

## RESEARCH AND POLICY NOTES 5

Jan Stráský:  
Optimal Forward-Looking Policy Rules in the Quarterly Projection Model  
of the Czech National Bank

5  
200

**RESEARCH AND POLICY NOTES**

**Optimal Forward-Looking Policy Rules in the Quarterly Projection  
Model of the Czech National Bank**

Jan Stráský

5/2005

## *CNB Research and Policy Notes*

The Research and Policy Notes of the Czech National Bank (CNB) are intended to disseminate the results of the CNB's research projects as well as the other research activities of both the staff of the CNB and collaborating outside contributors, including invited speakers. The Notes aim to present topics related to strategic issues or specific aspects of monetary policy and financial stability in a less technical manner than the CNB Working Paper Series. The Notes are refereed internationally. The referee process is managed by the CNB Research Department. The Notes are circulated to stimulate discussion. The views expressed are those of the authors and do not necessarily reflect the official views of the CNB.

Printed and distributed by the Czech National Bank. Available at <http://www.cnb.cz>.

Reviewed by: Jan Vlček (Czech National Bank)  
Julius Horváth (Central European University)  
Anton Nakov (Universitat Pompeu Fabra)

Project Coordinator: Juraj Antal

© Czech National Bank, December 2005  
Jan Stráský

# Optimal Forward-Looking Policy Rules in the Quarterly Projection Model of the Czech National Bank

Jan Stráský\*

## Abstract

This paper analyses the performance of the inflation forecast-based (IFB) monetary policy rules in the quarterly projection model of the Czech National Bank. The paper begins by reviewing the model and its parametrization, including the variance-covariance matrix of disturbances employed in simulations. The main part of the paper presents the results of an extensive grid search over various targeting horizons and coefficient values for a simple IFB rule with optimized coefficients, and suggests three possibilities for improvement: a shorter targeting horizon, a higher relative weight placed on inflation gap stabilization, and a lower coefficient on partial interest rate adjustment. These results are supported by an analysis of the impact of individual shocks on the optimal coefficients of the IFB rule. The last section of the paper argues for inclusion of the real exchange rate stabilization objective in the policy maker's loss function and repeats the grid search for an optimal rule allowing for the real exchange rate feedback term. The previous results are not dramatically altered and we conclude that the stabilization properties of the extended rules are comparable with the those of the original optimized IFB rules.

**JEL Codes:** E52, E58, F41.

**Keywords:** Exchange rates, inflation targeting, monetary policy rules, open economy.

---

\* Department of Economics, University of Oxford and Czech National Bank, (jan.strasky@economics.ox.ac.uk). This work was supported by the Czech National Bank Research Project No. C1/2002. I would like to thank, without implicating, Tibor Hlédik, Tomáš Holub and Terry O'Shaughnessy for helpful comments on earlier drafts, and Jan Vlček for providing some of the MATLAB codes.

## 1. Introduction

This paper analyzes results from simulations of the quarterly projection model of the Czech National Bank. Its aim is to determine the optimal forecast horizon for the class of inflation forecast-based (IFB) policy rules that are currently employed by the bank. The contribution of the paper is a grid search over a wide range of parameter values, which produces a simple optimal rule often discussed in the literature; see Söderlind (1999) and Dennis (2004). The robustness of our findings is then checked by the analysis of how the results based upon the historical variance-covariance matrix of model shocks change when we consider individual shocks separately. Finally, we consider the possibility of extending the policy rule for a feedback term in the real exchange rate. The search for the optimal inflation forecast-based rule is repeated with the new objective and the results compared with our previous findings.

The paper assesses the ways in which the baseline policy rule currently used by the Bank differs from the optimal simple rule and presents calculations of loss functions for optimal simple rules at various targeting horizons. We argue that there is room for improvement in the baseline policy rule and discuss some reasons why these improvements may be difficult to implement.

The paper is organized as follows. The next section presents an overview of the quarterly projection model and its parametrization, including the shock structure. Section three discusses the specification of the loss function and the technique employed in the calculation of unconditional variances. The next three sections provide the main results of the paper. Section four reports the results from a grid search based on the historical shock structure. Section five analyses the impact of individual shocks on the optimal inflation forecast-based rule coefficients. The next section discusses the role of real exchange rate stabilization and reports the results of calculations based on the extended version of the loss function. Section seven discusses the main findings and concludes.

## 2. The Quarterly Projection Model

The Czech National Bank's quarterly projection model is similar to the small open economy models of Svensson (1998), Batini and Haldane (1999), and Rudebusch and Svensson (1999). The model is formulated in "gap" form with the model variables entering as deviations from their long-run trends. The evolution of the gaps is motivated by standard macroeconomic theory and reflects all the transmission channels usually present in small open-economy models. In addition to the "gap" dynamics, the past trends for output, the real exchange rate and the interest rate are estimated simultaneously using a multivariate Kalman filter; see Beneš and N'Diaye (2003). This technique considerably improves the results obtained from a simple Hodrick-Prescott filter, especially the real-time updating properties of the filter. The basic inputs to the calculation are, however, made on the basis of past developments and current trends. The trend real appreciation is set to 1.26% p.a., which is the average value in the 1994 to 2004 sample, and the growth rate of potential output is assumed to be 3.5% p.a. These two values determine the results of the filtering exercise, although some provisions are made for co-movement and compatibility of the detrended variables. As far as model projections are concerned, the data are extrapolated to the future using expert judgement.<sup>1</sup>

---

<sup>1</sup> Since the projections will inevitably include some judgement about future developments, this is not a problem as such. It is the issue of the steady state equilibrium and the consistency of the long-run trend values that is of some concern here.

The model has been described in the literature before, see Beneš et al. (2002) and Coats et al. (2003) for a detailed exposition. The presentation here is very concise and aims at presenting the model and its basic mechanics to an unfamiliar reader. The core equations of the model can be written as

$$z_t = s_t + p_t - p_t^* \quad (2.1)$$

$$\pi_{core_t} = a_0(\pi_{4_t}^{MexE} + \Delta^4 z_{-eq_t}) + a_1 E_t^P \pi_{4_{t+1}} + (1 - a_0 - a_1) \pi_{core_{t-1}} + a_2 ygap_{t-1} + \epsilon_t^\pi \quad (2.2)$$

$$ygap_t = d_0 ygap_{t-1} - d_1 rc_{-gap_{t-1}} - d_2 r4_{-gap_{t-1}} - d_3 r4_{-gap_{t-1}}^* + - d_4 z_{-gap_t} + d_5 ygap_t^* + \epsilon_t^{ygap} \quad (2.3)$$

$$s_t = g_0 E_t s_{t+1} + (1 - g_0)[s_{t-1} - 2(E_t^P \pi_{t+1} - E_t^P \pi_{t+1}^*)/4 + 2\Delta z_{-eq_t}] + (i_{4_t} - i_{4_t}^*)/4 + \epsilon_t^s \quad (2.4)$$

$$i_{-eq_t} = r_{-eq_t} + E_t \pi_{4_{t+4}} \quad (2.5)$$

$$r4_{-eq_t} = r4_{-eq_t}^* - \Delta^4 z_{-eq_t} \quad (2.6)$$

$$i_{4_t} = m_0 + m_1 \left( \sum_{j=0}^3 E_t i_{t+j} \right) + (1 - m_1) i_t + \epsilon_t^i \quad (2.7)$$

$$rc_t = i_{4_t} + p_1 E_t ygap_{t+4} + \epsilon_t^{ic} \quad (2.8)$$

$$\pi_t^{MexE} = k_1(\pi_t^* - \Delta s_t) + (1 - k_1) \pi_{t-1}^{MexE} - k_2(p_{t-1}^{MexE} - p_{t-1}^* + s_{t-1} + k_0) + \epsilon_t^{MexE} \quad (2.9)$$

$$\pi_t^{ME} = h_1(\pi_t^{oil} - 4\Delta s_t) + (1 - h_1) \pi_{t-1}^{ME} - h_2(p_{t-1}^{ME} - p_{t-1}^{oil} + s_{t-1} + h_0) + \epsilon_t^{ME} \quad (2.10)$$

$$\pi_t^{EN} = n_0 \pi_{4_t}^{ME} + n_1 E_t^P \pi_{4_t} + (1 - n_0 - n_1) \pi_{t-1}^{EN} - n_2 ygap_{t-1} + \epsilon_t^{ME} \quad (2.11)$$

Equation (2.2) is the model's reduced-form equation for the dynamics of inflation. Equation (2.3) describes the dynamics of the output gap. Equation (2.4) is the model version of the uncovered interest rate parity condition, and equation (2.7) is the term-structure relationship linking the short-term and long-term interest rates. Equations (2.9) to (2.11) define components of inflation in the price of imported goods, while the rest of listed equations contain definitions of important model variables.

The variables describing the long-run trend position of the economy have the suffix ”*-eq*”. All other variables are in deviations from their long-run trends (the ”*gap*” form). Apart from interest rates they are all in logarithms; inflation rates are defined as logarithmic differences. The variables  $z$  and  $s$  denote the real and nominal exchange rates respectively;  $p$  is the CPI price level,  $ygap$  is the output gap;  $i$  and  $i_4$  are the short-term (3-months) and long-term (1-year) nominal interest rates respectively, and  $r4$  and  $rc$  are their respective real counterparts;  $\pi_t^{ME}$  is imported energy inflation and  $\pi_t^{oil}$  is (exogenous) oil price inflation, while  $p_{t-1}^{ME}$  and  $p_{t-1}^{oil}$  are the imported energy prices and oil prices respectively;  $\pi_t^{MexE}$  is imported inflation excluding energy inflation,  $\pi_{core_t}$  is core inflation (defined as a change in CPI adjusted for changes in energy and administered prices); and  $\pi_t$  is CPI inflation.

For all measures of inflation,  $\pi_4$  denotes the annualized change (fourth difference), while  $\pi$  is the change between quarters (first difference). The expectations of the private sector at time  $t$  of variable  $X$  are modelled as a weighted average of forward-looking model consistent expectations and backward-looking adaptive expectations

$$E_t^P X_{t+1} = b_0 E_t X_{t+1} + (1 - b_0) X_{t-1}.$$

**Table 2.1: Parametrization of the quarterly prediction model**

Parameter	Value	Parameter	Value
$a_0$	0.248	$m_0$	0.25
$a_1$	0.33	$m_1$	0.5
$a_2$	0.5	$p_1$	0.3
$d_0$	0.9	$k_0$	0.1
$d_1$	0.062	$k_1$	0.58
$d_2$	0.125	$k_2$	0.52
$d_3$	0.062	$h_1$	0.9
$d_4$	0.15	$h_2$	0.67
$d_5$	0.4	$n_0$	0.28
$b_0$	0.1	$n_1$	0.5
$g_0$	0.5	$n_2$	0.25

where  $b_0 = 0.1$ . Asterisked variables denote foreign counterparts and are proxied by German aggregates. The model is parametrized using econometric estimates from single-equation estimation on the Czech data.<sup>2</sup> The parameters of the model and their values are reported in Table 2.1.

The model is closed by an inflation forecast-based rule similar to those used in Clarida et al. (1997) and Woodford (2000)

$$i_t = \rho i_{t-1} + (1 - \rho) \{i_{eq} + 1.6[b(E_t \pi_{4t+j} - \pi^{tar}) + (1 - b)ygap_t]\} \quad (2.12)$$

where  $\rho$  is the interest rate smoothing parameter,  $i_{eq_t}$  is the equilibrium short-term nominal interest rate defined above,  $E_t \pi_{4t+4} - \pi^{tar}$  is the inflation gap, and  $b$  is the coefficient of the relative weight on inflation stabilization. The baseline policy rule currently used by the Czech National Bank sets the policy horizon  $j = 4$ ,  $\rho = 0.75$ , and  $b = 0.75$ . The rule includes a relatively high degree of interest rate smoothing, and, since the policy rate reacts to both the inflation and output gaps, it is an example of a flexible inflation targeting rule; see Svensson (1997) and Svensson (1999).

