Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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Expectations in Dynamic Macroeconomic Models

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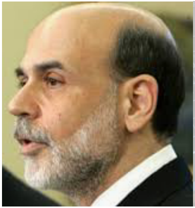
The views expressed are solely the views of the author and do not necessarily reflect the views of the ECB.

Anchoring

"Essential to anchor inflation expectations at some low level."

"We don't see a de-anchoring."

"Failure of the Fed to stably achieve its 2 percent target could de-anchor inflation expectations."

"Long-run inflation expectations [...] are not perfectly anchored in real economies; moreover, the extent to which they are anchored can change."

This paper

1. A model of unanchored expectations:

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   $\hookrightarrow$ sensitivity of long-run expectations to short-run fluctuations

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3. Key takeaway: optimal monetary policy
   - anchors expectations to inflation target
   - responds aggressively to movements in long-run expectations

Related literature

- **Optimal monetary policy in the New Keynesian model**
  Clarida, Gali & Gertler (1999), Woodford (2003)

- **Adaptive learning**
  Evans & Honkapohja (2001, 2006), Sargent (1999), Adam (2005), Primiceri (2006), Lubik & Matthes (2018), Bullard & Mitra (2002), Preston (2005, 2008), Evans & McGough (2015), Ferrero (2007), Molnár & Santoro (2014), Mele et al (2019), Eusepi & Preston (2011), Milani (2007, 2014), Marcet & Nicolini (2003), Eusepi, Giannoni & Preston (2018), Slobodyan & Wouters (2011)

- **Anchoring and the Phillips curve**
  Goodfriend (1993), Svensson (2015), Afrouzi & Yang (2020), Reis (2020), Hebden et al 2020, Hazell et al (2021), Gobbi et al (2019), Carvalho et al (2020)

MODEL OF ANCHORING EXPECTATIONS

QUANTIFICATION OF ANCHORING

OPTIMAL MONETARY POLICY

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right]$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_t C_t^i$$

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j(p_t(j)) \right]$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta}$$

Aggregate relationships

- New Keynesian core: standard IS and Phillips curves

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T)$$

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Observables: (π, x, i) inflation, output gap, interest rate

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Observables: (π, x, i) inflation, output gap, interest rate
Exogenous states: (r^n, u) natural rate and cost-push shock

Adaptive learning: long-run inflation expectations

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  $\Rightarrow$ Cannot compute aggregate relationships

- Instead: postulated inflation process

$$\hat{\mathbb{E}}_{t-1}\pi_t = \bar{\pi}_{t-1} + \mathbb{E}_{t-1}\pi_t$$

$\mathbb{E}$: rational (model-consistent) expectations
$\hat{\mathbb{E}}$: nonrational expectations $\rightarrow$ long-run inflation expectations $\bar{\pi}_{t-1}$

Evolution of long-run inflation expectations

One-period ahead inflation forecast:

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$$f_{t|t-1} = \pi_t - \hat{\mathbb{E}}_{t-1}\pi_t$$

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$k_t \in (0, 1)$ learning gain

Alternatives for the gain

1. Decreasing gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \frac{1}{t} f_{t|t-1}$$

2. Constant gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k \, f_{t|t-1}$$

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3. Endogenous gain:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \mathbf{g}(f_{t|t-1})\, f_{t|t-1}$$

Endogenous gain as a metric for unanchoring

$$k_t = \mathbf{g}(f_{t|t-1}), \qquad \mathbf{g}'' > 0$$

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Endogenous gain: time-varying sensitivity of long-run expectations

Evolution of long-run expectations:

$$\bar{\pi}_t = \bar{\pi}_{t-1}$$

- If $k_t = 1 \rightarrow$ short-run surprises translate 1:1 to long-run expectations
- If $k_t = 0 \rightarrow$ perfectly anchored expectations = rational expectations

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   Optimal monetary policy: -

Model of anchoring expectations

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Estimating form of gain function

- Calibrate parameters of New Keynesian core to literature

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$$\bar{\pi}_t = \bar{\pi}_{t-1} + \mathbf{g}(f_{t|t-1})\, f_{t|t-1}$$

- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead Survey of Professional Forecasters (SPF) inflation expectations at lags $0, \ldots, 4$

