices from steady state is

$$\widehat{q}_t^{u,P} = \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\beta_t} \widehat{\xi}_t^d + \frac{\beta(1 - \delta)(\beta_t - 1)}{1 - \beta(1 - \delta)\beta_t} \quad (50)$$

$$= \widehat{q}_t^{u,RE} + \frac{\beta(1 - \delta)(\beta_t - 1)}{1 - \beta(1 - \delta)\beta_t} + \frac{(1 - \beta(1 - \delta))\beta(1 - \delta)(\beta_t - \rho_\xi)}{(1 - \beta(1 - \delta)\beta_t)(1 - \beta(1 - \delta)\rho_\xi)} \widehat{\xi}_t^d, \quad (51)$$

where hatted variables denote percent deviations from steady-state values.

As $\rho_\xi \rightarrow 1$, this becomes

$$\widehat{q}_t^{u,P} = \widehat{q}_t^{u,RE} + (\beta_t - 1) \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)\beta_t} \left(1 + \widehat{\xi}_t^d\right) \quad (52)$$

The expected housing price under subjective beliefs can be written as the rational expectation forecast plus subjective belief terms

$$E_t^P \left[\widehat{q}_{t+1}^{u,P} \right] = E_t \left[\widehat{q}_{t+1}^{u,RE} \right] + (\beta_t - 1) \left[1 + \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)\beta_t} \right] + \frac{(1 - \beta(1 - \delta)\rho_\xi - (1 - \beta(1 - \delta)\beta_t)\rho_\xi)(1 - \beta(1 - \delta))}{(1 - \beta(1 - \delta)\beta_t)(1 - \beta(1 - \delta)\rho_\xi)} \widehat{\xi}_t^d,$$

which becomes

$$E_t^P \left[\widehat{q}_{t+1}^{u,P} \right] = E_t \left[\widehat{q}_{t+1}^{u,RE} \right] + (\beta_t - 1) \left[1 + \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)\beta_t} \left(1 + \widehat{\xi}_t^d\right) \right] \quad (53)$$

as $\rho_\xi \rightarrow 1$.

The price-to-rent ratio, in percent deviations from its steady state, is given by

$$\widehat{PR}_t^P = \widehat{q}_t^{u,P} - \widehat{\xi}_t^d.$$

²⁶The transversality condition may also hold under weaker condition, but actually showing this is mathematically tedious. The fact that agents will eventually hold rational housing and rental price expectations might be interpreted as agents learning to make rational predictions in the long-run.

Proof. See Appendix C. ■

Equation (49) shows that housing prices under subjective beliefs fluctuate because of fluctuations in the fundamental shocks $\widehat{\xi}_t^d$ and because of fluctuations in the subjective beliefs β_t . In the special case where subjective beliefs are given by $\beta_t = 1$ and with a unit root in the fundamental shocks ($\rho_\xi \rightarrow 1$), the housing price under subjective beliefs is equal to the one under rational expectations, see equation (52). For $\beta_t \gtrsim 1$, housing prices are also driven by subjective belief components. Since ρ_ξ will be close to one in the calibration used later on, the subjective belief setup generates close-to-rational housing prices whenever $\beta_t = 1$.²⁷

Furthermore, equation (53) shows that for ρ_ξ close to one, subjective housing price *expectations* can be expressed as the sum of the rational housing price expectation and an additional term that is zero whenever $\beta_t = 1$. This shows that subjective housing price expectations are affected by fundamentals in the same way as rational expectations, but contain an additional subjective component whenever $\beta_t \gtrsim 1$.

In general, fluctuations in housing price expectations generate additional housing price fluctuations, which then feed back into housing price expectations. Adam et al. (2016) show that these fluctuations take the form of persistent boom-bust cycles, in line with the observed dynamics of housing prices in the data.

As the steady-state natural interest rate falls, i.e., as the discount factor β moves closer to one, the subjective belief dynamics will display more instability and thereby generate more variable housing prices. This is illustrated in Figure 6, which depicts the standard deviation of the housing price q_t^u as a function of the steady-state natural interest rate, using the calibrated subjective belief model presented later on. The reason for the downward sloping pattern of housing price volatility is that housing prices become more sensitive to belief revisions as the term $\beta(1 - \delta)$ in the denominator of equation (49) moves closer to one: the denominator $1 - \beta(1 - \delta)\beta_t$ then fluctuates around numbers that are closer to zero, so that any given revision of beliefs has larger pricing implications. This again feeds back into larger belief revisions.

In contrast to the RE model, the subjective belief model will thus be able to generate a sufficiently strong increase in house price volatility, following a drop in the average natural rate, without having to resort to changes in the volatility of housing demand shocks ξ_t^d .²⁸

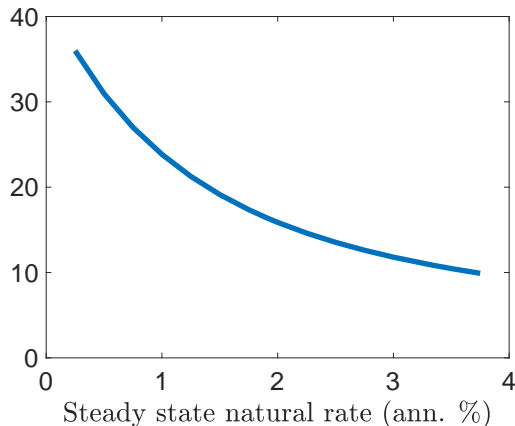
6.3 Empirical Evidence on Housing Price Expectations

This section shows that housing price expectations in the Michigan survey display predictable forecast errors and are thus inconsistent with the rational expectations hypothesis.

²⁷A setting with constant growth expectations $\beta_t = 1$ is in fact a special setting of the subjective belief setup. It requires that initial beliefs (1) assign probability one to $b_0 = 1$ and (2) probability one to $\sigma_v^2 = 0$. Bayesian updating then implies $1/\alpha = 0$ in equation (48). More generally, however, if prior beliefs are such that $\sigma_v^2 > 0$, then $\alpha < \infty$ and realized growth rates of housing prices will influence subjective expectations about future housing price growth.

²⁸Section 8 shows that the subjective belief model is indeed capable of generating the observed volatility in the data.

Figure 6: Unconditional standard deviation of housing prices q_t^u



The first section documents that expected housing capital gains in the Michigan survey covary positively with housing market valuation (price-to-rent ratio), while actual capital gains covary negatively and that the difference is statistically significant. The subsequent section shows that forecasts about the level of future housing prices are updated sluggishly: forecast errors about future housing prices are positively correlated with prior revisions in housing price expectations. Eventually, the third section considers the dynamic structure of housing-price forecasts and we show that an initial underreaction is followed by a delayed overshooting of housing expectations.

The subsequent sections also show that the subjective belief specification introduced in the previous section is quantitatively consistent with both of these findings, despite the fact that these survey features have not been targeted in our model calibration in section 8.

6.3.1 Cyclicity of Actuals versus Expected Capital Gains

We use data on expected capital gains from the Survey of Consumers from the University of Michigan for the time span these expectations are available (2007-2021).²⁹ Survey respondents report how much they expect housing prices to grow over the next four quarters.³⁰ For actual housing prices we use the S&P/Case-Shiller U.S. National Home Price Index.

To document the different cyclicity of expected and actual housing capital gains, we follow Adam et al. (2017) and run the following two regressions

$$E_t^P [CG_{t,t+4}] = a + c \cdot PR_t + u_t \quad (54)$$

$$CG_{t,t+4} = \mathbf{a} + \mathbf{c} \cdot PR_t + \mathbf{u}_t, \quad (55)$$

where PR_t denotes the price-to-rent ratio in quarter t , $E_t^P [CG_{t,t+4}]$ the (mean or median) survey expectation about capital gains between quarter t and $t + 4$, and $CG_{t,t+4} \equiv \frac{HP_{t+4}}{HP_t}$

²⁹Appendix D shows the result excluding the Covid crisis.

³⁰The exact variable we use to compute the expected gross capital gain is the answer to the question: "By about what percent do you expect prices of homes like yours in your community to go (up/down), on average, over the next 12 months?"

the realized capital gains over the same time span. We run the regressions using nominal housing prices and real housing prices, where we deflate nominal prices using the CPI. To transform the expected nominal capital gains into expected real capital gains, we deflated using the expected inflation rate from the Michigan Survey. As explained in Adam et al. (2017), the rational expectations hypothesis implies $c = \mathbf{c}$. Since the predictor variable in the equations above (the price-to-rent ratio) is highly persistent, we correct for small sample bias in the coefficient estimates (Stambaugh (1999)).³¹

Table 1 reports the regression results. It shows that expected capital gains are positively associated with the PR-ratio, while realized capital gains are negatively associated. Expected capital gains are thus pro-cyclical (are high when market valuation is high), while realized capital gains are counter-cyclical (are low when market valuation is high). The same pattern can be observed for capital gain (and return) expectations in stock markets, see Greenwood and Shleifer (2014) and Adam et al. (2017). The last column in table 1 performs a test of the rational expectations hypothesis that the cyclicalities of actual and expected returns are equal ($c = \mathbf{c}$). This test takes into account the presence of small sample bias (reported in the second to last column in the table). It shows that the difference in the cyclicalities of actual and expected capital gains is statistically significant.

Table 1: Expected vs. actual capital gains

	$\hat{c} \cdot 10^3$	$\hat{\mathbf{c}} \cdot 10^3$	bias $\cdot 10^3$ $-E(\hat{\mathbf{c}} - \hat{c})$	p -value $H_0 : c = \mathbf{c}$
<u>Nominal Housing Prices</u>				
Mean Expectations	0.648	-0.553	0.033	0.000
Median Expectations	0.193	-0.553	0.114	0.068
<u>Real Housing Prices</u>				
Mean Expectations	0.723	-0.396	-0.010	0.000
Median Expectations	0.243	-0.396	0.072	0.022

Note: \hat{c} is the estimate of c in equation (54) and $\hat{\mathbf{c}}$ the estimate of \mathbf{c} in equation (55). The small sample bias correction is reported in the second to last column and the last column reports the p -values for the null hypothesis $c = \mathbf{c}$ in the fifth column. Rows labeled "Mean" report results using the mean of the survey expectation. Rows labeled "Median" use the median expectation.

To compare these results with the model predictions, we re-run regressions (54) and (55) on model-simulated data using the calibrated subjective belief model from section 8.³² (The RE version of the model has - for obvious reasons - no chance to match the evidence in table 1.)

Table 2 reports the regression coefficients from the model for different levels of the steady-state natural rates that appear plausible for the considered sample period. The table also compares the model estimates to the ones obtained from the data. As in the data, model

³¹The small sample bias correction in table 1 follows the same approach as the one in table 1A in Adam et al. (2017).

³²Expected capital gains in the model are given by $E_t^P [CG_{t,t+4}] = (\beta_t)^4$ and actual capital gains by $CG_{t,t+4} = \frac{q_{t+4}^u}{q_t^u}$. The PR ratio is given by $PR_t = q_t^u / \xi_t^d$, as shown in Appendix C.

expectations of future capital gains are pro-cyclical and realized returns are counter-cyclical. The quantitative estimates are furthermore in the ballpark of values obtained from data. This is the case despite the fact that the calibration has not made use of the empirical evidence in table 1.

Table 2: Expected vs. actual capital gains

Specification	Model			Michigan Survey	
	$\underline{r}^n = 1\%$	$\underline{r}^n = 0.5\%$	$\underline{r}^n = 0.25\%$	Mean	Median
$\hat{c} \cdot 10^3$	0.395	0.289	0.220	0.614	0.196
$\hat{\mathbf{c}} \cdot 10^3$	-0.756	-0.611	-0.527	-0.483	-0.483

Note: \hat{c} is the coefficient estimated in regression (54), and $\hat{\mathbf{c}}$ the one from regression (55). The columns labelled "Model" report the estimated coefficients from model-simulated data for different average natural rates, \underline{r}^n . The column labelled "Mean" reports the result when using average expectations, and the column labelled "Median" the result using median expectations, both taken from the Michigan Survey. In both cases, we look at real capital gains.

6.3.2 Sluggish Adjustment of Housing Price Expectations

This section shows that expectations about the *level* of future housing prices are updated sluggishly. In particular, we find that past forecast revisions predict future forecast errors in the same direction, i.e., past forecast revisions are on average insufficiently strong.

Following Coibion and Gorodnichenko (2015) we consider the following regression

$$HP_{t+4} - E_t^{\mathcal{P}} [HP_{t+4}] = \alpha + \beta_{CG} \cdot (E_t^{\mathcal{P}} [HP_{t+4}] - E_{t-1}^{\mathcal{P}} [HP_{t+3}]) + \epsilon_t \quad (56)$$

where HP_t denotes the real housing price and $E_t^{\mathcal{P}} [HP_{t+4}]$ the expected housing real housing price implied by the Michigan survey.³³ The forecast error on the left-hand side of the previous equation is regressed on past forecast revisions on the right-hand side. Under the RE hypothesis, information that is in agents' information set, e.g., past forecasts, should not predict future forecast errors, i.e., one would expect $\beta_{CG} = 0$.

The second and third column in table 3 reports the regression results. It shows that β_{CG} is positive and highly statistically significant, even though the sample size is quite small.³⁴ Past upward revisions in housing price forecasts thus predict that future forecast errors are positive on average, which indicates that the observed past upward revisions are insufficiently strong to be consistent with the RE hypothesis.

The last three columns in table 3 report the model-implied coefficients from the subjective belief model for different levels of the steady-state natural rate.³⁵ While the RE model cannot

³³We compute real housing price expectations as $E_t^{\mathcal{P}} [HP_{t+4}] = \frac{E_t^{\mathcal{P}} [CG_{t,t+4}]}{E_t^{\mathcal{P}} [\Pi_{t,t+4}]} \cdot HP_t$, where expectations of nominal housing price growth and inflation are taken from the Michigan survey.

³⁴We control for serial correlation and heteroskedasticity in the error term by using the Newey-West estimator including four lags. Results are robust to using different lag lengths.

³⁵More precisely, it reports the population estimate of β_{CG} for the following regression:

$$q_{t+4}^u - E_t^{\mathcal{P}} [q_{t+4}^u] = \alpha + \beta_{CG} \cdot (E_t^{\mathcal{P}} [q_{t+4}^u] - E_{t-1}^{\mathcal{P}} [q_{t+4}^u]) + \epsilon_t.$$

match the empirical evidence, the subjective belief model generates regression estimates that are quantitatively close to the ones estimated in the data. This is again the case despite the fact that the model calibration in section 8 does not target the reported estimates.

Appendix D.3 show that our empirical findings are robust to using an instrumental-variable approach for estimating regression (56), in which forecast revisions are instrumented with monetary policy shocks obtained via high-frequency identification. Appendix D shows that results are also robust to using monthly data instead of quarterly data and similar patterns emerge when running quasi-panels in which survey respondents are grouped into income brackets.

