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Abstract

Monetary policy makers need to build two pillars for their inflation targeting strategy. Firstly, a methodology for producing the central forecast should be developed, since the whole decision process is more easily organised around a model forecast. Secondly, a methodology for dealing with uncertainties is equally important, because a poor evaluation of uncertainties can significantly reduce the quality of monetary policy decisions. Reflecting the further progress of the methodological debate inside the CNB, this paper aims to provide suggestions to policy makers as to which methods could be used to assess uncertainty during the monetary policy decision process. Suggestions for each stage of the process are summarised in the final chapter. These take into account the findings of surveys of three very distinct sources – the economic literature on monetary policy under uncertainty, the managerial literature on decision analysis, and the real-life strategies of five central banks. The lessons from these three surveys are presented in separate chapters.

Nontechnical summary

Monetary policy makers need to build two pillars for their inflation targeting strategy. Firstly, a methodology for producing the central forecast should be developed, since the whole decision process is more easily organised around a model forecast. Secondly, a methodology for dealing with uncertainties is equally important, because a poor evaluation of uncertainties can significantly reduce the quality of monetary policy decisions. Reflecting the further progress of the methodological debate inside the CNB, this paper aims to provide suggestions to policy makers as to which methods could be used to assess uncertainty during the monetary policy decision process. While there is a consensus that the decision process should be organised around the model forecast, it is much more difficult to develop an effective methodology for dealing with uncertainty. Since there are no easy solutions in this area, we formulate our suggestions taking into account the findings of surveys of three very distinct sources – the economic literature on monetary policy under uncertainty, the managerial literature on decision analysis, and the real-life strategies of five central banks.

There are two essential reasons why policy makers need a good methodology for dealing with uncertainty. Firstly, if an inadequate methodology is applied to dealing with uncertainty, it can lead to defects in the decision-making process. If uncertainty is neglected and policy interest rates are changed only according to the central forecast, changes in policy interest rates can be too large or too small with respect to the optimal policy response. Sub-optimal policy reactions can burden the economy with otherwise avoidable costs. If, on the other hand, the selected methodology tries to take in too many uncertainties during the decision-making process, it can produce interval forecasts that are too wide. Any comparison of these inflation forecasts with the inflation target then cannot give policy makers a clear picture of how to deal with interest rates. Secondly, a lack of communication about the uncertainty attached to the central forecast can lead the general public to think that any deviation of actual inflation from the forecast is the result of monetary policy errors. Hence, too little information on uncertainty can damage the credibility of the whole inflation targeting strategy, and this damage can be costly for the economy.

There is a whole stream of literature on monetary policy under uncertainty which illustrates that monetary policy makers are faced with different types of uncertainty, for example, parameter uncertainty, model uncertainty or data uncertainty, and each type has its specific implications for monetary policy decisions. In the typical research paper, a model of the economy with endogenous monetary policy is used to produce the baseline simulation. Then a selected type of uncertainty is introduced, and the outcome of the new simulation is compared to the baseline. Depending on the type of uncertainty selected, the new interest rate path is more or less volatile than the baseline one. Based on this analysis, the conclusion is derived that policy makers should react more aggressively or more cautiously if they are faced with uncertainty. Since the conclusion is dependent on the model used for the simulations as well as on other components of the selected methodology, one can find different conclusions for similar types of uncertainty. This is one source of discomfort to policy makers. In addition, in their papers, policy makers often emphasise that the uncertainty they face in real life when deciding about interest rates is different from the one analysed in the research papers. The former cannot be easily described with a probability distribution, whereas the latter is typically represented with a normal probability distribution.

The discomfort of monetary policy makers with the outcomes of economic research is similar to the discomfort of decision makers in other areas. All decision makers face uncertainty. They can use the outcomes of mathematical models and statistical techniques as important inputs into their decision process. However, they cannot rely solely on them. Decision makers need to combine various types of information, including various sensitivity tests and subjective judgements. Therefore, we suggest learning from other sources, not just from the economic research literature, about effective methodology for dealing with uncertainty. Since the necessity of deciding on important issues under uncertainty is not a problem solely related to monetary policy decisions, there is a very large demand for methods for overcoming this problem. For this purpose, decision theory and decision analysis offer various tools for all stages of the decision-making process. Some tools rooted in mathematics, such as forecasting models and stochastic optimisation, are designed to organise all the available data. Other decision analysis tools that are less mathematically rigorous, such as pay-off matrices and decision trees, work with other types of information. Specifically, these tools use subjective evaluation of uncertainties and are aimed at reaching good decisions in an uncertain world.

We confront the findings from the literature with the "real-life" methods that five inflation targeters employ in dealing with uncertainty in the internal decision-making process as well as in communication with the general public. The case studies, based on publicly available sources, document that monetary policy makers do not limit themselves to producing the central forecast when deciding about interest rates. They follow the methods recommended by decision analysis, however informal or implicit this may be. In the first step, policy makers use various forecasting techniques in order to organise the data. In the second step, they use intentionally a combination of several methods for dealing with uncertainty. Some of the methods are very close analogies to the methods recommended by decision analysis. For example, several central banks attach subjective distributions to the central forecast in order to produce fan charts. Several banks use alternative scenarios to deal with their uncertainty about important external factors such as commodity prices. Some central banks also let a group of experts vote prior to the meeting of the monetary policy committee on the optimal policy reaction.

We highlight the following points as far as dealing with uncertainty is concerned. It is important to employ a methodology that helps to organise very different types of information, for example the central projection and subjective probabilities, into one framework. For this purpose, decision analysis offers a very convenient tool in the form of the pay-off matrix. We suggest using the pay-off matrix informally. The process of constructing the matrix corresponds to the individual stages of the monetary decision process. It is possible to choose from various methods for constructing each particular component of the matrix. The central forecast is the first element of the matrix to be constructed. Then several alternative sets of assumptions are specified that illustrate key uncertainties about the model or some assumption of the central forecast. Also, all feasible policy reactions are spelled out. For all important combinations of alternative assumptions and feasible reactions, the model simulations are conducted and pay-offs are evaluated. In the next stage, subjective probabilities that can imply asymmetric distributions around the central forecast are attached to the alternative assumptions. Finally, monetary policy makers may choose between various decision strategies to determine the best policy reaction. It is worth noting that under certain circumstances an incomplete pay-off matrix can be sufficient for monetary policy decisions. However, in a period of considerable uncertainty, all components of the matrix should be informally constructed prior to the final verdict on policy interest rates.

1. Introduction

The rapidly increasing number of inflation targeters is proof that inflation targeting in the current environment is an effective strategy for stabilising inflation. However, everything comes at a price. This high efficiency goes hand in hand with increased demands on the quality of the decision-making process and communication with the general public. It is no coincidence that research on the implications of uncertainty for conducting monetary policy has increased significantly over the last decade, as illustrated in Hund, Orr (1999) or Salmon, Martin (1999). This was a period in which inflation targeters were developing their methodologies for producing the best possible inflation forecasts, given the constraints of imperfect knowledge about the current state of the economy and even less perfect knowledge about future economic events. Although the inflation targeting framework has reduced uncertainty about the goals and instruments of monetary policy, policy makers targeting inflation will always be decision makers reaching their verdicts about interest rates under considerable uncertainty, and thus will always search for methods that can reduce the negative consequences of uncertainty on the quality of decisions and communication.

There are two essential reasons why inflation targeters need good methodology for dealing with uncertainty. Firstly, monetary policy makers need to make qualified decisions about the policy interest rates. Unless a suitable methodology is applied to dealing with uncertainty, imperfect knowledge about the current and future state of the economy can lead to defects in the decision-making process. Specifically, uncertainty can be neglected, and the policy interest rates can be changed only according to the central inflation forecast. This implies, as illustrated in Brainard (1967) or in Leiderman (1999), that changes in policy interest rates can be too large or too small with respect to an optimal policy response, and that sub-optimal policy reactions can burden the economy with otherwise avoidable costs, for example with excessive output loss. If – on the other hand – the selected methodology tries to take in too many uncertainties during the decision-making process, it can produce too many alternative inflation forecasts or interval forecasts that are too wide. Any comparison of these inflation forecasts with the inflation target then cannot give policy makers a clear picture of how to deal with interest rates. For example, one alternative forecast can be above the target while the other one can be very close to the target, signalling the need to increase policy rates and to leave them unchanged at the same time.

The second important reason why central bankers need good methodology for dealing with uncertainty is to make the inflation targeting framework transparent. The credibility of the inflation targeting strategy depends on transparency, and the loss of monetary policy credibility can be costly for the economy, as analysed in Geraats (2001). The lack of communication about the uncertainty attached to the central inflation forecast can lead the general public to thinking that this forecast is fully unconditional and that any deviation of actual inflation from the forecast is the result of monetary policy errors. Hence, too little information on uncertainty can damage the credibility of the whole inflation targeting strategy. Similar damage can be caused by the disorganised way uncertainty, resulting from the forecasts of exogenous variables, the forecasting model or additional off-model information, is communicated. An unclear message about the methods of dealing with uncertainty and evaluation of the key uncertainties for a particular decision on policy interest rates can prevent financial markets from understanding the direction of

monetary policy and the general public from distinguishing the consequences of unforeseen external shocks from policy errors.

The Czech National Bank (CNB) started targeting inflation in 1998. During these five years, the Czech approach to inflation targeting has gone through several stages of development relating to in-house knowledge and surrounding conditions. During each stage of development, the method of dealing with uncertainty has been different. The experience gained is similar to that of "senior" inflation targeters, who have been gradually improving their methodologies since the beginning of the 1990s. Initially, the CNB described the inflation outlook verbally. In the subsequent stage, the CNB started publishing the year-end interval inflation forecast. After the modelling knowledge reached the required level of development, the year-end forecast was replaced with publication of a chart representing the inflation outlook for the whole period of transmission. The chart is based on the forecast that uses the model projection with endogenous monetary policy in combination with expert input. The bands surrounding the central forecast and the verbal description of risks illustrate that monetary decisions are subject to uncertainty. This description of the current stage implies that, while the methodology for producing the central forecast is comparable to the approaches of other inflation targeters, there is still a lot to be learnt as far as the methodology for dealing with uncertainty is concerned. Before an explicit choice for this methodology is made, we think that it is very important to compare all methods available either in the literature on monetary policy under uncertainty, in the literature on decision analysis or in case studies.