The term  $(1 - b)ygap_t$  in the rule (2.12) is strictly speaking redundant, even if we assume that the policy maker does care about the level of output. It can be shown that an inflation forecast-based rule of the type

$$i_t = \rho i_{t-1} + (1 - \rho) \{i_{eq} + \tilde{b}(E_t \pi_{4t+j} - \pi^{tar})\} \quad (2.13)$$

reacts due to its forecast form to all the state-space variables of the model including the output gap. It does so to the extent that the output gap and other model variables are important for stabilizing inflation at the given targeting horizon  $t + j$ . The inclusion of the output gap stabilization objective into the policy maker's objective function can be simply accommodated by making appropriate changes in the values of  $\rho$ ,  $\tilde{b}$  and  $j$ , although it is often written in a form such as (2.12) that introduces a separate output gap term; see Batini and Haldane (1999) for an early exposition of this point.

<sup>2</sup> The estimated values are in some cases slightly corrected by expert judgement. Details of the model parametrization can be found in Coats et al. (2003).

**Table 2.2: Variance of model shocks: per cent**

Variable	Variance
Imported energy price inflation	620.01
Imported food price inflation	53.29
Rest of import price inflation	17.64
Total energy price inflation	453.69
Total food price inflation	14.44
Core inflation excluding food	3.24
Output gap	0.49
Nominal exchange rate	2.25

**Table 2.3: Inflation, output and interest rate variability under the baseline policy rule: per cent**

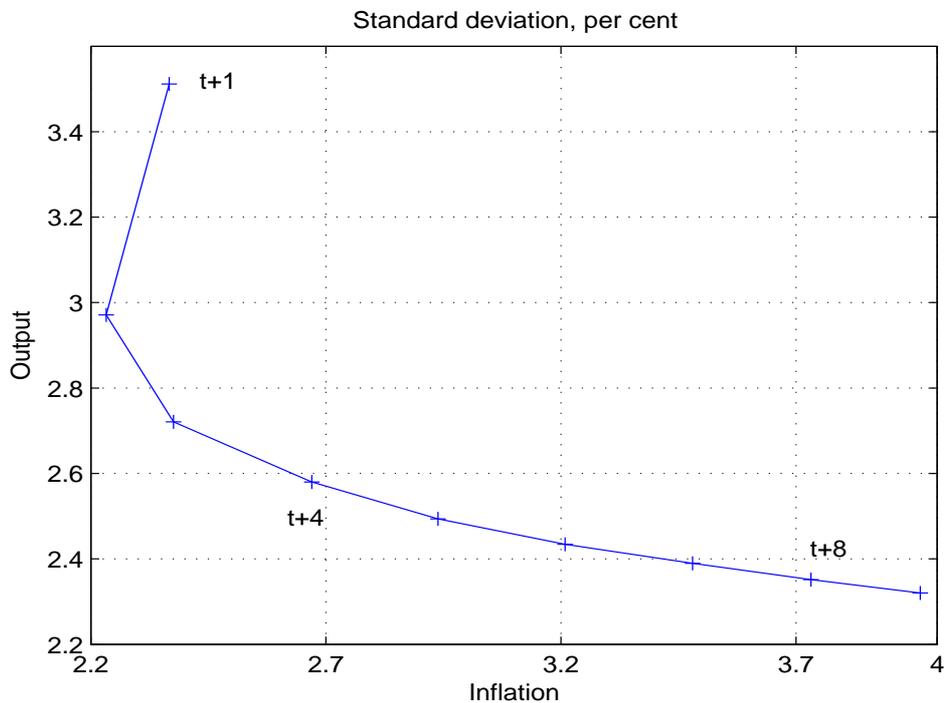
Horizon	Baseline rule		
	stdev(i)	stdev( $\pi$ )	stdev(y)
CPI+1	3.3473	2.3671	3.5118
CPI+2	2.9322	2.2329	2.9711
CPI+3	2.9053	2.3761	2.721
CPI+4	3.0588	2.6703	2.5798
CPI+5	3.2458	2.9386	2.4935
CPI+6	3.4576	3.2090	2.4341
CPI+7	3.6852	3.48	2.3894
CPI+8	3.9061	3.7318	2.3515
CPI+9	4.1161	3.9643	2.3201

This point illustrates the relative importance and precedence of the loss function in the determination of policy. In the class of inflation forecast-based rules, there seems to be no use in including a variable in the policy rule unless there is a reason to include it also in the policy maker's objective function. Even if the latter is true and an economic variable figures in the policy maker's loss function, it is not necessary for it to appear in the policy rule. The rule (2.13) under a loss function penalizing output gap would stabilize the economy just as well as the rule (2.12). We will return to these considerations in the last section where we make a case for a possible extending of the policy rule.

In the simulations reported below the model is perturbed using a variance-covariance matrix of historical shocks. The disturbances are independent normally distributed random variables with zero mean and variance  $E[\nu\nu'] = \Omega$  summarized in Table 2.2. The shocks are assumed to be uncorrelated,  $E[\nu_i\nu_j'] = 0$  for  $i \neq j$ , although this assumption is relaxed later.

A preliminary insight into the issue of the optimal horizon for inflation targeting can be derived from a simple monetary policy frontier that captures inflation and output variance under the various inflation-targeting horizons. Figure 2.1 shows the outcomes of CPI inflation targeting for the forecast horizons from 1 to 9 quarters under the baseline policy rule (2.12). The unconditional standard deviations of inflation, the output gap and the short-term interest rate under various horizons are summarized in Table 2.3.

**Figure 2.1: Inflation and output variability under the baseline policy rule at various targeting horizons (standard deviation of output and inflation, per cent)**



As the policy horizon increases, we observe a similar trade-off between the inflation and output variability, which is documented for Taylor rules under changing relative weight on inflation in the rule; see Ball (1999) and Taylor (1993). At longer horizons the inflation variability increases, while the output gap variability is gradually reduced. Targeting an inflation forecast further in the future allows the policy maker to become more accommodative in stabilizing the output gap today, even without substantially increasing the variability of the instrument. Due to this trade-off, consistent evaluation of policy choices requires the introduction of a loss function, to which task we now turn.

### 3. The Loss Function

The variability of the model variables that enter the policy maker's loss function is measured by unconditional variances. Since we do not explicitly solve for the optimal policy rule under either commitment or discretion and can easily invert the model into the "companion" form, unconditional variance is an obvious candidate often adopted in similar models.<sup>3</sup> The unconditional variances of the model variables depend on three factors: (i) the intertemporal dynamics defined by the structural parameters of the model, (ii) the monetary policy rule coefficients, and (iii) the variance of the economic shocks. In our search for the optimal targeting horizon and the "optimal" policy rule coefficients, we first consider the historical variance of the shocks reported in Table 2.2. In order to analyse the effects of the shock structure on the optimal rule,

<sup>3</sup> For example Soto (2003) and Parrado (2004). On the other hand, models that aim at deriving the optimal policy rule by explicitly minimizing a loss function with respect to the structural equations of the model, often discount future expected variability and optimize an intertemporal loss function; see Svensson (1998) and Hlédik (2002).

we also present separate calculations of unconditional variances under (i) a "cost-push" shock (a shock to inflation), (ii) a "demand" shock (a shock to the output gap) and (iii) an exchange rate shock. With the exception of the output gap shock, the shocks have no persistence (i.e. they are serially uncorrelated).

Unconditional variance provides a useful measure of the costs of macroeconomic stabilization in the quarterly projection model, even though it does not give a clear picture of the transitional dynamics. It tells us the average variance of a variable in the model which is constantly perturbed by stochastic shocks drawn from a given variance-covariance matrix  $\Omega$ .<sup>4</sup> The calculation is conducted in two steps. First, the structural model in  $n$  variables with  $p$  lags is transformed into the "companion" form, in which all endogenous variables (including shocks) are expressed as a vector AR(1) process, i.e. in terms of other once-lagged endogenous variables. This step also includes solutions for the expectational variables, and the outcome can be written as (Hamilton, 1994, p.259)

$$\xi_t = \mathbf{F}\xi_{t-1} + \nu_t$$

where  $\xi_t$  is the vector of stacked endogenous variables and their lags in deviations from the mean  $[\mathbf{y}_t - \mu, \mathbf{y}_{t-1} - \mu, \dots, \mathbf{y}_{t-p} - \mu]'$  and  $\mathbf{F}$  and  $\nu$  are the appropriately constructed stacked matrices of the structural parameters and residuals.

In the second step, the unconditional variance of the variables of interest is calculated using the recursive formula

$$\Sigma \equiv E(\xi_t \xi_t') = \mathbf{F}E(\xi_{t-1} \xi_{t-1}')\mathbf{F}' + E(\nu_t \nu_t')$$

where  $\Sigma$  is the unconditional variance,  $\mathbf{F}$  is the "companion matrix" of the model, and  $E(\nu_t \nu_t') \equiv \mathbf{Q}$  for  $t = \tau$  and  $\mathbf{0}$  otherwise. The matrix  $\mathbf{Q}$  is the stacked variance-covariance matrix of stochastic shocks.<sup>5</sup> In this paper, we calculate the unconditional variance recursively, summing up increments of variance over 100 quarters, which is enough for all values to converge.<sup>6</sup>

Since the quarterly projection model has no microfoundations and the decision rules of agents are not based on optimization, the model lacks a well-defined welfare measure.<sup>7</sup> The loss function is set by the policy maker in a way which is assumed to be consistent with the preferences of other agents. We follow the usual assumption that the central bank has both price stability and economic activity objectives. The former takes the form of an inflation target, while the

<sup>4</sup> Intuitively, unconditional variance can also be viewed as the variance from the time when the model is perturbed by shocks to the time when it returns to its equilibrium.

<sup>5</sup> The unconditional variances of all the endogenous variables can be calculated, given  $Q$  and a set of policy rule parameters, from the standard formula (Hamilton, 1994, p. 264-266):

$$vec(\Sigma) = [I_{r^2} - F \otimes F]^{-1}vec(Q),$$

where  $vec()$  denotes matrix vectorization,  $I_{r^2}$  is an identity matrix with  $r^2$  rows ( $r = np$ ;  $n$  is the number of variables and  $p$  is the number of lags in the structural form of the model), and  $\otimes$  denotes the Kronecker product.

<sup>6</sup> The computations were conducted in IRIS, a suite of MATLAB codes developed at the Czech National Bank; see Beneš (2004). A useful MATLAB code for the calculation of unconditional variabilities under different targeting horizons has been provided by the authors of Hurník and Vlček (2004).

<sup>7</sup> This is a drawback which recent contributions to the literature particularly strive to improve upon. Models with an explicit welfare measure derived from agents' preferences include those in Woodford (2003), Onatski and Williams (2004) and many more.

latter is implemented through a non-zero weight placed on stabilization of the output gap. The fact that the two terms explicitly enter the loss function reflects the inherent trade-off between the two when the central bank stabilizes the "cost-push" (inflationary) shock. Because reducing the variance of inflation requires setting the interest rate in the direction that increases the variance of output, a preference for lower inflation variance will always imply higher variance of the output gap. In our search for the optimal rule we thus consider the following version of the policy maker's loss function

$$L_t = w_i i_t^2 + (1 - w_i)[w_\pi(\pi_t - \pi^*)^2 + w_y(y_t - y^*)^2] \quad (3.14)$$

where  $w_j$  are the respective weights placed at various policy objectives in the assumed loss function, and the quadratic terms measure the variation in the variables of interest. In what follows we use the term "optimal rule" to refer to the inflation forecast-based rule which results in the lowest weighted sum of the unconditional variances of the variables of interest.