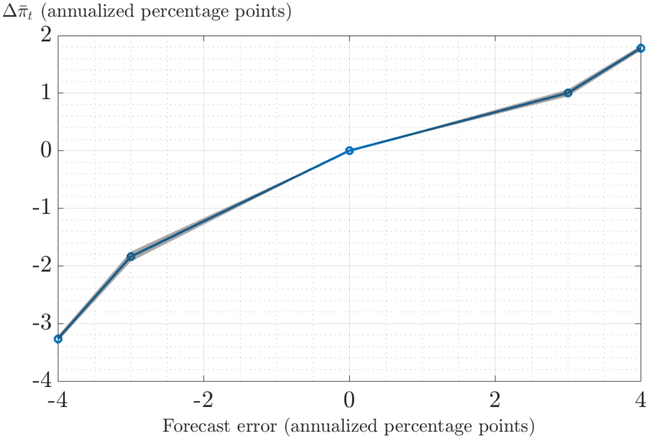
Calibration - parameters from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule
ψ_x	0.3	coefficient of the output gap in Taylor rule
σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock
σ_u	0.5	standard deviation, cost-push shock
$\bar{g}$	0.145	initial value of the gain

Chari et al 2000, Woodford 2003, Nakamura & Steinsson 2008
Carvalho et al 2019

Estimated expectations process

$$\bar{\pi}_t - \bar{\pi}_{t-1} = \hat{\mathbf{g}}(f_{t|t-1})\, f_{t|t-1}$$



Estimated change in long-run inflation expectations for various forecast errors

Model of anchoring expectations

Quantification of anchoring

Optimal monetary policy

Ramsey problem

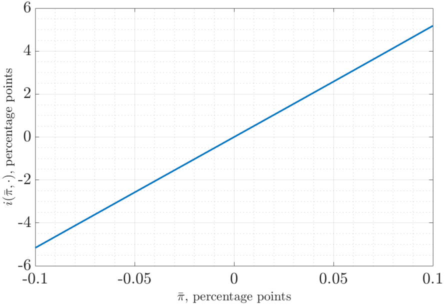
$$\min_{\{y_t,\bar{\pi}_{t-1},k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \lambda_x x_t^2)$$

s.t. model equations

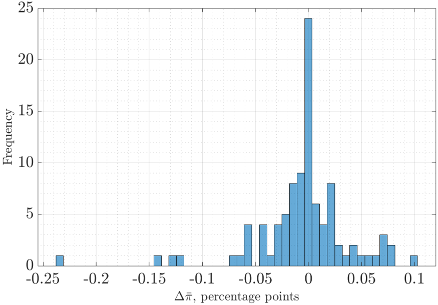
s.t. evolution of expectations

- $\mathbb{E}$ is the central bank's (CB) expectation
- Assumption: CB observes private expectations and knows the model

Optimal policy - responding to unanchoring

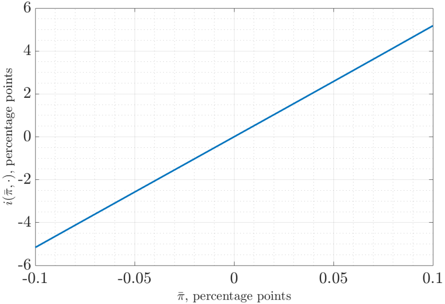


$i(\bar{\pi}$, all other states at their means)



Stabilizing $\bar{\pi}$

Optimal policy - responding to unanchoring



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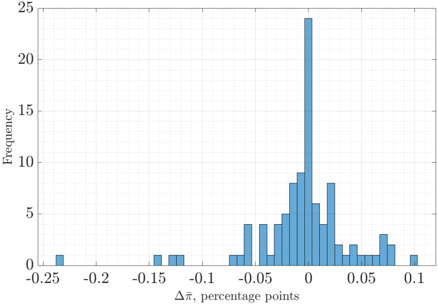
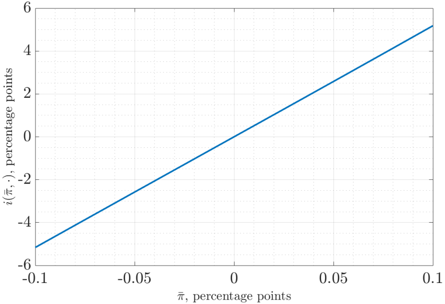


Stabilizing $\bar{\pi}$

$\downarrow \bar{\pi}$ by 5 bp $\quad \Rightarrow \quad \downarrow i$ by 250 bp

Optimal policy - responding to unanchoring



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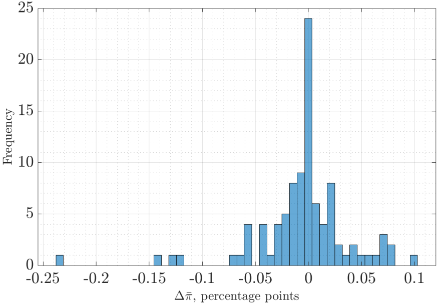