The rest of the paper is organised as follows. The second chapter reviews the conclusions of the literature on monetary policy under uncertainty. The conclusions show clearly that unless the world can be sufficiently well approximated with quadratic-linear models and additive, normally distributed shocks, monetary policy makers should not neglect uncertainty. In a more complicated world, policy reactions formed under the assumption of certainty are likely to departure from the optimal policy path. The literature also emphasises that monetary policy makers are faced with different types of uncertainty, for example, parameter uncertainty, model uncertainty or data uncertainty, and each type has its specific implications for monetary policy makers. In their papers, policy makers often explain that models and policy rules cannot explain fully monetary policy decisions due to their lack of ability to deal with uncertainty faced by policy makers. Policy makers emphasise that the economic research has focused primarily on implications of risks, not uncertainty. Although the literature on monetary policy under uncertainty is extensive, it does not offer explicit advice on which methodology is effective when deciding about interest rates under uncertainty. We suggest searching elsewhere for an effective methodology. Therefore, we present the findings of decision theory and decision analysis in the third chapter. The necessity of deciding on the important issues under uncertainty is not a problem solely related to monetary policy decisions. Every decision maker deals with uncertainty, and hence, demand for methods on how to overcome this problem is very large. For this purpose, decision theory and decision analysis offer various tools for all stages of the decision-making process. Some tools rooted in mathematics, such as forecasting models or stochastic optimisation, are designed to organise all available data. They are very similar to tools used by inflation targeters for producing the central projection. Other decision analysis tools that are less mathematically rigorous, such as a pay-off matrix or decision trees, work with other types of information. Specifically, these tools use the subjective evaluation of uncertainties and are aimed at reaching good decisions in an uncertain world. These less formal tools resemble some methods used by inflation targeters, such as subjective distributions on fan charts. In the fourth chapter, five case studies are briefly described.

These studies illustrate the "real-life" methods of five inflation targeters in dealing with uncertainty in the internal decision-making process as well as in communication with the general public. The case studies document that policy makers do not limit themselves to producing the central forecast when deciding about interest rates. In addition to the model forecast, they use intentionally several methods for dealing with uncertainty. Some methods are very close analogies to the methods recommended by decision analysis. In the conclusive part, we highlight several lessons for the Czech approach to inflation targeting as far as dealing with uncertainty is concerned. Specifically, we suggest using the pay-off matrix informally and propose various methods of constructing the matrix. The methodology provided by this tool from decision analysis can help in consistently evaluating all of the important uncertainties during monetary policy decisions.

2. Uncertainty in recent literature on monetary policy

Literature overview

During the 1990s, issues related to uncertainty played an increasingly more important role in monetary policy literature. A tremendous motivation for research was provided by the increasing number of inflation-targeting central banks. All research papers emphasise that the starting point of this debate is the analysis presented already in Brainard (1967). He claimed that policy makers faced by uncertainty about their model of the economy should react differently with policy interest rates than policy makers who are fully certain about their knowledge of the economy. For his analysis, he used a specific type of the model framework called "linear-quadratic" due to functional forms of model equations. Then he introduced uncertainty about one of the parameters in the model. He has concluded that monetary policy makers should react less with policy interest rates in the presence of parameter uncertainty than in the case of certainty. The framework used by the current research papers is analogous. One example of the quadratic-linear framework is described in Appendix I. In these papers, a specific type of uncertainty is usually introduced into the model in order to explain why outcomes of policy rules differ from the reactions of central bankers. Hence, the main emphasis is on explaining differences between the model and real life. However, policy makers have different motivation that has been illustrated in Battini, Haldane (1999). They show that if policy makers targeting inflation react too aggressively or are too cautious when changing policy interest rates, it is costly for the economy in terms of excessive output and inflation volatility. Policy makers should know what types of uncertainty they face and what the implications are for optimal policy reaction in comparison to the certainty case. Otherwise, their policy reactions based solely on the forecasts produced under the certainty assumption would be too costly for the economy.

Referring to Brainard's pioneering paper that has focused on one specific case of the parameter uncertainty, the recent literature has focused on classification of the alternative sources of uncertainty faced by monetary policy makers, and on analysing the consequences of different types of uncertainty for monetary policy decisions. This stream of literature has illustrated that it is not possible to use Brainard's rule as a rule of thumb, since a more aggressive reaction in comparison to the certainty case is required from policy makers under other types of uncertainty. In the late 1990s, the literature on monetary policy started to deal with the problem of

distinguishing between risk and uncertainty. The difference between risk and uncertainty has been defined by Knight (1921) in the second pioneering paper on uncertainty. According to him, if policy makers are able to estimate probability distribution of a certain event, this event is called a risk. If policy makers cannot obtain the probability distribution, they deal with uncertainty. In this period, the problem of uncertainty was also addressed by Hicks (1931) or by Keynes (1936). Policy makers were the first to emphasise that literature on policy rules and uncertainty, in fact, deals with risks only, for example, Blinder (1998) or Issing (1999). In the policy papers, one can repeatedly find comments stating that, while problems of risks are well covered by economic literature, it is, in fact, the problem of uncertainty that makes the lives of monetary policy makers difficult. After these claims had been made, the academic research on policy rules started to consider the difference between risk and uncertainty as well.

Methodology issues

The methodology of analysing uncertainty used in recent literature on monetary policy has been summarised in Srour (1999) or Taylor (1999) who has also given a name to it: "new normative macroeconomics". The typical paper consists of a (linear or non-linear) model of the economy and a policy rule representing policy reaction. The rule can be either simple, based on pragmatic observation, or it can be derived from the loss function of a central bank in the model. The case of certainty is usually taken as a benchmark for the analysis of the consequences of a selected case of uncertainty for monetary policy reactions. According to the selected model and the type of uncertainty, the comparison of the certainty benchmark to the uncertainty case suggests that monetary policy should be more aggressive under uncertainty or that monetary policy should be more cautious under uncertainty. Numerous studies, for example, Sgherri, Wallis (1997), Drew, Hunt (1999), Shuetrim, Thompson (1999), Srour (1999) or Cagliarini, Heath (2000), emphasise that monetary policy makers should not derive any rule of thumb from the academic research papers since the results are not independent from the model.

In this context, it is worth noting that academic research papers more frequently deal with the specific cases of uncertainties than with discussing the implications of the multiple appearance of uncertainty. The case would correspond more to a standard situation of policy makers who are usually faced with a combination of several different uncertainties. Authors who have introduced a specific type of uncertainty into their models in order to explain why policy rules differ from the reactions of central bankers have been aware of this problem. For example, Smets (1999) concludes that the introduction of output gap uncertainty in the optimal control exercise is not sufficient for explaining why the reactions of central bankers are more cautious than the reactions suggested by the policy rules in the model. Srour (1999) emphasises that if there are several uncertain parameters in the model, it is not possible to conclude *a priori* whether policy makers should react with interest rates more aggressively or be more cautious than in the certainty case, and that this multiple uncertainty can be so large that it leads to a policy of no interest rate changes.

One of the most interesting debates is about an appropriate methodology for dealing with model uncertainty. This debate employs rather technical terminology. For the purpose of this paper, several frequently used concepts are summarised in Appendix II. Some studies, for example Levin, Wieland, Williams (1999) or Tetlow, von zur Muehlen (2000), employ a robust control method. It relies on using several models rather than only one when searching for suitable policy

rules. On the other hand, Sims (2001) claims that this method does not offer a substitute for assessing probabilities. Cagliarini, Heath (2000) prefer an alternative methodology that uses the inertia assumption in a simple optimal control problem. According to them, the robust control method based on minimax principle leads incorrectly to selection of a model from the range of available models in which monetary policy is the least effective. As a result, under higher uncertainty, policy makers are recommended to react more aggressively than in the case of certainty. Cagliarini, Heath (2000) claim that this conclusion does not correspond well to the central bankers behaviour that is better approximated with the inertia assumption. Then the presence of Knightian uncertainty implies lower volatility of interest rates and lower frequency of interest rate changes than in the certainty case, since the algorithm is based on the "envelope" approach and considers all possible distributions. To sum up, due to the different methodologies employed, there is disagreement on the implications of model uncertainty for policy reactions.

Another interesting discussion deals with the scope of uncertainty faced by monetary policy makers. Specifically, for the UK case, Haldane, Salmon (1995) have conducted a study for the Bank of England in order to estimate how often it is likely to miss the inflation target. Their results showed that inflation can be outside the targeted range (1.5%–3.5%) 85% of the time. On the other hand, Sgherri, Wallis (1997) estimated that inflation uncertainty is much lower than the above-reported number. For their analysis, they employed the same model as Haldane, Salmon (1995) but have made several modifications such as adding an exchange-rate equation or recalculating the covariance matrix for shocks. The result is that inflation should be in a two-percent targeted range two-thirds of the time. In addition to the above-mentioned topic, there is another emerging stream of literature on monetary policy under uncertainty represented, for example, by Isard, Laxton (1999) or Tetlow, von zur Muehlen, Finan (1999*)*, related to modification of the standard framework by introducing learning elements.

The difference between risk and uncertainty

There is a significant difference between risk and uncertainty that has not been always expressed identically in all papers dealing with monetary policy under uncertainty. It is important to be aware of this difference, because policy makers use different methods to deal with risks and different methods to deal with uncertainties. According to Knight (1921), policy makers face a risk if they have a reliable estimate of its probability distribution available during their decisions. They are faced with uncertainty if a relevant probability distribution cannot be calculated. Following Knight's definition, one can conclude that many academic papers on monetary policy under uncertainty focus, in fact, on analysing the implications of risks. This inconsistency has been observed by policy makers, for example, by Blinder (1998) or Issing (1999). There are research papers such as Cagliarini, Heath (2000) or Tetlow, von zur Muehlen (2000) dealing with the problem of model uncertainty that is the closest approximation of Knight's definition of uncertainty but, as was mentioned already, agreement on an adequate methodology has not been reached yet.

In this paper, the Knightian concept of uncertainty is used because this concept allows us to group methods for policy makers into two categories. Specifically, forecasting models, including stochastic models, are very good tools for dealing with risks. Their value to policy makers is illustrated in Blinder (1998). We would like to put emphasis on the fact that other methods are more suitable for dealing with Knightian uncertainty. Specifically, some tools offered by decision

theory and decision analysis can be very useful. Recent economic literature has also looked in this direction for inspiration. Cagliarini, Heath (2000) use terminology very similar to that used in decision theory and decision analysis. They distinguish between risks for which expected values of pay-offs can be evaluated and between uncertainties to which policy makers need to attach subjective probabilities. Both concepts –pay-off and subjective probability – are parts of pay-off matrix. We would like to recommend this tool of decision analysis to attention of monetary policy makers. The concepts and tools of decision theory and decision analysis will be reviewed in the next chapter. The basic terminology is summarised in Appendix II.

