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Estimating market probabilities of future interest rate changes

Martin Hlušek^{*}

Abstract

The goal of this paper is to estimate the market consensus forecast of future monetary policy development and to quantify the priced-in probability of interest rate changes for different future time horizons. The proposed model uses the current spot money market yield curve and available money market derivative instruments (forward rate agreements, FRAs) and estimates the market probability of interest rate changes up to a 12-month horizon. The estimated probabilities and possible interest rate scenarios are consistent with the observed money market and FRA interest rates. Thus, the model's output has to be interpreted as a description of the current market consensus on future monetary conditions rather than a tool for predicting or setting the correct level of interest rates. The estimation method is based on standard money market data and thus is applicable to any developed financial market. The probability structure of expected interest rate changes in the future could serve as an indicator of the money market reaction to macroeconomic data releases and verbal interventions of monetary authorities. It also allows us to measure the extent of monetary policy predictability and thus to quantify the surprise effects of unexpected monetary policy changes.

JEL Codes: E52, E43

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Nontechnical Summary

The goal of this paper is to estimate the market consensus forecast of future monetary policy development and quantify the priced-in probability of interest rate changes for different future time horizons. The proposed model uses the current spot money market yield curve and available money market derivative instruments (forward rate agreements, FRAs) and estimates the market probability of interest rate changes up to a 12-month horizon. The estimated probabilities and possible interest rate scenarios are consistent with the observed money market and FRA interest rates. Thus, the model's output has to be interpreted as a description of the current market consensus on future monetary conditions rather than a tool for predicting or setting the correct level of interest rates. The estimation method is based on standard money market data and thus is applicable to any developed financial market. The probability structure of expected interest rate changes in the future could serve as an indicator of the money market reaction to macroeconomic data releases and verbal interventions of monetary authorities. It also allows us to measure the extent of monetary policy predictability and thus to quantify the surprise effects of unexpected monetary policy changes.

Interest rate adjustments by the central bank are modelled as a Poisson jump process with a certain frequency. The frequency is treated as a piecewise constant function on predefined time intervals. The extended expectation theory is applied to derive the whole money market yield curve based on the expected development of the central bank's monetary policy instrument. The jump frequencies are estimated in such a way that the model yield curve fits the observed money market and forward market curves, and the estimates allow us to restore the market probabilities of interest rate changes for different future time horizons. The derived probabilities of expected interest rate changes are depicted in the charts.

For example, the model's output using the Czech market data from 27 November 2001 (the central bank cut interest rates by 50bp on the following day) suggest that the market was almost sure before the CNB meeting that interest rates would be cut in a three-month horizon, but the outcome of the nearest monetary policy meeting was rather uncertain. The market was putting the highest 35% probability on one rate cut by 25bp, but the probabilities of an unchanged rate and two rate cuts (*i.e.* one by -50bp) were at non-negligible 30% and 22%, respectively. The most likely scenario in a three-month horizon was four rate cuts by 25bp, *i.e.* reducing the basic interest rate from 5.25% to 4.25%.

The main advantage of the model is its simplicity and wide applicability. It quantifies to what extent the market takes into account possible future monetary scenarios. While this could be intuitively clear to an active money market participant (a trader) by looking at the money market and FRA curves, portfolio managers or corporate financial planners need to rely on a salesman's recommendation or market rumours. This model provides them with an objective measure of "priced-in" scenarios and can serve as an alternative tool for practical financial decisions.

1. Introduction

There is not much research focused on estimating the market probabilities of future monetary policy changes. The majority of papers dealing with this problem focus on the US market, and market expectations are derived from Fed Fund Futures (FFFs). Boldin (2000) has shown that FFFs were a useful predictor of the Fed's monetary decisions in 1999 and 2000. He introduced a very simple model looking just one month forward. The expected future Fed fund rate was derived as the weighted average of possible interest rate changes in the following month in his model. Comparing the current FFFs and the expected Fed fund rate allowed him to derive the market probabilities of the Fed's possible moves. Robertson and Thomson (1997) explain that the prediction power of FFFs is much weaker if the simple model is tested over a longer time horizon. Testing the model on data between November 1997 and August 1998, it turned out that the model was successful just in 12 out of 38 of the Fed's interest rate changes. However, the prediction power of the model significantly increased when the probability was estimated from both one-month and two-month FFFs. Using the more sophisticated two-month forward-looking model, FFFs correctly predicted 26 out of 38 of the interest rate changes. The authors explain that FFFs have two drawbacks. First, FFFs are related to an average of the Fed fund rates during the month and not to the Fed's target which is directly affected by the central bank. Second, the market is often uncertain about the precise timing of the Fed's move. Therefore, the probability of an interest rate change can be sometimes included in both one-month and two-month futures. The authors raise a hypothesis that the FFF prediction power can differ in relation to the number of days remaining till the Fed's monetary policy meeting.