Taking unconditional expectations, the loss function (3.14) becomes

$$E[L_t] = w_i \text{var}(i_t) + (1 - w_i)[w_\pi \text{var}(\pi_t) + w_y \text{var}(y_t^{gap})] \quad (3.15)$$

where  $\text{var}(\cdot)$  are the unconditional variances of the variables of interest. Strictly speaking, the loss function penalizes variation in the level of the instrument, and does not assume an interest rate smoothing objective. Since the coefficient on the lagged short-term interest rate in the current policy rule is rather high, we assume that the policy maker wants to limit interest rate variability and set  $w_i = 0.2$ . This parametrization we retain in all the simulations reported below.

The values of the remaining parameters,  $w_\pi$ ,  $w_y$  sum up to one. The weight on inflation variability is varied between  $w_\pi = w_y = 0.5$  and  $w_\pi = 0.9$ ,  $w_y = 0.1$  with an increment of 0.2. The former is a loss function that places the same relative weight on inflation and output gap variability ( $w_\pi = 0.5$ ), whereas the latter corresponds to almost strict inflation targeting ( $w_\pi = 0.9$ ). We assume that the policy maker is unlikely to penalize output gap variation more than inflation variation and that, on the other hand, the central bank is unlikely to be only concerned about minimizing inflation variability.

#### 4. The Grid Search for the Optimal Simple Rule

So far we have restricted our analysis to a narrow class of monetary policy rules with a given degree of interest rate smoothing and fixed coefficients on the inflation and output gaps, and have only varied the inflation forecast horizon. However, finding a rule that is optimal also with respect to the degree of interest rate smoothing and the relative weight placed on the inflation and output gap would add substantially to our understanding. In this section of the paper we therefore focus on a simple grid search over some plausible coefficient values of the remaining monetary policy rule coefficients.

The analysis of policy rules in models with forward-looking variables has to take into account the issue of dynamic inconsistency. There are two ways of addressing this problem: (i) the assumption of precommitment, and (ii) explicit modelling of strategic interactions under discretion. The former approach implies that the policy maker commits itself to a single optimization and to application of the optimal policy in all subsequent periods. In the latter an equilibrium is

found whereby the policy maker reoptimizes every period, taking the other agents' expectations as given. This approach involves solving for some kind of "subgame-perfect Nash equilibrium"; see Dennis (2003).<sup>8</sup>

The difference between the two approaches in terms of solving for optimal policies is discussed in Svensson (1998). Under discretion, the forward-looking variables,  $x_t$ , are (linear) functions of  $X_t$ , the predetermined state variables:  $x_t = HX_t$ . The optimal reaction function,  $f$ , also becomes a linear function of the predetermined variables,  $i_t = fX_t$ . Both  $H$  and  $f$  are endogenously determined. In the precommitment case, the forward-looking variables and the optimal policy also depend on the shadow prices of the forward-looking variables, since the loss function is now minimized intertemporally (Svensson, 1998, p. 13-14).

The analysis in this paper focuses solely on *simple* optimal policy rules under precommitment discussed in Levine and Currie (1987) and Söderlind (1999). By this we mean that the policy maker is able to precommit to a simple rule which is a linear function of some state-space variables,  $i_t = -F[x_t, X_t]$ , and its coefficients,  $F$ , are based on minimization of a loss function. Although here the minimization is only approximated by a grid search, the outcome should in principle be the same as if the optimal values of  $F = F^*$  were derived from explicit minimization (Söderlind, 1999, p. 817-818).<sup>9</sup>

The grid search is carried out in two steps. In the first step, the standard deviations of the interest rate, inflation and output gaps are calculated for  $b$  and  $\rho$  both varying from 0.1 to 0.9. In the second step, an optimal targeting horizon is chosen from the best attainable losses at each  $t + k$  quarter ahead. (As previously, we consider targeting horizons from 1 to 9 quarters ahead.)

The degree of interest rate smoothing can be interpreted as a measure of the "aggressiveness" of the policy maker: the lower the  $\rho$  coefficient, the more aggressively the policy maker changes the policy rate in order to stabilize the economy. Shortening the targeting horizon has a similar effect: since the targeted inflation forecast lies closer in time, the policy maker is bound to react by changing the policy rate more to achieve this goal.

#### 4.1 Variability of the policy variables

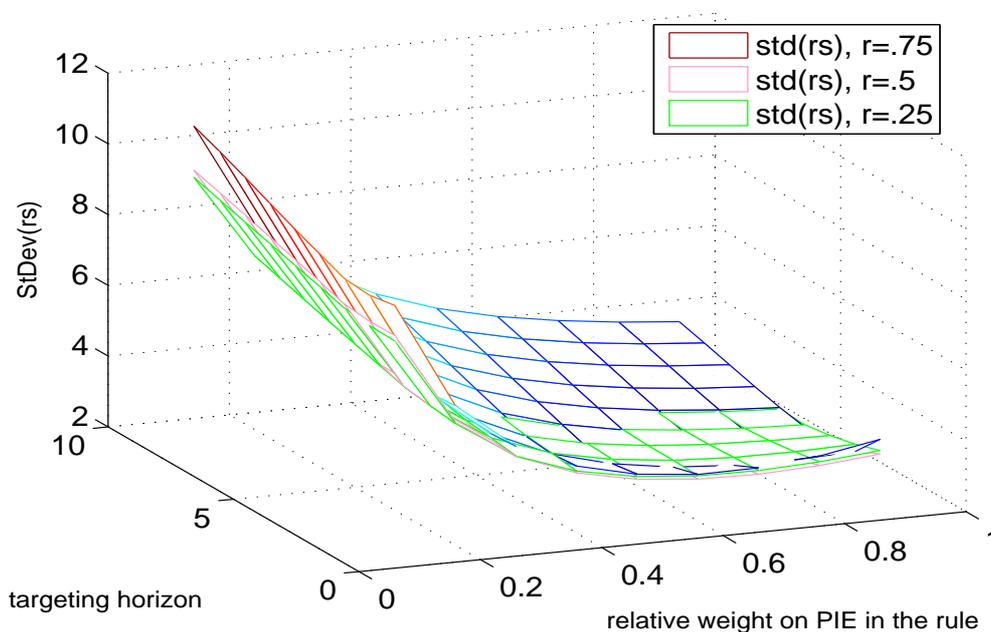
We first look at the variability of the policy instrument. As discussed above, we would like to restrict the choice to policies that produce a comparable variability of the instrument. The policy maker is likely to accept certain instrument variability according to its preferences, but is unlikely to change its operating procedures very often. That implies interest rates that are only changed by increments in a certain range.

The grid search suggests that the interest rate variability decreases as the weight on the inflation gap in the rule increases; see Figure 4.1. With the relative weight on inflation in the rule below 0.5, the instrument's standard deviation quickly increases well over 5%. On the other hand, under short targeting horizons the need to stabilize the inflation forecast quickly produces higher variability of the instrument, even with a high relative weight on the inflation gap. For most

<sup>8</sup> The "timeless perspective" analysis developed by Woodford is a recent alternative to the precommitment approach; see the discussion in Dennis (2003) and the references therein.

<sup>9</sup> Explicit minimization usually requires a solution using a non-linear optimization algorithm. It is also contingent on the initial state of the economy and the variance-covariance matrix of the shocks; see Söderlind (1999) and Dennis (2004).

**Figure 4.1:** Interest rate variability (standard deviation, per cent) for various degrees of interest rate smoothing ( $r$ )



parameter values, however, we get the intuitively expected result: the higher the weight on partial adjustment, the higher the instrument variability.

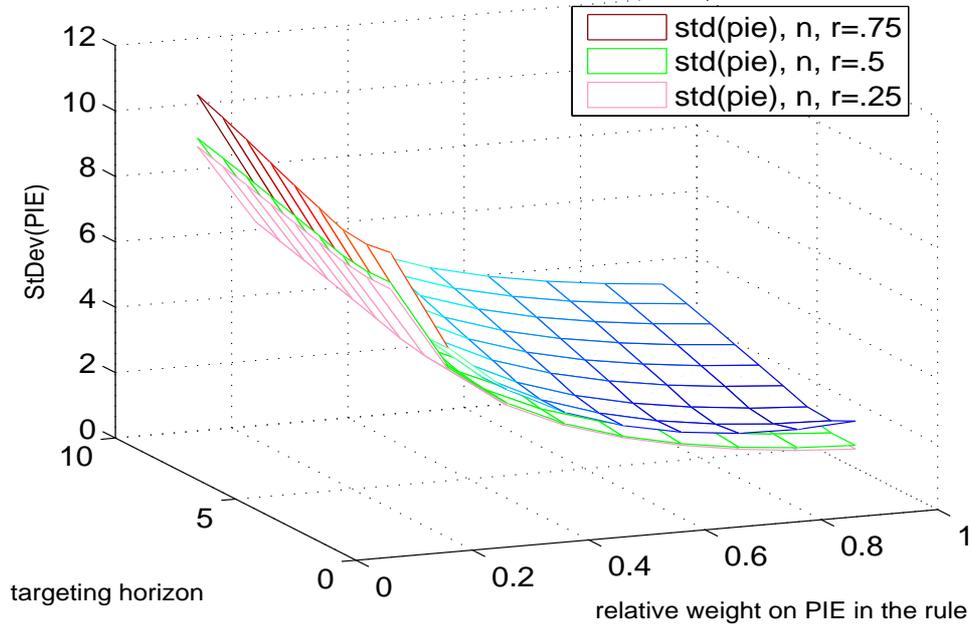
Inflation variability is graphed in Figure 4.2. As before, the variability increases considerably for targeting horizons over 5 quarters and a relative weight on inflation in the rule below 0.5. Since the inflation variability is as much as twice that of the output gap, the resulting optimal policy rule is likely to be one that delivers a low standard deviation of inflation. Higher  $\rho$  again does not seem to have much impact on the inflation variability; the difference is especially small for partial adjustment coefficients between 0.25 and 0.5.

Figure 4.3 graphs the output gap variability which is much smaller than the values calculated for interest rates and inflation. The output gap variability increases as the weight on the output gap in the rule decreases. The graph, however, also shows that the cost of decreasing the weight of the output gap in the rule is more costly at shorter horizons. Reducing interest rate smoothing seems to reduce output gap variability at its extremes, i.e. under short targeting horizons and high relative weights on inflation in the rule.