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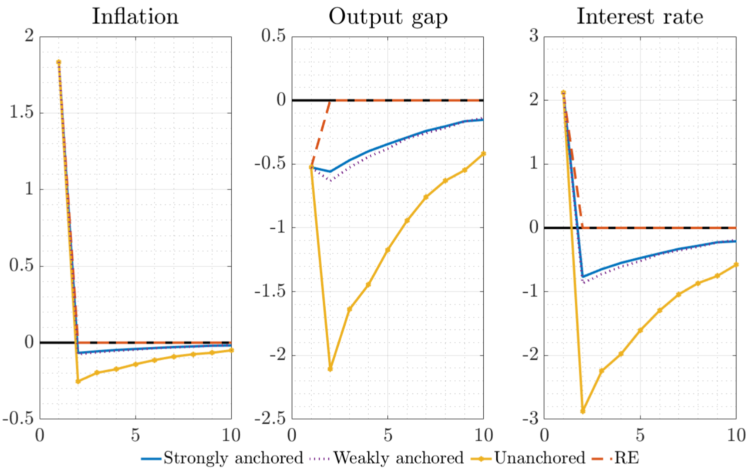
Mode: 0.3 bp movement in $\bar{\pi}$

Unanchoring amplifies shocks



Impulse responses after a cost-push shock when policy follows a Taylor rule

Conclusion

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First theory of monetary policy for potentially unanchored expectations

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Model-based notion of unanchoring

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Model-based notion of unanchoring

- Sensitivity of long-run expectations to short-run fluctuations

Optimal monetary policy

- Anchors expectations by responding aggressively to long-run expectations

Thank you!

Appendix

Long-run expectations: responsive to short-run conditions?

Individual-level Survey of Professional Forecasters (SPF): for 1991-Q4 onward, estimate rolling regression

$$\Delta \bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \tag{1}$$

$\bar{\pi}_t$ 10-year ahead inflation expectation

$f_{t|t-1} \equiv \pi_t - \mathbb{E}_{t-1}\,\pi_t$ individual one-year-ahead forecast error

w indexes windows of 20 quarters

Time-varying responsiveness

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w f_{t|t-1} + \epsilon_t \quad (1)$$

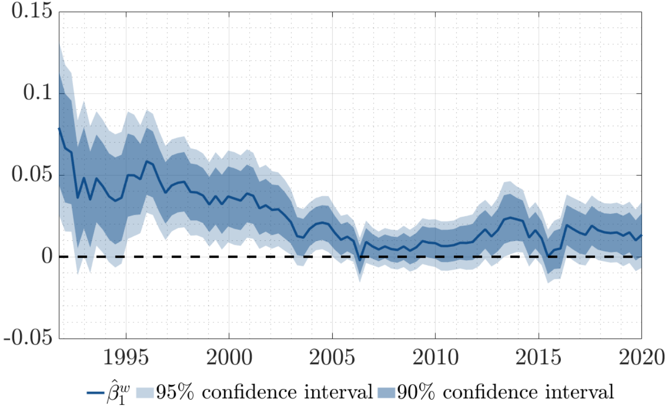


Figure: Time series of $\hat{\beta}_1^w$

Robustness

Breakeven inflation

Figure: Market-based inflation expectations, various horizons, %

Return

Correcting the TIPS from liquidity risk

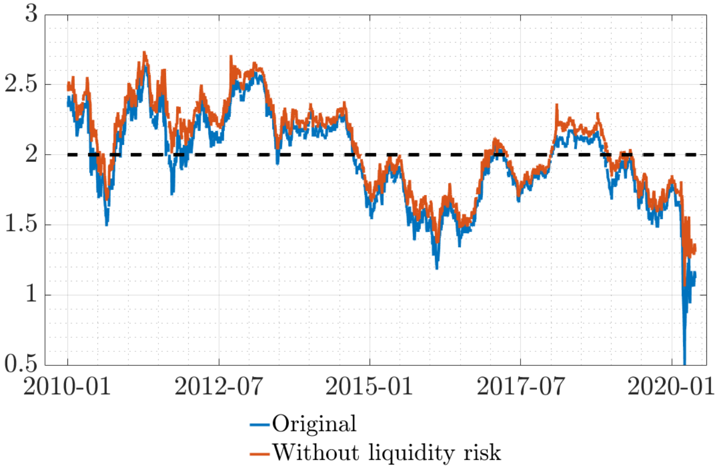
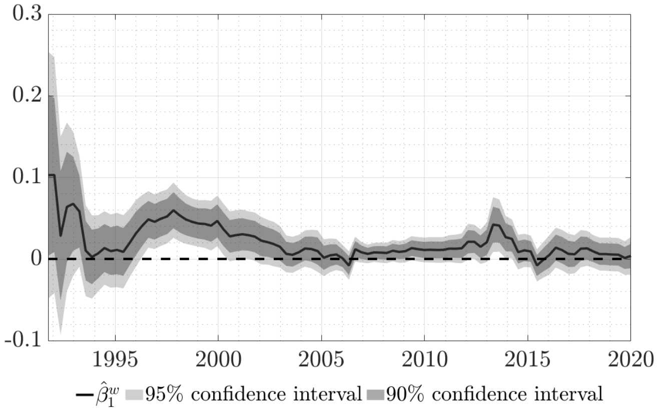