Only the most developed financial markets have available futures on the interest rate targeted by the central bank. Consequently, the FFF model's applicability to the Czech market is limited. However, empirical studies (Kotlán 2002) show that the Czech money market curve is a valuable predictor of future interest rates and thus contains information about the central bank's expected future behaviour. This paper presents a methodology similar to Boldin (2000), but uses standard money market instruments for estimating the money market probabilities of future monetary policy changes. Interest rate adjustments by the central bank are modelled as a Poisson jump process with a certain frequency. The frequency is treated as a piecewise constant function on predefined time intervals. The extended expectation theory is applied to derive the whole money market yield curve based on the expected development of the central bank's monetary policy instrument. The jump frequencies are estimated in such a way that the model yield curve fits the observed money market and forward market curves. The estimated frequencies allow us to restore the market probabilities of interest rate changes for different future time horizons.

The rest of the paper is organised as follows. The second section introduces the theoretical model in two stages. The first stage describes the very simple model with a constant jump-frequency, which clearly explains the basic idea of the model. The second stage describes the more complex and more realistic version with the non-constant frequency of possible monetary policy changes. The third section explains how to fit the model to real data and shows the model's performance on the Czech market. The last section summarises the achieved results, advantages and disadvantages of the model.

2. The Poisson-Jump Model of the Money Market Yield Curve

This section presents the theoretical model in two stages. For a better understanding, the simplest version of the model is derived in the first stage in order to clarify the basic idea how market expectations affect the money market instruments. The more sophisticated, realistic version is derived in the second stage. While the simplest model assumes that the market relies on the time-homogenous behaviour of the central bank, the more complex model allows for different monetary policy approaches in different future time periods. The basic assumptions are the same for both approaches.

2.1 Assumptions

Let us assume that there is a basic short-term interest rate r_B which is directly controlled by the central bank (usually the two-week repo rate). Let us further assume that the central bank can change the basic interest rate up or down by Δ with a certain time frequency. We treat Δ as a typical step used by the central bank for monetary policy adjustments (typically +/- 25 basis points (bp)). As for the frequency, let us assume that we can model market expectations by the Poisson jump process, *i.e.* that the central bank can change r_B by Δ with probability λdt for any time-interval dt (one day, for example). We assume that future interest rate changes are conditionally independent. Using the stochastic calculus notation, we can describe r_B as a continuous Poisson jump process following the equation,

$$dr_B = \Delta dq \tag{2.1}$$

Integrating the equation gives us a formula for future $r_{B,t}$,

$$r_{B,t} = r_{B,0} + \Delta q_t \tag{2.2}$$

where $r_{B,0}$ is the current basic interest rate and q_t the number of interest rate changes by time t . It is easy to derive the expected value of $r_{B,t}$, using the definition of the Poisson process

$$\Pr(q_t = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t},$$
$$Er_{B,t} = r_{B,0} + \Delta \lambda t \tag{2.3}$$

In order to build the whole money-market yield curve, we need to make an additional assumption, which postulates how longer-maturity interest rates depend on r_B . For this purpose, we will assume that the generalised expectation theory holds. In other words, we will assume that longer interest rates are weighted averages of the expected basic interest rate. This assumption relies on non-arbitrage markets, where longer maturity interest rates yield no advantage over the short-rate rollover strategy. Unlike the pure expectation theory, we will generalise this assumption by adding a maturity-dependent term premium r_{p1} . Mathematically written, an interest rate with maturity M , r_M can be derived as

¹ There are several theories explaining why the pure expectation hypothesis does not hold. Portfolio selection theory argues that long-term rates are riskier and thus must offer a premium. We can also argue that longer-term