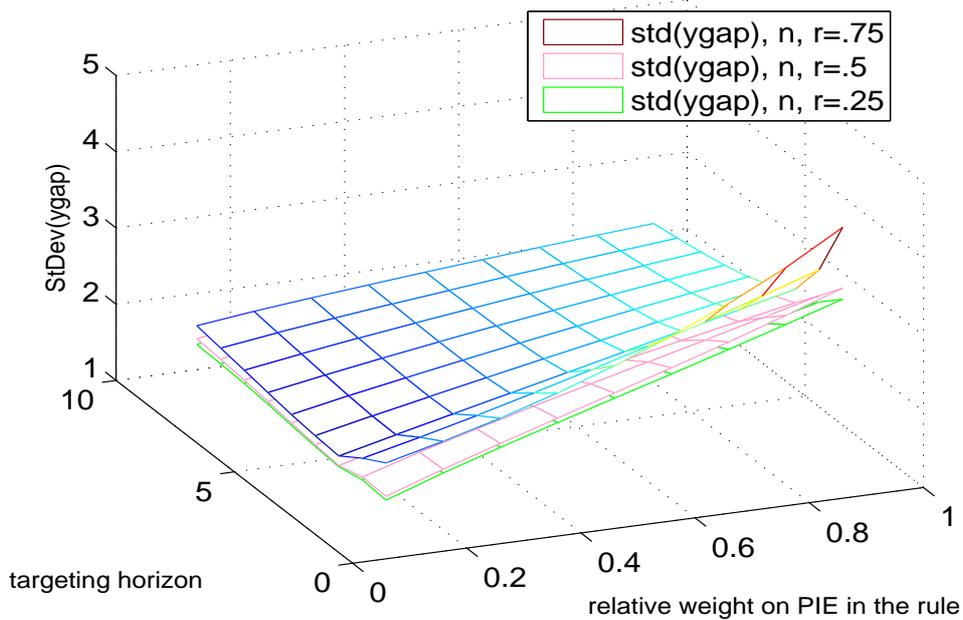
## 4.2 Optimized Taylor rules

Using the policy rule (2.12) nine variability surfaces were calculated for targeting horizons from 1 to 9 quarters ahead. The loss under various policy rules is driven by inflation variability; this is true even when the same relative weight of 0.5 is put on both output and inflation variability, and is even more the case as the weight on inflation variability increases; see Figure 4.4. The

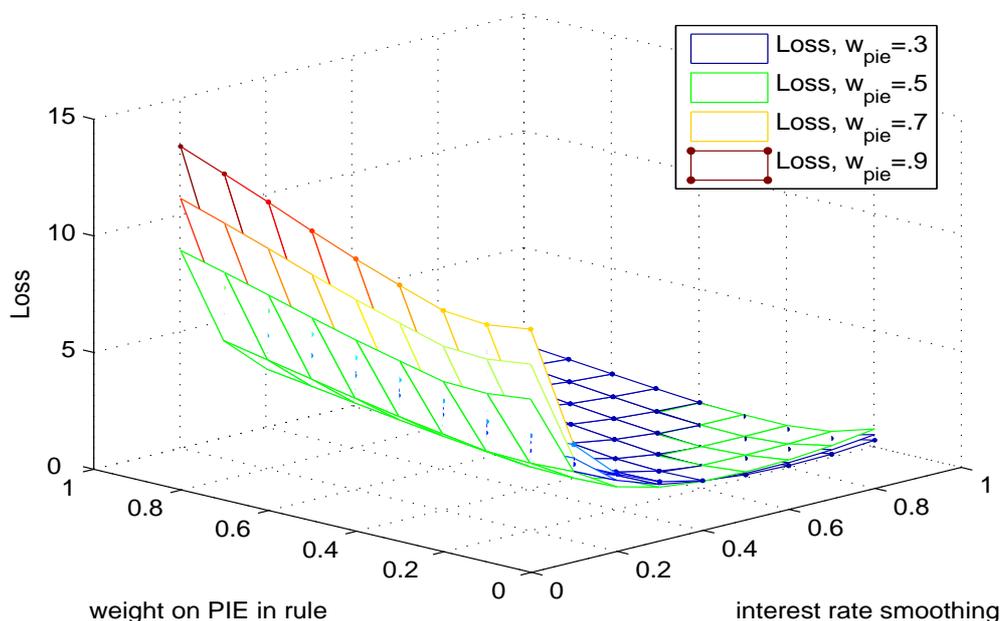
**Figure 4.2: Inflation variability (standard deviation, per cent) for various degrees of interest rate smoothing ( $r$ )**



**Figure 4.3: Output gap variability (standard deviation, per cent) for various degrees of interest rate smoothing ( $r$ )**



**Figure 4.4: Loss functions for different relative weights on inflation stabilization in the policy rule**



loss function has a minimum at the  $t + 3$  targeting horizon and a relative weight on the inflation gap in the rule of 0.8.

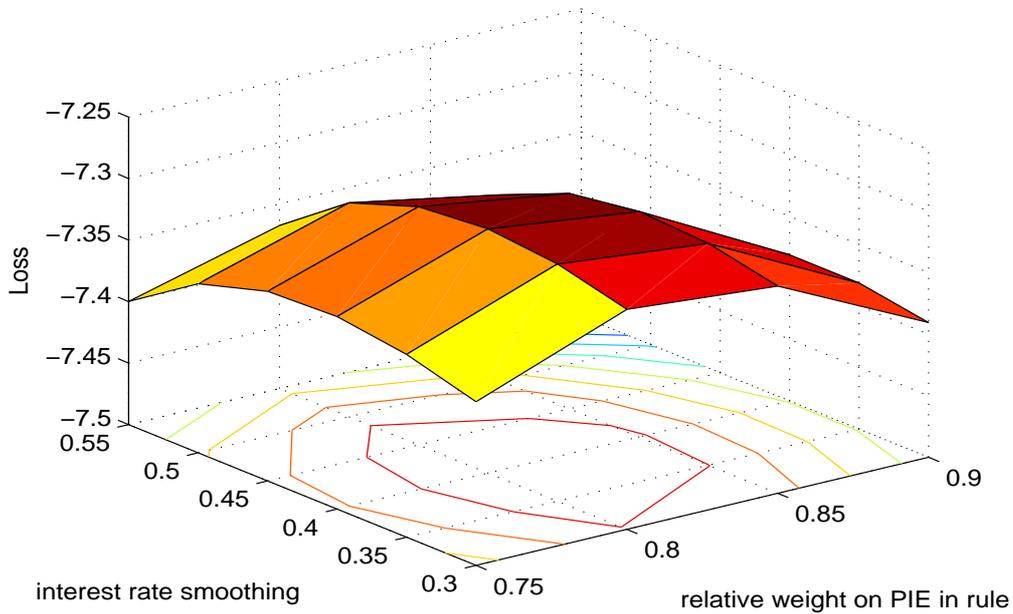
We have also varied the coefficients on the interest rate smoothing term from 0.1 to 0.9. Since the general shape of the loss function does not change, the graphs for these values of  $\rho$  are not reported. This allowed us to conduct a refined search for the optimal coefficients under the three different loss functions described above.

The loss function with  $w_\pi = 0.5$  is minimized at the targeting horizon of  $t + 3$  and a relative weight on the inflation gap in the rule of 0.8, irrespective of the value of  $\rho$ . A refined grid search confirms that the pair  $(\rho^*, b^*) = (0.5, 0.8)$  is a minimum of the given loss function. Figure 4.5 shows the result graphically (in terms of the negative of the loss function to facilitate visualization). As the contours show, the increments around the minimum are negligible for  $\rho$  between 0.35 and 0.45 and for a relative inflation coefficient between 0.7 and 0.85.

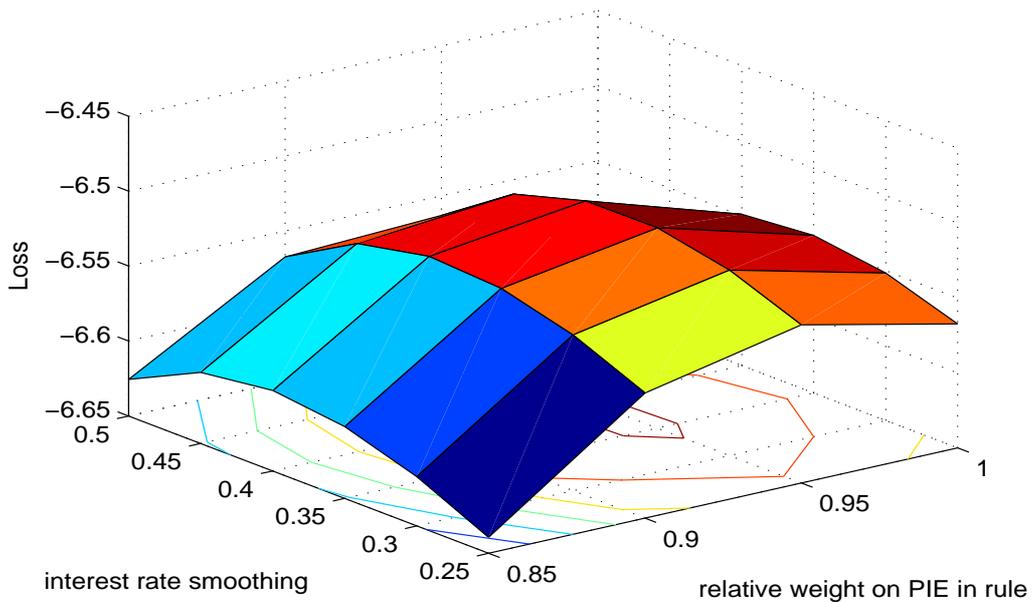
The loss function with  $w_\pi = 0.7$  is again minimized at the targeting horizon of  $t + 3$ , irrespective of the other parameters chosen. The refined grid search in this case shows that the optimal relative weight on inflation in the rule is very high,  $b^* = 0.95$ ; see Figure 4.6. The degree of interest rate smoothing that minimizes the loss function is now  $\rho^* = 0.35$ , well below the partial adjustment coefficient in the current policy rule.

The optimal simple rule under the loss function with  $w_\pi = 0.9$  differs from the two previous cases, because it is a *strict* inflation targeting rule, as  $b = 1$  minimizes the loss. This is most likely due to high persistence in the output gap equation. Despite the near coincidence of

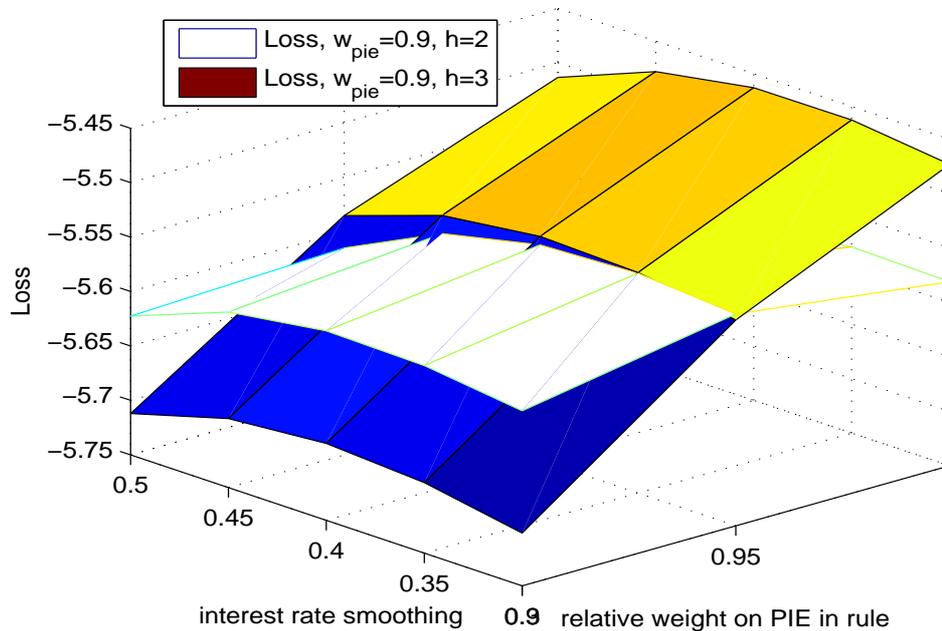
**Figure 4.5:** Minimum loss under 0.5 weight on inflation in the loss function and the  $t + 3$  targeting horizon



**Figure 4.6:** Minimum loss under 0.7 weight on inflation in the loss function and the  $t + 3$  targeting horizon



**Figure 4.7: Minimum loss under 0.9 weight on inflation in the loss function and the  $t + 2$  and  $t + 3$  targeting horizons**



the losses under two and three quarters ahead targeting at  $b = 0.9$ , the inclusion of higher relative weights clearly shows that  $t + 3$  is the optimal targeting horizon; see the negative of the loss function graphed in Figure 4.7. The optimal degree of interest rate smoothing for this specification of the loss function is 0.35.

### 4.3 Comparison with the baseline policy rule

We can now calculate the minimum losses for other targeting horizons, together with the optimal degree of interest rate smoothing and the relative weight on the inflation gap in the policy rule. For each assumed version of the loss function, it will be then possible to quantify how much is the outcome under the baseline policy rule differ from a minimum loss under the optimal policy rule.