Implications of alternative types of uncertainty

If monetary policy makers targeting inflation lived in a certain world that is similar to the world of linear models and quadratic loss functions, it would be easy for them to calculate the optimal path for policy interest rates with the help of model. However, policy makers are often not certain about the values of specific parameters, about the functional form of a specific model equation and – even worse – about the model, used for forecasting inflation, itself. Each type of uncertainty implies that the optimal policy reaction could differ from the one suggested by the benchmark model simulation. There are many types of uncertainty that one can find described and analysed in various papers. Specifically, Freedman (1999) presents one possible classification of uncertainties from the policy makers' perspective. He makes a distinction between additive uncertainty, multiplicative uncertainty, model uncertainty, uncertainty about data and about the output gap. Clements, Hendry (1994) put more emphasis on the econometric background of the models and decompose the forecast errors into structural shifts, model misspecifications, additive shocks affecting endogenous variables, data noise, and parameter estimation errors. Srour (1999) pays attention to uncertainty about coefficients, time lags, and the nature of shocks. Hall, Salmon, Yates, Batini (1999) suggest comparing two types of uncertainty in the quadratic-linear stochastic model. In their case, implications of additive uncertainty are analysed within the benchmark stochastic model and implications of multiplicative uncertainty with parameters that are all stochastic.

In order to describe the implications of alternative types of uncertainty for an optimal policy reaction, we find it useful to distinguish between three categories of uncertainty: additive, multiplicative and off-model uncertainty. Each category has a different implication for monetary policy makers, additive uncertainty being the easiest one to handle and off-model uncertainty being the most difficult one. As the following summary of research findings illustrates, even within one category, one cannot derive unambiguous conclusions for the optimal policy response, since the conclusions differ according to the employed methodology and also according to the model used for the particular research. Nevertheless, one can receive one very clear message from the literature on monetary policy under uncertainty. The uncertainty should not be neglected when decisions about policy interest rates are made since it can have a considerable impact on how the optimal path for interest rates is perceived.

Additive uncertainty, which is sometimes also called linear uncertainty, does not pose a serious problem for monetary policy makers. Various studies, for example Hall, Salmon, Yates, Batini (1999) or Srour (1999), illustrate that, under additive uncertainty, optimal policy reactions are identical to the certainty case. Monetary policy makers can be confronted with this type of uncertainty when they are unsure about time lags or when they need to attach normally distributed

error terms to individual equations, but only if their forecasting model is in a quadratic-linear form. The intuition behind this conclusion is explained, for example in Wallis (forthcoming). It is only in the linear models with quadratic loss functions and normally distributed additive shocks that the decision-maker– according to the certainty equivalence theorem – can reduce its decision problem from focusing on the forecast representing all uncertainties to focusing only on the point forecast based on expected values.

Multiplicative uncertainty, which is also referred to as non-linear, does create a problem for monetary policy makers. It can take the form of one uncertain coefficient, more uncertain coefficients or uncertainty about the form of a specific equation. Conclusions then depend on the model employed for analysis and the form of multiplicative uncertainty. Some studies, for example Hall, Salmon, Yates, Batini (1999), Salmon, Martin (1999) or Freedman (1999), have similar conclusions to Brainard (1967), i.e. monetary policy-makers should be more cautious in the face of multiplicative uncertainty when responding to shocks than in the certainty case. Other studies, for example Srour (1999) or Leiderman (1999), emphasise that conclusions depend on which coefficient is uncertain. For example, uncertainty related to the elasticity of demand to interest rates should lead to more cautious reactions from monetary policy makers, while uncertainty related to the effect of inflation surprise on inflation should lead to more aggressive policy reactions. Similarly, the cases of uncertainty related to expectations should lead to more aggressive policy reactions, because a more cautious approach could damage the credibility of monetary policy.

Several studies dealing with multiplicative or non-linear uncertainty pay specific attention to the output gap and the NAIRU. Isard, Laxton, Eliasson (1999) analyse the consequences of the non-linear convex Phillips curve and give several other examples of non-linearities that should be subject to further research, such as the non-linear response of inflation expectations to the track record of hitting the target, floors on nominal interest rates or asymmetry in hysteresis on the labour market. Mishkin, Estrella (1999) study various uncertainties related to the NAIRU. Smets (1999) analyses output gap uncertainty, and Drew, Hunt (1999) analyse the implications of potential output uncertainty that, according to them, should lead to more aggressive policy reactions than in the certainty case. All authors agree that implications of multiplicative and non-linear uncertainties related to the output gap or the NAIRU are important for monetary policy makers. However, any simple rule of thumb on whether the reaction should be more cautious or more aggressive than in the certainty case is difficult to assert.

The third category of uncertainties monetary policy makers can face relates to uncertainties that cannot be represented and analysed within the model used for forecasting inflation. The most serious uncertainty in this category is whether the model provides a satisfactory representation of the economy. In this case, monetary policy makers are uncertain about the model. They are in a very difficult position, which can be illustrated by a broad variety of suggestions on how to overcome this problem. On the one hand, Tetlow, von zur Muehlen (2000) use robust control to analyse the implications of model uncertainty. Their results suggest that under uncertainty, policy makers should react more aggressively than in the certainty case. On the other hand, the same issue is analysed in Cagliarini, Heath (2000). However, a different methodology is used, and implementation of the inertia assumption brings the opposite conclusion that, under model uncertainty, policy makers should react more cautiously. Levin, Wieland, Williams (1999) test the performance of various monetary policy rules for a range of structural models and show that

simple rules work well with the whole range while complicated rules affected by forecasted values are less robust to model uncertainty. They conclude that policy makers, if uncertain about the model, should use simple rules based on the current output gap, current inflation and the lagged policy rate. Freedman (1999) and Issing (1999) argue that model uncertainty should be overcome by using several models.

Sometimes monetary policy makers may be uncertain only about a specific part of the model, such as one particular assumption or equation. For example, the works of Sgherri, Wallis (1997), Ball (2000) or Leitemo, Söderström (2001) illustrate that exchange-rate uncertainty and exchange-rate model uncertainty are very important for small, open economies. Blinder (1999) emphasises that it is important to further research a better specification of the equation linking long- and short-term interest rates. The last important uncertainty, which is very difficult to resolve, is data uncertainty. Orphanides (1998) explains that distorted data should be taken into account during monetary policy decisions, and according to his analysis, policy reactions should be more cautious under data uncertainty. Table 1 summarises the implications of all three categories of uncertainties faced by policy makers when forecasting inflation.

Table 1: Implications of alternative types of uncertainty

Uncertainty	Example	Knightian classification	Implication for policy reactions relative to the certainty case
Additive Linear	Equation error in linear model	Risk	None
	Uncertain time lag in linear model	Risk	None
Multiplicative Non-linear	Uncertain coefficient(s)	Risk	More aggressive or cautious (depending on the study)
	Uncertain functional form of Phillips curve	Risk	More cautious
Off-model	Model uncertainty	Uncertainty	More aggressive or cautious (depending on the study)
	Assumption or equation (fixed exchange rate)	Uncertainty	More aggressive or cautious (depending on the model)
	Noise in data	Uncertainty	More cautious

Note: *The described Knightian classification corresponds to the case of an estimated model. If the model is calibrated, probability distributions cannot be derived.*

Academic research versus policy papers

Until very recently, academic researchers have taken a different perspective on risk and uncertainty than policy makers. In their papers, policy makers focus more on Knightian uncertainty and on related issues, such as the role of forecast in the decision-making process, while academic researchers, in most cases, analyse the implications of risks. Policy makers feel that, since all their decisions are taken under Knightian uncertainty, it is not possible to automatically follow the suggestions of policy rules from model simulations. We suggest that

their reactions correspond more to the conclusions of literature on decision analysis than to literature on monetary policy under uncertainty. Another request made from the policy makers' perspective has been to analyse the consequences of the multiple-case of uncertainty. Policy makers are rarely so lucky to have doubts only about one coefficient in the model they use for forecasting or about the functional form of only one equation.

Academic researchers admit that it has been difficult to explain the difference between the reactions of policy makers that are usually cautious and between policy rules that suggest more frequent and larger changes in policy interest rates. This difficulty has been an important motivation for introducing uncertainty into academic research papers. However, the model extensions used for this purpose have usually been too specific and too close to the Knightian concept of risk to explain fully the observed difference between reactions suggested by model and true reactions of policy makers. Mainly papers dealing with model uncertainty have come closer to the Knightian uncertainty concept that explains the pattern of policy makers' reactions better. However, there have been studies, for example Blinder (1999) or Sims (2001), remarking that the types of uncertainty analysed by academic research papers are not as important as some key model assumptions that have not yet been subject to more stringent academic debate. Specifically, the issues of the long-run trade-off between inflation and output, the mechanism of deflationary spirals or new specifications of equations with well-known defects, such as an uncovered interest parity equation or an equation linking long- and short-term interest rates, remain unanswered. Summing up, the gap between the policy papers and academic research papers still prevails in the new millennium, and agreement on methodology suitable for dealing with Knightian uncertainty has not been reached yet in academic debate.

3. Uncertainty in literature on decision analysis

Decision analysis and monetary policy makers

It should not be surprising that monetary policy rules implemented into the core model do not generate the same paths of policy interest rates that are then observed in reality. Their actual paths are determined by policy makers who work under considerable uncertainty. The number and magnitude of the risks and uncertainties that can be accounted for in the core model is only limited. When policy makers feel that it is not safe to neglect uncertainties that could not be accounted for in the core model, they must combine the information provided by the model with other types of information, and consequently, react differently than their approximation in the model. The need of monetary policy makers to combine various types of information, for example the core model forecast with their own intuition, is not a problem solely related to monetary policy decisions. In this chapter, we would like to draw attention to findings from the literature on decision analysis (DA). DA has been developed to help decision-makers in various spheres - from engineers developing new technology to army executive officers - to make qualified decisions under uncertainty. Since the findings of DA are general, they can be applied to decisions on monetary policy as well. We would like to use these findings to illustrate that, when deciding about policy interest rates and communicating the outcome of policy decisions, it is not enough to work with the central inflation forecast based on one model, and that it is important to invest the

same amount of effort used in developing the core forecasting model when developing a methodology for dealing with risks and uncertainties, both internally and externally.

As was illustrated in the previous chapter, while typical risks can be taken care of within the forecasting model framework, uncertainties are more difficult to handle. DA offers general methods on how to make decisions under uncertainty. The DA terminology has mathematical roots. The basic terminology used in this paper is explained in Appendix II. Although a direct connection to DA literature has been only rarely made in literature on monetary policy under uncertainty, monetary policy makers have been using DA methods, sometimes explicitly, sometimes implicitly. For example, Blinder (1998) recommends that monetary policy makers should use methods similar to dynamic programming when making their decisions. This is an approach not very far removed from employing a decision tree as a tool for making decisions under uncertainty. Blinder (1998) also recommends working with dynamic multipliers when deriving their point estimates. Freedman (1999) recommends using a successive approximation when making policy decisions on interest rates. Blinder (1998), Freedman (1999) and Issing (1999) suggest that model uncertainty should be overcome by using several models and that a more cautious approach to decisions on interest rates is a typical outcome of considering model uncertainty during policy decisions. Budd (1998) and Issing (1999) emphasise that due to Knightian uncertainty, simple policy rules do not present sufficient information for monetary policy makers. Issing (1999) suggests that simple communication devices such as a single fan chart cannot be used to present the full story to the general public.