The positive sign for β_{CG} is consistent with previous findings on the behavior of survey expectations of output, inflation and unemployment (see, e.g., Coibion and Gorodnichenko (2015) and Angeletos et al. (2020)). Bordalo et al. (2018) provide evidence of sluggish belief adjustments in consensus forecasts for a number of variables, including residential investment and new housing starts.

Table 3: Sluggish Adjustment of Housing Price Expectations

	Survey Mean		Survey Median		Model		
	2007-21Q1	2010-21Q1	2007-21Q1	2010-21Q1	$\underline{r}^n = 1\%$	$\underline{r}^n = 0.5\%$	$\underline{r}^n = 0.25\%$
β_{CG}	2.00*** (0.332)	1.47*** (0.480)	2.47*** (0.366)	1.87*** (0.591)	1.71	2.40	2.96

Note: The first two columns show the empirical estimates of regression (56) for different samples. The last four columns show the model-implied regression coefficients for different levels of the average natural rates. The reported standard errors are robust (Newey-West with four lags). Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

Dynamic Forecast Error Structure

We just saw that households tend to update their housing-price expectations sluggishly. In this section, we look at the dynamics of forecast errors after an increase in housing prices. To do so, we estimate the following local projection à la Jordà (2005):

$$FE_{t+h} = \alpha^h + \beta^h \left(\frac{RHP_t}{RHP_{t-1}} \right) + u_t^h, \quad (57)$$

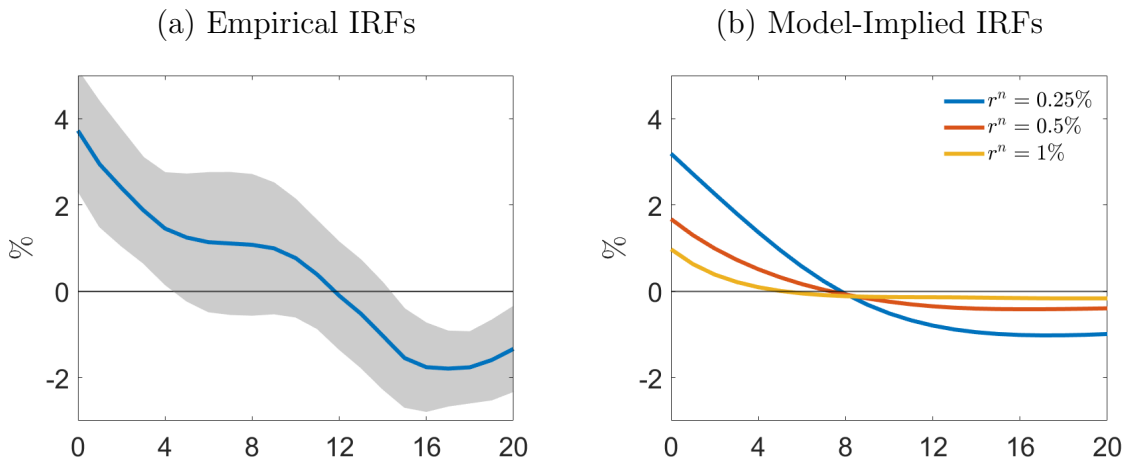
where

$$FE_{t+h} \equiv \frac{RHP_{t+4+h}}{RHP_{t+h}} - E_{t+h}^{\mathcal{P}} \left[\frac{RHP_{t+4+h}}{RHP_{t+h}} \right]$$

denotes the one-year ahead forecast error, and h the horizon.

Instead of regressing forecast errors on an exogenous shock, we regress the errors on the gross housing capital gain in period t , $\left(\frac{RHP_t}{RHP_{t-1}} \right)$. As we show in Appendix D, real housing

Figure 7: Dynamic Forecast Error Structure



Notes: Panel (a) shows impulse-response functions of housing-price forecast errors of one-year ahead expectations from the Survey of Consumers to a one standard deviation innovation in the housing capital gain. The shaded area shows the 90%-confidence intervals. Panel (b) reports the model-implied impulse-response functions of housing-price forecast errors of one-year ahead expectations to a one standard deviation innovation in the housing capital gain for different steady-state natural rates.

capital gains in fact coincide with the structural housing demand shock if we assume that the economy is in the steady state when the shock occurs. That is,

$$\frac{q_t^u}{q_{t-1}^u} = e^{\varepsilon_t^d}, \quad (58)$$

where $\frac{q_t^u}{q_{t-1}^u}$ is the model equivalent of $\left(\frac{RHP_t}{RHP_{t-1}}\right)$.

Panel (a) in Figure 7 shows the impulse-response functions of the one-year ahead forecast errors to a one-standard deviation innovation in the real housing capital gain for twelve periods. The shaded area reports the 90% confidence intervals, with standard errors that are robust with respect to heteroskedasticity and serial correlation. We see that forecast errors are initially positive and then turn negative after about twelve quarters. Thus, housing-price expectations initially underreact, followed by a delayed overshooting. This is in line with findings in Angeletos et al. (2020) who document a similar behavior of inflation and unemployment expectations.

In the model-implied IRF, the dependent variable is given by

$$FE_{t+h}^{model} \equiv \frac{q_{t+4+h}^u}{q_{t+h}^u} - \beta_{t+h}^4.$$

The independent variable is again given by the change in house prices, $\frac{q_t^u}{q_{t-1}^u}$, to make it comparable to the empirical IRF.³⁶

³⁶Instead of regressing forecast errors on capital gains, we could directly use the structural housing-demand shock. This barely affects the results.

Panel (a) of Figure 7 shows the model-implied impulse-response functions for three different steady-state natural rates. We observe the same pattern as in the data: positive forecast errors are followed by negative ones. Especially for $\underline{r}^n = 0.25\%$, the model implies a dynamic forecast error structure that is quantitatively close to its empirical counterpart, even though this is not targeted in the calibration, outlined in Section 8. In Appendix D, we show that the results hold when looking at forecast errors about the *level* of the house price rather than its growth rate.

7 Quadratic Approximation of the Policy Problem

One can obtain analytic insights into the nature of the nonlinear optimal monetary policy problem by considering a quadratic approximation to the objective (34) and a linear approximation to the constraints (35)-(40), keeping the lower-bound constraint on nominal interest rates in its nonlinear form.³⁷

Appendix A shows that under the considered belief setting, the Lagrangian of the optimal policy problem can be approximated as follows:

$$\max_{\{\pi_t, y_t^{gap}, \hat{q}_t^u, i_t \geq \underline{i}\}} \min_{\{\varphi_t, \lambda_t\}} \quad (59)$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y (y_t^{gap})^2 \right) \right. \\ \left. + \varphi_t \left[\pi_t - \kappa_y y_t^{gap} - \kappa_q (\hat{q}_t^u - \hat{q}_t^{u*}) - u_t - \beta E_t \pi_{t+1} \right] \right. \quad (60)$$

$$\left. + \lambda_t \left[y_t^{gap} - \lim_T E_t y_T^{gap} + \varphi E_t \sum_{k=0}^{\infty} \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n, RE} \right) + \frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}) \right] \right. \quad (61)$$

$$\left. - \varphi_{-1} \pi_0 - \lambda_{-1} \left(\varphi \pi_0 - y_0^{gap} - \frac{C_q}{C_Y} (\hat{q}_0^u - \hat{q}_0^{u*}) \right) \right\}.$$

The variable π_t denotes inflation, y_t^{gap} the output gap, i_t nominal interest rates, and $\hat{q}_t^u - \hat{q}_t^{u*}$ the housing price gap. The housing price gap is the difference between the housing price \hat{q}_t^u and its efficient welfare-maximizing level \hat{q}_t^{u*} , which (in nonlinear terms) is given by³⁸

$$q_t^{u*} = \bar{\xi}_t^d. \quad (62)$$

Nominal interest choices are subject to an effective lower bound $i_t \geq \underline{i}$, where the lower bound $\underline{i} < 0$ is expressed in terms of deviation from the interest rate in a zero-inflation steady state. For the case with a zero lower bound on nominal rates, we have $\underline{i} = -(1 - \beta)/\beta$.

³⁷This will deliver a valid second-order approximation to the problem for small shock disturbances, whenever (i) the steady-state Lagrange multipliers associated with the constraints are of order $O(1)$, which is the case when the steady state output distortion $\Theta \equiv \log \left(\frac{\eta}{\eta-1} \frac{1-g}{1-\bar{\tau}} \right)$ is of order $O(1)$, and (ii) the gap between the steady-state interest rate and the lower bound, i.e., $\frac{1}{\beta} - 1$, is also of $O(1)$. Eggertsson and Singh (2019) compare the exact solution of the New Keynesian model with lower bound to the solution of the linear-quadratic approximation with lower bound and show that the quantitative deviations are modest, even for extreme shocks of the size capturing the 2008 recession in the U.S..

³⁸See the derivation in appendix C.2.

Constraint (60), which features the Lagrange multiplier φ_t , is the New Keynesian Phillips Curve for our model with housing and depends on the housing price gap. The coefficients $\kappa_q < 0$ and $\kappa_y > 0$ are defined in appendix A.3 and imply that a positive housing price gap has a negative cost-push effects. This is so because high housing prices increase housing investment. For a given output gap, higher housing investment raises the marginal utility of non-housing consumption, thereby depressing wages and marginal production costs.

Constraint (61), which features the Lagrange multiplier λ_t , is the linearized (and forward-iterated) IS equation. The coefficients $C_q < 0$ and $C_Y > 0$ are the derivatives of the function $C(\cdot)$ defined in (29) with respect to q^u and Y , respectively, evaluated at the efficient steady state. The long-run output gap expectations $\lim_T E_t y_T^{gap}$ in the IS curve are the ones associated with a setting in which agents hold rational housing expectations.³⁹ The initial Lagrange multipliers $(\varphi_{-1}, \lambda_{-1})$ capture initial pre-commitments.

Interestingly, the expectations showing up in the monetary policy problem (59) are all rational. Therefore, subjective housing price expectations affect the monetary policy problem solely through their effects on housing price gaps. These gaps are determined differently under RE and under subjective beliefs, see section 6.

An interesting novel feature of our setup is that the housing price gap also enters the IS equation (61), which is key for understanding the quantitative results presented later on. The RE natural interest rate $r_t^{n,RE}$ entering the IS equation is thereby defined in the usual way: it is the real interest rate consistent with the optimal consumption level in a setting with flexible prices and rational expectations.⁴⁰ Importantly, the restrictions that the IS constraint (61) imposes on monetary policy choices depend on the belief setting, as we discuss next.

Rational Housing Price Expectations. With fully rational expectations, it follows from lemma 1 and equation (62) that

$$\widehat{q}_t^u = \widehat{q}_t^{u*}. \quad (63)$$

The housing price gap is thus always zero. In particular, the housing price gap is independent of monetary policy and economic shocks. While policy can affect the level of raw housing prices (\widehat{q}_t), it cannot affect the housing price in marginal utility units (\widehat{q}_t^u). This will also be true for the setting with subjective housing beliefs and explains why one can ignore utility contributions from the housing price gap in the objective function of problem (59), despite the fact that the nonlinear utility function (34) depends on q_t^u .

Equation (63) implies that under RE the IS equation simplifies to

$$y_t^{gap} = \lim_T E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) \right),$$

so that setting real interest rates equal to natural rates each period, i.e., choosing

$$i_t - E_t \pi_{t+1} = r_t^{n,RE} \quad \text{for all } t, \quad (64)$$

³⁹Recall that we assume housing expectations to be rational in the long-run for all our belief settings. Our numerical solution approach solves for the long-run expectations together with the state-contingent optimal policy functions.

⁴⁰See appendix A.4 for the precise definition.

causes the IS equation to be consistent with a constant output gap $y_t^{gap} = \lim_T E_t y_T^{gap}$ for all t , as in the simple New Keynesian model without housing. In the presence of a zero lower bound constraint on nominal rates, however, it will generally not be optimal (or even impossible) to implement (64) at all times.

The RE housing investment gap $\widehat{k}_t - \widehat{k}_t^*$ is given by⁴¹

$$\widehat{k}_t - \widehat{k}_t^* = \frac{\tilde{\sigma}^{-1} C_Y}{1 - \tilde{\alpha}} y_t^{gap}, \quad (65)$$

which shows that housing investment under RE is purely driven by movements in the output gap. This is so because the house price gap is 0, so that additional output will be allocated with constant shares to housing and non-housing consumption. As can be seen from (65), the output gap comoves positively with the housing investment gap.

Subjective housing price expectations. With subjective housing price beliefs, the efficient level of housing prices continues to be given by equation (62), but housing prices are now jointly determined by equations (48) and (50).⁴² As a result, the housing price gap $\widehat{q}_t^u - \widehat{q}_t^{u*}$ will generally differ from zero, with the housing price gap exceeding (falling short of) zero, whenever agents' subjective capital gain expectations, β_t are larger than one (smaller than ρ_ξ).⁴³

With subjective housing price beliefs, a policy that sets real interest rates equal to the RE natural real rate $r_t^{n,RE}$ ceases to deliver a constant output gap. In particular, the IS equation then implies

$$y_t^{gap} = \lim_T E_t y_T^{gap} - \frac{C_q}{C_Y} (\widehat{q}_t^u - \widehat{q}_t^{u*}),$$

which shows that high house price gap ($\widehat{q}_t^u - \widehat{q}_t^{u*} > 0$) will then be associated with a more positive output gap (recall that $C_q/C_Y < 0$): high housing prices stimulate housing investment and thus output.

The following lemma derives the natural rate $r_t^{n,P}$ that - in a setting with subjective beliefs - causes the IS equation to be consistent with a constant output level. As in the case with RE, it will generally not be optimal (or not even feasible) to set interest rates equal to this natural rate level at all times in the presence of a lower bound constraint on nominal rates:

Lemma 3 *Define the natural rate under subjective beliefs as*

$$r_t^{n,P} \equiv r_t^{n,RE} - \frac{1}{\varphi} \frac{C_q}{C_Y} ((\widehat{q}_t^u - \widehat{q}_t^{u*}) - E_t (\widehat{q}_{t+1}^u - \widehat{q}_{t+1}^{u*})) \quad \text{for all } t, \quad (66)$$

⁴¹Equation (65) follows from linearizing equation (13) and using the linearized version of equation (29) to substitute consumption.

⁴²Since the equations do not depend on policy, one can again treat the housing price gap as an exogenous process, as in the case with RE.