$$r_{M,0} = \frac{1}{M} E \int_0^M r_{B,s} ds + r_P(M) \quad (2.4)$$

The assumption that r_B can jump by Δ does not mean that the central bank is expected to cut ($\Delta < 0$) or hike ($\Delta > 0$) the interest rate during one session by exactly Δ . Δ must be viewed as the smallest possible interest rate adjustment. If the market anticipates a more aggressive monetary policy change, it should be reflected by λ . For example, if the market is confident that the central bank will move the interest rate on its next meeting exactly by Δ , λ must be such that $\lambda T = 1$, for T being the time of the central bank meeting. Conversely, if the market is betting on a 2Δ adjustment, it should be reflected by double λ , such that $\lambda T = 2$. It will be obvious from the model that Δ and λ cannot be treated separately, as the interest rate formula always contains the multiple $\Delta\lambda$, which can be interpreted as an expected interest rate change per unit of time. Let us call the multiple *interest rate change potential*, as it describes the market feeling about possible interest rate changes in the future. The disadvantage of this model is that it does not distinguish between the different timing of interest rate changes. From the point of view of this model, two changes by Δ in one-month intervals have the same impact on the money market as a single 2Δ change during two months. On the other hand, frequency λ very intuitively describes “how surely” the market bets on an interest rate change. The more complex version of the model tries to cope with the disadvantage by introducing finer subintervals with different frequencies. This allows application of different market expectations for different time-periods in the future and thus better describes the time heterogeneity of interest rate change expectations.

2.2 The Simplest Version

Using the assumptions mentioned above, we can derive the shape of the money market yield curve using the current interest rate r_B , fixed rate change Δ , and the market probability of interest rate changes described by frequency λ . From (2.3) and (2.4) we have,

$$r_{M,0} = \frac{1}{M} \int_0^M (r_{B,0} + \Delta\lambda s) ds + r_P(M) = r_{B,0} + \frac{\Delta\lambda}{2} M + r_P(M) \quad (2.5)$$

Formula (2.5) corresponds with our intuition. If the market expects interest rate hikes ($\Delta > 0$), the money market yield curve is increasing with maturity M . The slope of the curve (apart from the term-premium effect) depends on the interest rate hike potential $\Delta\lambda$, which also makes sense.

Apart from money market interest rates, there are also forward interest rates, which directly reflect the market expectation of future interest rates. Loosely speaking, the forward interest rate $f_{M \times N}$ ($N > M$) is the current market guess, what the N - M maturity interest rate will be at time M . Putting 1 unit of money on an M -maturity deposit and buying $f_{M \times N, 0}$ allows us to deposit money for time M at the rate of $r_{M,0}$ and re-deposit the receipt for time interval $(N-M)$ at the rate of $f_{M \times N, 0}$. Unlike in the simple roll-over strategy, the market participant does not bear any risk, since both interest

deposits are less liquid and therefore, according to the liquidity-preference theory, must yield a compensation (see Keynes, 1936 or Hlušek, 1999).

rates $r_{M,0}$ and $f_{M \times N,0}$ are known *ex-ante*. Similarly as there is a market for simple interest rates $r_{M,0}$, the money market participants trade $f_{M \times N,0}$ for different maturities M, N on a daily basis².

If there is no arbitrage in the market, then using the $r_{M,0}$ and $f_{M \times N,0}$ interest rates for time-period N must yield the same interest as the currently available interest rate $r_{N,0}$. This idea brings us to the mathematical definition of $f_{M \times N,0}$,

$$(N - M)f_{M \times N,0} + Mr_{M,0} = Nr_{N,0} \quad (2.6)$$

Substituting for $r_{M,0}$ and $r_{N,0}$ from (2.5) allows us to derive a formula for $f_{M \times N,0}$,

$$f_{M \times N,0} = \frac{N}{N - M}r_{N,0} - \frac{M}{N - M}r_{M,0} \quad (2.7)$$

$$= r_{B,0} + \Delta\lambda M + \frac{\Delta\lambda}{2}(N - M) + \frac{Nr_p(N) - Mr_p(M)}{N - M} \quad (2.8)$$

Formula (2.8) confirms the importance of the interest rate change potential $\Delta\lambda$ as it happens to be the slope of the FRA curve. Note that the only unknown element in the model is the frequency λ , which is for quantifying the market expectations of basic interest rate changes. If we use market data for $r_{M,0}$ and $f_{M \times N,0}$, set $\Delta = \pm 0.25\%$ (choosing the sign according to the shape of the money market and forward rate curve), we can estimate λ and compute the probabilities of an interest rate change for different time intervals. Of course, the model yields only very simple interest rate structures (linear if we neglect the premium effect), which seems to be very restrictive and unrealistic. Nevertheless, the simple model describes the basic idea of how we can obtain the market probabilities of expected interest rate changes from the current money market and forward market yield curves.