These suggestions of monetary policy makers resemble very much the DA finding that it is important to consider the whole set of information, not only some parts of it, when deciding under uncertainty. According to DA, the whole information set consists of very different types of information – ranging from a rigorous mathematical simulation to a purely subjective assessment of event probabilities. We would like to emphasise this finding because DA offers a tool for organising very different types of information into one framework that is very suitable for monetary policy decisions. The tool is called the pay-off matrix. Although the name sounds familiar, it does not have the same meaning as in the game theory. In the DA context, the pay-off matrix is a rather pragmatic tool that helps to find optimal decisions under uncertainty. For example, the pay-off matrix can be used prior to the actual decision in order to identify important pieces of information that are missing and should be collected or approximated. According to DA, after the pay-off matrix is filled with all available information, a selected decision rule determines the outcome of the decision-process. Since there are various decision rules, decision makers can select an adequate decision rule according to the problem they solve. One important feature of this framework is that if the pay-off matrix is incomplete, some decision rules are not possible to apply.

Illustration of the proposed methodology

Let us illustrate the methodology we propose for dealing with uncertainty faced by monetary policy makers. Table 2 shows a pay-off matrix that represents all information considered typically by policy makers: the central forecast, implications of alternative scenarios and alternative policy reactions, and subjective probabilities. Various parts of the pay-off matrix correspond to various stages of monetary decision process. In the first step, the core forecasting model and expert inputs are used in order to produce the central forecast. For the sake of simplicity, we assume that the

core forecasting model works with a fixed-rate assumption and that the central forecast is based on the assumption of unchanged policy interest rates. The pay-off of the central forecast is calculated according to subjective criteria selected by policy makers. In our example, the pay-off is equal to the distance of the forecast from the inflation target in the transmission horizon. Hence, the lower the pay-off, the better. The pay-off of the central forecast is located in the middle of the pay-off matrix. In the second step, two alternative sets of assumptions that correspond to situations in which significant deflation or inflation pressures prevail are specified. For these two alternative sets, the pay-offs of the policy of unchanged interest rates are derived from the core model simulations. It is worth emphasising that our example only illustrates the proposed methodology. The pay-offs can be defined differently. Also, the issue of specifying the alternative sets of assumptions is not an easy one. For example, under special circumstances, a completely different model from the core one can be employed in order to produce the alternative simulations. These issues will be discussed in more detail further in the text.

Table 2: The pay-off matrix of monetary policy makers

Alternative sets of assumptions **Possible reactions**	**Deflation pressures**	**Neutral pressures**	**Inflation pressures**
Subjective probabilities	**0.4**	**0.5**	**0.1**
Reduction in interest rates	0	3	5
No change in interest rates	-1	1 *	4
Increase in interest rates	-5	0.5	2.5

Note: *Pay-offs give a measurement in percentage points of the distance of the inflation forecast from the inflation target cumulated over a selected time horizon.*
     [*)] *This is the pay-off of the central forecast.*

In the third step, consequences of all other possible policy reactions are considered in a similar way. For the sake of simplicity, we consider three basic options - to leave policy interest rates unchanged, to reduce them or to increase them. As a result of the second and the third step, there are nine possibilities that are available to policy makers to choose from according to the pay-offs and subjective probabilities. The policy makers may (or may not) attach subjective probabilities to alternative sets of assumptions in order to derive their policy reaction. If the subjective probabilities are not attached, it is not possible to find the optimal policy reaction since policy makers need to calculate the expected pay-offs of all possible policy reactions in order to do so. In our example, the expected pay-off of reducing interest rates is 2. The expected pay-off of leaving the interest rates unchanged is 0.5 and the , the expected pay-off of increasing interest rates is -1.5. Hence, the optimal policy reaction is to leave interest rates unchanged. The example illustrates that the framework of the pay-off matrix allows policy makers to compare the outcomes of possible policy reactions under all relevant alternative sets of assumptions. It is worth noting that the incomplete pay-of matrix would have implied other conclusion. Specifically, if only one set of neutral assumptions was used in order to avoid a necessity to attach subjective probabilities, the option of increasing interest rates would be the most attractive.

Traces of decision analysis in research on monetary policy

Our suggestion to use more systematically the framework of the pay-off matrix during monetary policy decision process is not isolated. There are various traces of DA recognisable in research on

monetary policy. In the previous chapter, we have already summarised the findings of two research papers that have explicitly applied concepts and algorithms from DA in order to analyse the implications of model uncertainty for monetary policy decisions. Cagliarini, Heath (2000) and Tetlow R. J., von zur Muehlen P. (2000) debate about the suitable methods to overcome model uncertainty and compare outcomes of using the minimax approach to outcomes of using the inertia assumption. Cagliarini, Heath (2000) argue that the latter approach corresponds better to the behaviour of central bankers, since in reality, they are more likely to consider all possible probability distributions than to react with their interest rates according to the one model that represents monetary policy as the least effective from all available models. This debate can be transformed into the debate about decision rules that are used to select one alternative from the pay-off matrix in the DA framework.

From the survey on methods used by central bankers to forecast important economic variables presented in Sims (2002), one can conclude that a "mechanical" method based only on forecasting with the core model is outperformed by a "combination" method based on using a suite of models in a combination with expert forecasts and subjective judgements of monetary policy makers. Sims (2002) emphasises that central bankers use more suitable methods than those used in academic research to discuss monetary policy decisions under uncertainty since central bankers have employed elements of the Bayesian decision theory, such as subjective probability distributions. He illustrates that non-Bayesian methods, such as textbook econometrics or the calibration of coefficients, used by the majority of the forecasting models are not adequate. He gives examples of events that have to be included into monetary policy analysis but that cannot be treated as drawn from statistical distributions, for example oil crises, data collection error or an attack on a currency. Again, these conclusions are not very far from the DA findings that, as we will see in next paragraphs, suggest using models to illustrate outcomes of potential alternatives, using expert knowledge to specify all viable alternatives and leaving it to decision makers to attach probabilities to these alternatives and to select a decision rule adequate for a specific problem.

Monetary policy makers need to deal with uncertainty not only internally but also externally, since they need to justify their decisions made under uncertainty to the general public. Wallis (forthcoming) compares the various possibilities of how to report uncertainty, such as a forecast interval, event probability, a histogram or density forecast. Similarly to the decision theory, he views the problem of a decision maker in the case of certainty, where a decision maker can decide optimally knowing the model and its loss function, very differently from the case of uncertainty, where a decision maker needs to know all distributions related to the decision problem in order to decide optimally. As a result, according to Wallis (forthcoming), monetary policy makers should not rely exclusively on the point forecasts of important economic variables or on those representations of uncertainty that do not provide full information about all distributions related to decision problem. This is a strong argument for reporting the density forecasts for all economic variables (not just inflation) that are crucial to monetary policy decisions. Similarly, DA findings suggest that, without a complete pay-off matrix, it is not possible to apply all types of decision rules. In addition, Wallis (forthcoming) also points out that it is important to evaluate the quality of the forecast regularly, because this evaluation helps to improve the forecasting tools. This proposition corresponds well to the weighing method proposed by DA for the case in which parallel analytical tools are available. The method derives relative weights of alternative analytical tools according to their track record of past successes.

Dealing with uncertainty in decision analysis

Since the majority of decisions are based on imperfect knowledge, the issue of decision-making under uncertainty has been extensively addressed in the several branches of literature. Specifically, probability theory and expected utility theory have been combined in decision theory. The original elements of decision theory are mapped, for example, in Laplace (1795), von Neumann, Morgenstern (1944) or Wald (1950). Decision theory itself has been applied in several fields of economics, for example finance or game theory, as illustrated in Laffont (1989) or Reny (1998). However, there is a specific discipline that focuses on applying decision theory to decision-making in practice. DA can offer supporting tools that aim at improving the decision-making process at various stages, such as problem identification, collecting and evaluating information, and actual decision or sensitivity analysis. There are numerous DA textbooks that explain all of the important concepts, for example Beroggi (1998) or Skinner (1999). The techniques that DA can offer to decision makers include pay-off matrices, decision rules, decision trees, dynamic programming, cost-benefit analysis, Bayesian methods, and Monte Carlo simulations.

Central banks targeting inflation have already overcome all the major obstacles of the earlier stages of the decision process. The decision problem has been clearly identified with announcements of explicit inflation targets, granting independence to central banks and with their subsequent effort to be highly transparent and accountable. Also, the tools for collecting and evaluating information available to monetary policy makers are already extensive. It is worth focusing more on the decision stage and on the subsequent stages of the decision process. According to DA, policy makers are in a decision situation if there are at least two possible reactions available to them. The decision is produced after the outcomes of all possible reactions are evaluated and compared with respect to the final goal and various uncertainties faced by policy makers. In this context, a "good" decision offers the best chance of success to meet the final goal in the time of the decision. As a consequence, a good decision does not guarantee a good outcome, because there is uncertainty. This definition of a "good" decision implies that a decision should be based on evaluation of the broader context of a problem, including the risks and uncertainties related to applied analytical tools

Inflation targeting central banks do not have any problems when listing all the possible reactions, since they can either to reduce policy interest rates, leave them unchanged or increase them. Central banks usually develop a very good core model to be used for forecasting inflation and other key economic variables. Hence, evaluation of the outcomes of these possible policy reactions in some well-defined analytical framework is possible. In this framework, it is usually also possible to deal with the risks attached to the central forecast, for example, by assuming that shocks in the core model have normal distributions and then by using stochastic simulations. It is much more difficult to develop a methodology for dealing with Knightian uncertainties. For this stage of the decision-making process, DA textbooks recommend detecting which assumptions of the analytical framework are the most influential, for example, by running sensitivity tests, and which assumptions are most uncertain, for example, by regular *ex post* analysis of forecasting tools, regular review of the reliability of data sources, or by attaching subjective probabilities to various assumptions. Then several alternative situations can be specified, for which the evaluation of outcomes of all (three) possible policy reactions will be conducted. This methodology should guarantee that the major sources of uncertainty, such as the most influential and most doubtful

forecast assumptions, are illustrated not within the central forecast but with alternative simulations. As a result, policy makers will have a pay-off matrix before them, explicitly or implicitly, that compares the outcomes of possible policy reactions under several alternative sets of assumptions and will be ready to make their decision. As we will see in the next chapter, several inflation targeting central banks have selected subjective probabilities as a method of dealing with Knightian uncertainties, and publish fan charts to communicate with the general public. It is interesting to note that, in comparison to DA recommendations, fan charts are not reported for all three possible policy reactions.