⁴³In the intermediate range $\beta_t \in (\rho_\xi, 1)$, the ranking depends also on other parameters. ρ_ξ will be close to one in our calibration.

where $E_t[\cdot]$ denotes the rational expectations operator. When real interest rates are equal to $r_t^{n,\mathcal{P}}$ for all t , the IS equation is consistent with

$$y_t^{gap} = \lim_T E_t y_T^{gap} \quad \text{for all } t.$$

Proof. See Appendix C. ■

Equation (66) generalizes the natural interest rate definition under RE to a setting with potentially subjective beliefs. In the special case with a constant housing price gap, we have $r_t^{n,\mathcal{P}} = r_t^{n,RE}$. More generally, predictable fluctuations in the housing price gap will contribute to fluctuations in the natural rate of interest. To the extent that housing prices and thus the housing price gap becomes more volatile as the average natural rate falls, natural rate volatility will go up in line with housing price volatility, as is the case in the data.

Consider, for instance, a setting where the housing price gap is high but expected to go down over time. We then have $E_t(\hat{q}_{t+1}^u - \hat{q}_{t+1}^{u*}) < (\hat{q}_t^u - \hat{q}_t^{u*})$ and the natural rate under subjective beliefs will exceed its RE level. Conversely, if the housing price gap is low or even negative, but expected to rebound over time, then natural rates will be lower than under RE.

With subjective beliefs, the housing investment gap is given by

$$\hat{k}_t - \hat{k}_t^* = \frac{\tilde{\sigma}^{-1} C_Y}{1 - \tilde{\alpha}} y_t^{gap} + \frac{1 + \tilde{\sigma}^{-1} C_q}{1 - \tilde{\alpha}} (\hat{q}_t^u - \hat{q}_t^{u*}). \quad (67)$$

Unlike in the case with RE, the housing investment gap is also driven by housing prices. Given the calibration considered later on, which implies $\frac{1 + \tilde{\sigma}^{-1} C_q}{1 - \tilde{\alpha}} > 0$, a housing price boom will go hand in hand with a housing investment boom. Therefore, larger housing price volatility translates into larger housing investment volatility, as tends to be the case in the data.

In order to solve the problem in (59), we recursify the problem as proposed in Marcet and Marimon (2019) and solve for the associated value functions and optimal policies. Details of the recursive formulation can be found in Appendix C.4.

8 Calibration

We calibrate the model to the pre-1990 period in the U.S., matching salient features of the behavior of natural interest rates and housing prices. We then test the model by considering its predictions for lower natural rate levels, as observed post 1990.

Table 4 summarizes the model parameterization. The quarterly discount factor β is chosen such that the steady-state natural rate equals the pre-1990 average of the U.S. natural rate of 3.34%, as estimated by Holston et al. (2017). The interest rate elasticity of output φ , the slope of the Phillips curve κ_y , and the welfare weight $\frac{\Lambda_y}{\Lambda_\pi}$ are taken from table 2 in Adam and Billi (2006). The Phillips Curve coefficient κ_q and the ratio C_q/C_y are set as in Adam and Woodford (2020).⁴⁴ Details of the calibration are spelled out in Appendix C.6.

⁴⁴The calibration target for the ratio C_q/C_y is the ratio of residential fixed investment over the sum of nonresidential fixed investment and personal consumption expenditure, which is on average approximately equal to 6.3% in the US. This and the remaining parameters then imply $\kappa_q = -0.0023$.

Table 4: Model Parameterization

Parameter	Value	Source/Target
<i>Preferences and technology</i>		
β	0.9917	Average U.S. natural rate pre 1990
φ	1	Adam and Billi (2006)
κ_y	0.057	Adam and Billi (2006)
$\frac{\Lambda_y}{\Lambda_\pi}$	0.007	Adam and Billi (2006)
κ_q	-0.0023	Adam and Woodford (2020)
$\frac{C_q}{C_Y}$	-0.29633	Adam and Woodford (2020)
δ	0.03/4	Adam and Woodford (2020)
<i>Exogenous shock processes</i>		
ρ_{r^n}	0.8	Adam and Billi (2006)
σ_{r^n}	0.2940% (RE) 0.1394% (subj beliefs)	Adam and Billi (2006)
ρ_ξ	0.99	Adam and Woodford (2020)
σ_{ξ^d}	0.0233 (RE) 0.0165 (subj. beliefs)	Std. dev. of price-to-rent ratio pre 1990
<i>Subjective belief parameters</i>		
α	1/0.007	Adam et al. (2016)
β^U	1.0031	Max percent deviation of PR-ratio from mean

We now discuss parameterization of the exogenous shock processes. We assume that the RE natural rate follows an AR(1) process with

$$r_t^{n,RE} = \rho_r r_{t-1}^{n,RE} + \epsilon_t^n, \quad (68)$$

where $\epsilon_t^n \sim iiN(0, \sigma_{r^n}^2)$. We set $\rho_{r^n,RE} = 0.8$ following Adam and Billi (2006) and the persistence of housing demand shocks $\rho_\xi = 0.99$, following Adam and Woodford (2020). The mark-up shock is set to a constant value, so as to economize on the number of state variables in the model.⁴⁵ This is justified by the fact that - given their size and persistence - mark-up shocks tend to be quantitatively irrelevant for driving the economy towards the lower bound constraint (Adam and Billi (2006)). Specifically, setting $u_t = 0$ implies that the model collapses under RE to the setup in Adam and Billi (2006) for the case where one abstracts from mark-up shock disturbances.

The standard deviations of the innovations to the housing preference and natural rate shocks are set differently under RE and subjective beliefs, because housing volatility and natural rate volatility differ across the two specifications.

Under RE, we set σ_{ξ^d} so that the model matches the unconditional standard deviation of the percent deviation of the price-to-rent ratio of 6% observed over the period 1970-1990 in the U.S..⁴⁶ The standard deviation of the innovation to the natural rate, $\sigma_{r^n,RE}$, is set equal

⁴⁵Solving for optimal policy still requires solving an optimization problem featuring six state variables.

⁴⁶This calibration is based on equation (45). The reported standard deviation refers to ratio of the housing price over quarterly rent.

to the value in Adam and Billi (2006). The implied annual unconditional standard deviation of the natural rate is then 1.96%.⁴⁷

For the subjective belief model, we choose again ε_t^d to match the unconditional standard deviation of the price-to-rent ratio. This is achieved by simulating equations (48) and (49) and using $PR_t^P = q_t^{u,P} / \xi_t^d$ to compute the price-to-rent ratio. Doing so requires specifying the subjective belief parameters α and β^U , which enter equation (48). We set $\alpha = 1/0.007$ following Adam et al. (2016) and determine σ_{ξ^d} and β^U jointly such that (1) the volatility of the price-to-rent ratio is 6% and (2) the simulated data matches the maximum deviation of the price-to-rent ratio from its sample mean, which is a statistic that identifies β^u . This yields $\beta^U = 1.0031$ and $\sigma_{\xi^d} = 0.165$. Note that the innovations to the housing demand disturbance are less volatile than under RE because part of the fluctuations in housing prices are now generated by fluctuations in subjective beliefs.

It only remains to determine $\sigma_{r,n,RE}$ for the subjective belief model. We choose its value, such that the generalized natural rate for the subjective belief model, as defined in equation (66), has the same volatility as the natural rate in the RE model. This yields $\sigma_{r,n,RE} = 0.1393\%$, which is lower than under RE, because belief fluctuations also contribute to fluctuations in the natural rate.

Figure 8 compares the predictions of the RE and subjective belief model for various steady-state levels of the natural rate of interest. Since the steady-state level of the natural rate is purely a function of the time discount factor, we vary the discount factor accordingly. As discussed before, variations in the discount factor may well be driven by variations in the long-term growth rate of the economy.

Panel (a) in figure 8 depicts the standard deviation of the price-to-rent ratio and panel (b) the standard deviation of the natural rate.⁴⁸ The dots in the figures report the data values for the pre- and post 1990 U.S. sample, in which the average natural rate was equal to 3.34% and 1.91%, respectively.⁴⁹ Since the model has been calibrated to the pre-1990 period, the RE and subjective belief model both match the pre-1990 data point.

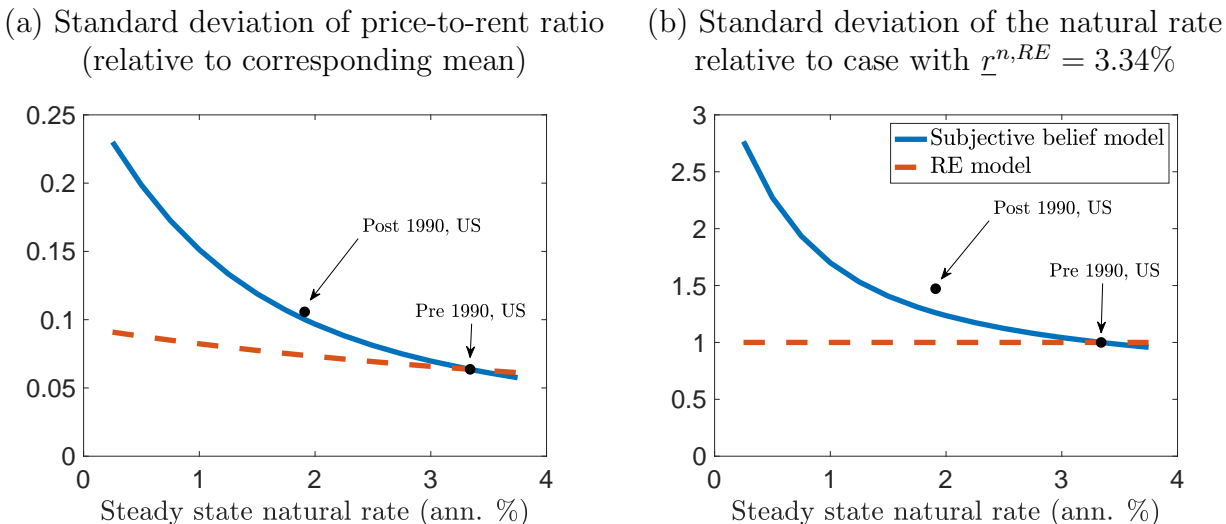
The subjective belief model also performs quite well in matching the post-1990 outcome, despite the fact that it has not been targeted in the calibration. In particular, the standard deviation of the price-to-rent ratio and the standard deviation of the natural rate endogenously increase as the natural rate falls, with the magnitudes roughly matching the increase observed in the data. In contrast, the RE model produces no increase in the volatility of the natural rate and an insufficiently strong increase in the volatility of the price-to-rent ratio. Matching the increase in housing price volatility under RE would require increasing the volatility of housing demand shocks. However, under RE these shocks are irrelevant for monetary policy and inflation outcomes, allowing us to ignore volatility increases in housing preference shocks. Matching the increase in the natural rate volatility would require increasing the $\sigma_{r,n}$.

⁴⁷We prefer this calibration approach to matching the standard deviation estimated in Holston et al. (2017), because Adam and Billi (2006) identify the standard deviation of the natural rate in a way that is consistent with our structural model, while Holston et al. (2017) use an empirically motivated model.

⁴⁸These predictions follow directly from the calibration and are independent of policy.

⁴⁹The reported increase in the standard deviation of the natural rate is based on the estimates in Holston et al. (2017).

Figure 8: Standard deviation of Price-to-Rent Ratio and Natural Rate



9 Quantitative Findings

This section discusses the quantitative implications of falling natural rates for optimal monetary policy. We start with a discussion of the inflation target implications. This is followed by a stylized experiment in which the economy is pushed to the lower bound in order to illustrate the different behavior of the economy under subjective beliefs compared to the model with rational expectations. Then we discuss the effects of shocks for optimal stabilization policy under subjective beliefs and RE.

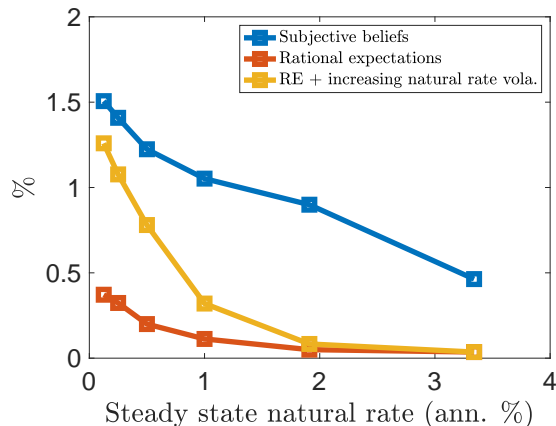
9.1 Optimal average inflation

Figure 9 depicts the optimal inflation target for different steady state levels of the natural rate of interest, where changes in the natural rate are brought about by varying the discount factor β .⁵⁰ The figure shows that a fall in the natural rate leads only to a small increase in the optimal inflation under rational expectations, as indicated by the red line. Even though lower natural rates trigger larger housing price fluctuations, these increased fluctuations are efficient, thus do not require a monetary policy response. In addition, the natural rate of interest is - under RE - independent of housing fluctuations. Therefore, even at very low levels of the natural rate, the annualized optimal inflation rate is barely above zero. This shows that the findings in Adam and Billi (2006) are surprisingly robust towards assuming a lower average natural rate of interest.

Under subjective beliefs, on the other hand, the optimal inflation target is substantially higher and also rises more strongly as the natural rate falls. In fact, a fall in the steady-state

⁵⁰Recall that a falling steady-state growth rates is associated with a rise in the discount factor and thus a decrease in the steady-state real interest rate.

Figure 9: Average Inflation under Optimal Monetary Policy



Notes: The figure reports the optimal inflation target (y-axis) for different average levels of the natural rate (x-axis) in the presence of a zero lower bound constraint. The red line depicts the optimal target for the case with rational housing price beliefs and the blue line the one with subjective housing price beliefs. The yellow line shows the optimal average inflation under RE where the exogenous volatility of the natural rate is adjusted such that it matches the endogenous volatility increase under subjective beliefs.