Figure: Market-based inflation expectations, 10 year, %

Return

Robustness checks

$$\Delta\bar{\pi}_t = \beta_0 + \beta_1^w \pi_t + \epsilon_t \tag{1}$$



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Robustness checks - PCE core

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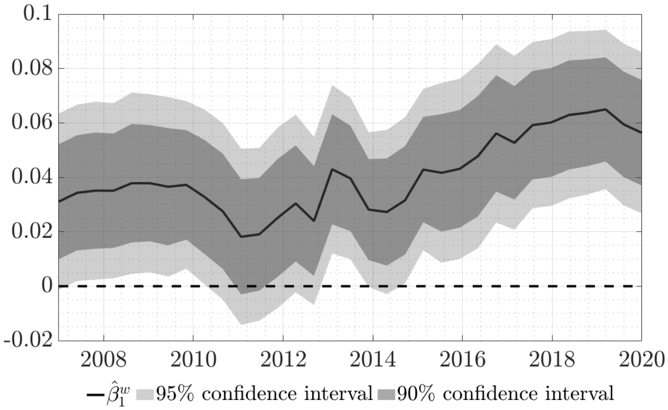

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Return

Robustness checks - controlling for inflation levels

$$\Delta\bar{\pi}_t = \beta_0^w + \beta_1^w f_{t|t-1} + \beta_2^w \pi_t + \epsilon_t \quad (1)$$

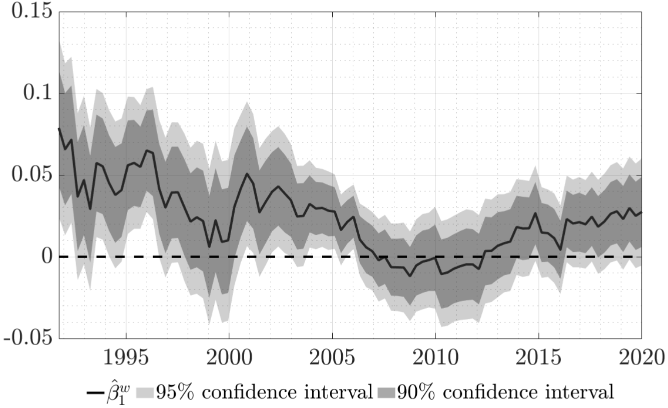
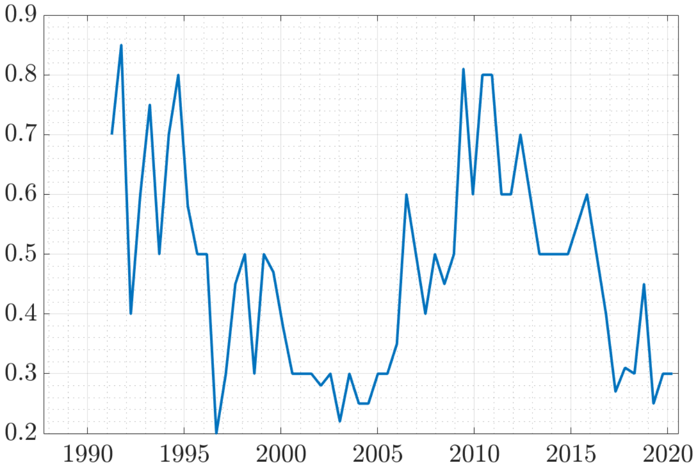