2.3 The More Complex Version

The disadvantage of the simple model described above is that we assume time-homogenous market expectations. According to the model, the probability of an interest rate change during the next month is the same as the probability of a change during one month any time in the future. Consequently, the simple model predicts only linear money market yield curves. Market participants are more sophisticated. Usually, the market expects some inflation development and, based on the inflation scenario, some monetary policy changes. Moreover, the market has typically a much stronger view about the nearest one-month horizon, as all the central banks' comments and indications are associated with the nearest monetary policy meeting (usually taking place once per month). The market expectations beyond this meeting are rather vague.

Based on the assumption of different market expectations for different future-time horizons, we can derive the model for a time-dependent frequency $\lambda(t)$. The formula for interest rates $Er_{B,t}$ (2.3) now becomes

² The forward rates are called FRAs (forward rate agreements). They are available in any developed money market, and they are traded (similarly as money market interest rates) with a certain bid-ask spread. The liquidity of FRAs can vary for different maturities M, N .

$$Er_{B,t} = r_{B,0} + \Delta \int_0^t \lambda(u) du \quad (2.9)$$

and (2.5) for $r_{M,0}$ translates into

$$\begin{aligned} r_{M,0} &= \frac{1}{M} \int_0^M (r_{B,0} + \Delta \int_0^s \lambda(u) du) ds + r_p(M) \\ &= r_{B,0} + \frac{\Delta}{M} \int_0^M (M-u) \lambda(u) du + r_p(M) \end{aligned} \quad (2.10)$$

In order to exploit the information about the market expectations of future monetary policy included in FRA rates, we need to derive the FRA curve implied by the extended model. Using the formula for $r_{M,0}$ (2.10) and substituting into (2.7), we obtain

$$f_{M \times N,0} = r_{B,0} + \Delta \int_0^M \lambda(u) du + \frac{\Delta}{N-M} \int_M^N (N-u) \lambda(u) du + \frac{Nr_p(N) - Mr_p(M)}{N-M} \quad (2.11)$$

Obviously, formulae (2.10) and (2.11) allow us to fit a wider variety of money market and forward market yield curve shapes than the linear structure implied by the constant λ approach. It is worth mentioning that expectations affect the money market curve again via interest rate potentials $\Delta\lambda$, which now can be different over time. For example, if the market gives zero probability ($\lambda=0$) to interest rate changes by time T , the model money market curve remains flat by maturity T , but has a non-zero slope thereafter (if the long-horizon λ s are non-zero). Consequently, this model can better reflect a possible time-non-homogenous market expectation of interest rate changes in the future.

3. The Model Fitted to Czech Money Market Data

This section introduces a slightly modified version of the advanced theoretical model, which is applicable to real data. The second part explains in detail the estimation methodology and the output of the model. The model is fitted to several past snapshot Czech money market data in the last part of this section.

3.1 Modification Appropriate for Data Fitting

The model derived in the previous chapter is interesting from the theoretical point of view, but it has no practical use, as we cannot estimate the (unrestricted) function $\lambda(t)$ from a limited set of market data. Limiting $\lambda(t)$ to a piecewise constant function seems to be a good compromise yielding a more general model than the simplest version, but with a bounded degree of freedom. For this purpose, let us assume that

$$\lambda(t) \equiv \begin{matrix} \lambda_1 & 0 < t \leq T_1 \\ \vdots & \vdots \\ \lambda_N & T_{N-1} < t \leq T_N \end{matrix} \quad (3.1)$$

Substituting for $\lambda(u)$ from (3.1) into (2.10) and integrating separately for each subinterval on which λ is a constant, we obtain a new generalised formula for money market rate $r_{M,0}$ and $T_{M-1} < M \leq T_M$,

$$\begin{aligned} r_{M,0} = & r_{B,0} + \frac{\Delta}{2M} \lambda_1 [M^2 - (M - T_1)^2] + \frac{\Delta}{2M} \sum_{i=2}^{M-1} \lambda_i [(M - T_{i-1})^2 - (M - T_i)^2] + \\ & + \frac{\Delta}{2M} \lambda_M (M - T_{M-1})^2 + r_P(M) \end{aligned} \quad (3.2)$$

Similarly, using the definition (3.1), we can modify the formula for forward interest rates (2.11) and for $M < N$, $T_{M-1} < M \leq T_M$ and $T_{N-1} < N \leq T_N$ derive

$$\begin{aligned} f_{M \times N,0} = & r_{B,0} + \Delta \lambda_1 T_1 + \Delta \sum_{i=2}^{M-1} \lambda_i (T_i - T_{i-1}) + \Delta \lambda_M (M - T_{M-1}) + \\ & + \frac{\Delta}{2(N-M)} \lambda_M [(N-M)^2 - (N - T_{M+1})^2] + \frac{\Delta}{2(N-M)} \sum_{i=M+1}^{N-1} \lambda_i [(N - T_{i-1})^2 - (N - T_i)^2] + \\ & + \frac{\Delta}{2(N-M)} \lambda_N (N - T_{N-1})^2 + \frac{Nr_P(N) - Mr_P(M)}{N-M} \end{aligned} \quad (3.3)$$