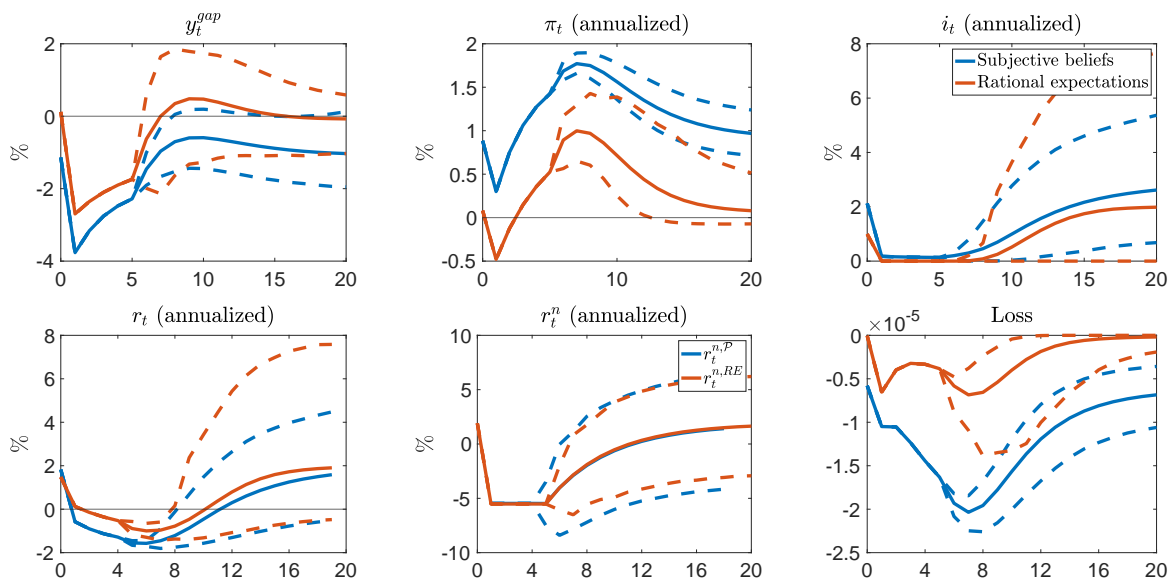
natural rate to 0.125% causes the optimal inflation target to increase to 1.5%. Already at the pre-1990 average of the natural rate (3.34%), the inflation target with subjective beliefs is larger than under RE for any considered level of the natural rate. This is the case even though the volatility of the natural rate is calibrated at this point to be equal across the RE and subjective belief models. This shows that fluctuations in the natural rate that are induced by belief fluctuations affect the inflation target more than fluctuations in the natural rate induced by other disturbances. This is mainly the case because a binding ZLB induces larger welfare losses under subjective beliefs, compared to rational expectations. We discuss this in detail in the next section.

To illustrate the point that it is not only the volatility of the natural rate that matters, but the source of these fluctuations, the yellow line in figure 9 shows the optimal inflation rate under rational expectations, when we set the volatility of the (exogenous) natural rate in the RE model such that it matches the volatility implied by the subjective belief model at each considered natural rate level. While the optimal inflation rate increases relative to the benchmark RE setting, the level of the optimal inflation target still falls short of the one implied by subjective beliefs. Under subjective beliefs, housing price fluctuations distort the optimal allocations, which leads to larger welfare losses, compared to the model under RE. This is why the inflation target is higher under subjective beliefs, even with the same natural rate volatility.

9.2 Optimal Policy Responses at the Lower Bound

To understand why the optimal inflation target is higher under subjective beliefs than under rational expectations, we consider a specific shock episode that pushes the economy towards the zero-lower-bound constraint. We do so by considering a steady-state natural rate of

Figure 10: Zero lower bound episode



Notes: The figure shows the evolution of the economy after a natural rate shock that brings the economy to the lower bound and lasts for six quarters (see figure in the bottom row, second column). The solid lines show the average response and the dashed lined the 1% and 99% percentiles. The red lines show the evolution under rational expectations and the blue lines the one under subjective beliefs.

1.91%, which is the post-1990 average for the United States.

We initialize the state variables of the economies with RE and subjective beliefs at their respective ergodic means. We then cause an initial fall in the natural rate and keep the natural rate at this level for 6 quarters, with no other shocks occurring during this period, see the panel in the center in the lower row of figure 10. After quarter 6, all shocks operate again as usual. We then report the mean response of the economy (solid lines in figure 10), as well as the 1st and 99th percentiles of the response distribution (dashed lines in figure 10).

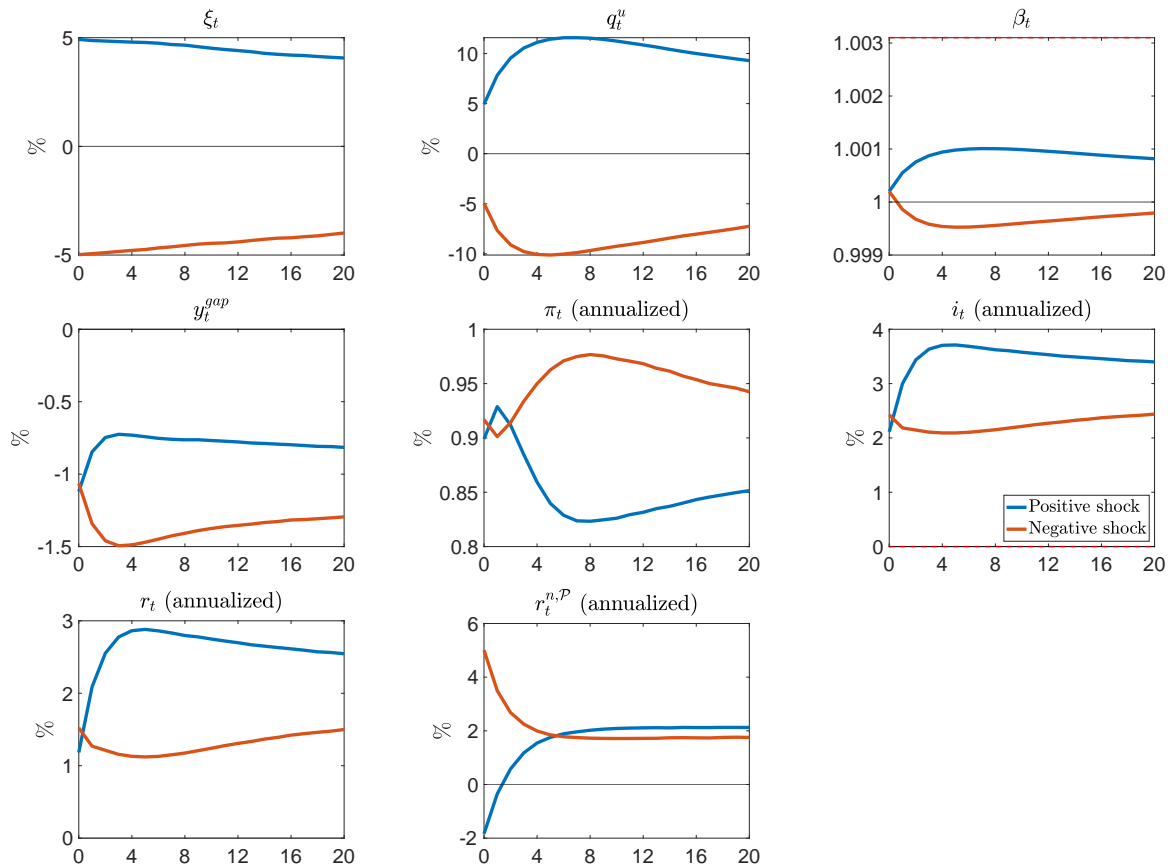
The fall in the natural rate depresses the output gap and inflation. The monetary policymaker lowers the nominal rate to counteract this and indeed, the ZLB starts binding in both economies. Note, that due to the higher average nominal interest rate under subjective beliefs—due to the higher average inflation—the fall in the nominal rate is much more pronounced compared to the RE case. The loss, measured by $-(\Lambda_\pi \pi_t^2 + \Lambda_y (y_t^{gap})^2)$, increases in both cases. After that, however, the two economies evolve quite differently. First of all, the higher inflation rate under subjective beliefs induces a lower real rate. This allows the economy to escape the lower bound faster. Especially in a “worst-case scenario”, indicated by the lower dashed lines, the optimal policy response in the economy under subjective beliefs is to raise interest rates shortly after the natural rate starts recovering. The economy under rational expectations, on the other hand, is kept at the lower bound for a longer time. This is true even though the potential worst-case scenario is worse under subjective beliefs as housing demand shocks may exacerbate the situation, as indicated by the evolution of r_t^n .

While the economy escapes the lower bound faster under subjective beliefs, the hike in nominal rates is more pronounced under rational expectations if the recovery is supported by

favorable shocks, indicated by the upper dashed lines. This is mirrored in the faster recovery of the real rate under rational expectations, which is especially driven by the large increase if future shocks are positive which moves the economy away from the bound.

The slower recovery of the real rate under subjective beliefs is nevertheless accompanied by a greater loss than in the RE setting. This illustrates that the lower bound poses a more severe problem under subjective beliefs and that it is optimal to allow for a substantially higher inflation rate. Otherwise, the strong initial decrease in the nominal rate would not have been possible, and the corresponding loss would be even larger.

Figure 11: Impulse responses to a housing preference shock



Notes: The figure reports the average impulse responses of the economy under subjective beliefs (at $\underline{r}^n = 1.91\%$) after a three-standard-deviation housing demand shock. The blue lines show the responses after a positive shock and the red lines after a negative shock.

9.3 Leaning Against Housing Demand Shocks

We now examine the optimal monetary policy response to housing demand shocks.⁵¹

Under RE, the housing demand shocks only affect the housing price, but leave the nominal rate, the output gap and inflation unaffected. In contrast, under subjective beliefs, it becomes optimal to lean against housing demand shocks, with the optimal response being asymmetric when faced with positive and negative shocks.

The top row in figure 11 shows the response of housing-related variables. A positive housing demand shock increases house prices and housing price expectations. As anticipated, our subjective belief formulation generates momentum in housing prices. In this specific scenario, the initial shock pushes housing prices up by about 5%, with belief momentum generating another 5% approximately. As *actual* housing price increases start to fall short of the *expected* housing price increases, the housing boom slows down and house prices eventually revert direction.

Higher housing prices push up housing investment, which increases the output gap. The increase in housing prices and investment increases marginal utility of consumption, hence, dampens wages and marginal costs. The result of a positive housing demand shock is thus a disinflationary housing boom episode.

Optimal monetary policy leans strongly against the housing price increase. In particular, the policy response is much stronger than when policy faces a negative housing demand shock. Under rational expectations, on the other hand, policy does not lean against housing prices as these price movements are fully efficient.

10 Conclusion

This paper documents new facts about the changing volatility patterns in housing markets and the natural rate. In advanced economies, the standard deviation of the price-to-rent ratio and of the natural rate both increased as the average levels of the natural rate fell.

We examine the implications of these macroeconomic trends for monetary policy in a New Keynesian model featuring a housing market and a zero lower bound on nominal interest rates. Lower natural rates trigger larger volatility in house prices and investment. The policy implications of these developments depend on the source of the increased housing price volatility. If agents hold rational house price expectations, house price fluctuations are driven by efficient housing demand shocks and optimal policy implies that average inflation should rise only by very little, following a fall in natural rates. Instead, if housing price volatility is driven by speculative beliefs about future housing prices, then falling natural rates require a much stronger increase in average inflation under optimal policy. Larger housing price fluctuations endogenously trigger more volatility in natural interest rates, which exacerbates the lower bound problem. Ameliorating the adverse effects of the lower bound then requires implementing on average higher inflation rates, as well as a policy that leans against housing prices.

⁵¹As before, we initialize the economy at its ergodic mean. We then hit the economy with a one-time shock of three standard deviations and average the subsequent response over the possible future shock realizations. As before, the impulse responses assume a steady state natural rate of 1.91%, which is the post-1990 mean for the United States.

A Quadratic Approximation of the Policy Problem

This appendix derives the linear-quadratic approximation to the nonlinear policy problem in section 5.

A.1 Optimal Dynamics and the Housing Price Gap

It will be convenient to determine the welfare-maximizing level of output and the welfare-maximizing housing price under flexible prices, so as to express output and housing prices in terms of gaps relative to these maximizing values. We thus define (Y_t^*, q_t^{u*}) as the values (Y_t, q_t^u) that maximize $U(Y_t, 1, q_t^u; \xi_t)$, which are implicitly defined by⁵²

$$U_Y(Y_t^*, 1, q_t^{u*}; \xi_t) = U_{q^u}(Y_t^*, 1, q_t^{u*}; \xi_t) = 0.$$

In particular, we have

$$q_t^{u*} = \bar{\xi}_t^d, \quad (69)$$

as shown in Appendix C.2. We have

$$\widehat{q}_t^{u, RE} = \widehat{q}_t^{u*}, \quad (70)$$

which shows that housing price fluctuations are indeed efficient under RE.

Under subjective beliefs, it follows from equations (44) and (50) that

$$\widehat{q}_t^{u, \mathcal{P}} - \widehat{q}_t^{u*} = \left(\frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\beta_t} - \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)\rho_\xi} \right) \widehat{\xi}_t^d + \frac{\beta(1 - \delta)(\beta_t - 1)}{1 - \beta(1 - \delta)\beta_t}. \quad (71)$$

Again, for the case where $\beta_t = 1$ and with persistent housing demand shocks ($\rho_\xi \rightarrow 1$), the housing price gap under subjective beliefs is equal to the housing price gap under RE. Belief fluctuations, however, now contribute to fluctuations in the housing price gap.

For the real house price gap, $\widehat{q}_t - \widehat{q}_t^*$, this implies

$$\widehat{q}_t - \widehat{q}_t^* = (1 + \tilde{\sigma}^{-1}C_q) (\widehat{q}_t^u - \widehat{q}_t^{u*}) + \tilde{\sigma}^{-1}C_Y y_t^{gap}. \quad (72)$$

A.2 Quadratically Approximated Welfare Objective

A second-order approximation to the utility function delivers⁵³

$$\frac{1}{2}U_{\widehat{Y}\widehat{Y}}(\widehat{y}_t - \widehat{y}_t^*)^2 + \frac{1}{2}U_{\widehat{q}^u\widehat{q}^u}(\widehat{q}_t^u - \widehat{q}_t^{u*})^2 + \frac{1}{2}\underline{\gamma}^* h_{22}\pi_t^2 + t.i.p.,$$

where *t.i.p.* denotes terms independent of policy and $\underline{\gamma}^*$ is the Lagrange multiplier associated with equation (38) at the optimal steady state. The dependence of the objective function on inflation follows from a second-order approximation of the constraint (38), which allows

⁵²The optimal path for $\{Y_t^*, q_t^{u*}\}$ can then be used to determine optimal dynamics for the remaining variables. In particular, equation (29) determines C_t^* , equation (13) determines k_t^* and thus D_t^* , and equation (7) determines H_t^* .

⁵³See Appendix C.3 for a detailed derivation.

expressing the second-order utility losses associated with price distortions Δ_t as a function of squared inflation terms.

Since the fluctuations in the housing price gap, $\widehat{q}_t^u - \widehat{q}_t^{u*}$, are either constant (with RE) or determined independently of policy (under subjective beliefs, see (71)), the endogenous part of the loss function can be written as

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\Lambda_{\pi} \pi_t^2 + \Lambda_y (y_t^{gap})^2 \right),$$

where $y_t^{gap} \equiv \widehat{y}_t - \widehat{y}_t^*$ denotes the output gap; the log-difference of output from its dynamically optimal value.