Figure: Time series of $\hat{\beta}_1^w$

Return

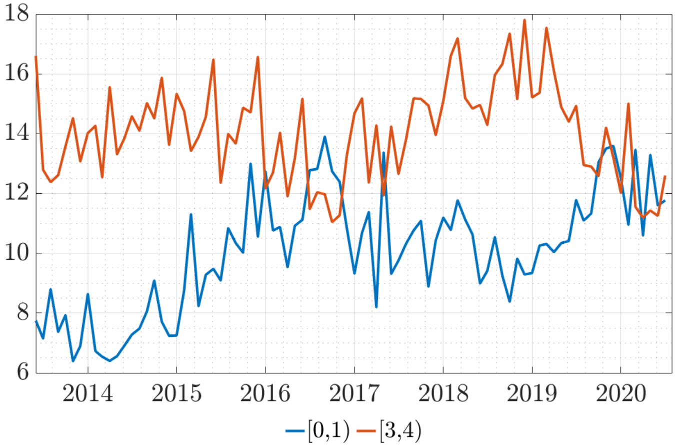
Further evidence: disagreement

Figure: Livingston Survey of Firms:
Interquartile range of 10-year ahead inflation expectations



Return

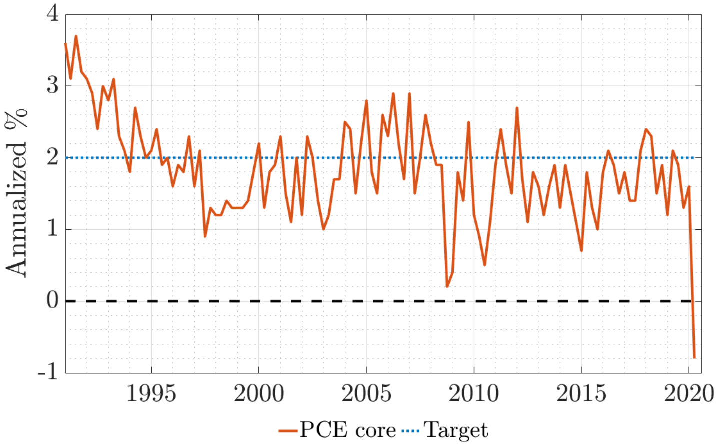
Figure: New York Fed Survey of Consumers:
Percent of respondents indicating 3-year ahead inflation will be in a particular range



◂ Return

Further evidence: introspection

Figure: PCE core inflation against the Fed's target



◂ Return

Households: standard up to $\hat{\mathbb{E}}$

Maximize lifetime expected utility

$$\hat{\mathbb{E}}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \right] \tag{2}$$

Budget constraint

$$B_t^i \leq (1 + i_{t-1})B_{t-1}^i + \int_0^1 w_t(j)h_t^i(j)dj + \Pi_t^i(j)dj - T_t - P_tC_t^i \tag{3}$$

Return

Firms: standard up to $\hat{\mathbb{E}}$

Maximize present value of profits

$$\hat{\mathbb{E}}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_t^j (p_t(j)) \right] \tag{4}$$

subject to demand

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta} \tag{5}$$

Return

Oscillatory dynamics in adaptive learning

Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta f_t + u_t \tag{6}$$

$$f_t = f_{t-1} + k(\pi_t - f_{t-1}) \tag{7}$$

Solve for the time series of expectations f_t

$$f_t = \underbrace{\frac{1 - k^{-1}}{1 - k^{-1}\beta}}_{\approx 1} f_{t-1} + \frac{k^{-1}}{1 - k^{-1}\beta} u_t \tag{8}$$

Solve for forecast error $f_t \equiv \pi_t - f_{t-1}$:

$$f_t = \underbrace{-\frac{1 - \beta}{1 - k\beta}}_{\lim_{k \to 1} = -1} f_{t-1} + \frac{1}{1 - k\beta} u_t \tag{9}$$

Return

Functional forms for **g** in the literature

- Smooth anchoring function (Gobbi et al, 2019)

$$p = h(y_{t-1}) = A + \frac{BCe^{-Dy_{t-1}}}{(Ce^{-Dy_{t-1}} + 1)^2} \tag{10}$$

$p \equiv Prob$(liquidity trap regime)
y_{t-1} output gap

- Kinked anchoring function (Carvalho et al, 2019)

$$k_t = \begin{cases} \frac{1}{t} & \text{when} \quad \theta_t < \bar{\theta} \\ k & \text{otherwise.} \end{cases} \tag{11}$$

θ_t criterion, $\bar{\theta}$ threshold value

Return

Choices for criterion θ_t

- Carvalho et al. (2019)'s criterion

$$\theta_t^{CEMP} = \max|\Sigma^{-1}(\phi_{t-1} - T(\phi_{t-1}))| \tag{12}$$

Σ variance-covariance matrix of shocks
$T(\phi)$ mapping from PLM to ALM

- CUSUM-criterion

$$\omega_t = \omega_{t-1} + \kappa k_{t-1}(f_{t|t-1}f'_{t|t-1} - \omega_{t-1}) \tag{13}$$

$$\theta_t^{CUSUM} = \theta_{t-1} + \kappa k_{t-1}(f'_{t|t-1}\omega_t^{-1}f_{t|t-1} - \theta_{t-1}) \tag{14}$$

ω_t estimated forecast-error variance

Return

General updating algorithm

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \left(y_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \right)' \right)' \tag{15}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right) \tag{16}$$

Return

Assumptions on $\mathbf{g}(\cdot)$

$$\mathbf{g}_{ff} \geq 0 \tag{17}$$

$\mathbf{g}(\cdot)$ convex in forecast errors.

