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The aim of the paper

- To examine roles of extreme shocks and non-linearities during extreme events in the economy.

  - Extreme/rare shocks:
    - Does occurrence of large shocks corresponds to normal distribution?
    - t-distributed shocks (fat tails)

  - Non-linearities:
    - regime switch in shock propagation mechanism and shock volatility

- Extreme events:
  - crises, downturns, crashes
  - focus on dynamics of such events

- Need to take into account credit/financial markets (and real economy).
Motivation

Figure: Reduced form residuals (absolute value) in standard deviation units.
Motivation

- Model used in estimation of residuals: linear, small-scale, Gaussian, with constant shock volatility.
- Each characteristic can represent a wrong assumption.
- Need to account for all types of non-linearity and fat tails simultaneously - ignoring one can falsely suggest presence of the other.
- Important for policy makers:
  - shocks are unexpected - cannot be dealt with by some preemptive measures
  - non-linearities reflect structure of the economy - can be affected by regulation
- Paper does not provide a complex answer, just another piece of empirical evidence.
t-distributed shocks:
- DSGE models: Chib and Ramamurthy (2014), Cúrdia et al. (2014)
- VARs: Chiu et al. (2014)

Non-linearities:
- DSGE models: financial accelerator (Bernanke et al., 1996)

Models with t-dist. shocks linear/linearized, the only non-linearity is represented by stochastic volatility of shocks.

Non-linear models usually assume normal distribution of shocks.
Contribution and results

- Accounting for t-distributed shocks, regime switch in shock propagation mechanism and regime switch in shock volatility (and addressing concerns related to 'small-scale’ model).

Results:
- strong evidence of fat tails
- fat tails more important than non-linearities in terms of model data fit
- role for non-linearities in density forecasting
Flexible enough to distinguish between regime switch in shock propagation mechanism, shock volatility and at the same time allow for $t$-distributed shocks.

Simple enough to estimate all model parameters (important for density forecasting!) and avoid overfitting.

Threshold VAR:

$$y_t = \sum_{i=1}^{R} x_{t,p_i} B_i I \left[ r_{i-1} < y_{t-d}^{TR} < r_i \right] + u_t,$$

$$u_t \sim MN \left( 0, \sum_{i=1}^{R} I \left[ r_{i-1} < y_{t-d}^{TR} < r_i \right] \Sigma_i \right)$$

or $$u_t \sim MT \left( 0, \sum_{i=1}^{R} I \left[ r_{i-1} < y_{t-d}^{TR} < r_i \right] \Sigma_i, n \right)$$
Random variable distributed as multivariate $t$ distribution can be viewed as normally distributed with stochastic volatility:

\[
\begin{align*}
\omega_t & \sim \Gamma \left( \frac{n}{2}, \frac{2}{n} \right) \\
\mathbf{u}_t & \sim \mathcal{MN} \left( \mathbf{0}, \omega_t^{-1} \sum_{i=1}^{R} I \left[ r_{i-1} < y_{t-d}^{TR} < r_i \right] \Sigma_i \right)
\end{align*}
\]
Data and Estimation

- Data set: quarterly data 1984Q1-2013Q4 (1964Q1-2013Q4, 1984Q1-2008Q2)
- Output growth, inflation, federal funds rate, measure of credit/financial market conditions.
- Credit/financial market conditions: BAA spread (Mix variable, Financial conditions index - FCI). Robustness wrt this indicator important to address the concern about small-scale model.
- Estimation: Gibbs sampler (Chen and Lee, 1995) with Metropolis step (Koop and Potter, 2003) and adaptive rejection sampling (Gilks and Wild, 1992).
- Priors: independent Normal-inverse Wishart, Beta for thresholds, Gamma for degrees of freedom, multinomial for delay parameter.
- 100 000 iterations for inference, 50 000 burn-in period
- One regime or two regimes assumed.
Results

In-sample fit measured by the Deviance Information Criterion (DIC).