A.3 New Keynesian Phillips Curve

We now linearize equations (35)-(37) to derive the linearized Phillips curve. The condition for the equilibrium wage (36) in period T in industry j in which firms last updated their prices in period t is given by

$$w_T(j) = \tilde{w}_T(j) \left(\frac{p_t^j}{P_t} \right)^{-\eta\phi\nu} \left(\frac{P_T}{P_t} \right)^{\eta\phi\nu},$$

where

$$\tilde{w}_T(j) \equiv \lambda \frac{\bar{H}_T^{-\nu}}{\bar{C}_T^{\bar{\sigma}-1}} \left(\frac{Y_T}{A_T} \right)^{\phi\nu} C(Y_T, q_T^u, \xi_T)^{\bar{\sigma}-1}.$$

Since the firms' expectations about $w_T(j)$ and P_T are rational, their expectations about $\tilde{w}_T(j)$ are rational as well.

Using the expression for $w_T(j)$, noting that $p_t(i) = p_t^j = p_t^*$, and writing out $Q_{t,T}$, it follows that

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\eta}{\eta-1} \phi \bar{C}_T^{\bar{\sigma}-1} C_T^{-\bar{\sigma}-1} \tilde{w}_T(j) \left(\frac{Y_T}{A_T} \right)^{\phi} \left(\frac{P_T}{P_t} \right)^{\eta(1+\omega)}}{E_t^{\mathcal{P}} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \bar{C}_T^{\bar{\sigma}-1} C_T^{-\bar{\sigma}-1} (1 - \tau_T) Y_T \left(\frac{P_T}{P_t} \right)^{\eta-1}} \right)^{\frac{1}{1+\omega\eta}}. \quad (73)$$

Log-linearizing equation (73) delivers⁵⁴

$$\widehat{p}_t^* - \widehat{P}_t = \frac{1 - \alpha\beta}{1 + \omega\eta} \left\{ \widehat{w}_t(j) + \phi (\widehat{y}_t - \widehat{A}_t) - \widehat{\tau}_t - \widehat{y}_t + \alpha\beta E_t^{\mathcal{P}} \left[\frac{1 + \omega\eta}{1 - \alpha\beta} (\widehat{p}_{t+1}^* - \widehat{P}_{t+1} + \pi_{t+1}) \right] \right\}. \quad (74)$$

⁵⁴This follows from the the fact that in steady state, we have $p^* = P$, so that

$$\frac{\eta}{\eta-1} \phi \bar{C}^{\bar{\sigma}-1} C^{-\bar{\sigma}-1} \tilde{w}(j) \left(\frac{Y}{A} \right)^{\phi} = \bar{C}^{\bar{\sigma}-1} C^{-\bar{\sigma}-1} (1 - \tau) Y.$$

The steady state value of the numerator in (73) is thus given by $\frac{1}{1-\alpha\beta} \frac{\eta}{\eta-1} \phi \bar{C}^{\bar{\sigma}-1} C^{-\bar{\sigma}-1} \tilde{w}(j) \left(\frac{Y}{A} \right)^{\phi}$ and the steady state value of the denominator by $\frac{1}{1-\alpha\beta} \bar{C}^{\bar{\sigma}-1} C^{-\bar{\sigma}-1} (1 - \tau) Y$.

As the expectation in (74) is only about variables about which the private agents hold rational expectations, we can replace $E_t^P[\cdot]$ with $E_t[\cdot]$.⁵⁵ Therefore, (37) can be used in period t and $t + 1$, which in its linearized form is given by

$$\widehat{p}_t^* - \widehat{P}_t = \frac{\alpha}{1 - \alpha} \pi_t.$$

Substituting $\widehat{w}_t(j)$ with the linearized version of the equilibrium condition (36) delivers the linearized New Keynesian Phillips Curve:

$$\pi_t = \kappa_y y_t^{gap} + \kappa_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) + \beta E_t \pi_{t+1} + u_t, \quad (75)$$

where the coefficients κ are given by

$$\begin{aligned} \kappa_y &= \frac{1 - \alpha}{\alpha} \frac{1 - \alpha\beta}{1 + \omega\eta} (k_y - f_y) > 0 \\ \kappa_q &= -\frac{1 - \alpha}{\alpha} \frac{1 - \alpha\beta}{1 + \omega\eta} f_q < 0, \end{aligned}$$

with $k_y = \partial \log k / \partial \log y$, $f_y = \partial \log f / \partial \log y$, $f_q = \partial \log f / \partial \log q^u$, such that

$$\begin{aligned} k_y - f_y &= \omega + \tilde{\sigma}^{-1} \frac{(1 - g) Y}{\underline{C} + \frac{\tilde{\sigma}^{-1}}{1 - \alpha} k} = \omega + \tilde{\sigma}^{-1} C_Y > 0 \\ f_q &= \tilde{\sigma}^{-1} \frac{\frac{k}{1 - \alpha}}{\underline{C} + \frac{\tilde{\sigma}^{-1}}{1 - \alpha} k} = -\tilde{\sigma}^{-1} C_q > 0, \end{aligned}$$

where $C_q \equiv \frac{q^u}{C} \frac{\partial C}{\partial q^u}$ and $C_Y \equiv \frac{Y}{C} \frac{\partial C}{\partial Y}$, and where the functions $f(Y, q^u; \xi) \equiv (1 - \tau) \bar{C}^{\tilde{\sigma}^{-1}} Y C(Y, q^u; \xi)^{-\tilde{\sigma}^{-1}}$ and $k(y; \xi) \equiv \frac{\eta}{\eta - 1} \lambda \phi \frac{\bar{H}^{-\nu}}{A^{1+\omega}} Y^{1+\omega}$ are the same as in Adam and Woodford (2020), for the current period in which markets clear and the internally rational agents observe this.

The cost-push shock u_t is given by

$$u_t = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha(1 + \omega\eta)} (\Theta + \widehat{\tau}_t - \widehat{g}_t),$$

where

$$\begin{aligned} \widehat{\tau}_t &= -\log \left(\frac{1 - \tau_t}{1 - \bar{\tau}_t} \right) \\ \widehat{g}_t &= -\log \left(\frac{1 - g_t}{1 - \bar{g}_t} \right) \end{aligned}$$

define deviations of τ_t and g_t from their second-best steady state values.

As in the standard New Keynesian model, a linearization of (38) implies that the state variable Δ_t is zero to first order under the maintained assumption that initial price dispersion satisfies $\Delta_{-1} \sim O(2)$. This constraint, together with the assumption that the Lagrange multipliers are of order $O(1)$, thus drops out of the quadratic formulation of the optimal policy problem. The second-order approximation of (38) is, however, important to express the quadratic approximation of utility in terms of inflation.

⁵⁵The subjective consumption plans showing up in the stochastic discount factor drop out at this order of approximation.

A.4 Linearized IS Equation with Potentially Non-Rational Housing Price Beliefs

We now linearize constraint (39). One difficulty with this constraint is that it features the limiting expectations of the subjectively optimal consumption plan on the right hand side. Generally, this would require solving for the subjectively optimal consumption paths, which is generally difficult.

Under our beliefs specifications, housing prices beliefs are rational in the limit. This insures that we do not have to solve for the subjectively optimal consumption plan, instead can derive the IS equation directly in terms of the output gap.

We can now define the natural rate of interest:

Definition 2 *The natural rate $r_t^{n,RE}$ is the one implied by the consumption Euler equation (12) or (39), rational expectations, and the welfare-maximizing consumption levels under flexible prices $\{C_t^*\}$. It satisfies*

$$\tilde{u}_C(C_t^*; \xi_t) = \beta E_t \left[u_C(C_{t+1}^*; \xi_t) (1 + r_{t+k}^{n,RE}) \right]. \quad (76)$$

Using the previous definition, we obtain the linearized Euler equation under potentially subjective housing prices beliefs:

Lemma 4 *For the considered belief specifications, the log-linearized household optimality condition (39) implies for all t*

$$y_t^{gap} = \lim_T E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) \right) - \frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}), \quad (77)$$

where $\lim_T E_t y_T^{gap}$ is the (rational) long-run expectation of the output gap, and $\varphi \equiv -\frac{\tilde{u}_c}{\tilde{u}_{cc} C} \frac{1}{C_Y} > 0$. The coefficients $C_q < 0$ and $C_Y > 0$ are the ones defined in the derivation of the linearized Phillips Curve.

Proof. See Appendix C. ■

B Online Appendix – Not for publication

B.1 Robustness of Empirical Results

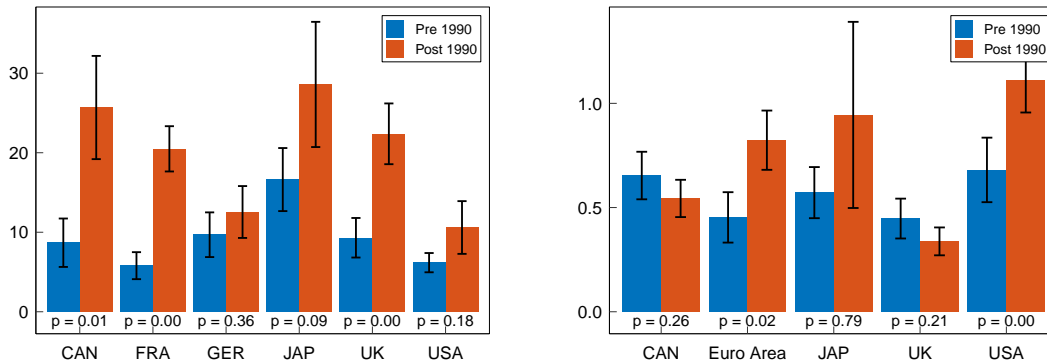
Table 5: Annual Persistence of Price-to-Rent Ratios

Sample	USA	Japan	Germany	France	UK	Canada
All Periods	0.90	0.95	0.97	0.98	0.97	0.99
	[0.79, 1.01]	[0.85, 1.05]	[0.92, 1.03]	[0.92, 1.05]	[0.88, 1.07]	[0.95, 1.04]
Pre 1990	0.81	0.89	0.88	0.89	0.84	0.97
	[0.52, 1.11]	[0.59, 1.19]	[0.63, 1.13]	[0.68, 1.10]	[0.47, 1.21]	[0.80, 1.15]
Post 1990	0.91	0.99	0.98	0.97	0.96	0.99
	[0.75, 1.06]	[0.93, 1.06]	[0.87, 1.08]	[0.86, 1.07]	[0.86, 1.07]	[0.90, 1.08]

Notes: This table shows the estimated persistence of the annual price-to-rent ratios for the whole sample as well as the periods before and after 1990, respectively. 95%-confidence intervals are reported in brackets.

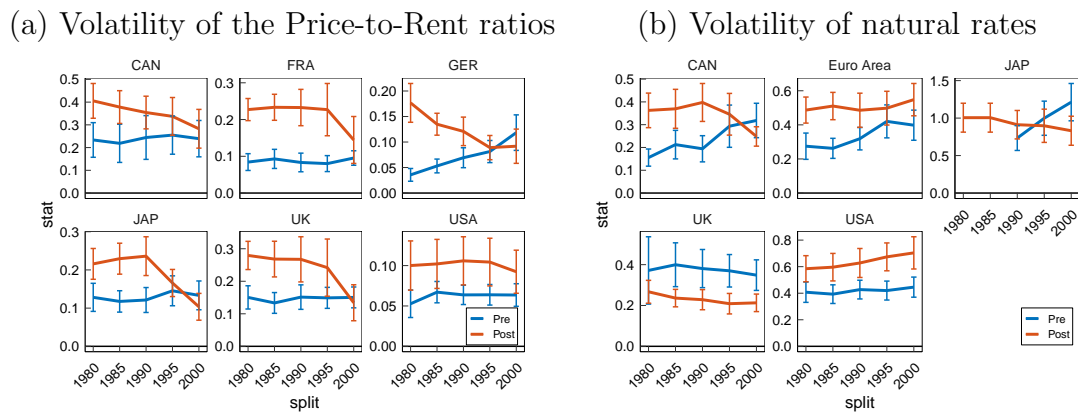
Figure 12: Absolute Standard Deviation of the Price-to-Rent Ratio, Standard deviation of the undetrended Natural Rate

(a) Absolute Standard Deviation of the Price-to-Rent ratio pre- and post-1990. (b) Standard Deviation of undetrended Natural Rate pre- and post-1990.



Notes: Panel (a) shows the *absolute* standard deviation of the Price-to-Rent ratio (i.e., not relative to the respective sample mean) for different advanced economies. Panel (b) shows the standard deviation of the *undetrended* natural rate. The blue bars show the estimates for the pre-1990 period, and the red bars for the post-1990 period. The black whiskers denote 90%-confidence bands. The reported *p*-values are for the null hypothesis that the volatility has not changed from pre- to post-1990.

Figure 13: Robustness of housing and natural rate volatility increases with different sample splits



Notes: Panel (a) shows the standard deviation of the price-to-rent ratio, and panel (b) shows the standard deviation of the natural rate for different advanced economies, computed for varied subsamples. The blue lines show the estimates for the pre-period, and the red lines for the post-period, when the sample is split at the year marked on the horizontal axis. The whiskers denote 90%-confidence bands.