◂ Return

Estimating form of gain function

- Calibrate parameters of New Keynesian core to literature

- Estimate flexible form of expectations process via simulated method of moments
  (Duffie & Singleton 1990, Lee & Ingram 1991, Smith 1993)

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \mathbf{g}(f_{t|t-1})\, f_{t|t-1} \tag{18}$$

- Moments: autocovariances of inflation, output gap, federal funds rate and 1-year ahead Survey of Professional Forecasters (SPF) inflation expectations at lags $0, \ldots, 4$

Return
Details

Estimated expectations process

$$\bar{\pi}_t - \bar{\pi}_{t-1} = \mathbf{g}(f_{t|t-1})\, f_{t|t-1} \tag{18}$$

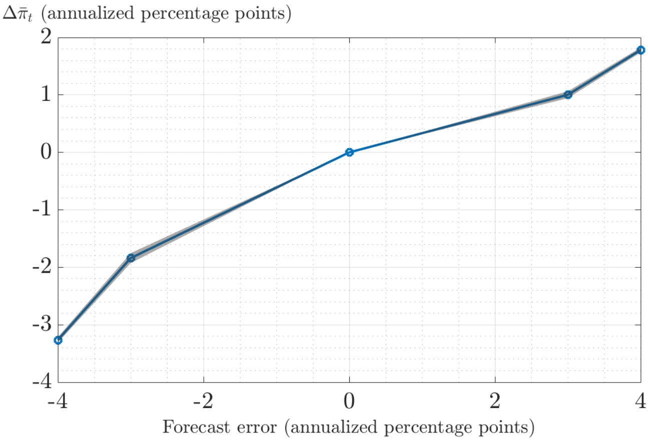


Figure: Changes in long-run inflation expectations as a function of forecast errors

Return

Details on households and firms

Consumption:

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{18}$$

$\theta > 1$: elasticity of substitution between varieties

Aggregate price level:

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}} \tag{19}$$

Profits:

$$\Pi_t^j = p_t(j)y_t(j) - w_t(j)f^{-1}(y_t(j)/A_t) \tag{20}$$

Stochastic discount factor

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{21}$$

Return

Derivations

Household FOCs

$$\hat{C}_t^i = \hat{\mathbb{E}}_t^i \hat{C}_{t+1}^i - \sigma(\hat{i}_t - \hat{\mathbb{E}}_t^i \hat{\pi}_{t+1}) \tag{22}$$

$$\hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{C}_t^i = \omega_t^i + \hat{\mathbb{E}}_t^i \sum_{s=0}^{\infty} \beta^s \hat{Y}_t^i \tag{23}$$

where 'hats' denote log-linear approximation and $\omega_t^i \equiv \frac{(1+i_{t-1})B_{t-1}^i}{P_t Y^*}$.

1. Solve (22) backward to some date t, take expectations at t
2. Sub in (23)
3. Aggregate over households i

→ Obtain (**??**)

◂ Return

Actual laws of motion

$$y_t = A_1 f_{a,t} + A_2 f_{b,t} + A_3 s_t \tag{24}$$

$$s_t = h s_{t-1} + \epsilon_t \tag{25}$$

where

$$y_t \equiv \begin{pmatrix} \pi_t \\ x_t \\ i_t \end{pmatrix} \qquad s_t \equiv \begin{pmatrix} r_t^n \\ u_t \end{pmatrix} \tag{26}$$

and

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{T+1} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} y_{T+1} \tag{27}$$

Return

Piecewise linear approximation to gain function

$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}) \tag{28}$$

- $b_i(f_{t|t-1})$ = piecewise linear basis
- γ_i = approximating coefficient at node i