This version of the model has a limited number of degrees of freedom and thus can be fitted to real data using a sufficient number of money market interest rates $r_{M,0}$ and forward market interest rates $f_{M \times N,0}$. For a fixed Δ and time periods T_1, \dots, T_N , the model has unknown parameters $\lambda_1, \dots, \lambda_N$ and the unobservable term premiums $r_P(M)$. The number of the premiums is exactly equal to the number of money market interest rates, which we decide to use. Consequently, in order to be able to estimate the frequencies $\lambda_1, \dots, \lambda_N$ we need at least N FRA interest rates. It should be kept in mind that the model is not estimated from time series, but from snapshot market data. As such, it reflects the current market mood and its immediate expectation concerning future monetary policy development with no ambition to predict the actual interest rate changes. The purpose of the model is to quantify market expectations rather than to predict future central bank behaviour.

3.2 Fitting to the Observed Money Market and Forward Market Curves

In order to fit the derived model to real data, we need to fix the time-intervals on which λ is assumed to be a constant. It is reasonable to set the times T_1, \dots, T_N equal to the most liquid maturities of money market instruments, as they best reflect the market consensus on future monetary conditions and the prices are not distorted by a temporary excess of supply or demand. The most actively traded maturities up to one year are typically one, three, six, nine, and twelve months ($N=5$) with money market interest rates $r_{1,0}$, $r_{3,0}$, $r_{6,0}$, $r_{9,0}$, and $r_{12,0}$, respectively. Other maturity prices are derived as weighted averages of the prices of the five benchmarks. The one-month maturity is crucial to our model. Central banks typically discuss monetary policy on a monthly basis. Consequently, the one-month interest rate (and thus λ_1) reflects the market expectations of the monetary policy decision at the latest meeting.

The number of time intervals ($N=5$) defines the minimum number of forward rates necessary to estimate the unknown parameters $\lambda_1, \dots, \lambda_5$ and $r_P(1)$, $r_P(3)$, $r_P(6)$, $r_P(9)$, and $r_P(12)$. The criterion

for choosing appropriate forward rate maturities (at least five) is the same as for the time intervals, the highest liquidity. Not surprisingly, the most liquid forward rates are those related to the benchmark money market interest rates. Consequently, the best choice seems to be forwards on the three-month money market interest rate f_{1x4} , f_{3x6} , f_{6x9} , and f_{9x12} and forwards on the six-month money market interest rate f_{1x7} , f_{3x9} , and f_{6x12} . The number of forward interest rates exceeds five, which insures that the unknown coefficients can be identified.

As for the choice of the constant Δ , it should be equal to a typical interest rate adjustment made by the central bank. In most developed financial markets with one-digit interest rates, the step will be equal to +25bp or -25bp³. The sign can be easily distinguished from the shape of the money market curve. While increasing interest rates with maturity indicate expected monetary tightening (positive Δ), a flat or downward sloping curve agrees with expected monetary easing (negative Δ). As it has been explained in section 2, the magnitude of Δ is not important since it enters the model via multiple $\lambda\Delta$. In other words, Δ defines the scale of the interest rate change potential. λ itself measures the “number of events” per unit of time, while $\lambda\Delta$ measures the number of bp per unit of time.

The least problematic is the basic interest rate $r_{B,0}$. This is the interest rate affected directly (offered rate by the central bank as a money market facility) or indirectly (targeted interest rate using open market operations) by the monetary authority, which is adjusted and announced on a regular basis. This interest rate defines the level of very short-term money market interest rates, and it is understood as one of the basic measures used to quantify monetary policy prudence.