The pay-off matrix and decision rules

As was said, we propose to follow the logic of the pay-off matrix during the monetary policy decision process in order to deal adequately with uncertainty. Table 2 illustrated the proposed methodology. Although the framework looks simple, the difficulty of the decision process is reflected in the fact that the elements of the pay-off matrix are not always easily identified. Firstly, the evaluation of the pay-offs need not be defined as the distance of the expected inflation from the target. Depending on the actual implementation of the inflation targeting strategy, the pay-off can be equal to the cumulated differences between the forecasted inflation and the target for a certain period. If both the target and the forecast are announced in the form of an interval, the event probability, as suggested in Wallis (forthcoming), can be used to evaluate the pay-offs. In addition, the pay-offs can be constructed analogously to the loss function, and certain weights can be attached to other factors such as output gaps of interest rate volatility. Alternatively, policy makers can choose to work with a less formal, for example graphical, representation of the pay-offs.

Secondly, it is not easy to identify ex ante all the major weaknesses of the set of assumptions that corresponds to the central forecast and that we called a "neutral" set. Hence, the columns of the pay off-matrix are more difficult to construct than the rows, since the three basic possible reactions of monetary policy makers – to increase or decrease interest rates or to leave them unchanged – are ex ante well known. This decision on which columns the pay-off matrix should have is difficult – though often implicit because it cannot count on a methodology as elaborate as the one used to produce the central forecast. Monetary policy makers can be uncertain about some parts of the model – the paths of exogenous variables, future shocks or the model itself – and they can be uncertain about different factors in different times. However, decisions about interest rates must be taken in real time, and so policy makers do not want to debate every small uncertainty separately. In other words, they do not want to work with a pay-off matrix that has too many columns. Hence, they use their judgment and intuition to select the most influential factors that are uncertain and to analyse the consequences of several sets of alternative assumptions (each of which may group several uncertainties together) for the central forecast. Although there are methods available to provide input information for decisions about the columns of the pay-off matrix, such as sensitivity analysis or impulse response functions, this stage of the monetary policy decision process relies enormously on the intuition of individual policy makers.

Thirdly, it was said that the rows of the pay-off matrix are more straightforward to construct than the columns. In this context, it is worth noting that if the core model works with a policy rule instead of a fixed-rate assumption, the three basic possible policy reactions cannot be represented by three different assumptions about the level of the policy interest rates. However, it is always

important for policy makers – according to DA findings – to have the option of comparing the outcomes of all the possible actions they consider during their policy meeting. This implies that the core model with endogenous monetary policy should offer at least three possible reaction functions that would produce *ceteris paribus* three different paths of policy interest rates. These three paths should correspond to the three basic policy options mentioned earlier. In this modelling set-up, the rows of the pay-off matrix would correspond to switching between three possible policy rules, for example the baseline policy rule and more cautious and more aggressive rules. Otherwise, the framework of the pay-off matrix would remain equivalent to the one described in Table 2.

Finally, after the pay-off matrix is completed, policy makers are able to use it to find the best reaction. DA offers various decision rules to determine which reaction from the possibilities represented as the rows of the pay-off matrix is the best one. Let us illustrate the problem of selecting the appropriate decision rule. Policy makers can, for example, compare the alternative sets of assumptions before the central forecast is produced and select the set that is the most probable. Let us say that the neutral set is the most likely. Then policy makers only need to compare the pay-offs of the three possible reactions under the neutral set of assumptions and select the policy reaction with the best pay-off. Table 3 illustrates that this decision strategy results in increasing policy interest rates in our example from Table 2. In DA, this decision rule is called the rule of the best of the most probable. It is also called the optimist's rule, because it neglects the other sets of assumptions that both have non-zero probability.

Table 3: Pay-off matrix: "The best of the most probable" rule

Alternative assumptions Possible reactions	Deflation pressures	Neutral pressures	Inflation pressures
Reduce rates		3	
No change		1	
Increase rates		0.5	
The most likely assumptions			

On the one hand, the rule of the best of the most probable saves on the costs of evaluating the pay-offs for the whole matrix. On the other hand, if the uncertainties are high, the application of this rule can lead to a sub-optimal decision. In this case, policy makers may prefer to invest more and construct the whole pay-off matrix. Not only will they work out nine forecasts in order to evaluate the pay-offs, they will also attach subjective probabilities to the three alternative sets of assumptions. Then policy makers will derive the expected pay-off values for each of the three possible policy reactions and select the policy reaction with the best expected pay-off. Table 4 shows that this decision strategy will lead to unchanged policy interest rates. In DA, this decision rule is called the rule of the best expected value and is often referred to as the rational rule. It is worth noting that if policy makers do not attach subjective probabilities to all the alternative sets of assumptions or do not fill the whole pay-off matrix, they can use the rational rule for their decisions.

Table 4: Pay-off matrix: "The best expected value" rule

Probabilities	0.4	0.5	0.1	
Alternative assumptions Possible reactions	Deflation pressures	Neutral pressures	Inflation pressures	Expected pay-off
Reduce rates	0	3	5	2
No change	-1	1	4	0.5
Increase rates	-5	0.5	2.5	-1.5

As illustrated in DA textbooks, the outcome of the decision process will always depend on the decision rule selected. There are many ways of deciding which policy reaction is the best. Some decision rules, such as the "best of the most probable", require attaching probabilities to the alternative sets of assumptions. Some decision rules, such as "maximin", do not require this. One of the decision rules in the latter category has been used, as was said, in Tetlow R. J., von zur Muehlen P. (2000). It is worth noting that the decision rules in the first category work with the mathematical concept of expected values, and so alternative sets of assumptions that yield very high pay-offs or that are very likely to will be considered more during the actual decision. For these rules, it is important to have prepared a clear methodology for attaching subjective probabilities to the alternative sets of assumptions. They are called subjective because in the case of important assumptions, decision makers often do not have enough historical information to calculate mathematical distributions, and consequently they have to make their best educated guesses about them. Table 5 illustrates which policy reaction has been selected by various decision rules in our example from Table 2. It is worth noting that – depending on the decision rule selected – it is possible to reach three different conclusions about the "good" policy response with the same central forecast. For example, the Laplace rule suggests increasing interest rates. The rule working with the best of the expected values suggests leaving rates unchanged. The rule based on the principle of the "best of the most probable" suggests increasing interest rates. More detailed information about the algorithms behind the decision rules is provided in Appendix III.

Table 5: Outcome of the alternative decision rules

Decision Rule	Description	Outcome
Alternative Sets of Assumptions without Probability		
Laplace	Each alternative situation equally probable	Increase in interest rates
	Select the reaction with the best pay-off	
Maximin	Find the worst pay-off for each reaction	No change
	Select the reaction with the minimal worst pay-off	
Maximax	Find the best pay-off for each reaction	Reduction in interest rates
	Select the reaction with the best of best pay-off	
Minimum regret	Construct a regret matrix	No change
	Select the reaction with the smallest regret	
Probabilities of Alternative Sets of Assumptions Are Needed		
Best expected value	Attach probabilities for alternative situations	No change
	Select the reaction with the best expected pay-off	
Best of the most probable	Find the most likely alternative situation	Increase in interest rates
	Select the reaction that produces the best pay-off	

Note: See Appendix III for illustrative calculations of outcomes under the individual decision rules.

Lessons for inflation targeting from decision analysis

DA analysis offers analytical tools for combining various types of information important for monetary policy decisions. Specifically, the structure of the pay-off matrix illustrates well that the central forecast obtained with the core forecasting model is not enough to decide whether the assumed path of policy interest rates is optimal. In our example, the forecasting exercise that produced the central forecast filled in only one ninth of the pay-off matrix. The remaining 88 percent of information necessary for a decision on policy interest rates came from debating alternative sets of assumptions, attaching subjective probabilities to them, and comparing the outcomes of all possible policy reactions under all alternative sets of assumptions. It is worth noting that, unless there is perfect model certainty, this conclusion is true even if the forecasting model is stochastic. Monetary policy makers may not employ the pay-off matrix as a formal analytical tool when deciding about policy interest rates, but in order to make monetary policy decisions, they informally combine all types of information present in the pay-off matrix, such as a model forecast, off-model information or a subjective judgement. As was mentioned, this is well documented, for example, in Blinder (1998), Budd (1998), Freedman (1999), Issing (1999) or Sims (2002).

To conclude, according to DA, it is a very significant simplification to represent decisions of an inflation targeting central bank with a forward-looking rule based on the point inflation forecast, as illustrated, for example, in Svensson (1996), or with simple policy rules as illustrated, for example, in Taylor (1993). These representations are very useful when the central forecast is produced, but should not be used as a substitute for monetary policy decisions. This does not imply that there will not be periods of time when policy rules do not predict monetary policy decisions well. If policy makers face *ex ante* very low Knightian uncertainty, their (implicit) pay-off matrix can be reduced to one element containing the central forecast. However, under less favourable circumstances, the central forecast can play a much smaller role in the actual decisions

than other types of information, including which decision rule is mimicked by the institutional settings in the central bank.

The decision rule is an important part of the decision-making process. Table 5 documented that, given the pay-off matrix, each of the three possible policy reactions can be the best reaction depending on which rule has been used for the decision. Consequently, it is important for monetary policy makers to try to coordinate their decisions with a decision rule that is adequate to their situation. Specifically, if it is feasible, the probabilities of alternative sets of assumptions should be attached so that the rule of the highest expected value can be followed. There are several methods for attaching probabilities to alternative sets of assumptions. It is worth recalling that these probabilities are subjective since they cannot be estimated for various reasons (e.g. due to Knightian uncertainty). Policy makers can attach subjective probabilities verbally during an informal discussion, or they can vote anonymously in order to create distribution with votes. Both approaches are observed in the case studies, as we will see in the next chapter. If – for some reason – it is necessary to compare the outcomes of alternative sets of assumptions without expressing their probabilities, then it is preferable to decide according to the minimum regret rule, which is more cautious than, for example, the maximin rule. The maximin rule is more suitable for spheres of decision-making (e.g. defence) in which leaders need to prevent at all costs the largest possible damage.