$\hookrightarrow$ Estimate $\hat{\gamma}$ via simulated method of moments

Return

The expectation process over time

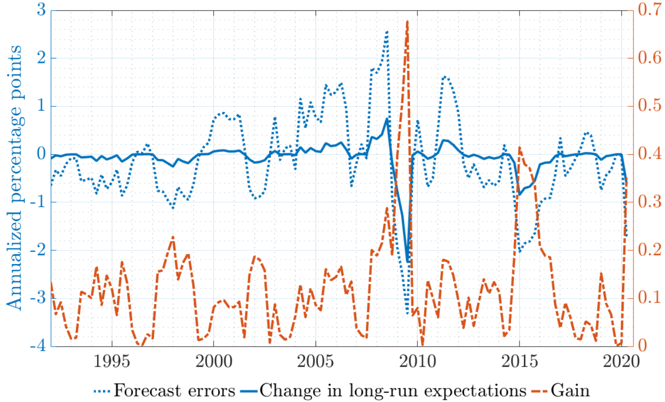


Figure: Time series of forecast errors, changes in long-run expectations and gain

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*. Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t = -\frac{\lambda_x}{\kappa} x_t$$

RE (discretion): move π_t and x_t to offset cost-push shocks

Return

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t*. Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t - \Gamma(k)\, \mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} = -\frac{\lambda_x}{\kappa} x_t$$

Adaptive learning: can move $\mathbb{E}_t\, x_{t+i}$ too if $k > 0$

▸ $\Gamma(k)$

◂ Return

Target criterion

Proposition

Let $\mathbf{g}_{z,t} \equiv \frac{\partial \mathbf{g}}{\partial z}$ *at* t. *Then monetary policy optimally brings about the following target relationship between inflation and the output gap*

$$\pi_t - \Omega\left(k_t + f_{t|t-1}\mathbf{g}_{\pi,t}\right)\left(\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1}(1 - k_{t+1+j} - f_{t+1+j|t+j}\mathbf{g}_{\bar{\mathbf{ß}},\mathbf{t+j}})\right) = -\frac{\lambda_x}{\kappa} x_t$$

Endogenous gain: ability to move $\mathbb{E}_t\, x_{t+i}$ depends on present and future degree of unanchoring

Full expression, Ω
No commitment
Return

Lemma

The discretion and commitment solutions of the Ramsey problem coincide.

▸ Why no commitment?

Corollary

Optimal policy under adaptive learning is time-consistent.

No commitment - no lagged multipliers

Simplified version of the model: planner chooses $\{\pi_t, x_t, f_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ \pi_t^2 + \lambda x_t^2 + \varphi_{1,t}(\pi_t - \kappa x_t - \beta f_t + u_t)$$

$$+ \varphi_{2,t}(f_t - f_{t-1} - k_t(\pi_t - f_{t-1})) + \varphi_{3,t}(k_t - \mathbf{g}(\pi_t - f_{t-1})) \Big\}$$

$$2\pi_t + 2\frac{\lambda}{\kappa} x_t - \varphi_{2,t}(k_t + \mathbf{g}_{\text{ß}}(\pi_t - f_{t-1})) = 0 \quad (29)$$

$$-2\beta\frac{\lambda}{\kappa} x_t + \varphi_{2,t} - \varphi_{2,t+1}(1 - k_{t+1} - \mathbf{g}_{\mathbf{f}}(\pi_{t+1} - f_t)) = 0 \quad (30)$$

Return

Target criterion system for anchoring function as changes of the gain

$$
\begin{aligned}
\varphi_{6,t} &= -cf_{t|t-1}x_{t+1} + \left(1 + \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1}) - f_{t|t-1}\mathbf{g}_{\bar{\pi},t}\right)\varphi_{6,t+1} \\
&\quad - \frac{f_{t|t-1}}{f_{t+1|t}}(1-k_{t+1})\varphi_{6,t+2} \qquad (31) \\
0 &= 2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \left(\frac{k_t}{f_{t|t-1}} + \mathbf{g}_{\pi,t}\right)\varphi_{6,t} + \frac{k_t}{f_{t|t-1}}\varphi_{6,t+1} \qquad (32)
\end{aligned}
$$

$\varphi_{6,t}$ Lagrange multiplier on anchoring function

The solution to (32) is given by:

$$
\varphi_{6,t} = -2\,\mathbb{E}_t \sum_{i=0}^{\infty}\left(\pi_{t+i} + \frac{\lambda_x}{\kappa}x_{t+i}\right)\prod_{j=0}^{i-1}\frac{\frac{k_{t+j}}{f_{t+j|t+j-1}}}{\frac{k_{t+j}}{f_{t+j|t+j-1}} + \mathbf{g}_{\pi,t+j}} \qquad (33)
$$