Having defined the time intervals and money market and forward market instruments, we can use equations (3.2) and (3.3) and estimate the unknown parameters in such a way that the model values best fit⁴ the observed interest rate benchmarks $r_{1,0}$, $r_{3,0}$, $r_{6,0}$, $r_{9,0}$, $r_{12,0}$, f_{1x4} , f_{3x6} , f_{6x9} , f_{9x12} , f_{1x7} , f_{3x9} , and f_{6x12} . The estimated frequencies $\lambda^*_1, \dots, \lambda^*_5$ allow us to derive the Poisson-type probabilities of k interest rate changes (by Δ) until time T_n

$$P_k(T_n) = \frac{(\Lambda_n T_n)^k e^{-\Lambda_n T_n}}{k!}$$

$$\text{for } \Lambda_n T_n = \lambda^*_1 T_1 + \sum_{i=2}^n \lambda^*_i (T_i - T_{i-1}) \quad (3.4)$$

As an example, we can apply the model to the Czech market data from 27 November 2001 (the central bank cut interest rates by 50bp on the following day). Table 1 shows the used and fitted data, the estimated frequencies λ^* and the term premiums r^*_p . Both money market and FRA curves were downward sloping indicating expected monetary policy easing ($\Delta=-25$ bp). The basic interest rate $r_{B,0}$ (two-week repo rate) was at 5.25% p.a. The strong monetary easing expectation is reflected by high λ^*_1 , λ^*_2 , λ^*_3 , each of which predicts on average more than 12 “events” per unit of time (year). In other words, the market was pricing in higher than 25bp interest rate adjustments in one step.

³ 100 bp (basis points) is 1%.

⁴ Any curve fitting method can be used to identify the coefficients. The standard SSE criterion is used in the applications below.

Table 1: Data and estimated coefficients

(market and model interest rate values and the estimated frequencies λ^* and the term premiums r^*_P)

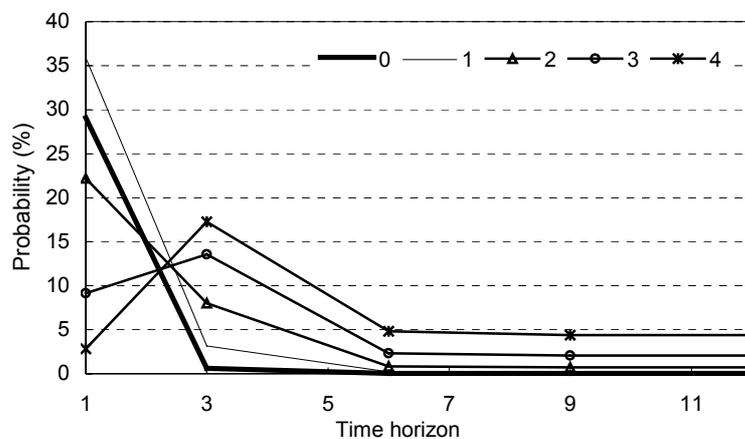
	Market Values	Model Values	λ^*	r^*_P
$r_{1,0}$	5.09	5.10	14.82	0.00
$r_{3,0}$	5.01	5.02	19.98	0.30
$r_{6,0}$	4.83	4.83	13.42	0.63
$r_{9,0}$	4.82	4.78	5.38	0.94
$r_{12,0}$	4.82	4.75	8.07	1.21
$f_{1 \times 4}$	4.86	4.91		
$f_{3 \times 6}$	4.58	4.64		
$f_{6 \times 9}$	4.65	4.67		
$f_{9 \times 12}$	4.68	4.67		
$f_{1 \times 7}$	4.84	4.77		
$f_{3 \times 9}$	4.67	4.66		
$f_{6 \times 12}$	4.63	4.67		

The derived probabilities of expected interest rate changes P_k are typically decreasing relatively fast for higher k reflecting the market belief in a limited number of interest rate cuts in a one-year horizon. The curves $P_k(T)$ for $k=0,1,2,3$ and 4 are depicted on Chart 1. The bold line indicates the market assessment of maintained stable interest rates. It is obvious that the market was almost sure that interest rates would be cut in a three-month horizon, but the outcome of the nearest monetary policy meeting was rather uncertain. The market was putting the highest 35% probability on a one-rate cut by 25bp, but the probabilities of an unchanged rate and two rate cuts (*i.e.* one by -50bp) were at a non-negligible 30% and 22%, respectively. The most likely scenario in the three-month horizon was four rate cuts by 25bp, *i.e.* a reduction in the basic interest rate from 5.25% to 4.25%.

Figure 1: Market probabilities of 0, 1, 2, 3 and 4 interest rate cuts by 25bp

Czech data, 28 November 2001

(scenario probability on y-axis, forecasting time horizon in months on x-axis)



3.3 The Czech Market Reaction to Central Bank Monetary Policy Changes

In order to demonstrate the usefulness of the model and to show how to interpret the estimated results, the model has been applied to the Czech data the day before and the day after the meetings of the Czech Central Bank (CNB) from November 2001 to March 2002. Comparing the two probability charts, we can learn how surprising the CNB monetary policy changes were. It turns out that the shapes of the probability curves can describe the money market mood before the monetary policy meeting and thus can be a helpful tool for the central bank's interest rate decision process. The charts related to the five meetings are in the Appendix.