Table 4.1 reports results for targeting horizons of one to five quarters,<sup>10</sup> showing that the  $t + 3$  horizon results in the smallest losses for all three loss functions considered. The optimal degree of interest rate smoothing,  $\rho^*$ , decreases as the targeting horizon increases from  $t + 2$ , while the optimal weight on the inflation gap in the policy rule,  $b^*$ , increases (in some cases as much as for strict inflation targeting, as shown above).

Table 4.1 also reports the loss function values for the baseline policy rule. The results suggest three departures of the baseline policy rule from the calculated optimal simple rules. First, the

<sup>10</sup> The results reported here are calculations from a refined grid search with  $r$  and  $b$  running from 0.1 to 1 for the horizon  $t + 3$ . For other horizons, the results are for  $m_0$  and  $b$  between 0.1 and 0.9 only. The unreported horizons are those where the loss function is monotonically increasing.

**Table 4.1: Optimized IFB Taylor rule coefficients for some targeting horizons**

Targeting horizon	Loss ( $w_\pi = 0.5$ ) $\rho^*$ , $b^*$ , loss	Loss ( $w_\pi = 0.7$ ) $\rho^*$ , $b^*$ , loss	Loss ( $w_\pi = 0.9$ ) $\rho^*$ , $b^*$ , loss
$t + 1$	0.1 0.7 8.08	0.1 0.8 7.199	0.2 0.8 5.923
$t + 2$	0.3 0.8 7.851	0.4 0.8 6.849	0.4 0.9 5.576
$t + 3^*$	0.4 0.8 7.301	0.35 0.95 6.516	0.35 1 5.462
$t + 4$	0.2 0.9 7.39	0.2 0.9 6.877	0.2 0.9 6.365
$t + 5$	0.1 0.9 7.64	0.1 0.9 7.355	0.1 0.9 7.07
Baseline rule ( $t + 4$ )	0.75 0.75 8.975	--- --- 9.259	--- --- 9.542

optimal targeting horizon seems to be 3 rather than 4 quarters ahead. The difference can perhaps be explained by the presence of an information lag in monetary policy. There is no uncertainty in the model, while it is likely that the policy maker may need some time to recognize the shocks as they arrive.

Second, the baseline policy rule has a substantially higher coefficient of partial adjustment and hence it is less "aggressive" than the optimal Taylor rule for all three formulations of the loss function. Again, this departure from preferable policy can be viewed as a precaution against uncertainty. There may also exist credibility concerns, which prevent the policy maker from reversing its policy too often.

The grid search also suggests that the relative weight on the inflation gap in the rule should be increased. This may be easier to do, since inflation can be measured more precisely and with a smaller lag than the output gap. On the other hand, this result depends strongly on the historical shock structure that we employed in the simulations. It is difficult to make recommendations for the future based on the transition period experience.

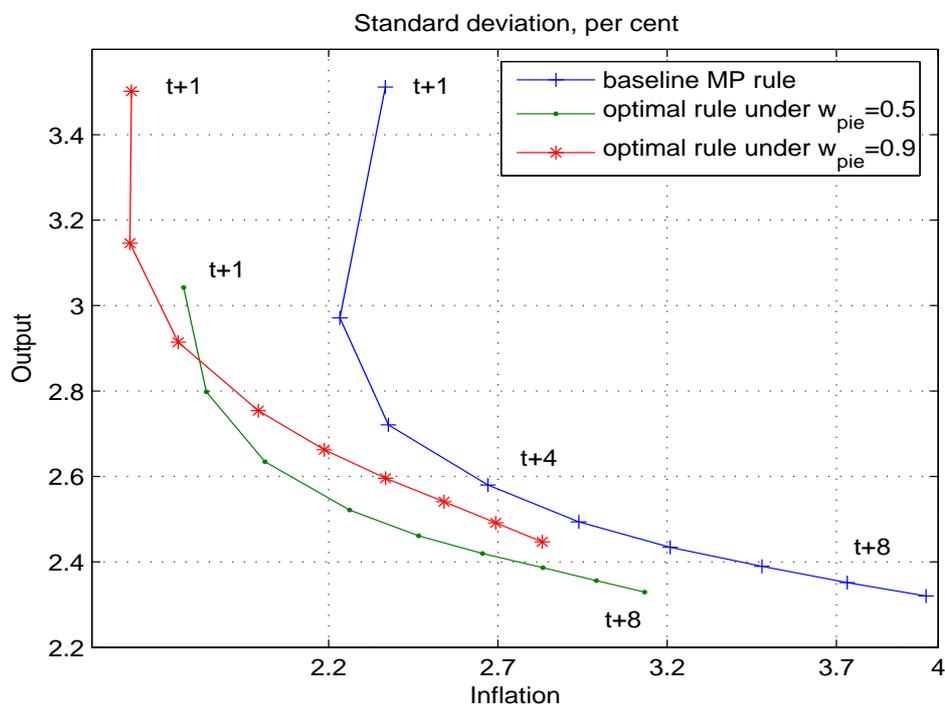
Figure 4.8 summarizes the results of the grid search by plotting the two optimized policy rules (with  $w_\pi = 0.5$  and  $0.9$  respectively) against the baseline policy rule. As can be seen from Table 4.1, when  $w_\pi = 0.5$ , and the weight on inflation and output stabilization is the same, the optimal interest rate smoothing parameter is 0.4 and the relative weight on inflation in the policy rule is 0.8. When the weight on inflation in the loss function increases to  $w_\pi = 0.9$ , the optimal interest rate smoothing parameter becomes 0.35, and the relative weight on inflation in the policy rule increases to 1 (i.e. the output gap term disappears from the optimal rule).

We see that the outcomes of the optimized policy rules in terms of inflation stabilization do not change very much, while the variability of the output gap increases rapidly. As far as the baseline policy rule is concerned, it seems that the gains from optimization lie mainly in inflation gap stabilization and mostly at very short policy horizons.

## 5. Analysis of Individual Shocks

In this section we consider the effects of the demand shock, the cost-push shock, and the shock to the nominal exchange rate separately. We assume that in each case the given shock is the only one present in the model. This allows us to draw some conclusions about how the coefficients of

**Figure 4.8: Optimized IFB Taylor rules for two relative weights on inflation in the loss function and the baseline rule at various targeting horizons**



the optimal rule and the length of the optimal targeting horizon change when the shock structure of the model changes.

It is clear that the outcome under the optimal rule will differ from that of the baseline policy rule at each targeting horizon. In order to check how important the effect of the coefficient optimization is, we report at each targeting horizon the loss values for the rule with optimized coefficients together with the losses under the baseline policy rule that sets  $m_0 = 0.75$  and  $b = 0.75$ . At higher targeting horizons than those reported in the tables below the loss function is monotonically increasing and it is not optimal to use them in the policy rule. The minima of the loss functions for both the optimized coefficient rules and the baseline rule are denoted by an asterisk.

## 5.1 The demand shock

Table 5.1 shows the loss function values when the only shock present in the model is a demand shock implemented as a 1% shock to the output gap. Under the baseline policy rule, the results suggest that when the same weight is attached to output gap and inflation stabilization,  $t + 4$  is the appropriate targeting horizon. For  $w_\pi = 0.7$  and  $0.9$ , the baseline policy rule produces the smallest loss at the horizon  $t + 3$ . It is, however, possible to improve on these outcomes by moving to the optimal rules. Under the loss function with  $w_\pi = 0.5$  and  $0.7$ , the targeting horizon  $t + 2$  seems optimal, and if we increase the weight placed on inflation stabilization to  $0.9$ , the  $t + 1$  horizon becomes optimal. In terms of the rule coefficients, the relative weight on inflation stabilization changes from  $0.7$  to  $0.8$ , which is consistent with the value of  $0.75$  implied

**Table 5.1: Optimized IFB Taylor rule coefficients for a demand shock with zero persistence**

	Loss ( $w_\pi = 0.5$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )	Loss ( $w_\pi = 0.7$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )	Loss ( $w_\pi = 0.9$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )
t+1	0.1 0.6 1.126 (1.643)	0.1 0.7 0.831 (1.189)	0.1 0.8 0.511* (0.735)
t+2	0.1 0.7 1.123* (1.416)	0.1 0.8 0.829* (1.007)	0.4 0.8 0.512 (0.599)
t+3	0.1 0.7 1.128 (1.345)	0.25 0.8 0.841 (0.962*)	0.5 0.8 0.526 (0.58*)
t+4	0.1 0.7 1.142 (1.341*)	0.3 0.7 0.859 (0.979)	0.4 0.8 0.557 (0.616)
t+5	0.1 0.6 1.165 (1.399)	0.25 0.7 0.893 (1.054)	0.4 0.7 0.607 (0.709)
t+6	0.1 0.6 1.180 (1.469)	0.1 0.7 0.921 (1.143)	0.25 0.7 0.646 (0.816)

by the baseline rule. The main difference is in the interest rate smoothing coefficient  $\rho^*$ , which is only 0.1 under all three optimal rules.

The result is not surprising, since we consider a case where only demand shocks are present in the model. As the shock to demand does not imply any trade-off between inflation and output gap stabilization, and the shock has no persistence, it is reasonable for the policy maker to stabilize the inflation gap rather "aggressively". This policy at the same time stabilizes the output gap.

The historical data, however, suggest that output gap shocks are correlated, i.e. their evolution can be described as an autoregressive process of order 1. We thus repeat our analysis of a 1% shock to the output gap assuming that

$$\epsilon_t^{ygap} = \alpha \epsilon_{t-1}^{ygap} + \nu_t$$

where  $\epsilon_t^{ygap}$  is the output gap shock and  $\nu_t$  is an independent identically distributed random variable with zero mean and the variance given in Table 2.2. Consistently with the data, we set the autocorrelation coefficient  $\alpha$  to 0.6. The results reported in Table 5.2 show that the outcomes for horizons longer than  $t+3$  are identical to those for the demand shock with zero persistence.<sup>11</sup> The optimal targeting horizon for the rule with optimized coefficients now becomes  $t+3$  for all three loss functions considered. The relative weight on the inflation gap is again comparable to that under the baseline rule, but a much lower degree of interest smoothing seems optimal; the coefficient values are between 0.1 and 0.5 rather than 0.75. The persistent demand shock results in a longer optimal forecast horizon, because it is more costly to stabilize the demand shock quickly.

<sup>11</sup> In other words, the effects of the persistent demand shock seem to vanish after two quarters.

**Table 5.2: Optimized IFB Taylor rule coefficients for a demand shock with persistence**

	Loss ( $w_\pi = 0.5$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )	Loss ( $w_\pi = 0.7$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )	Loss ( $w_\pi = 0.9$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )
t+1	0.1 0.5 2.675 (4.487)	0.1 0.6 1.965 (3.135)	0.1 0.7 1.124 (1.784)
t+2	0.1 0.5 2.634 (3.894)	0.1 0.6 1.931 (2.664)	0.25 0.7 1.111 (1.435)
t+3	0.1 0.7 1.128* (1.345)	0.25 0.8 0.841* (0.962*)	0.5 0.8 0.526* (0.58*)
t+4	0.1 0.7 1.142 (1.341*)	0.3 0.7 0.859 (0.979)	0.4 0.8 0.557 (0.616)

**Table 5.3: Optimized IFB Taylor rule coefficients for a cost-push shock**

	Loss ( $w_\pi = 0.5$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )	Loss ( $w_\pi = 0.7$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )	Loss ( $w_\pi = 0.9$ ) $\rho^*$ , $b^*$ $L$ ( $L_{QPM}$ )
t+1	0.6 0.7 0.317 (0.506)	0.6 0.7 0.339 (0.477)	0.5 0.8 0.358 (0.447)
t+2	0.6 0.8 0.316* (0.399*)	0.6 0.9 0.332* (0.404*)	0.6 0.9 0.344* (0.409*)
t+3	0.6 0.9 0.329 (0.401)	0.6 0.9 0.352 (0.434)	0.5 0.9 0.375 (0.467)
t+4	0.6 0.9 0.371 (0.457)	0.5 0.9 0.407 (0.521)	0.4 0.9 0.437 (0.585)

## 5.2 The inflation shock

We model the "cost-push" shock in the quarterly projection model as a 1% shock to all components of CPI inflation. Although macroeconomic stabilization of the "cost-push" shock always requires the real interest rate to move in the direction which destabilizes the output gap, the trade-off is swamped by the relative strength of the inflation shock. The optimal rules summarized in Table 5.3 in all cases suggest a relative weight on inflation stabilization between 0.6 and 0.9. The policy horizon  $t + 2$  minimizes all three considered loss functions under the baseline rule. The same horizon is optimal under the three rules with optimized coefficients. If cost-push shocks are predominant in the economy, a targeting horizon shorter than four quarters seems advisable. The optimal horizon for cost-push shock stabilization is the same as that for the demand shock with zero persistence. When, however, we take into account persistence, which makes stabilization at short horizons more costly, the optimal horizon for the demand shock becomes longer than that for the cost-push shock. This reflects the fact that the demand shock only affects inflation with a lag, whereas the cost-push shock affects inflation directly.