- Specifications with $t$-distributed shocks preferable.
- For normal shocks regime switch helps to explain data for Mix variable and FCI.
- So, imposing normality can suggest presence of non-linearity which is a consequence of ignored fat tails.

Table: DIC (quarterly data, 1984Q1-2013Q4)

<table>
<thead>
<tr>
<th>Shocks:</th>
<th>BAA spread</th>
<th>Mix variable</th>
<th>FCI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of regimes</td>
<td>Number of regimes</td>
<td>Number of regimes</td>
</tr>
<tr>
<td>Normal</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>666.26</td>
<td>-154.30</td>
<td>744.37</td>
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<tr>
<td></td>
<td>1258.36</td>
<td>-190.61</td>
<td>701.13</td>
</tr>
<tr>
<td>t-dist.</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>476.23</td>
<td>-337.05</td>
<td>530.95</td>
</tr>
<tr>
<td></td>
<td>519.17</td>
<td>-288.22</td>
<td>639.90</td>
</tr>
</tbody>
</table>
Excluding the Great Recession provides similar results (lower need for fat tails).

Extending data set (start by 1964Q1) provides similar results.

Strong need for fat tails ($n$ close to 5), allowing for regime change in shock volatility does not lower fat-tailedness.

Results

In-sample fit

![Graphs showing BAA spread, MA(2), Mix variable, MA(6), and Financial condition index.](image-url)
Results

Note on robustness

- Three regimes (specification with quarterly data and BAA spread).
- Table suggests that models with more regimes (TVP-VAR) are perhaps not necessary:

<table>
<thead>
<tr>
<th>Number of regimes</th>
<th>Shocks: 1</th>
<th>Shocks: 2</th>
<th>Shocks: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>666.26</td>
<td>1258.36</td>
<td>1319.29</td>
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<tr>
<td>t-dist.</td>
<td>476.23</td>
<td>519.17</td>
<td>597.00</td>
</tr>
</tbody>
</table>

Table: DIC

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Results
Out-of-sample fit

- Pseudo out-of-sample point/density forecasting exercise based on windows ending at 2002Q4 - 2013Q4.
- One-period-ahead forecasts compared with 45 ex-post observations, two-period-ahead forecasts with 44 observations, etc.
- Point/density forecasts simulated within the run of the Gibbs sampler - iterated forecasts for up to 7 quarters.
- Measure of point forecasting accuracy: root mean square error.
- As a measure of accuracy of density forecasts is used Kullback-Leibler Information Criterion.
- So, looking for model that yields the highest average logarithmic score \( \frac{1}{N} \sum \ln f_{t+h,t}(\bar{x}_{t+h}) \).
Results
Out–of-sample fit

- Basically forecasting accuracy measured for 4 variables and 7 forecasting horizons (28 cases).

- Density forecasts: for majority of cases the specification with regime switch and/or t-distributed shocks are preferred.

- When focusing on point forecasts, linear model with normally distributed errors preferred approximately in half of the cases.

- **So, non-linearities and fat tails improve tails of density forecasts (i.e. we get more accurate forecasts of extreme events).**

- Results are not driven by the Great Recession (evaluation on sub-sample 2002Q4-2008Q2 results in shift of preference towards non-linear models and t-distributed shock distributions).
Results
Out-of-sample fit

![Graphs showing output growth, inflation, FF rate, and credit market condition over the years 2002Q2 to 2009Q4. The graphs compare t-dist. shocks, 2 regimes vs. nominal shocks, 1 regime.](image-url)
Conclusions

- First attempt to account for all possible reasons of extreme events.
- Econometric/computational reasons force us to use small-scale model with a simple way how to account for non-linearities.
- Non-linearities and fat-tails in error distributions lead to more accurate tails of density forecasts.
- Application: probabilistic evaluation of macro scenarios in stress testing.