28 November 2001 – interest rate cut by 50bp

As it has been discussed in the example above, the market was waiting for a 25bp interest rate cut before this meeting (35%), but an interest rate cut of 50bp was not totally unexpected (22%). The situation in the money market rapidly changed after the interest rate cut. Since monetary easing of 50bp was an unusually fast interest rate reduction (and less expected than a 25bp rate cut), the market expected interest rates to be steady for at least one month (83%) after the November's meeting. Even in a three-month horizon, stable interest rates seemed to be more likely than a further interest rate cut of 25bp. However, the market remained in an easing bias as reflected by the prevailing (38%) probability of the 25bp interest rate cut in a twelve-month horizon.

20 December 2001 – unchanged interest rates

The market mood shifted towards interest rate cuts at the end of December for two reasons. First, the local market was under the influence of global economic development. The interest rate cut by the US Fed on 10 December indicated a gloomy economic outlook and low inflationary pressures. Second, the Czech market was surprised by fast local currency appreciation at the end of 2001. The strong currency indicated room for the interest cuts to offset the currency-related tightening of monetary conditions. Nevertheless, the market did not expect the CNB to cut rates at the December meeting (expected unchanged interest rates with 60% probability), but in a three-month horizon. After the November experience, the market assessed 25bp and 50bp interest rate cuts as being equally likely, near 30%. The CNB met market expectations and left interest rates unchanged. Consequently, the market was insured that its expectations were correct and did not need to adjust its outlook. The shape of the probability curves remained almost unchanged after announcement of the meeting's outcome.

31 January 2002 – interest rate cut by 25bp

This market expectation was affected by the interim meeting in mid-January when the CNB cut rates by 25bp in response to continuing currency appreciation. Given the short time passed since the interim meeting, the market did not expect the CNB to cut rates once more at the end of January and counted on stable interest rates (with a probability 60%). The longer-term outlook remained biased towards further easing, but the market expected the CNB to come back to a cautious 25bp rate reduction. The probability of a 50bp rate cut during the following three months was about one-third of that associated with only the 25bp rate cut. Similarly as in November, the surprising CNB interest rate decision triggered an adjustment in market expectations. After the

meeting, the market switched into neutral bias and assumed that the monetary-easing period would be finished with a stable rates scenario probability near 90%. This outlook was also supported by the changing global economic environment.

28 February 2002 – unchanged interest rates

Money market and FRA yield curves were flat up to six months before the meeting, indicating a sustained neutral monetary policy outlook. Only nine-month and twelve-month maturities indicated possible interest rate hikes, but the increase could also indicate higher term premiums as monetary policy uncertainty increased. The shape of the probability curves did not change very much compared to the previous months. It seems that the market became surer about stable interest rates in the three-month horizon after the CNB left interest rates unchanged. However, the February charts should be interpreted with caution, as the estimation procedure did not converge very well. The reason is that the shape of the curves on the long end was fitted by an adjustment of the term premiums on nine and twelve months, but the distinction between r_p^* and λ^* was probably very weak.

28 March 2002 – unchanged interest rates

This meeting is interesting since the market changed its bias from easing to tightening ($\Delta=+25\text{bp}$), in line with the leading world money market yield curves. The market remained short-term neutral (100% probability of stable interest rates in a three-month horizon), but found monetary tightening by 25bp possible in a six-month horizon. The probability of an interest rate hike of 25bp or 50bp in a nine-month horizon was higher than the probability of stable interest rates and in a twelve-month horizon. The market did not exclude the possibility of a 100bp interest rate increase (*i.e.* coming back to November's 5.25%). The CNB left interest rates unchanged as expected, but indicated "equally weighted risks". As a reaction, the market adjusted the probability of an interest rate hike in a six-month horizon downwards. The long-term monetary tightening outlook remained unchanged.

4. Conclusions

The presented model allows us to extract the estimated market probability of future monetary policy changes. Unlike the previous models applied to developed markets, this model relies on standard money market data and thus can be applied to the Czech market. The case studies of the Czech monetary policy changes show that the model is able to quantify market expectation before the central bank meeting and thus indicate if the monetary policy decision will be surprising or not. It turns out that the market does not adjust its monetary policy outlook if the central bank's decision meets its expectation. In contrast, a surprising interest rate cut leads to a substantial decrease in the probability of additional cuts in the future. Further, the case studies also showed that the Czech money market is sensitive to global money market trends (March 2002) indicating that the market expects the CNB to act in accordance with the ECB and the Fed.