## 5.3 The exchange rate shock

The exchange rate shock is modelled as a 1% shock to the nominal exchange rate, and we again assume that the shock has zero persistence; see Table 5.4. The relative weight on the

**Table 5.4: Optimized IFB Taylor rule coefficients for an exchange rate shock**

	Loss ( $w_\pi = 0.5$ )		Loss ( $w_\pi = 0.7$ )		Loss ( $w_\pi = 0.9$ )	
	$m_0^*$	$b^*$	$m_0^*$	$b^*$	$m_0^*$	$b^*$
	$L$	$(L_{QPM})$	$L$	$(L_{QPM})$	$L$	$(L_{QPM})$
t+1	0.1	0.9	0.1	0.9	0.1	0.9
	0.361	(0.874)	0.381	(0.862)	0.401	(0.85)
t+2	0.1	0.9	0.1	0.9	0.1	0.9
	0.333*	(0.562)	0.361*	(0.588)	0.389*	(0.613)
t+3	0.3	0.9	0.3	0.9	0.3	0.9
	0.348	(0.508*)	0.384	(0.551*)	0.42	(0.594*)
t+4	0.3	0.9	0.25	0.9	0.25	0.9
	0.388	(0.562)	0.434	(0.621)	0.481	(0.681)

stabilization of inflation suggested by the rules with optimized coefficients is  $b = 0.9$  for all the combinations of parameters considered. The exchange rate shock seems predominantly to affect imported inflation, and its effects on domestic output seem negligible. The secondary importance of output gap stabilization is illustrated by the fact that the optimal policy horizon does not change, although the relative weight on inflation in the loss function changes from  $w_\pi = 0.5$  to 0.9.

The optimal targeting horizon for rules with optimized coefficients is  $t + 2$ , while the policy horizon of the baseline rule is  $t + 3$  for all three loss functions. This result, however, can only be interpreted in favour of the shorter policy horizon provided that the other parameters of the rule are brought closer to their optimized values. It seems that given the relative historical magnitude of inflation and exchange rate shocks, both variables can be reasonably well stabilized using the parametrization implied by the inflation shock analysis.

## 6. The Role of the Real Exchange Rate

In this section we present two arguments why the policy maker may want to stabilize the real exchange rate. The first one focuses on the fact that the central bank in an open economy faces a trade-off between controlling the interest rate and controlling the exchange rate, and the optimal choice of policy instrument is conditional upon the shock structure. The second argument takes on the issue of current account stabilization and the possible ways of implementing this policy objective in the framework of the quarterly projection model.

### 6.1 Motivation for real exchange rate stabilization

The case for real exchange rate stabilization may be justified by the structure of the shocks hitting the domestic economy. Parrado (2004) presents a microfounded dynamic neo-Keynesian model of Chile which includes optimizing consumers and producers, monopolistic competition and nominal price rigidities.<sup>12</sup> He argues that the motivation for real exchange rate stabilization can be traced back to the seminal analysis of Poole (1970), whose model shows that the optimal

<sup>12</sup> This is a slightly more sophisticated framework than that of the quarterly projection model. Although microfounded, the model lacks the current account equation completely. The motivation for exchange rate stabilization is restricted to the fixed versus flexible exchange rate debate.

policy instrument (and rule) depends on the nature of the shocks predominant in the economy. Under real shocks the flexible exchange rate dominates the managed one, whereas the reverse seems to be true when the shocks are predominantly nominal. The relative importance of shocks is an empirical question which can be analysed in an empirical macroeconomic model such as the quarterly projection model.

In the case of the Czech Republic and the other new EU members, this argument can be taken even further, since stabilization of the nominal exchange rate is required by the ERM stage II criteria for membership in the Eurozone. Natalucci and Ravenna (2002) and Ghironi and Rebucci (2002) recently used similar reasoning for including an exchange rate feedback term in the inflation forecast-based rules in microfounded models parametrized for emerging market economies.<sup>13</sup> Under the policy of inflation targeting, the exchange rate stabilization objective can enter the policy rule as either a nominal or real exchange rate "gap" term. Although a precise quantification of the link between the exchange rate feedback term and the ERM stage II criterion for exchange rate stability would require further calculation, it can be viewed as an additional reason for real exchange rate stabilization.

The second argument stresses the case for current account stabilization. The usual reasoning emphasizes the negative effect of current account deficits on a country's risk premium and worsening access to external financing. The Central Bank of Chile even has stability and smooth functioning of the external payments system among its formal objectives (Medina and Valdés, 2000, p.2). In practice, this objective has been interpreted as a sustainable current account deficit of up to 5% of GDP, reflecting the idea that an excessive deficit can result in a balance of payments crisis.

Medina and Valdés (2000) analyse an estimated macroeconomic model of the Chilean economy in which the variables are formulated in terms of deviations from their long-run trends. The fact that the model has no microfoundations and is formulated in "gap" form makes it very similar to the Czech quarterly projection model, including the criticism about the unmodelled "secular" trends underlying the "gap" variables. The central bank in this model has four policy objectives: it strives to keep (i) inflation close to the inflation target (ii) the output gap close to zero and (iii) the current account deficit close to its pre-announced value, formulated as a ratio to current output, and (iv) it dislikes frequent and sudden changes in policy instrument. The current period loss function can thus be written as

$$L_t = \lambda_\pi(\pi_t - \pi^*)^2 + \lambda_{ca}(ca_t - ca^*)^2 + \lambda_y(y_t - y^*)^2 + \lambda_r(\Delta r_t - \Delta r^*)^2 \quad (6.16)$$

where the variables with an asterisk now denote the inflation and current account targets, the long-run average values of potential output and the short-term real interest rate respectively.<sup>14</sup>

The difference between the quarterly projection model and the approach in Medina and Valdés (2000) is that the latter model includes an explicit current account equation. The dependent variable is the deviation of the ratio of the current account deficit to output from its target value,

<sup>13</sup> Natalucci and Ravenna (2002) report results from simulations of a microfounded two-sector model with optimizing agents calibrated on Czech data from 1994 to 2002. Their policy rules include the nominal exchange rate stabilization required by the ERM stage II, but they do not consider inflation forecast-based rules. The likely reason for this is that under the ERM stage II requirements, inflation, the exchange rate and all other variables of interest must stay within the prescribed range at all times.

<sup>14</sup> Note that the current account stabilization objective here is symmetric. It is, however, often argued that only current account deficits are potentially dangerous for the economy and that the objective should be made asymmetric.

which is a decreasing function of the domestic output gap, and an increasing function of foreign output and the real exchange rate (competitiveness).<sup>15</sup>

Although explicit introduction of the current account into the quarterly projection model is beyond the scope of this paper, the issue can be addressed indirectly. Note that the current account variable does not feed back into any other equation of the model. The model dynamics are not affected by the introduction of the current account, which is only a useful summarization of the real exchange rate and output gap effects that the central bank may wish to take into account. In other words, it is not necessary for our purposes to model the evolution of the current account explicitly. In order to describe the effect of this additional objective on the optimal policy horizon, it is enough to set a reasonable weight that current account stabilization should have in the policy maker's objective function and link this underlying objective to the modelled variability of the determinants of the current account.

One point in our implementation remains open to criticism. In what follows we assume that the real exchange rate is the most important determinant of the current account. We thus restrict our attention to the relative price and neglect the domestic and foreign income effects on the determination of the current account. Two points can be made in defence of such a simplification. First, the parametrization suggested below shows that the real exchange rate is considerably more important than domestic output in determining the current account. Second, the loss function already allows for some weight on domestic output stabilization.

## 6.2 Parametrization of the extended loss function

The proposed link between the current account balance and the real exchange rate has to be quantified. Since the current account is left unmodelled, the weight placed on its stabilization in the loss function (6.16),  $\lambda_{ca}$ , has to be replaced by a meaningful weight placed on the real exchange rate variability. The value of this parameter,  $\lambda_q$ , depends on the elasticity of the current account with respect to the real exchange rate and is indeed country-specific.

Empirical evidence reviewed in Goldstein and Khan (1985) suggests that the relative price elasticity of import demand for a typical country lies between  $-0.5$  and  $-1$ , while the relative price elasticity of export demand should lie between  $-1.25$  and  $-2.50$  (Goldstein and Khan, 1985, p.1076). The Marshall-Lerner condition seems to be met. Due to their high openness to trade, the transitional economies are, however, likely to have higher price elasticities of imports and exports. Šmídková et al. (2002) report coefficients obtained from panel data estimation for five new EU member countries and put the real exchange rate elasticity of exports at 3.15 and that of imports at  $-0.62$ . A one per cent improvement in competitiveness should thus improve the trade balance by roughly 2.5%. Hence, if the current account stabilization objective enters the loss function with weight  $x$  per cent, the real exchange rate stabilization objective should enter with a weight of  $\frac{1}{2.5}x = 0.4x$ . The current period loss function, in which the real exchange rate stabilization objective replaces the current account stabilization objective, can thus be re-written as a variant of the loss function (3.14), with an appropriately selected parameter  $w_q$ . For the Chilean economy, Medina and Valdés (2000) suggest a weight on the current account objective of 0.3, which implies that the weight of the real exchange rate variability in the loss function should be around 0.12. In the context of the Czech economy, the weights suggested for the real exchange rate variability in Hlédik (2002) are between 0.2 and 0.4, which in the light of

---

<sup>15</sup> In Medina and Valdés (2000), the estimation version of the trade balance equation proxies income effects with the short- and long-term real interest rates,  $CA/Y = f(\bar{r}, \bar{R}, \bar{q})$ .

the previous argument seems on the high side, since the Czech National Bank does not have an explicit current account objective.

We consider the following extension of the loss function

$$L_t = w_i i_t^2 + (1 - w_i)[w_\pi(\pi_t - \pi^*)^2 + w_y(y_t - y^*)^2 + w_q q_t^2] \quad (6.17)$$

where we also allow for a term penalizing variation in the level of the real exchange rate. After taking unconditional expectations we get

$$E[L_t] = w_i \text{var}(i_t) + (1 - w_i)[w_\pi \text{var}(\pi_t) + w_y \text{var}(y_t^{gap}) + w_q \text{var}(q_t)] \quad (6.18)$$

The weight put on interest rate variability is again set to 0.2. The weight on the real exchange rate  $w_q$  is set to 0.1, a value implying slightly less emphasis on current account stabilization than the number reported for the Central Bank of Chile. We consider two possible parametrizations of the remaining coefficients: the case of a "dovish" real exchange rate targeter, who sets  $w_\pi = w_y = 0.45$ , and the case of a "hawkish" real exchange rate targeter, who strongly emphasizes inflation gap stabilization,  $w_\pi = 0.8$  and  $w_y = 0.1$ .