Return

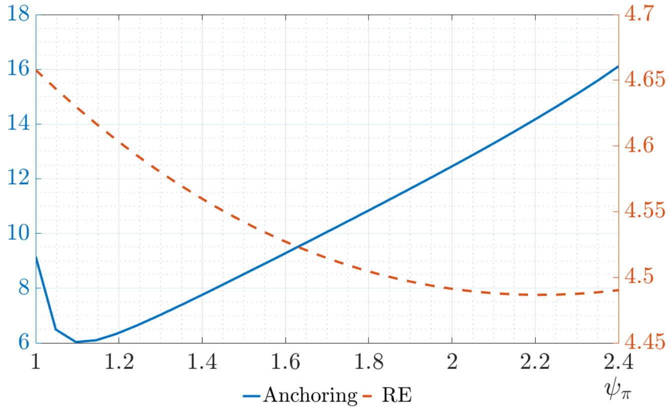
Optimal Taylor-coefficient on inflation

$$i_t = \psi_\pi \pi_t + \psi_x x_t \tag{34}$$

Optimal Taylor-coefficient on inflation

$$i_t = \psi_\pi \pi_t + \psi_x x_t \tag{34}$$

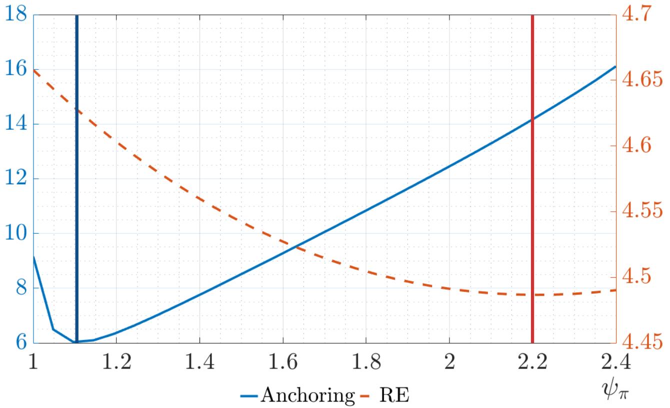
Figure: Central bank loss as a function of ψ_π



Optimal Taylor-coefficient on inflation

$$i_t = \psi_\pi \pi_t + \psi_x x_t \quad (34)$$

Figure: Central bank loss as a function of ψ_π



Anchoring-optimal coefficient: $\psi_\pi^A = 1.1$ RE-optimal coefficient: $\psi_\pi^{RE} = 2.2$

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- Current interest rate i_t: one channel of policy

Return

IRFs for various ψ_π

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- Current interest rate i_t: one channel of policy
- Taylor rule implies interest rate expectation

$$\hat{\mathbb{E}}_t i_{t+k} = \psi_\pi \hat{\mathbb{E}}_t \pi_{t+k} + \psi_x \hat{\mathbb{E}}_t x_{t+k} \tag{35}$$

Return

IRFs for various ψ_π

Why less aggressive? Future interest rate expectations

IS curve:

$$x_t = -\sigma i_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_{T+1} - \pi_{T+1}) + \sigma r_T^n)$$

- Current interest rate i_t: one channel of policy
- Taylor rule implies interest rate expectation

$$\hat{\mathbb{E}}_t i_{t+k} = \psi_\pi \hat{\mathbb{E}}_t \pi_{t+k} + \psi_x \hat{\mathbb{E}}_t x_{t+k} \tag{35}$$

- If private sector understands and believes Taylor rule, expected future interest rates additional channel of policy
(Eusepi, Giannoni & Preston 2018)

Return

IRFs for various ψ_π

Respond but not too much

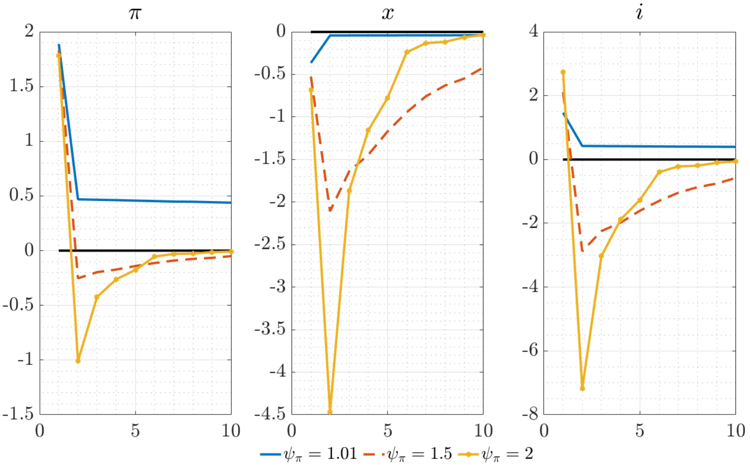


Figure: Impulse responses for unanchored expectations for various values of ψ_π

Return