C Proofs

Proof of Lemma 1

Proof. Result (41) follows from iterating forward on (15). Log linearizing (41), we have

$$\widehat{q}_t^u = \widehat{\xi}_t^d,$$

and log-linearizing (6) delivers

$$\widehat{\xi}_t^d = \rho_\xi \widehat{\xi}_{t-1}^d + \varepsilon_t^d.$$

Since the steady-state value of $\underline{\xi}^d$ is

$$\underline{\xi}^d = \frac{\xi^d}{1 - \beta(1 - \delta)},$$

the log-linearization of (42) delivers

$$\begin{aligned} \widehat{\xi}_t^d &= (1 - \beta(1 - \delta)) \left[\widehat{\xi}_t^d + \beta(1 - \delta) E_t \widehat{\xi}_{t+1}^d + \dots \right] \\ &= (1 - \beta(1 - \delta)) \left[\widehat{\xi}_t^d + \beta(1 - \delta) \rho_\xi \widehat{\xi}_t^d + \dots \right] \\ &= (1 - \beta(1 - \delta)) \sum_{T=t}^{\infty} (\beta(1 - \delta) \rho_\xi)^{T-t} \widehat{\xi}_t^d \\ &= \widehat{\xi}_t^d \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta) \rho_\xi}. \end{aligned}$$

The results for the price-to rent ration follow by noticing that equation (14) implies

$$PR_t \equiv \frac{q_t}{R_t} = \frac{q_t^u}{\xi_t^d}. \quad (78)$$

■

Proof of Lemma 2

Proof. From equation (15), which has to hold with equality in equilibrium, and equation (47) we get

$$q_t^{u,\mathcal{P}} = \frac{1}{1 - \beta(1 - \delta)\beta_t} \xi_t^d$$

The percent deviation of housing prices from the steady state, in which $\beta_t = 1$ and $\xi_t^d = \underline{\xi}^d$, is then given by

$$\begin{aligned}
\widehat{q}_t^{u,\mathcal{P}} &= \frac{\frac{1}{1-\beta(1-\delta)\beta_t}\xi_t^d - \frac{1}{1-\beta(1-\delta)}\underline{\xi}^d}{\frac{1}{1-\beta(1-\delta)}\underline{\xi}^d} \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t} \frac{\xi_t^d}{\underline{\xi}^d} - 1 \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t} \left(1 + \widehat{\xi}_t^d\right) - 1 \\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\beta_t} \widehat{\xi}_t^d + \frac{\beta(1-\delta)(\beta_t-1)}{1-\beta(1-\delta)\beta_t}
\end{aligned} \tag{79}$$

Note, that we can decompose the housing price under subjective beliefs into the housing price under RE and terms that are driven by beliefs:

$$\widehat{q}_t^{u,\mathcal{P}} = \widehat{q}_t^{u,RE} + \frac{\beta(1-\delta)(\beta_t-1)}{1-\beta(1-\delta)\beta_t} + \frac{(1-\beta(1-\delta))(\beta(1-\delta)(\beta_t-\rho_\xi))}{(1-\beta(1-\delta)\beta_t)(1-\beta(1-\delta)\rho_\xi)} \widehat{\xi}_t^d. \tag{80}$$

Note, that

$$E_t^{\mathcal{P}} \left[\widehat{q}_{t+1}^{u,\mathcal{P}} \right] = \beta_t \widehat{q}_t^{u,\mathcal{P}}.$$

Therefore, a log-linear approximation around the optimal steady state, in which $\underline{\beta} = 1$, yields

$$E_t^{\mathcal{P}} \left[\widehat{q}_{t+1}^{u,\mathcal{P}} \right] = \widehat{q}_t^{u,\mathcal{P}} + (\beta_t - 1).$$

From this, we can add and subtract on the right-hand side

$$E_t \left[\widehat{q}_{t+1}^{u,RE} \right] = \rho_\xi \widehat{\xi}_t^d \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\rho_\xi},$$

which, after plugging in the expression from (79), delivers

$$\begin{aligned}
E_t^{\mathcal{P}} \left[\widehat{q}_{t+1}^{u,\mathcal{P}} \right] &= E_t \left[\widehat{q}_{t+1}^{u,RE} \right] + (\beta_t - 1) \left[1 + \frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t} \right] \\
&\quad + (1-\beta(1-\delta)\rho_\xi - (1-\beta(1-\delta)\beta_t)\rho_\xi) \frac{(1-\beta(1-\delta))}{(1-\beta(1-\delta)\beta_t)(1-\beta(1-\delta)\rho_\xi)} \widehat{\xi}_t^d.
\end{aligned}$$

In the limit $\rho_\xi \rightarrow 1$, this boils down to

$$E_t^{\mathcal{P}} \left[\widehat{q}_{t+1}^{u,\mathcal{P}} \right] = E_t \left[\widehat{q}_{t+1}^{u,RE} \right] + (\beta_t - 1) \left[1 + \frac{\beta(1-\delta)}{1-\beta(1-\delta)\beta_t} \left(1 + \widehat{\xi}_t^d\right) \right]$$

Log-linearizing equation (78), which holds true independent of the belief specification, yields

$$\widehat{PR}_t^{\mathcal{P}} = \widehat{q}_t^{u,\mathcal{P}} - \widehat{\xi}_t^d.$$

This proves the Lemma. ■

Proof of Lemma 4

Proof. Log-linearizing equation (39) around the optimal steady state delivers

$$\tilde{u}_{CC}C\widehat{c}_t + \tilde{u}_{C\xi}\widehat{\xi}_t = E_t^{\mathcal{P}} \sum_{k=0}^{\infty} \tilde{u}_C (i_{t+k} - \pi_{t+1+k}) + \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left(\tilde{u}_{CC}C\widehat{c}_T + \tilde{u}_{C\xi}\widehat{\xi}_T \right),$$

and log-linearizing (76) gives

$$\tilde{u}_{CC}C\widehat{c}_t^* + \tilde{u}_{C\xi}\widehat{\xi}_t^* = E_t \sum_{k=0}^{\infty} \tilde{u}_C r_{t+k}^{n,RE} + \lim_{T \rightarrow \infty} E_t \left(\tilde{u}_{CC}C\widehat{c}_T^* + \tilde{u}_{C\xi}\widehat{\xi}_T^* \right).$$

Subtracting the previous equation from (81) delivers

$$\widehat{c}_t - \widehat{c}_t^* = E_t^{\mathcal{P}} \sum_{k=0}^{\infty} \frac{\tilde{u}_C}{\tilde{u}_{CC}C} \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) + \lim_{T \rightarrow \infty} E_t^{\mathcal{P}} \left(\widehat{c}_{T+1} - \widehat{c}_{T+1}^* \right), \quad (81)$$

where we used $E_t^{\mathcal{P}}\xi_T = E_t\xi_T$ and $E_t^{\mathcal{P}}\widehat{c}_{T+1}^* = E_t\widehat{c}_{T+1}^*$, which hold because agents hold rational expectations about fundamentals.

In all periods in which the subjectively optimal plan is consistent with market clearing in the goods sector, the plan satisfies equation (29). Log-linearizing equation (29) delivers

$$\widehat{c}_t = C_Y \widehat{y}_t + C_q \widehat{q}_t^u + C_\xi \widehat{\xi}_t, \quad (82)$$

where $\widehat{\xi}_t$ is a vector of exogenous disturbances (involving A_t^d, \bar{C}_t, g_t). Evaluating this equation at the optimal dynamics defines the optimal consumption gap \widehat{c}_t^* :

$$\widehat{c}_t^* \equiv C_Y \widehat{y}_t^* + C_q \widehat{q}_t^{u*} + C_\xi \widehat{\xi}_t^*.$$

Subtracting the previous equation from (82) delivers

$$\begin{aligned} \widehat{c}_t - \widehat{c}_t^* &= C_Y (\widehat{y}_t - \widehat{y}_t^*) + C_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) \\ &= C_Y y_t^{gap} + C_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) \end{aligned} \quad (83)$$

Since the current consumption market in period t clears, equation (83) holds in period t and can be used to substitute the consumption gap on the l.h.s. of equation (81). Similarly, since housing price expectations are rational in the limit, the consumption market also clears in the limit under the subjectively optimal plans, i.e., equation (29) holds for $t \geq T'$. We can thus use equation (83) also to substitute the consumption gap on the r.h.s. of equation (81). Using the fact that housing price expectations are rational in the limit ($\lim_T E_t^{\mathcal{P}} (\widehat{q}_t^u - \widehat{q}_t^{u*}) = 0$), we obtain

$$y_t^{gap} = \lim_T E_t^{\mathcal{P}} y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) \right) - \frac{C_q}{C_Y} (\widehat{q}_t^u - \widehat{q}_t^{u*}).$$

Since we assumed that agents' beliefs about profits and taxes are given by equations (25) and (26), respectively, evaluated using rational income expectations, the household holds rational expectations about total income. This can be seen by substituting (25) and (26) into the budget constraint (2). We thus have $\lim_T E_t^{\mathcal{P}} y_T^{gap} = \lim_T E_t y_T^{gap}$ in the previous equation, which delivers (77). ■

Proof of Lemma 3

Proof. Under the proposed policy that sets $i_t - E_t \pi_{t+1}$ equal to the natural rate defined in equation (66), we have

$$\begin{aligned}
y_t^{gap} &= \lim_T E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(i_{t+k} - \pi_{t+1+k} - r_{t+k}^{n,RE} \right) \right) - \frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}) \\
&= \lim_T E_t y_T^{gap} - E_t \left(\sum_{k=0}^{\infty} \varphi \left(r_{t+k}^{n,RE} - \frac{1}{\varphi} \frac{C_q}{C_Y} \left((\hat{q}_{t+k}^u - \hat{q}_{t+k}^{u*}) - E_{t+k} (\hat{q}_{t+k+1}^u - \hat{q}_{t+k+1}^{u*}) \right) - r_{t+k}^{n,RE} \right) \right) \\
&\quad - \frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}) \\
&= \lim_T E_t y_T^{gap} + E_t \left(\sum_{k=0}^{\infty} \left(\frac{C_q}{C_Y} \left((\hat{q}_{t+k}^u - \hat{q}_{t+k}^{u*}) - (\hat{q}_{t+k+1}^u - \hat{q}_{t+k+1}^{u*}) \right) \right) \right) - \frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}) \\
&= \lim_T E_t y_T^{gap} + E_t \left(\frac{C_q}{C_Y} \left((\hat{q}_t^u - \hat{q}_t^{u*}) - \lim_k E_t (\hat{q}_{t+k+1}^u - \hat{q}_{t+k+1}^{u*}) \right) \right) - \frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}) \\
&= \lim_T E_t y_T^{gap} + \left(\frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}) \right) - \frac{C_q}{C_Y} (\hat{q}_t^u - \hat{q}_t^{u*}) \\
&= \lim_T E_t y_T^{gap},
\end{aligned}$$

which proves that with this policy, the output gap is indeed constant, and $r^{n,P}$ is the real rate that implies a constant output gap. ■

C.1 Transversality Condition Satisfied with Subjective Housing Price Beliefs

This appendix shows that under the considered subjective belief specifications, the optimal plans satisfy the transversality constraint (17). Since $D_t \in [0, D^{\max}]$ and $E_t^P q_T^u = E_t \bar{\xi}_T^d$ for $T \geq T'$, we have $\lim_{T \rightarrow \infty} \beta^T E_t^P (D_T q_T^u) = 0$. We thus only need to show that $\lim_{T \rightarrow \infty} \beta^T E_t^P \frac{\bar{C}_T^{\bar{\sigma}-1}}{\bar{C}_T^{\bar{\sigma}}} B_T = 0$. Combining the budget constraint (2) with (25) and (26) we obtain

$$C_t + B_t + \left(D_t - (1 - \delta) D_{t-1} - \tilde{d}(k_t; \xi_t) \right) q_t^u \frac{C_t^{\bar{\sigma}-1}}{\bar{C}_t^{\bar{\sigma}-1}} + k_t = (1 - g_t) Y_t + B_{t-1}.$$

For $t \geq T'$ the subjectively optimal plans satisfy market clearing in the housing market, i.e.,

$$D_t - (1 - \delta) D_{t-1} - \tilde{d}(k_t; \xi_t) = 0$$

so that the budget constraint implies

$$C_t + B_t + k_t = (1 - g_t) Y_t + B_{t-1}. \quad (84)$$

Furthermore, for $t \geq T'$ subjectively optimal plans also satisfy market clearing for consumption goods, i.e.,

$$C_t + k_t = (1 - g_t) Y_t.$$

It thus follows that the subjectively optimal debt level B_t in the budget constraint (84) is constant under the subjectively optimal plan, after period $t \geq T'$. Furthermore, the expectations about Y_t in the budget constraint (84) is rational under the assumed lump sum transfer expectations, so that the household's subjective consumption expectations are the same as in a rational expectations equilibrium. (The subjectively optimal investment decisions k_t are driven by rational housing price expectations). Since the limit expectations $\bar{C}_T^{\tilde{\sigma}-1}/C_T^{\tilde{\sigma}-1}$ are bounded in the rational expectations equilibrium, it follows that $\lim_{T \rightarrow \infty} \beta^T E_t^P \frac{\bar{C}_T^{\tilde{\sigma}-1}}{C_T^{\tilde{\sigma}-1}} B_T = 0$.

C.2 Optimal House Price Absent Price Rigidities

The following derivation closely follows Adam and Woodford (2020). We obtain $U_{q^u}(Y_t, \Delta_t, q_t^u, \xi_t)$ from differentiating equation (33) with respect to q_t^u and set it equal to 0:

$$\begin{aligned} U_{q^u}(Y_t, \Delta_t, q_t^u, \xi_t) &= \bar{C}_t^{\tilde{\sigma}-1} C_{q^u}(Y_t, q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{-\tilde{\sigma}-1} \\ &\quad + A_t^d \bar{\xi}_t^d \frac{\partial \Omega(q_t^u, \xi_t)}{\partial q_t^u} \Omega(q_t^u, \xi_t)^{\tilde{\alpha}-1} C(Y_t, q_t^u, \xi_t)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\sigma}-1} \\ &\quad + \frac{\tilde{\sigma}}{1-\tilde{\alpha}} A_t^d \bar{\xi}_t^d \Omega(q_t^u, \xi_t)^{\tilde{\alpha}} C(Y_t, q_t^u, \xi_t)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\sigma}-1-1} C_{q^u}(Y_t, q_t^u, \xi_t) = 0, \end{aligned}$$

where

$$\frac{\partial \Omega(q_t^u, \xi_t)}{\partial q_t^u} = \frac{1}{q_t^u} \frac{1}{1-\tilde{\alpha}} \Omega(q_t^u, \xi_t),$$

and when defining $\chi \equiv \frac{\tilde{\sigma}-1}{1-\tilde{\alpha}} - 1$, we get

$$C_{q^u}(Y_t, q_t^u; \xi_t) \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u} \frac{1}{1-\tilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1 + \chi) \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^\chi}.$$

Taking everything together, we get

$$U_{q^u}(Y_t, \Delta_t, q_t^u, \xi_t) = \frac{\frac{1}{q_t^u} \frac{1}{1-\tilde{\alpha}} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1 + \chi) \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^\chi} \bar{C}_t^{\tilde{\sigma}-1} \left(\frac{\bar{\xi}_t^d}{q_t^u} - 1 \right).$$

In order for U_{q^u} to be zero, we need to have that

$$q_t^{u*} = \bar{\xi}_t^d,$$

as stated in equation (69).

C.3 Quadratically Approximated Welfare Objective

This derivation follows Adam and Woodford (2020). In the optimal steady state, we have $U_Y = U_{q^u} = U_{Y^{q^u}} = 0$, as well as $U_\Delta + \underline{\gamma}(\beta h_1 - 1) = 0$. Given the assumption $\Delta_{-1} \sim O(2)$, it follows $\Delta_t \sim O(2)$ for all $t \geq 0$. Additionally, we have $h_2 \equiv \frac{\partial h(\Delta, \Pi)}{\partial \Pi} = 0$ at the optimal

steady state. Therefore, a second-order approximation of the contribution of the variables $(Y_t, \Delta_t, q_t^u, \Pi_t, \xi_t)$ to the utility of the household yields

$$\frac{1}{2}U_{\hat{Y}\hat{Y}}(\hat{y}_t - \hat{y}_t^*) + \frac{1}{2}U_{\hat{q}^u\hat{q}^u}(\hat{q}_t^u - \hat{q}_t^{u*}) + \frac{1}{2}\underline{\gamma}^*h_{22}\pi_t^2 + t.i.p.,$$

where *t.i.p.* contains all terms independent of policy. Under rational expectations, we have that $(\hat{q}_t^u - \hat{q}_t^{u*}) = 0$ and is thus constant and independent of (monetary) policy. Under subjective beliefs, $(\hat{q}_t^u - \hat{q}_t^{u*})$ is purely driven by beliefs β_t and housing demand shocks ξ_t^d , see equation (71), both independent of policy. Therefore, we include $\frac{1}{2}U_{\hat{q}^u\hat{q}^u}(\hat{q}_t^u - \hat{q}_t^{u*})$ in *t.i.p.*.