The last lesson to learn from DA is that it is very important to invest considerable analytical effort into creating what we can call a "map of uncertainties", since the specified alternative sets of assumptions need to illustrate well the major uncertainties faced by monetary policy makers. There is no easy method available showing how this should be done. As a rule of thumb, given the structure of the forecasting model, all very influential elements of the model, such as coefficients, variables or shocks, as well as all major concerns related to the model itself, should be on the map together with very likely dramatic changes in exogenous variables. The influence of the various parts of the model can be judged with the help of various analytical tools such as impulse response functions or variance decompositions. It is then possible to group all uncertainties on the map into several sets and attach probabilities to these sets, as we did in our example. For the sake of simplicity, it is possible to use deliberately only two sets of assumptions that can be called "favourable" and "cautious", as illustrated in Wallis (forthcoming). Alternatively, the "envelope" subjective probability distribution can be derived around the central forecast. In this case, the implicit pay-off matrix will have, in fact, an infinite number of columns.

4. How inflation targeters deal with uncertainty

Short case studies on dealing with uncertainty

The subject of five short case studies has been to document how inflation targeting central banks deal with uncertainty. Specifically, we wanted to learn which information pillars are used for monetary policy decision under uncertainty. Our working hypothesis was that most central banks targeting inflation would need at least two information pillars, as was suggested in Šmídková (1999). Firstly, the core forecasting model or a suite of models is used in order to produce the central inflation forecast. It is worth noting that deciding whether the core model should be

calibrated or estimated has further implications for the methodology of dealing with uncertainty. Secondly, a methodology for processing all off-model information, including subjective judgements of policy makers, is needed. This second pillar is added on to the central forecast in order to facilitate decisions on policy interest rates and to produce the final inflation forecasts presented to the general public. The second information pillar can have various forms such as a verbal description of risks, alternative policy simulations produced by the core forecasting model or a subjective probability distribution for a fan chart. Some of these forms have methods of dealing with uncertainty already built-in. Various procedures can be employed to select which risks or uncertainties should be analysed in a specific time period.

The following case studies are described in chronological order according to the year in which the respective central banks have started targeting inflation: the Reserve Bank of New Zealand, the Bank of Canada, the Bank of England, the Swedish Riksbank and then the Czech National Bank. Only publicly available sources have been used to produce the case studies. A general overview of the early experience with inflation targeting can be found in Haldane (1995) and an overview of the experience of the first decade in Mishkin, Schmitt-Hebbel (2001). A comparison of the forecasting methodology of central banks has been presented, for example, in Sims (2002) or Pagan (2002). Comments on how central banks have exchanged modelling know-how can be found in Poloz, Rose, Tetlow (1994) and Amano, Coletti, Macklem (1999). Additional country-specific information has been obtained from Inflation Reports, Monetary Policy Statements or Monetary Policy Reports and research and policy papers published by the five central banks, see Blix, Sellin (1999), Britton, Fisher, Whitley (1998), Hrnčíř, Šmídková (2001), Macklem (2002), RBNZ (2000) and Vickers (1998).

The Reserve Bank of New Zealand

The Reserve Bank of New Zealand started targeting inflation in 1990, and Monetary policy statements have been published since that time. Economic projections are presented to the general public in the form of graphs that show the paths for inflation, output and other variables - such as exports - produced by the central projection for a three-year horizon. In addition, a graph with a projected path for 90-day interest rates is also published, but only one quarter in advance. It has been stressed that the inflation forecast is not a policy projection as in the cases of other central banks, because monetary policy is endogenous in the forecasting model in order to approximate the strategy of inflation forecast targeting as closely as possible. Uncertainties and risks related to the central projection and decisions on interest rates are described verbally. The central economic projection and a description of risks and uncertainties correspond to the Governor's decision. The MPC has only an advisory role, and the minutes from the monetary meetings are not published.

The internal policy debate has three stages. The first stage is focused on broadening the policy debate. Important issues that should be discussed during the policy debate are identified. In order to capture the whole range of feasible policy options and to ensure that the policy debate is robust, selected MPC members are asked to prepare brief notes advocating a "hawkish" and "dovish" policy stance. The second stage consists of a presentation of the central projection with a sensitivity analysis and risk scenarios requested by the MPC. The forecasting model FPS is similar to the Canadian QPM in its structure. It is calibrated and works with the reaction function. Alternative reaction functions that are slower or faster than the benchmark reaction function are sometimes used to produce policy simulations. The model can be adjusted – both in its dynamic

part as well as in the long-run equilibria – to correspond better to new data or other off-model information. In the third stage, the debate is narrowed in order to reach conclusions, and the policy statement is drafted. The central projection plays a key role in the policy debate, but it is not the only factor affecting policy decisions.

The Bank of Canada

The Bank of Canada started targeting inflation in 1991, and monetary policy reports have been published since 1995. The inflation projection presented to the general public takes the form of a one-year point estimate with an accompanying verbal description of the general economic outlook, including the risks of the projection. The economic projection is produced as the most likely path for the economy. Although the projection is based on the model working with monetary policy rules, it is not called a forecast in order to emphasise that the point estimate of future inflation is conditional on various assumptions. In comparison to the Bank of England or the Riksbank, the Bank of Canada communicates about uncertainty in a much less formal way using a verbal description of important risks and their potential impact on the inflation outlook and a verbal description of alternative scenarios. It is often emphasised that a multiple-model approach has been taken in order to overcome model uncertainty.

The internal process starts with the staff producing an economic projection that will be the reference points for further steps of the decision-making process. Although a suite of models is used, there is a core model for medium-term projections (QPM) that has been calibrated. The core model combines three parts: a steady-state model, a dynamic part, and expert input reflected in residuals. In the second step, the main risks are then selected by the Council, partly reflecting the staff's suggestions, and alternative scenarios are specified for alternative simulations. In addition to risk analyses, alternative policy scenarios are also considered and represented with alternative policy rules. As a result, the Council is presented with a range of projections and a range of policy simulations that can be compared to the reference projection. Then all projections and policy simulations and additional off-model information are discussed during the meeting where directors and experts can recommend a particular policy reaction. Finally, the Council meets to decide on an appropriate policy reaction, which must be consensual.

The Bank of England

The Bank of England started targeting inflation in 1992, and Inflation reports have been published since that time. The Minutes of the MPC meeting are published with the names of the MPC members and their specific votes. Economic projections presented to the general public have the form of two-year fan charts for inflation and output growth. Fan charts are constructed around the central projection. The central projection corresponds to the mode of a subjective probability distribution that reflects the MPC subjective assessments of risks. However, the whole content of the Inflation reports is important since the official documents often stress that decisions on interest rates are based on a broader set of information than the one used for the construction of fan charts, and that monetary policy is not automatically derived solely from fan charts. While the core model works with 150 variables, information discussed on the pre-MPC briefing typically consists of 500 charts and tables and 1000 variables. Fan charts are attributed two roles. Internally, their creation facilitates monetary policy debates. Externally, they help to communicate monetary

policy bias through the difference between mode and median and- through the subjective distribution variance - to illustrate the degree of uncertainty faced by policy makers.

The Bank of England organises several rounds of meetings in order to produce the final projection. In the first step, the suite of models is used to produce the draft projection. The core model has around twenty equations, and it is estimated. The projections are run under the fixed interest rate assumption, and residual adjustments are made when necessary to correspond better to expert views or off-model information. Sometimes, alternative projections are prepared conditioned on the higher and lower values of interest rates. In the subsequent rounds, the draft projection, off-model information and assessments of risks are discussed using both "bottom up" and "top down" approaches in order to ensure that the final version represents the MPC views. After individual risks are evaluated by the MPC and the staff, they are aggregated to give the overall balance of risks that is then used to produce fan charts. As a starting point, historical distributions of exogenous variables and equation error terms are taken, but the final version represents a subjective assessment of risks. The final projection is not interpreted as the most likely path for inflation, because not all information has been included in the draft projection and subsequent risk assessment and because the projection is conditional.

The Swedish Riksbank

The Swedish Riksbank started targeting inflation in 1993, and Inflation reports have been published since that time. The Riksbank also publishes Minutes of the monetary policy meetings. Inflation projections in the form of a two-year fan chart have been a part of the inflation reports since 1997. The Riksbank communicates about uncertainty related to monetary policy with the help of the main scenario and a verbal description of the risk spectrum around the main scenario. The fan chart probability distribution and the table with percentage probability of different outcomes illustrate the risk spectrum graphically. The main scenario has been defined as the most likely inflation path, because this definition avoids any impact of extreme events on the decision even though the Riksbank is aware that there is a disadvantage with this choice since the whole distribution is not considered. The risk spectrum need not be symmetrical due to the introduction of skewness of the composed probability distribution. For example, there is a downside risk if the inflation forecast is more likely to overestimate future inflation.

Internally, the main scenario and the risk spectrum are created in several steps. The "bottom-up" process is used, and the main scenario and risks are mostly prepared by experts. If the Board disagrees with the outcome of the process, the main scenario or inflation forecast distribution can be adjusted. The main scenario of the inflation projection is based on the model forecast obtained from a suite of models. The core forecasting model is similar to the Canadian QPM but it is run under the fixed-rate assumption. Then uncertainty intervals for important inflation factors are constructed. The assessments of risk factors are subjective because the stochastic approach would not allow several models, off-model information and subjective judgments on future risks to be considered. Finally, the inflation forecast distribution is composed of the uncertainty intervals with a methodology that has been developed especially for this purpose by the Riksbank. The weights of individual factors for aggregation of distributions are derived from the underlying forecasting model. The methodology is similar to that of the Bank of England – for example, it uses a two-piece normal distribution – but it works with uncertainty intervals differently.

The Czech National Bank

The CNB started targeting inflation in 1998, and Inflation Reports have been published since that time. The CNB publishes Minutes of the Monetary Policy Meetings that include the voting pattern without names if the Board members. Inflation projections take the form of a chart representing the inflation outlook for the whole horizon of monetary policy transmission, approximated to be eighteen months. In order to illustrate the scope of uncertainty about the central forecast, the bands are attached to it, and the chart represents the band forecast. As a result, the public can compare the inflation target with the inflation forecast on one chart, both in the form of a band. The projection of GDP growth is published as an interval forecast for the end of the year. The forecast is unconditional with respect to interest rates, and their future path, consistent with the forecast for the forthcoming twelve-month horizon, is verbally indicated on the press conference. Risks and uncertainties are assessed verbally in the Inflation report. In addition, important exogenous assumptions of the central forecast, such as development of import prices or external demand, are explicitly published in order to emphasise that if external conditions change, the path of policy interest rates will be different from the one indicated by the forecast.