### 6.3 The monetary policy rule under the real exchange rate objective

In theory, it is not necessary to change the policy rule when the policy objective changes. Since the inflation forecast-based rule reacts to all the state-space variables of the model, it feeds back upon the real exchange rate as well, in a way which is consistent with the policy objectives reflected in the loss function. When we search for the optimal coefficients of the policy rule, the loss function values should penalize coefficient values that result in excessive real exchange rate variability.

In practice, however, we often assume that when the exchange rate stabilization objective is implemented, the policy maker will introduce new instrument variable(s) into her reaction function. In our case, the nominal or real exchange rate gap is the most obvious candidate. Parrado (2004) complements a loss function of the form (6.16) with an inflation forecast-based rule of the same structure as the one employed in the quarterly prediction model

$$i_t^{gap} = \rho i_{t-1}^{gap} + (1 - \rho)[\alpha_1 E_t \pi_{t+k}^{gap} + \alpha_2 y_t^{gap} + \alpha_3 s_t^{gap}] \quad (6.19)$$

where  $\rho = 0.7$ ,  $\alpha_1 = 1.5$ ,  $\alpha_2 = 0.5$ , and  $\alpha_3 = 3.34$ , and the "gap" variables are per cent deviations from the inflation target, potential output and the steady state value of the nominal exchange rate. A similar policy rule is used in Soto (2003), where the exchange rate feed back term is specified in terms of the real exchange rate gap,  $q_t^{gap}$ . The coefficient values for  $\rho$ ,  $\alpha_1$  and  $\alpha_2$  are identical, and  $\alpha_3$  is now set to 2.5.

The coefficients of both policy rules are calibrated for the Chilean economy in the 1990s and hence are of only partial interest as far as the Czech macroeconomic model is concerned. They also put a relatively high weight on exchange rate stabilization. The relative weights on inflation and the output gap in the former are 0.28 and 0.1 respectively, leaving more than 60% for nominal exchange rate stabilization. The respective values for the latter are 0.33, 0.11 and 0.55, which is still rather high considering that the Czech National Bank is not explicitly stabilizing the exchange rate. We thus conduct a search for the optimal policy rule coefficients using the

**Table 6.1: Relative weights for the coefficients of the extended policy rule**

	$\alpha_\pi$	$\alpha_y$	$\alpha_q$		$\alpha_\pi$	$\alpha_y$	$\alpha_q$
<b>1.</b>	0.8	0.1	0.1	<b>5.</b>	0.5	0.1	0.4
<b>2.</b>	0.6	0.1	0.3	<b>6.</b>		0.2	0.3
<b>3.</b>		0.2	0.2	<b>7.</b>		0.25	0.25
<b>4.</b>		0.3	0.1	<b>8.</b>		0.3	0.2
				<b>9.</b>		0.4	0.1

**Table 6.2: Optimized IFB Taylor rule coefficients under the real exchange rate objective**

	"Dovish" RER targeter				"Hawkish" RER targeter			
	$\rho^*$	$\alpha_\pi$	$\alpha_y$	Loss	$\rho^*$	$\alpha_\pi$	$\alpha_y$	Loss
t+1	0.25	0.6	0.3	8.01	0.1	0.6	0.3	6.834
t+2	0.3	0.6	0.3	7.455	0.25	0.8	0.1	6.399
t+3	0.3	0.6	0.3	7.349	0.3	0.8	0.1	6.189*
t+4	0.1	0.8	0.1	7.282*	0.1	0.8	0.1	6.47
t+5	0.1	0.8	0.1	7.452	0.1	0.8	0.1	6.96
t+6	0.1	0.8	0.1	7.745	0.1	0.8	0.1	7.539

baseline formulation of the policy rule in the quarterly projection model, to which we add the real exchange rate feedback term in the following form

$$i_t = \rho i_{t-1} + (1 - \rho)(i^{eq} + 1.6[\alpha_\pi(E_t\pi_{4t+k} - \pi_{4t}^*) + \alpha_y y_t^{gap} + \alpha_q q_t^{gap}]) \quad (6.20)$$

where  $\rho$  is the coefficient on interest rate smoothing,  $E_t\pi_{4t+k} - \pi_{4t}^*$  is the inflation gap, and  $y_t^{gap}$  and  $q_t^{gap}$  are per cent deviations of output and the real exchange rate respectively from their long-run trends. The relative weight on the inflation gap,  $\alpha_\pi$  is increased from 0.5 to 0.8, with the relative weights on the output and exchange rate gaps spanning from 0.1 to 0.4 respectively. The full grid of the combinations  $(\alpha_\pi, \alpha_y, \alpha_q)$ , which in all the rules sum up to one, is reported in Table 6.1. As for the degree of interest rate smoothing, for each of the nine policy rules considered we vary the parameter  $\rho$  from 0.1 to 0.9.

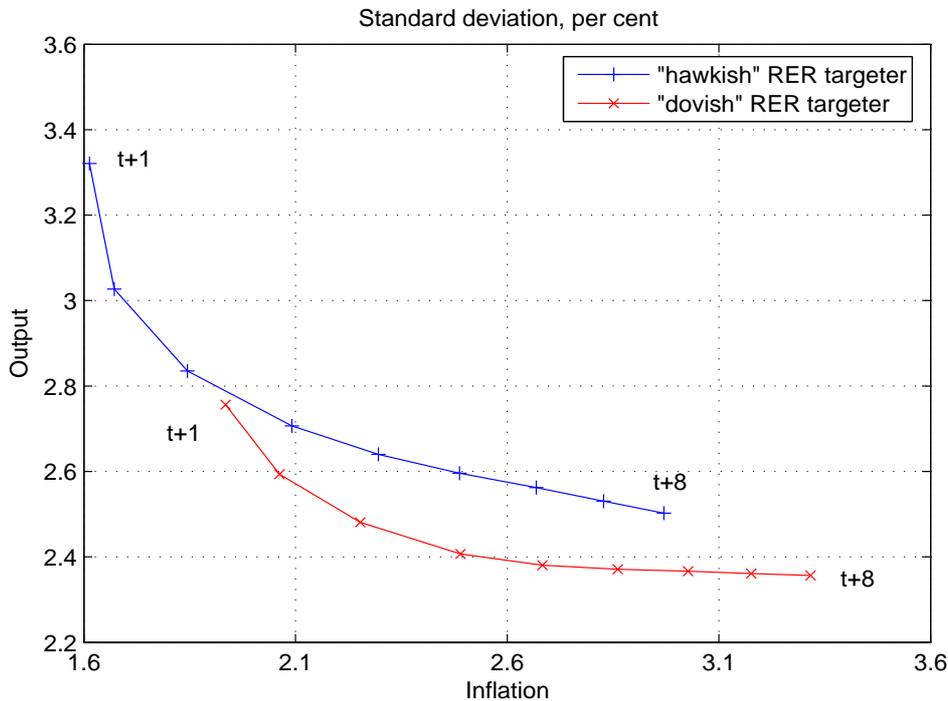
This is arguably a rather simple grid of coefficient values, and the results reported below should be treated with caution. As before, the non-reported horizons are those where the loss function is monotonically increasing. The minimum loss over all considered horizons is denoted by an asterisk.

#### 6.4 Optimal simple rule with the real exchange rate term

In the search for an extended optimized rule we return to the full variance-covariance matrix in Table 2.2. The rules reported in Table 6.2 are thus derived under the assumption of the same shock structure as the optimized IFB Taylor rules. The optimal policy horizon in the case of the "hawkish" targeter remains at  $t + 3$ , while the higher relative weight put on output gap stabilization results in a longer optimal horizon,  $t + 4$ .

Out of the nine possible specifications of the policy rule considered in the search, only rules (1) and (4) were found optimal. The optimal relative weight on the real exchange rate hence

**Figure 6.1: Optimized IFB rules with the real exchange rate feedback term at various targeting horizons (standard deviation of output and inflation, per cent)**



equals 0.1 for all the policy horizons of the two loss functions considered. The degree of interest rate smoothing can, however, act as a substitute for the insufficient coefficient variability in the search. The "hawkish" targeter predominantly uses rule (1), which has a relative weight on inflation of 0.8, while the minimum of the loss for the "dovish" targeter is attained with rule (4), where  $\alpha_\pi = 0.6$ .

The results are similar in structure to the findings from the previous grid search. The optimal policy horizon is longer when the policy maker gives more weight to output gap stabilization. Moreover, the loss function with the real exchange rate objective requires only a relatively low feedback coefficient on the real exchange rate term. Figure 6.1 plots the inflation and output variabilities of the two optimized policy rules with the real exchange rate term. Although the coefficients  $\rho$  and  $\alpha_\pi$ ,  $\alpha_y$  are only optimized for one of the nine targeting horizons ( $t + 3$  and  $t + 4$  respectively), the reduction in variability with respect to the baseline rule is clearly visible.

## 7. Conclusions

In this paper we have used the quarterly projection model of the Czech National Bank and a variance-covariance matrix of historical shocks to derive a set of inflation forecast-based policy rules with optimized coefficients. The simulation results presented in this paper suggest that the baseline policy rule of the Czech National Bank targets CPI inflation close to the optimal targeting horizon of three quarters. There are, however, at least two other ways in which the performance of the rule could be improved. We have seen that the optimal simple rule under all versions of the loss function considered in this paper suggests that a higher relative weight

should be placed on inflation gap stabilization. Moreover, the coefficient of the partial interest rate adjustment seems to be set too high in the current version of the policy rule. Although these two findings seem to imply a simple way of improving upon the current rule, it should be noted that the results were derived from a model which does not allow for uncertainty.

The results from the analysis of individual shocks show that a demand shock with zero persistence shortens the optimal policy horizon even further. This finding is however not robust and vanishes when we introduce persistent demand shocks. The optimal policy horizon under cost-push shocks becomes shorter, since cost-push shocks feed directly in to inflation. The optimal inflation forecast-based rule now has a similar degree of interest rate smoothing and a similar relative weight on the inflation gap as the non-optimized baseline policy rule. It seems that a baseline rule with a shorter policy horizon would be a good approximation of the optimized inflation forecast-based rule when cost-push shocks dominate.

The analysis of the loss functions with a real exchange rate term does not alter the previous results, although it provides some evidence against policy horizons shorter than three quarters. It also suggests that the extended optimal policy rules should provide a degree of inflation and output gap stabilization comparable with the optimal simple rules.