The term $U_{\hat{Y}\hat{Y}}$ is given by $U_{\hat{Y}\hat{Y}} \equiv Y \frac{\partial}{\partial Y} (U_Y) \equiv Y \frac{\partial}{\partial Y} (Y U_Y) = \underline{Y}^* U_Y + (\underline{Y}^*)^2 U_{YY}$. At the optimal steady state, we have

$$\begin{aligned}\Lambda_\pi &= -\frac{1}{2}\underline{\gamma}^*h_{22} > 0 \\ \Lambda_y &= -\frac{1}{2}(\underline{Y}^*)^2 U_{YY} > 0,\end{aligned}$$

where

$$\begin{aligned}U_{YY} &= -\tilde{\sigma}^{-1}(1-g)\bar{C}^{\tilde{\sigma}^{-1}}C(\underline{Y}, \underline{q}^u, \underline{\xi})^{-\tilde{\sigma}^{-1}-1}C_Y \frac{\underline{Y}^*}{C(\underline{Y}, \underline{q}^u, \underline{\xi})} \\ &\quad - \frac{\lambda}{1+\nu}(1+\omega)\omega \frac{\bar{H}^{-\nu}}{A^{1+\omega}}\underline{Y}^{\omega-1} < 0 \\ h_{22} &= \frac{\alpha\eta(1+\omega)(1+\omega\eta)}{1-\alpha} > 0 \\ \underline{\gamma}^* &= \frac{U_\Delta}{1-\alpha\beta} < 0,\end{aligned}$$

with

$$U_\Delta = -\frac{\underline{Y}^*(1-g)}{1+\omega} \left(\frac{\bar{C}^{\tilde{\sigma}^{-1}}}{C(\underline{Y}^*, \underline{q}^{u*}, \underline{\xi})} \right)^{\tilde{\sigma}^{-1}} < 0.$$

C.4 Recursifying the Optimal Policy Problem with Lower Bound

We apply the techniques of Marcet and Marimon (2019) to recursify the optimal policy problem with forward-looking constraints (59). We thereby assume that the Lagrangian defined by problem (59) satisfies the usual duality properties that allow interchanging the order of maximization and minimization, which we then show numerically to hold. The value function for $t = T'$ is given by the RE value function $W^{RE}(\cdot)$. For $t \leq T'$ we have a value

function $W_t(\cdot)$ which satisfies the following recursion:

$$\begin{aligned}
& W_t(\varphi_{t-1}, \mu_{t-1}, u_t, r_t^{n,RE}, \beta_t, \xi_t^d, q_{t-1}^u) \\
= & \max_{(\pi_t, y_t^{gap}, i_t \geq \underline{i})} \min_{(\varphi_t, \lambda_t)} -\frac{1}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y (y_t^{gap})^2 \right) \\
& + (\varphi_t - \varphi_{t-1}) \pi_t - \varphi_t (\kappa_y y_t^{gap} + \kappa_q (\widehat{q}_t^u - \widehat{q}_t^{u*}) + u_t) \\
& + \lambda_t \left[y_t^{gap} - \lim_T E_t y_T^{gap} + \varphi \left(i_t - E_t \sum_{k=0}^{\infty} r_{t+k}^{n,RE} \right) + \frac{C_q}{C_Y} (\widehat{q}_t^u - \widehat{q}_t^{u*}) \right] \\
& + \mu_{t-1} \varphi (i_t - \pi_t) + \gamma_t (i_t - \underline{i}) \\
& + \beta E_t \left[W_{t+1}(\varphi_t, \underbrace{\beta^{-1} (\lambda_t + \mu_{t-1})}_{=\mu_t}, u_{t+1}, r_{t+1}^{n,RE}, \beta_{t+1}, \xi_{t+1}^d, q_t^u) \right] \tag{85}
\end{aligned}$$

where the next period state variables (β_{t+1}, q_t^u) are determined by equations (48) and (49) and $(\widehat{q}_t^u - \widehat{q}_t^{u*})$ is determined by equation (71). Here we assume that $r_t^{n,RE}$ follows a Markov process, such that the term $E_t \sum_{k=0}^{\infty} r_{t+k}^{n,RE}$ showing up in the current-period return can be expressed as a function of the current state $r_t^{n,RE}$. The future state variables $(\varphi_t, \mu_t, \beta_{t+1}, q_t^u)$ are predetermined in period t . The expectation about the continuation value is thus only over the exogenous states $(u_{t+1}, r_{t+1}^{n,RE}, \xi_{t+1}^d)$. The endogenous state variable φ_{t-1} is simply the lagged Lagrange multiplier on the NK Phillips curve with housing. The endogenous state variable μ_{t-1} is given for all $t \geq 0$ by

$$\mu_t = \beta^{-(t+1)} (\lambda_0 + \mu_{-1}) + \beta^{-t} \lambda_1 + \dots + \beta^{-1} \lambda_t.$$

The initial values (φ_{-1}, μ_{-1}) are given at time zero and equal to zero in the case of time-zero-optimal monetary policy.

For periods $t < T'$, where T' is the period from which housing price expectations are rational and the lower bound constraint ceases to bind, the value functions depend on time, thereafter they are time-invariant. Likewise for sufficiently large T' , the value functions $W_t(\cdot)$ and $W_{t+1}(\cdot)$ will become very similar.

We can numerically solve for the value function $W_t(\cdot)$ by value function iteration, starting with $W_{T'}$ which is the value function associated with the LQ problem with RE and without lower bound.

C.5 Optimal Targeting Rule

Differentiating (85) with respect to $\{\pi_t, y_t^{gap}, i_t\}$ yields:

$$\begin{aligned}
\frac{\partial W_t}{\partial \pi_t} &= -\Lambda_\pi \pi_t + (\varphi_t - \varphi_{t-1}) - \mu_{t-1} \varphi = 0 \\
\frac{\partial W_t}{\partial y_t^{gap}} &= -\Lambda_y y_t^{gap} - \varphi_t \kappa_y + \lambda_t = 0 \\
\frac{\partial W_t}{\partial i_t} &= \gamma_t + \lambda_t \varphi + \mu_{t-1} \varphi = 0 \text{ and } \gamma_t (i_t - \underline{i}) = 0.
\end{aligned}$$

Combining these first-order conditions, we can derive the following targeting rule which characterizes optimal monetary policy

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y} (y_t^{gap} - y_{t-1}^{gap}) + \frac{\lambda_{t-1}}{\kappa_y} + \mu_{t-1} \left(\varphi + \frac{1}{\kappa_y} \right) + \frac{\gamma_t}{\varphi \kappa_y} = 0,$$

where γ_t is the Lagrange multiplier associated with the lower bound on interest rates. If the lower bound on the nominal interest rate does not bind in the current period, we have $\gamma_t = 0$. Furthermore, if the lower bound has not been binding up to period t , the IS equation has not posed a constraint for the monetary policymaker. Thus, $\lambda_{t-1} = \lambda_{t-k} = 0$ for all $k = 0, 1, \dots, t$. For an initial value of $\mu_{-1} = 0$, it follows that $\mu_{t-1} = 0$. The targeting rule then collapses to

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y} (y_t^{gap} - y_{t-1}^{gap}) = 0,$$

which is the same as in Clarida et al. (1999).

The Lagrange multiplier $\gamma_t \leq 0$ captures the cost of a currently binding lower bound. If $\gamma_t < 0$, the optimal policy requires a compensation in the form of a positive output gap or inflation. The multipliers λ_{t-1} and μ_{t-1} capture promises from past commitments when the lower bound was binding.

Another way to express equation (86) is to write it as

$$\Lambda_\pi \pi_t + \frac{\Lambda_y}{\kappa_y} (y_t^{gap} - y_{t-1}^{gap}) + \frac{1}{\varphi \kappa_y} \left[\gamma_t - \frac{1 + \beta + \varphi \kappa_y}{\beta} \gamma_{t-1} + \frac{\gamma_{t-2}}{\beta} \right] = 0. \quad (86)$$

House prices do not enter the optimal target criterion directly but larger fluctuations in house prices make the lower bound bind more often and for a longer period of time. The optimal policy, thus, requires larger compensations in terms of positive output gaps and inflation. To implement this, the nominal interest rate needs to be kept longer at the lower bound.

C.6 Calibration of C_q/C_Y

To calibrate C_q/C_Y , the ratio of the consumption elasticities to housing prices and income, respectively, note that from appendix "Second-Order Conditions for Optimal Allocation" in Adam and Woodford (2020), we have

$$C_{q^u}(Y_t, q_t^u; \xi_t) \equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q^u} = \frac{-\frac{1}{q_t^u} \frac{1}{1-\alpha} \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^{\chi+1}}{1 + (1 + \chi) \Omega(q_t^u, \xi_t) C(Y_t, q_t^u, \xi_t)^\chi}$$

where $\chi \equiv \frac{\tilde{\sigma}^{-1}}{1-\alpha} - 1$. In our formulation, we have defined

$$\begin{aligned} C_q &\equiv \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial \ln q_t^u} \\ &= \frac{\partial C(Y_t, q_t^u; \xi_t)}{\partial q_t^u} \frac{\partial q_t^u}{\partial \ln q_t^u} \\ &= C_{q^u}(Y_t, q_t^u; \xi_t) \frac{q_t^u}{C_t} \end{aligned}$$

so that we have

$$C_q = -\frac{\frac{1}{1-\tilde{\alpha}}\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{C(Y_t, q_t^u, \xi_t) + (1+\chi)\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

From the appendix in Adam and Woodford (2020) we also have

$$C_Y(Y_t, q_t^u, \xi_t) \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial Y_t} = \frac{1-g_t}{1+\Omega(q_t^u, \xi_t)(1+\chi)C(Y_t, q_t^u, \xi_t)^\chi}$$

so that in our notation

$$C_Y \equiv \frac{\partial C_Y(Y_t, q_t^u, \xi_t)}{\partial \ln Y_t} = \frac{(1-g_t)Y_t}{C(Y_t, q_t^u, \xi_t) + \Omega(q_t^u, \xi_t)(1+\chi)C(Y_t, q_t^u, \xi_t)^{\chi+1}}.$$

We then have

$$\begin{aligned} \frac{C_q}{C_Y} &= \frac{\frac{-\frac{1}{1-\tilde{\alpha}}\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{C(Y_t, q_t^u, \xi_t) + (1+\chi)\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}}{\frac{(1-g_t)Y_t}{C(Y_t, q_t^u, \xi_t) + \Omega(q_t^u, \xi_t)(1+\chi)C(Y_t, q_t^u, \xi_t)^{\chi+1}}} \\ &= -\frac{1}{1-\tilde{\alpha}} \frac{\Omega(q_t^u, \xi_t)C(Y_t, q_t^u, \xi_t)^{\chi+1}}{(1-g_t)Y_t}. \end{aligned}$$

In the steady state, we have $\bar{Y}(1-\bar{g}) = \bar{C} + \bar{\Omega}\bar{C}^{\chi+1}$, which says that privately consumed output $\bar{Y}(1-\bar{g})$ is divided up into consumption \bar{C} and resources invested in the housing sector, $\bar{\Omega}\bar{C}^{1+\chi}$. We thus have that

$$\begin{aligned} \frac{\bar{\Omega}\bar{C}^{\chi+1}}{\bar{Y}(1-\bar{g})} &= 1 - \frac{\bar{C}}{\bar{Y}(1-\bar{g})} \\ &= 1 - \frac{\bar{C}}{\bar{C} + \bar{\Omega}\bar{C}^{\chi+1}} \\ &= 1 - \frac{1}{1 + \bar{\Omega}\bar{C}^\chi}. \end{aligned}$$

Following Adam and Woodford (2020), we set this to the share of housing investment to total consumption, $\bar{\Omega}\bar{C}^\chi$, equal to 6.3%, so that in steady state we have

$$\frac{C_q}{C_Y} = -\frac{1}{1-\tilde{\alpha}} \left(1 - \frac{1}{1.063} \right)$$

Finally, following Adam and Woodford (2020), we set the long-run elasticity of housing supply equal to five, which implies $\tilde{\alpha} = 0.8$, so that

$$\frac{C_q}{C_Y} = -5 \left(1 - \frac{1}{1.063} \right) \approx -0.29633.$$

From this, it follows that

$$\begin{aligned}
C_Y &= \frac{(1-g)Y}{C + (1+\chi)\Omega C^{x+1}} \\
&= \frac{C+k}{C + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}}k} \\
&= \frac{1 + \frac{k}{C}}{1 + \frac{\tilde{\sigma}^{-1}}{1-\tilde{\alpha}} \frac{k}{C}} \\
&= \frac{1 + 0.063}{1 + 5 \cdot 0.063} \\
&= 0.80836
\end{aligned}$$

and hence,

$$C_q = -0.29633 \cdot 0.80836 = -0.23954.$$

D Survey Expectations and Model Expectations

D.1 Forecast Revisions

To see how we construct $Revisions_t$ from the model, given our belief-formation, note that we have $E_t^{\mathcal{P}} [q_{t+1}^u] = \beta_t q_t^u$ and

$$E_{t-1}^{\mathcal{P}} [q_{t+1}^u] = E_{t-1}^{\mathcal{P}} [E_t^{\mathcal{P}} [q_{t+1}^u]]$$

by the law of iterated expectations. It then follows that

$$\begin{aligned}
E_{t-1}^{\mathcal{P}} [q_{t+1}^u] &= E_{t-1}^{\mathcal{P}} [\beta_t q_t^u] \\
&= \beta_{t-1} E_{t-1}^{\mathcal{P}} [q_t^u] \\
&= \beta_{t-1}^2 q_{t-1}^u,
\end{aligned}$$

where the second line follows from β_t being a random walk. Similarly, for four and five periods ahead expectations, we have

$$\begin{aligned}
E_t^{\mathcal{P}} [q_{t+4}^u] &= \beta_t^4 q_t^u \\
E_{t-1}^{\mathcal{P}} [q_{t+4}^u] &= \beta_{t-1}^5 q_{t-1}^u.
\end{aligned}$$

Revisions in expectation in the model are thus given by

$$Revisions_t \equiv E_t^{\mathcal{P}} [q_{t+4}^u] - E_{t-1}^{\mathcal{P}} [q_{t+4}^u] = \beta_t^4 q_t^u - \beta_{t-1}^5 q_{t-1}^u. \quad (87)$$

D.2 Capital Gains and Housing Demand Shocks

In the empirical analysis of the dynamics of housing-price forecast errors, we regress one-year ahead forecast errors at different horizons on gross changes in real house prices, defined as

$$\frac{RHP_t}{RHP_{t-1}}.$$

If we assume that the economy is in the steady state before the shock hits, this change in the real house price in fact coincides exactly with a housing demand shock in the model. To see this, recall that

$$q_t^u = \frac{\xi_t^d}{1 - \beta(1 - \delta)\beta_t}.$$

Therefore, the capital gain equals

$$\frac{q_t^u}{q_{t-1}^u} = \frac{\xi_t^d}{\xi_{t-1}^d} \frac{1 - \beta(1 - \delta)\beta_{t-1}}{1 - \beta(1 - \delta)\beta_t}.$$

If the shock hits in period t and the economy was in steady state before, it follows that $\beta_t = \beta_{t-1}$, such that

$$\begin{aligned} \frac{q_t^u}{q_{t-1}^u} &= \frac{\xi_t^d}{\xi_{t-1}^d} \\ &\Leftrightarrow = \frac{\left(\frac{\xi_{t-1}^d}{\xi_t^d}\right)^{\rho_\xi} e^{\varepsilon_t^d}}{\frac{\xi_{t-1}^d}{\xi_t^d}} \\ &= e^{\varepsilon_t^d}, \end{aligned}$$

where the last equality follows from the steady state assumption. Thus, the structural housing shock in the model coincides with the gross change in housing prices which we use as our empirical shock proxy.