The internal process of forecasting inflation has several stages. Initially, the Board meets with a group of directors, advisers and experts in order to discuss the initial assumptions of the forecast. Attention is paid to model assumptions as well as to the forecasts of external variables, and several alternative scenarios are specified. Then the central forecast emerges from the interaction of the core model with expert inputs. The structure of the model is similar to that of the RBNZ. It consists of long-run and dynamic parts, and endogenous monetary policy. The Czech version of the model does not include stock variables nor does it include a representation of fiscal policy. The methodology of producing the forecast is similar to that used by the RBNZ. The assumptions as well as residuals are adjusted in order to produce forecast that would be in line with the expert views. In the next stage, the central forecast and outcomes of alternative policy simulations are presented to the Board and the group of directors, advisers and experts, who can then require further adjustments or to suggest leaving out specific alternative policy simulations. After this meeting, the Inflation report is drafted. For the purpose of Inflation report, the bands, whose widths are calibrated in a relatively simple way, are attached to the central forecast in order to illustrate the risks. During the policy meeting, the new forecast is presented. Afterwards, the Board discusses the forecast and votes on policy decision in a closed meeting, and one of the advisers records the Minutes. The Minutes focus on describing the reasons behind the decision and on illustrating the uncertainties related to the forecast. The votes of the Board members are independent, it is not necessary to reach a consensus. If no consensus is reached, the voting pattern is published in order to give additional information about uncertainty to the general public.

Several findings from the five case studies

According to publicly available sources, the central banks targeting inflation do not seem to be strict inflation forecast targeters. Inflation forecasts have two important roles for central banks. Internally, they provide a framework for policy discussion. They help to structure the decision process and ensure its consistency. Externally, inflation forecasts are communication tools that illustrate reasons for the decision and the uncertainty faced by policy makers. However, other communication tools are used as well. Specifically, central banks include verbally described risk profiles in their Inflation reports or publish minutes of their policy meetings with the voting

pattern. All five central banks emphasise that they work with the whole range of information when deciding about policy interest rates: the suite of models, off-model forecasts by experts, other off-model information, preliminary voting by a group of advisers, and the subjective judgment of the board members. These findings are in line with our working hypothesis that most central banks targeting inflation need at least two information pillars. While the methodology for producing the first pillar – the central forecast – has been extensively covered by literature, the methodology for composing the second pillar - that should evaluate uncertainty attached to the central forecast - from various information sources certainly deserves more attention.

The central projection represents in all five cases the most likely trajectory of the economy, conditioned by the information available at the time of the policy decision. In the cases of central banks that subsequently use asymmetric distribution of risks, this explicitly implies that the central forecast is not the mean or the median of the possible trajectories. At the same time, all five central banks declare that, although the central forecast is the most visible part of the information presented to the general public, it need not be a decisive piece of information. This brings us back to the second information pillar. In addition to the central forecast, off-model information is analysed as well, such as financial sector development. Then the whole distribution of risks is considered during policy debate. However, this distribution is not necessarily symmetric. The evidence suggests that it is rather the expected trajectory of the economy derived from the subjective probability distributions that form the final outcome of the decision process than the model forecast itself. It is important to understand that subjective probability distributions are a broader concept than the distributions used for the construction of fan charts. Specifically, they are formed by the board members with the help of managerial methods, such as confronting two opposite views (e.g. "hawkish" and "dovish" stance), preliminary indicative voting in a group of experts or the requirement to reach consensus within the Board.

The second information pillar is always represented in Inflation or Policy reports for the benefit of the general public. However, the way of representing it is not as straightforward as in the case of the central forecast. Even the two central banks that illustrate the scope of uncertainty and bias of the monetary policy stance with fan charts use additional tools, such as a verbal description of the risk profile or minutes of the meeting with a voting pattern. Fan charts are aggregating subjective assessments of the board members and experts of important uncertainties. The weights of various risk factors usually correspond to their impact on inflation in the core forecasting model. Several banks use alternative scenarios to communicate their uncertainty about important external factors such as commodity prices. One bank calculates bands around the central inflation forecast without attaching the probability distributions to the interval forecast and verbally communicates when the outcome is more likely to be close to one of the bands than to the central point of the interval. To sum up, due to the complexity of the decision-making process, it is very difficult to represent the whole content of the second information pillar with only one communication tool, and hence, it is not possible for the external observer to easily reconstruct the subjective probability distribution built around the central forecast.

There are several sources of uncertainties that are prominent in policy debates of inflation targeting central banks. Specifications of the long-run equilibrium exchange rate and estimates of potential output are two important sources of model uncertainty. It is worth noting that they are important in our case studies, because four out of five central banks follow the Canadian modelling school when producing their central projections. The school requires a well-defined,

long-run part of the model, and the results depend heavily on the values of the output gap. On the other hand, errors in estimates are not a matter of great concern, because the Canadian school prefers calibrated models. All five central banks conduct monetary policy in open economies. Consequently, one of the stages of the forecasting process is usually fully devoted to forecasting external variables and evaluating the quality of alternative sources of forecasts for the world economy. In addition, specific attention is paid to financial markets and their impact on the exchange rate. All three central banks that use endogenous monetary policy in their models produce alternative policy simulations for different values of risk premium.

5. Conclusions

It is important for monetary policy makers to work with the both information pillars during the decision process. A high-quality central projection of economic development without a careful evaluation of uncertainties does not guarantee that a good decision will be taken regarding the level of policy interest rates. Hence, similar attention should be paid to developing a forecasting framework as well as a methodology for dealing with uncertainty. The second information pillar representing the assessment of uncertainties should not be neglected during monetary policy decisions especially when:

- there is Knightian uncertainty related to the forecasting model, for example, specification of the influential equation is problematic,
- there is Knightian uncertainty related to some assumption of the central projection, for example, the probability distribution of the future values of important exogenous variable is unknown,
- there are signals that risks around the central forecast are not distributed symmetrically, for example, off-model information or new data may change the balance of risks.

Under these conditions, monetary policy decisions that are based solely on the central forecast, even if the forecast is the outcome of stochastic simulations, are likely to depart from the optimal policy reaction. Since it is usually difficult to judge *ex ante* whether one of these conditions is fulfilled, monetary policy makers should use general methods developed for all decision makers dealing with uncertainty rather than be strict "inflation forecast targeters". However, it should be noted that monetary policy makers targeting inflation do have a considerable advantage over decision makers in other spheres due to the well-defined framework, provided by the inflation targeting strategy, that makes evaluation of the outcomes of possible policy reactions under alternative sets of assumptions straightforward.

While literature on monetary policy under uncertainty has developed an excellent background necessary for building the first information pillar, the question of how to build the second pillar has not yet received a sufficient amount of attention. We suggest working with the pay-off matrix as a suitable methodology for dealing with uncertainty during the monetary decision process. As was confirmed by the five case studies, this tool, which has been developed within the context of DA literature, is used by central banks in real-life situations, however informal or implicit it may be. DA tools are also referred to in literature on monetary policy under uncertainty, though the pay-off matrix is not explicitly quoted. We do not recommend using the matrix in a formal, mathematically rigorous manner that would require calculating the exact value of each element of

the pay-off matrix. Rather we find it useful to think in the logic of the pay-off matrix and pay equal attention to all its components when making monetary-policy decisions and when communicating with the general public.

Table 6 presents the alternative methods that are referred to by the literature or are used by central banks to develop the pay-off matrix from the individual components. The components represent various stages of the monetary-policy decision process. While the methodology of producing the central forecast has already been developed in the CNB, examples of methodologies applied at subsequent stages of the decision process may provide interesting inspiration to the CNB. For example, the core model could be used to produce on a quarterly basis five projections that would illustrate the implications of using three variants of the reaction function (baseline, faster, slower) for the neutral set of assumptions and the implications of two alternative sets of assumptions (deflation risks, inflation risks) under the baseline reaction function. The remaining four corner elements of the pay-off matrix would be commented on only verbally, if needed. Then expert judgement or preliminary voting of a group of experts could be systematically used to attach subjective probabilities to three alternative sets of assumptions. The pay-off could be initially represented with inflation and output growth forecast charts, then when the stochastic forecasting model is available, the pay-offs could correspond to the probability that inflation will be inside the targeted range. As shown in Table 6, there are other alternatives feasible for each stage of the decision process, and their advantages or disadvantages should be subject to further debate.

We suggest that using a framework corresponding to the pay-off matrix has two advantages. Firstly, the pay-off matrix organises very different types of information that is important for monetary policy decisions into one framework. The outcomes of model simulations, such as the central projection or policy simulations, are combined with more subjective inputs, such as specifications of the alternative sets of assumptions or probabilities attached to them. During the process of combining these inputs, the previously mentioned problems with uncertainty are taken care off. Alternative sets of assumptions can illustrate key uncertainties without restricting the assumptions of the linear forecasting model or standard stochastic shocks. Subjective probabilities can define asymmetric distributions around the central forecast and evaluate the potential impact of asymmetric risks on the expected values. This is a very important advantage when monetary policy makers wish to use an analogy to the "rational" decision rule due to large asymmetry in probabilities attached to the alternative sets of assumptions, for example, with deflation being considered a serious threat. In this case, policy makers need to select from possible policy reactions the one with the best expected pay-off. If probabilities attached to the alternative sets of assumptions do not produce large asymmetries with respect to the central case, policy makers may wish to apply the "optimist's" decision rule. In this case, they select the policy reaction that guarantees the best pay-off under the most likely set of assumptions. The framework corresponding to the pay-off matrix can easily accommodate both cases.

Secondly, the structure of the pay-off matrix provides a very good background for communicating the decision to the general public. Various communication tools used by central banks can be viewed as a subset of information organised in the pay-off matrix. For example, the central forecast represents one element of the pay-off matrix. The subjective probabilities attached to alternative sets of assumptions can be illustrated with the voting pattern of the board members if it is a part of the publicly available minutes of the meeting. The fan chart corresponds to one row of the pay-off matrix since it combines the central forecast with subjective distribution implied by

probabilities attached to alternative sets of assumptions. Similarly, comments on implications of the interaction of the forecasting model with different policy rules reveal information found in one column of the pay-off matrix. The more the implicit pay-off matrix is revealed to the general public in a consistent, well-organised way, the higher the transparency of the monetary policy decisions is. External communication explaining all key inputs of monetary policy decisions that are represented by various parts of the pay-off matrix convinces the general public more easily that much more is needed to generate good decisions than producing the central forecast, and that monetary policy decisions must not be a mechanical response to the point forecasts. As a result, the public will not expect – as it is often the case – interest rates to change only if the forecast changes. It will follow that even if input represented by elements of the pay-off matrix is not revised, new probabilities attached to alternative sets of assumptions will change the optimal monetary policy stance.