## References

- BALL, L. (1999): *Policy Rules for Open Economies*. In John B. Taylor, editors, *Monetary Policy Rules*, pages 127–156. University of Chicago Press, Chicago.
- BATINI, N. AND HALDANE, A. (1999): *Forward-Looking Rules for Monetary Policy*. In John B. Taylor, editors, *Monetary Policy Rules*, pages 157–192. University of Chicago Press, Chicago.
- BENEŠ, J. (2004): “IRIS: A Toolbox for the Quarterly Projection Model of the Czech National Bank.” Unpublished manual, Czech National Bank, Prague.
- BENEŠ, J. AND N’DIAYE, P. (2003): *A Multivariate Filter for Measuring Potential Output and the NAIRU*. In Coats, W., Laxton, D. and Rose, D., editors, *The Czech National Bank’s Forecasting and Policy Analysis System*, pages 99–118. Czech National Bank, Prague.
- BENEŠ, J., VÁVRA, D., AND VLČEK, J. (2002): “Střednědobá makroekonomická predikce.” *Finance a úvěr*, 52(4):197–231.
- CLARIDA, R., GALÍ, J., AND GERTLER, M. (1997): “Monetary Policy Rules in Practice: Some International Evidence.” *European Economic Review*, 42(2):1033–1067.
- COATS, W., LAXTON, D., AND ROSE, D. editors (2003): *The Czech National Bank’s Forecasting and Policy Analysis System*, Prague. Czech National Bank.
- DENNIS, R. (2003): “Optimal Policy Rules in Rational-Expectations Models: New Solution Algorithms.” Working paper, Federal Reserve Bank of San Francisco, San Francisco.
- DENNIS, R. (2004): “Solving for Optimal Simple Rules in Rational Expectations Models.” *Journal of Economic Dynamics and Control*, 28:1635–1660.
- GHIRONI, F. AND REBUCCI, A. (2002): “Monetary Policy Rules for Emerging Market Economies.” Working Paper WP/02/34, International Monetary Fund, Washington, D.C.
- GOLDSTEIN, M. AND KHAN, M. (1985): *Income and Price Effects in Foreign Trade*. In Jones, R. W. and Kenen, P. B., editors, *Handbook of International Economics*, chapter 20, pages 1041–1105. Elsevier Science Publishers, Amsterdam.
- HAMILTON, J. D. (1994): *Time Series Analysis*. Princeton University Press, Princeton.
- HLÉDIK, T. (2002): “Optimální pravidla měnové politiky v dynamickém modelu České ekonomiky.” *Finance a úvěr*, 52(3):164–185.
- HURNÍK, J. AND VLČEK, J. (2004): “Horizont měnové politiky v QPM.” Unpublished paper, Czech National Bank, Prague.
- LEVINE, P. AND CURRIE, D. (1987): “The Design of Feedback Rules in Linear Stochastic Rational Expectations Models.” *Journal of Economic Dynamics and Control*, 11:1–28.
- MEDINA, J. P. AND VALDÉS, R. (2000): “Optimal Monetary Policy Rules When the Current Account Matters.” Working paper 77, Central Bank of Chile, Santiago.
- NATALUCCI, F. AND RAVENNA, F. (2002): “The Road to Adopting the Euro: Monetary Policy and Exchange Rate Regimes in EU Candidate Countries.” International Finance Discussion Paper 741, Board of Governors of the Federal Reserve System

- ONATSKI, A. AND WILLIAMS, N. (2004): “Empirical and policy performance of a forward-looking monetary model.” *Proceedings of the Federal Reserve Bank of San Francisco*. Available at <http://ideas.repec.org/a/fip/fedfpr/y2004imarx6.html>
- PARRADO, E. (2004): “Inflation Targeting and Exchange Rate Rules in an Open Economy.” Working paper 04/21, International Monetary Fund, Washington, D.C.
- POOLE, W. (1970): “Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model.” *Quarterly Journal of Economics*, 84(2):197–216.
- RUDEBUSCH, G. AND SVENSSON, L. E. O. (1999): *Policy Rules for Inflation Targeting*. In John B. Taylor, editors, *Monetary Policy Rules*, pages 203–246. University of Chicago Press, Chicago.
- ŠMÍDKOVÁ, K., BARRELL, R., AND HOLLAND, D. (2002): “Estimates of Fundamental Real Exchange Rates for the Five EU Pre-Accession Countries.” Working paper 03/2002, Czech National Bank, Prague.
- SÖDERLIND, P. (1999): “Solution and estimation of RE macromodels with optimal policy.” *European Economic Review*, 43(4-6):813–823.
- SOTO, C. (2003): “Monetary Policy, Real Exchange Rate, and the Current Account in a Small Open Economy.” Working paper 253, Central Bank of Chile
- SVENSSON, L. E. O. (1997): “Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets.” *European Economic Review*, 41:1111–1146.
- SVENSSON, L. E. O. (1998): “Open-Economy Inflation Targeting.” Working paper 6545, National Bureau of Economic Research, New York.
- SVENSSON, L. E. O. (1999): “Inflation Targeting as a Monetary Policy Rule.” *Journal of Monetary Economics*, 43:607–654.
- TAYLOR, J. B. (1993): “Discretion versus Policy Rules in Practice.” *Carnegie Rochester Conference Series on Public Policy*, 39:195–214.
- WOODFORD, M. (2000): “Pitfalls of Forward-Looking Monetary Policy.” *American Economic Review*, 90(2):100–104.
- WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, New Jersey.

## CNB RESEARCH AND POLICY NOTES

5/2005	Jan Stráský:	<i>Optimal forward-looking policy rules in the quarterly projection model of the Czech National Bank</i>
4/2005	Vít Bárta:	<i>Fulfilment of the Maastricht inflation criterion by the Czech Republic: Potential costs and policy options</i>
3/2005	Helena Šůvová: Eva Kozelková David Zeman Jaroslava Bauerová	<i>Eligibility of external credit assessment institutions</i>
2/2005	Martin Čihák: Jaroslav Heřmánek	<i>Stress testing the Czech banking system: Where are we? Where are we going?</i>
1/2005	David Navrátil: Viktor Kotlán	<i>The CNB's policy decisions – Are they priced in by the markets?</i>
4/2004	Aleš Bulíř:	<i>External and fiscal sustainability of the Czech economy: A quick look through the IMF's night-vision goggles</i>
3/2004	Martin Čihák:	<i>Designing stress tests for the Czech banking system</i>
2/2004	Martin Čihák:	<i>Stress testing: A review of key concepts</i>
1/2004	Tomáš Holub:	<i>Foreign exchange interventions under inflation targeting: The Czech experience</i>
2/2003	Kateřina Šmídková:	<i>Targeting inflation under uncertainty: Policy makers' perspective</i>
1/2003	Michal Škořepa: Viktor Kotlán	<i>Inflation targeting: To forecast or to simulate?</i>

## CNB WORKING PAPER SERIES

14/2005	Stephen G. Cecchetti:	<i>The brave new world of central banking: The policy challenges posed by asset price booms and busts</i>
13/2005	Robert F. Engle: Jose Gonzalo Rangel	<i>The spline GARCH model for unconditional volatility and its global macroeconomic causes</i>
12/2005	Jaromír Beneš: Tibor Hlédik Michael Kumhof David Vávra	<i>An economy in transition and DSGE: What the Czech national bank's new projection model needs</i>
11/2005	Marek Hlaváček: Michael Koňák Josef Čada	<i>The application of structured feedforward neural networks to the modelling of daily series of currency in circulation</i>
10/2005	Ondřej Kameník:	<i>Solving DSGE models: A new algorithm for the Sylvester equation</i>
9/2005	Roman Šustek:	<i>Plant-level nonconvexities and the monetary transmission mechanism</i>
8/2005	Roman Horváth:	<i>Exchange rate variability, pressures and optimum currency area criteria: Implications for the central and eastern European countries</i>
7/2005	Balázs Égert: Luboš Komárek	<i>Foreign exchange interventions and interest rate policy in the Czech Republic: Hand in glove?</i>

6/2005	Anca Podpiera: Jiří Podpiera	<i>Deteriorating cost efficiency in commercial banks signals an increasing risk of failure</i>
5/2005	Luboš Komárek: Martin Melecký	<i>The behavioural equilibrium exchange rate of the Czech koruna</i>
4/2005	Kateřina Arnoštová: Jaromír Hurník	<i>The monetary transmission mechanism in the Czech Republic (evidence from VAR analysis)</i>
3/2005	Vladimír Benáček: Jiří Podpiera Ladislav Prokop	<i>Determining factors of Czech foreign trade: A cross-section time series perspective</i>
2/2005	Kamil Galuščák: Daniel Münich	<i>Structural and cyclical unemployment: What can we derive from the matching function?</i>
1/2005	Ivan Babouček: Martin Jančar	<i>Effects of macroeconomic shocks to the quality of the aggregate loan portfolio</i>
10/2004	Aleš Bulíř: Kateřina Šmídková	<i>Exchange rates in the new EU accession countries: What have we learned from the forerunners</i>
9/2004	Martin Cincibuch: Jiří Podpiera	<i>Beyond Balassa-Samuelson: Real appreciation in tradables in transition countries</i>
8/2004	Jaromír Beneš: David Vávra	<i>Eigenvalue decomposition of time series with application to the Czech business cycle</i>
7/2004	Vladislav Flek, ed.:	<i>Anatomy of the Czech labour market: From over-employment to under-employment in ten years?</i>
6/2004	Narcisa Kadlčáková: Joerg Keplinger	<i>Credit risk and bank lending in the Czech Republic</i>
5/2004	Petr Král:	<i>Identification and measurement of relationships concerning inflow of FDI: The case of the Czech Republic</i>
4/2004	Jiří Podpiera:	<i>Consumers, consumer prices and the Czech business cycle identification</i>
3/2004	Anca Pruteanu:	<i>The role of banks in the Czech monetary policy transmission mechanism</i>
2/2004	Ian Babetskii:	<i>EU enlargement and endogeneity of some OCA criteria: Evidence from the CEECs</i>
1/2004	Alexis Derviz: Jiří Podpiera	<i>Predicting bank CAMELS and S&amp;P ratings: The case of the Czech Republic</i>
12/2003	Tibor Hlédik:	<i>Modelling the second-round effects of supply-side shocks on inflation</i>
11/2003	Luboš Komárek: Zdeněk Čech Roman Horváth	<i>ERM II membership – the view of the accession countries</i>
10/2003	Luboš Komárek: Zdeněk Čech Roman Horváth	<i>Optimum currency area indices – how close is the Czech Republic to the eurozone?</i>
9/2003	Alexis Derviz: Narcisa Kadlčáková Lucie Kobzová	<i>Credit risk, systemic uncertainties and economic capital requirements for an artificial bank loan portfolio</i>
8/2003	Tomáš Holub: Martin Čihák	<i>Price convergence: What can the Balassa–Samuelson model tell us?</i>

7/2003	Vladimír Bezděk: Kamil Dybczak Aleš Krejdl	<i>Czech fiscal policy: Introductory analysis</i>
6/2003	Alexis Derviz:	<i>FOREX microstructure, invisible price determinants, and the central bank's understanding of exchange rate formation</i>
5/2003	Aleš Bulíř:	<i>Some exchange rates are more stable than others: Short-run evidence from transition countries</i>
4/2003	Alexis Derviz:	<i>Components of the Czech koruna risk premium in a multiple-dealer FX market</i>
3/2003	Vladimír Benáček: Ladislav Prokop Jan Á. Víšek	<i>Determining factors of the Czech foreign trade balance: Structural Issues in Trade Creation</i>
2/2003	Martin Čihák: Tomáš Holub	<i>Price convergence to the EU: What do the 1999 ICP data tell us?</i>
1/2003	Kamil Galuščák: Daniel Münich	<i>Microfoundations of the wage inflation in the Czech Republic</i>
4/2002	Vladislav Flek: Lenka Marková Jiří Podpiera	<i>Sectoral productivity and real exchange rate appreciation: Much ado about nothing?</i>
3/2002	Kateřina Šmídková: Ray Barrell Dawn Holland	<i>Estimates of fundamental real exchange rates for the five EU pre-accession countries</i>
2/2002	Martin Hlušek:	<i>Estimating market probabilities of future interest rate changes</i>
1/2002	Viktor Kotlán:	<i>Monetary policy and the term spread in a macro model of a small open economy</i>

#### **CNB ECONOMIC RESEARCH BULLETIN**

November 2005	<i>Financial stability</i>
May 2005	<i>Potential output</i>
October 2004	<i>Fiscal issues</i>
May 2004	<i>Inflation targeting</i>
December 2003	<i>Equilibrium exchange rate</i>



Czech National Bank  
Economic Research Department  
Na Příkopě 28, 115 03 Praha 1  
Czech Republic  
phone: +420 2 244 12 321  
fax: +420 2 244 14 278  
<http://www.cnb.cz>  
e-mail: [research@cnb.cz](mailto:research@cnb.cz)