Figure 14 shows that the model also does a good job in matching the dynamic forecast error structure in the *level* of the housing price. The only difference to the baseline specification in Section 6.3 (Figure 7) is that the forecast error at horizon h is now defined as

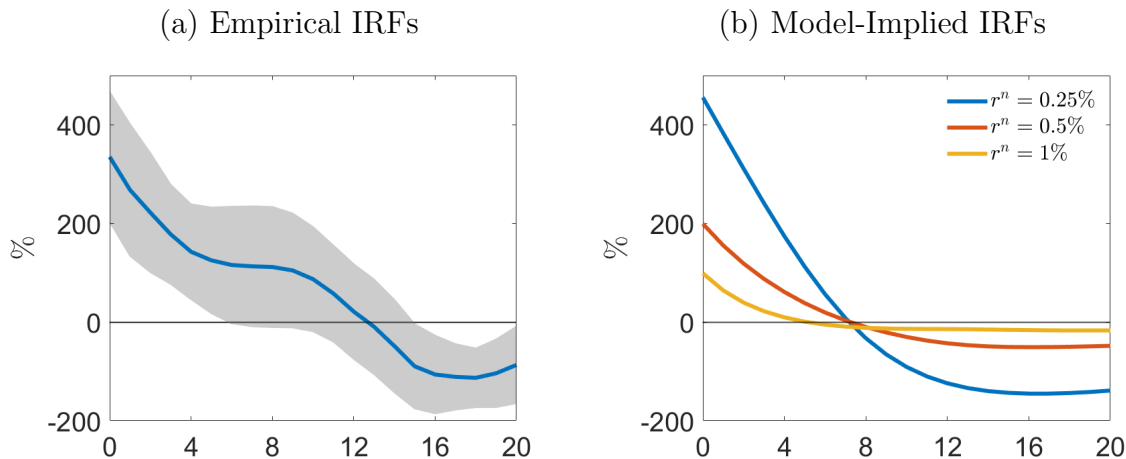
$$FE_{t+h} \equiv RHP_{t+4+h} - E_{t+h}^{\mathcal{P}}[RHP_{t+4+h}]. \quad (88)$$

Figure 14 shows that the model, especially at low levels of the steady-state natural rate, matches the empirical IRFs quite well. It turns negative somewhat earlier, but overall the model-implied results align well with their empirical counterparts even though none of these features was targeted in the calibration.

D.3 Robustness of Survey Expectations and Model-Implied Expectations

AMB Regressions up to 2019

Figure 14: Dynamic Forecast Error Structure



Notes: Panel (a) shows impulse-response functions of housing-price forecast errors of one-year ahead expectations from the Survey of Consumers to a one standard deviation innovation in the housing capital gain. The shaded area shows the 90%-confidence intervals. Panel (b) reports the model-implied impulse-response functions of housing-price forecast errors of one-year ahead expectations to a one standard deviation innovation in the housing capital gain for different steady-state natural rates.

Forecast Errors and Housing Prices An equivalent version of the test from Adam et al. (2017) that we presented in Section 6.3 is proposed by Kohlhas and Walther (2018). In this case, we regress forecast errors about real housing prices on the level of the current real housing price. We therefore estimate

$$Error_t = \alpha + \beta_{AMB} \cdot HP_t + \epsilon_t. \quad (89)$$

Table 7 shows the results. In the data, we find a negative, statistically significant coefficient when considering the whole period from 2007 to 2019. If we restrict the sample to start in 2010, we still find a negative, but not significant, estimate. In general, we can state that consumers tend to become too optimistic when they observe high housing prices, inconsistent with rational expectations (as in Adam et al. (2017), Kohlhas and Walther (2018) and Angeletos et al. (2020)). The last four columns show the model-implied coefficients. We see that the model predicts, consistent with the data, a negative sign.

Instrumental Variable Regression To make sure that the forecast revisions are not correlated with the error term in regression (56), we follow Coibion and Gorodnichenko (2015) and use an IV approach to estimate it, using monetary policy shocks as an instrument. We identify daily monetary policy shocks as changes of the current-month federal funds future in a 30-minute window around scheduled FOMC announcements. We aggregate these shocks to quarterly frequency by assigning daily shocks partly to the current quarter and partly to the consecutive quarter, based on the number of remaining days in the current quarter. Table 8 shows the first-stage F -statistic is about 20. The results show that our results are

Table 6: Expected vs. actual capital gains

	$\hat{c} \cdot 10^3$	$\hat{\mathbf{c}} \cdot 10^3$	bias $\cdot 10^3$ $-E(\hat{\mathbf{c}} - \hat{c})$	p -value $H_0 : c = \mathbf{c}$
<u>Nominal Housing Prices</u>				
Mean	0.580	-0.647	0.036	0.000
Median	0.176	-0.647	0.118	0.042
<u>Real Housing Prices</u>				
Mean	0.614	-0.483	-0.009	0.000
Median	0.196	-0.483	0.076	0.017

Note: \hat{c} is the estimate of c in equation (54) and $\hat{\mathbf{c}}$ the estimate of \mathbf{c} in equation (55). The small sample bias correction is reported in the second to last column and the last column reports the p -values for the null hypothesis $c = \mathbf{c}$ in the fifth column. Rows labeled "Mean" report results using the mean of the survey expectation. Rows labeled "Median" use the median expectation.

Table 7: Forecast Errors and Housing Prices

	Survey Mean		Survey Median		Model		
	2007-19	2010-19	2007-19	2010-19	$\underline{r}^n = 1\%$	$\underline{r}^n = 0.5\%$	$\underline{r}^n = 0.25\%$
β_{AMB}	-0.35** (0.140)	-0.09 (0.109)	-0.30** (0.155)	-0.01 (0.118)	-0.07	-0.08	-0.11

Note: The first four columns show the empirical estimates of regression (89) once for average expectations, and once for median expectations, for different samples. The last three columns show the model-implied regression coefficients for different levels of the average natural rates. The reported standard errors are robust (Newey-West with four lags). Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

robust to using this IV approach. The estimated coefficients are positive and significantly different from 0.

Table 8: Instrumental Variable Regression

	Survey Average		Survey Median	
	b	p -value	b	p -value
Michigan, 1yr	2.615	0.000	3.454	0.000
1 st stage F -stat.	44.49		34.13	

The columns labelled "b" report the results from estimating (56), instrumenting *Revisions* using monetary policy shocks, obtained via high-frequency identification. Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

Monthly Data In the following, we show that our results are robust if we move from quarterly frequency to monthly frequency. The revisions will, consistent with our main

exercise, be the change in the expectations from one quarter to the next. Table 9 shows the results.

Table 9: Survey Estimates using Monthly Data

Data:	RHP 2007-21Q1	RHP 2010-21Q1
<u>Mean Expectations</u>		
β_{CG}	1.72*** (0.312)	1.44*** (0.388)
β_{AMB}	-0.302*** (0.098)	-0.076 (0.0664)
<u>Median Expectations</u>		
β_{CG}	2.10*** (0.336)	1.69*** (0.383)
β_{AMB}	-0.254** (0.100)	-0.002 (0.071)

The rows labelled β_{CG} report the estimation results from regression (56) for different samples. The rows labelled β_{AMB} report the estimation results from regression (89) for different samples. The reported standard errors are robust (Newey-West with four lags).

Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

Nominal Forecast Errors We now show that our results are robust to using nominal forecast errors and revisions in the expectations about nominal housing prices instead of real forecast errors and revisions in expectations about real housing prices. Given the trend in the level of the nominal house prices, however, we still regress the forecast error on the real house price to estimate β_{AMB} . Table 10 shows the results at the quarterly and monthly frequency.

Expectations across the income distribution. We now divide the survey respondents into three groups, according to their reported income. Table 11 shows the results. The first column gives the estimated coefficients from a panel regression, using income-tertile-specific fixed effects. The next three column looks at the three groups separately, in increasing order. We see that our results are robust, and additionally that the results do not systematically differ along the income distribution. In fact, we cannot reject in any case the null hypothesis that the regression coefficients differ across income groups.

Optimists vs. pessimists In the following, we test whether our results are driven by either optimists or pessimists. To do so, we re-run the CG and the AMB regressions as before, but instead of considering mean or median expectations, we look at the 25th and the 75th percentile, respectively. For this, we use monthly data and consider real and nominal house prices. Table 12 shows the results. We observe the expected difference in the magnitude of the coefficients (higher β_{CG} and less negative β_{AMB} for the pessimists). Nevertheless, the signs are the same in all cases, and we reject the rational expectations hypothesis.

Table 10: Survey Estimates using Nominal Housing Price Data

Data:	NHP 2007-21Q1 (quarterly)	NHP 2010-21Q1 (quarterly)	NHP 2007-21Q1 (monthly)	NHP 2010-21Q1 (monthly)
<u>Mean Expectations</u>				
β_{CG}	2.22*** (0.507)	1.55*** (0.511)	2.02*** (0.418)	1.24*** (0.387)
β_{AMB}	-0.34** (0.156)	-0.05 (0.121)	-0.288*** (0.090)	-0.06 (0.068)
<u>Median Expectations</u>				
β_{CG}	2.85*** (0.513)	2.02*** (0.647)	2.63*** (0.423)	1.78*** (0.452)
β_{AMB}	-0.29 (0.175)	0.04 (0.131)	-0.23** (0.100)	0.03 (0.073)

The rows labelled β_{CG} report the estimation results from regression (56) for different samples. The rows labelled β_{AMB} report the estimation results from regression (89) for different samples. The reported standard errors are robust (Newey-West with four lags).

Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

Table 11: Survey Estimates for Different Income Groups

	Panel (monthly)	Bottom 33% (monthly)	Middle 33% (monthly)	Top 33% (monthly)
<u>Mean Expectations</u>				
β_{CG}	1.61*** (0.086)	1.60*** (0.318)	1.69*** (0.324)	1.53*** (0.305)
β_{AMB}	-0.29*** (0.023)	-0.30*** (0.104)	-0.30*** (0.101)	-0.27*** (0.096)
<u>Median Expectations</u>				
β_{CG}	2.12*** (0.092)	2.31*** (0.321)	2.12*** (0.322)	1.94*** (0.296)
β_{AMB}	-0.24*** (0.025)	-0.22** (0.113)	-0.24** (0.107)	-0.25** (0.102)

The rows labelled β_{CG} report the estimation results from regression (56). The rows labelled β_{AMB} report the estimation results from regression (89). The first column reports the results from a panel regression, controlling for income-group-specific fixed effects. For columns 2-4, we split the sample into three groups, depending on their reported income. The reported standard errors are robust (Newey-West with four lags). Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

Information set concerns A potential concern with the stated result that capital gain expectations are higher in times of higher current house prices might be that households are not aware of the level of current house prices. In order to account for this, we use the household's perceived change of the value of its own home over the last year instead the

Table 12: Survey Estimates: Pessimists (25th percentile) vs. Optimists (75th percentile)

	25 th pct, Real HP (monthly)	25 th pct, Nom. HP (monthly)	75 th pct, Real HP (monthly)	75 th pct, Nom. HP (monthly)
β_{CG}	1.93*** (0.360)	2.49*** (0.453)	1.54*** (0.308)	1.80*** (0.341)
β_{AMB}	-0.23** (0.104)	-0.20* (0.105)	-0.36*** (0.090)	-0.35*** (0.083)

The rows labelled β_{CG} report the estimation results from regression (56). The rows labelled β_{AMB} report the estimation results from regression (89). We split the sample into three groups, depending on how optimistic they are about future housing price growth. The reported standard errors are robust (Newey-West with four lags). Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

current house price as the independent variable. More specifically, we include the share of households that state that their own home increased in value as well as the share of households that say their house value decreased over the last 12 months as independent variables. We use monthly data from the Survey of Consumers by region. The four regions are *North Central*, *North East*, *South*, and *West*. This allows us to control for time- and region-specific fixed effects. That is, we estimate

$$E_t^P[CG_{i,t,t+4}] = \alpha_i + \delta_t + \beta_{Up}ShareUp_{i,t} + \beta_{Down}ShareDown_{i,t} + u_{i,t}, \quad (90)$$

where $ShareUp_{i,t}$ and $ShareDown_{i,t}$ are the shares of survey respondents in region i at time t who say that they think their house value went up, or down, respectively. α_i and δ_t denote region- and time-specific fixed effects.

The first column of Table 13 shows the results. Table 13 shows that even if consumers do not observe current housing prices, when they perceive past growth to be high (low), they expect future growth to be high (low). This extrapolative behavior is consistent with the results from regression (54) and our subjective belief model.

Table 13: Expected and Perceived House Prices

Dependent Variable: $E_t^P [\Delta HP_{t,t+4}]$		
Data	Panel	Panel
$\hat{\beta}_{Up}$	0.026*** (0.0038)	0.044*** (0.0055)
$\hat{\beta}_{Down}$	-0.041*** (0.0042)	-0.037*** (0.0062)
N	636	636
Time FE	×	✓
Region FE	✓	✓

The table reports the results of regressing expected future housing price growth on the share of consumers who believe house prices increased over the last year (β_{Up}) and on the share of consumers who believe that house prices decreased (β_{Down}). Our panel consists of four different regions in the US. The first column reports the results from a panel regression, controlling for income-group-specific fixed effects. For columns 2-4, we split the sample into three groups, depending on their reported income. The reported standard errors are robust (Newey-West with four lags). Significance levels: *** : $p < 0.01$, ** : $p < 0.05$, * : $p < 0.1$

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