Table 6: Alternative Methods for Developing a Pay-Off Matrix

Method	Description	Examples
Benchmark Case		
Model forecast	Theory-based model, parameters calibrated Monetary policy endogenous (reaction function)	BoC, RBNZ, CNB
Halfway	Theory-based model, parameters calibrated Interest rates fixed (before or after decision)	SR, DA
Model projection	Model based on weak theory, parameters estimated Interest rates fixed (before or after decision)	BoE, DA
Possible Policy Reactions		
Interest rates	Consequences of possible reactions considered (e.g. interest rate increase, no change, decrease)	DA, BoE*
Reaction functions	Consequences of possible reactions considered (e.g. interest rate increase, no change, decrease)	DA, BoC*, RBNZ*
Alternative Sets of Assumptions		
Two extreme sets	Two sets of assumptions are specified "hawkish" and "dovish", central forecast is not emphasised	RBNZ*, KW
Benchmark +2	Three sets of assumptions are specified for central forecast and surrounding bands	CNB*
Several alternatives	Several alternative sets of assumptions specified, Central forecast is not emphasised	DA, CNB*
Benchmark + +	Several alternative sets of assumptions specified with respect to central forecast	DA, BoC, RBNZ*
Distribution	Distribution approximates numerous alternative sets of assumptions	DA, KW, BoE*, SR*
Subjective Probabilities		
Not attached	Alternative sets do not have attached probabilities. Decision rules are used that do not require them.	DA
Expert judgment	Experts attach subjective probabilities to alternative sets of assumptions.	DA, BoE*, SR*
Preliminary voting	Experts vote and distribution of votes defines probabilities of alternative sets of assumptions.	DA, RBNZ, BoC
Benchmark most likely	Probabilities attached verbally. Central forecast is the most likely. Risks have no probability attached.	All five CBs
Distribution	Distribution approximating numerous sets of assumptions show probabilities	DA, KW, BoE*, SR*
Pay-Offs		
Implicit	Various tables or charts illustrate divergence of inflation from target under specific conditions	All five CBs*
Strict targeting	Each element of the matrix contains single number (e.g. deviation of projection from target)	DA
Event probability	Elements of the matrix contain probabilities of inflation being inside targeted range	KW
Decision		
Governor	Governor decides, no explicit voting	RBNZ
Voting by members	Board decides by voting, votes are independent	BoE, CNB
Consensus method	Board needs to reach consensus, no explicit voting	BoC
Decision rule	Decision rule is applied, outcome depends on rule	DA

Note: *Abbreviations: BoE – Bank of England, BoC – Bank of Canada, CB – central banks, CNB –Czech National Bank, DA – decision analysis, KW – Wallis (forthcoming), RBNZ – Reserve Bank of New Zealand, SR – Swedish Riksbank.*
*) *In this case, the method has not been applied to the full extent (e.g. applied in combination with some other method or not before every decision).*

Appendix I – Example of the Quadratic-Linear Framework

The Basic Model and the Certainty Benchmark

Srour (1999) describes the following baseline model for the closed economy:

(1a) $\pi_{t+1} = \pi_t + d.(y_t - y^*)$

(2a) $y_{t+1} - y^* = b.(y_t - y^*) - c.(r_t - r^*)$

where π_t is the inflation rate, y_t is the aggregate output (in logs), y^* is the potential output (in logs), is r_t is the real interest rate and r^* is the average real interest rare.

In this type of models, the economy is represented with the two linear equations - the Phillips curve and IS curve. The central bank is represented with the quadratic loss function:

(3) $L(\pi, y) = \alpha(y_t - y^*)^2 + (1-\alpha)(\pi - \pi^*)^2$

where π^* is the inflation target and α represent the weight attached to the output gap. If α is equal to zero, the central bank in the model is a strict inflation targeter.

The expected value of the loss function is minimised. Then the optimal policy rule takes the following form:

(4a) $r_t - r^* = (1+b)/c.(y_t - y^*) + 1/(c.d).(\pi_t - \pi^*)$.

Since there is a full certainty about the model of the economy, this case is called a certainty benchmark. In other cases that are usually compared to the benchmark case some parts of the baseline model are uncertain. The uncertainty is represented with white noise shocks.

The Case of Linear Uncertainty

In this case, the uncertainty takes a linear form. For example, normally distributed additive shocks are introduced into both equations:

(1b) $\pi_{t+1} = \pi_t + d.(y_t - y^*) + \varepsilon_{t+1}$

(2b) $y_{t+1} - y^* = b.(y_t - y^*) - c.(r_t - r^*) + \eta_{t+1}$

where εt and ηt are white noise shocks.

Due to the nature of the shocks, the optimal policy rule is the same as in the baseline case:

(4b) $r_t - r^* = (1+b)/c.(y_t - y^*) + 1/(c.d).(\pi_t - \pi^*)$.

The Case of Non-linear Uncertainty

In this case, the uncertainty takes a non-linear form. For example, as explained in Srour (1999), parameters of IS curve are uncertain:

(1b) $\pi_{t+1} = \pi_t + d.(y_t - y^*) + \varepsilon_{t+1}$

(2b) $y_{t+1} - y^* = b.(y_t - y^*) - c_{t+1}.(r_t - r^*) + \eta_{t+1}$

where c_t is the white noise shock.

Since the uncertainties now combine in a non-linear manner, the optimal policy rule is different from the baseline case:

(4c) $r_t - r^* = (1+b)/(c.Sc).(y_t - y^*) + 1/(c.Sc.d).(\pi_t - \pi^*)$

where Sc is the ratio of the standard deviation of the shock c_t to its mean.

In this specific case, interest rates will move less due uncertain elasticity of demand to real interest rates.

Appendix II – Basic Terminology Used by Decision Analysis

Bayesian theory derives the probability from both data samples and prior knowledge in a consistent manner, and thus uses broader set of information than the textbook econometrics.

Decision rules select one of possible actions of decision makers according to its outcomes in different states of the world and their subjective probabilities. See Appendix III.

Decision tree is a tool for analysing decision problems with high degree of uncertainty. Outcomes of decisions are represented in the tree, and – according to the expected values – the best sequence of decisions is determined.

Dynamic programming is an approach to solving a sequence of decision problems. The algorithms selecting optimal decisions work in a backward way, from the final decision to the first one.

Good decision offers the best chance of success to meet the target in the time of the decision. Due to uncertainty, the good decision does not guarantee a good outcome.

Inertia assumption states that a decision maker chooses an alternative to the current state only if this alternative is better.

Minimax algorithm selects the action of a decision-maker that gives the best outcome in the least favourable state of the world.

Monte Carlo simulations provide probability distributions for output variables by generating random numbers as values of uncertain variables in the model.

Pay-off is an evaluation (usually numerical) of the outcome of a selected action of a decision maker under specific state of the world.

Pay-off matrix is a tool of decision analysis showing a pay-off for each combination of possible states of the world with possible actions of a decision-maker.

Robust control methods are mathematical methods that search for optimum solutions under uncertainty.

Subjective probability is a probability attached to a certain event by a decision maker according to her intuition without using statistical methods.

Appendix III – Examples of Decision Rules

The Laplace Rule

According to this decision rule, each set of alternative assumptions is treated as equally probable. Probability attached to each of the three alternatives in our example is therefore 1/3. For each of the three possible policy reactions, the expected pay-off value is then calculated. See Table III.1. The rule selects the policy reaction with the best expected pay-off, which in our example, is the lowest expected distance of the inflation from the target. We assume that the inflation target is symmetrical, and that both overshooting and undershooting the target imply an equal loss for policy makers.

Table III.1 – Pay-off Matrix: Laplace Rule

Alternative assumptions Possible reactions	Deflation pressures	Neutral pressures	Inflation pressures	Expected pay-off values
Reduce rates	0	3	5	8/3
No change	-1	1	4	4/3
Increase rates	-5	0.5	2.5	-2/3

The Maximin Rule

This decision rule is called the pessimist's rule because it aims at avoiding large losses. It is an analogy to the rule discussed in Cagliarini, Heath (2000) and Tetlow, von zur Muehlen (2000). The rule finds the worst pay-off for each of the three possible policy reactions. See Table III.2. Then the policy reaction with the minimal worst pay-off is selected.

Table III.2 – Pay-off Matrix: Maximin Rule

Alternative assumptions Possible reactions	Deflation pressures	Neutral pressures	Inflation pressures	The worst pay-off
Reduce rates	0	3	5	5
No change	-1	1	4	4
Increase rates	-5	0.5	2.5	-5

The Maximax Rule

This decision rule is called the optimist's rule because it selects the policy reaction that can lead to the best pay-off under specific circumstances. The rule finds the best pay-off for each of the three possible policy reactions. See Table III.3. Then the policy reaction with the best of the best pay-offs is selected.

Table III.3 – Pay-off Matrix: Maximax Rule

Alternative assumptions / Possible reactions	Deflation pressures	Neutral pressures	Inflation pressures	The best pay-off
Reduce rates	0	3	5	0
No change	-1	1	4	1
Increase rates	-5	0.5	2.5	0.5

The Minimum Regret Rule

This decision rule is sometimes called the most cautious rule. In the first step, it is necessary to construct the regret matrix from the pay-off matrix. Each element of the regret matrix is equal to the best pay-off for a given alternative set of assumptions *minus* the pay-off from the original pay-off matrix. See Table III.4. In the second step, the highest regret is selected for each possible policy reaction. Then the policy reaction with the smallest regret (out of the highest regrets) is selected.

Table III.4 – Regret Matrix: Minimum Regret Rule

Alternative assumptions / Possible reactions	Deflation pressures	Neutral pressures	Inflation pressures	The highest regret
Reduce rates	0-0	0.5-3	2.5-5	-2.5
No change	0-(-1)	0.5-1	2.5-4	-1.5
Increase rates	0-(-5)	0.5-0.5	2.5-2.5	5

The Rule of the Best Expected Value

This decision rule is often referred to as the rational rule. It requires attachment of relative probabilities to all alternative sets of assumptions. See Table III.5. This rule is similar to the Laplace rule. The expected pay-off value is calculated for each of the three possible policy reactions. The rule selects the policy reaction with the best expected pay-off.

Table III.5 – Pay-off Matrix: The Best Expected Value

Probabilities	*0.4*	*0.5*	*0.1*	
Alternative assumptions / Possible reactions	Deflation pressures	Neutral pressures	Inflation pressures	Expected pay-off
Reduce rates	0	3	5	2
No change	-1	1	4	0.5
Increase rates	-5	0.5	2.5	-1.5

The Best of the Most Probable Rule

This decision rule is also sometimes called the optimist's rule, because it only works with the most likely set of assumptions. This rule also requires attachment of probabilities to all alternative sets of assumptions. The most likely set of assumptions is selected. See Table III.6. Then the policy reaction that produces the best pay-off under the most likely set of assumptions is selected.

Table III.6 – Pay-off Matrix: The Best of the Most Probable

Probabilities	*0.4*	*0.5*	*0.1*
Alternative assumptions Possible reactions	Deflation pressures	Neutral pressures	Prevail inflation shocks
Reduce rates	0	3	5
No change	-1	1	4
Increase rates	-5	0.5	2.5
The most likely situation		*0.5*	

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