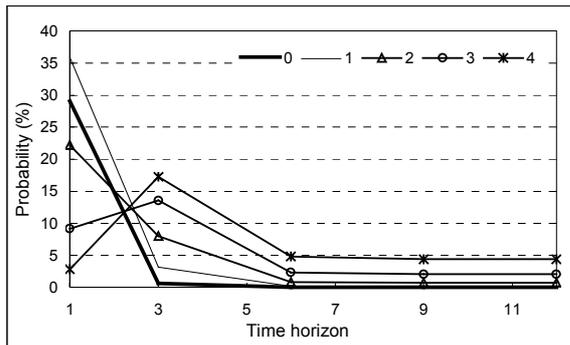
The main advantage of the model is its simplicity and wide applicability. It quantifies to which extent the market takes into account possible future monetary scenarios. While this could be intuitively clear to an active money market participant (a trader) by looking at the money market and FRA curves, portfolio managers or corporate financial managers need to rely on a salesman's recommendation or market rumours. This model provides them with an objective measure of "priced-in" scenarios and can serve as an alternative tool for practical financial decisions. Of course, the simplicity is counterbalanced by disadvantages. The main disadvantage is that the model is able to identify only one-direction interest rate changes. The fixed adjustment step Δ is either positive or negative, and therefore, ignores the possibility of a monetary policy stance change during 12 months. Another disadvantage is that the model ignores a shorter time period than one month. According to the model, the first monetary policy change can take place anytime during the following one month. However, real expectation is almost always focused on one particular date when the regular monetary policy meeting is planned. Consequently, the model provides us with biased probabilities. In order to make the estimates comparable in time (make the time bias always the same), the model should be fitted to market data on the same day of the month.

The model can be extended in several ways. The first way is to add more market data in order to make the estimation process more precise. For example, the Poisson-type expectation allows us to derive *ex ante* interest rate volatility. This could be fitted to interest rate option-implied volatility data. The second way is to use a model for term premiums $r_p(1)$, $r_p(3)$, $r_p(6)$, $r_p(9)$, and $r_p(12)$ (estimates from historical data, for example) and thus limit the number of unknowns and improve the model's convergence characteristics. The presented model converges very badly if both the money market and FRA curves are flat and the model has a tendency to explain the shapes solely by term premiums (whose estimates do not necessarily make sense). The third way of possible modification is to redesign the underlying probability distribution measure. Binomial distribution (rates changed versus unchanged), for example, would allow us to model the expected interest rate change timing precisely. However, such an approach would require introducing random Δ , as the "changed rates" scenario does not fully describe the extent of monetary policy adjustment.

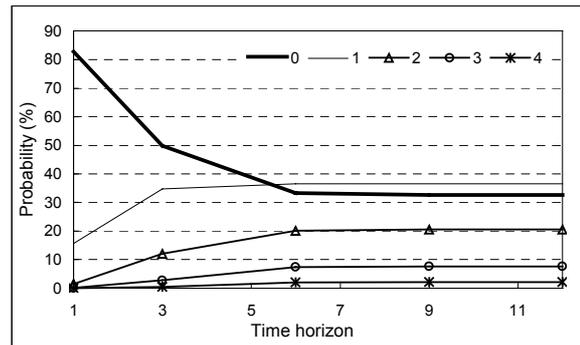
Appendix: Market probabilities of future interest rate changes

Derived probabilities from the estimated parameters one day before and one day after the regular CNB monetary policy meetings
(scenario probability on y-axis, forecasting time horizon in months on x-axis)

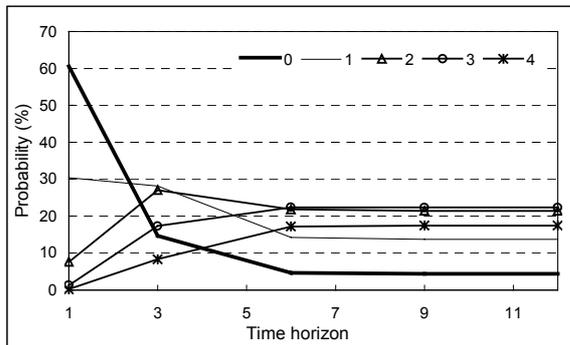
27 November 2001 ($\Delta = -0.25$)



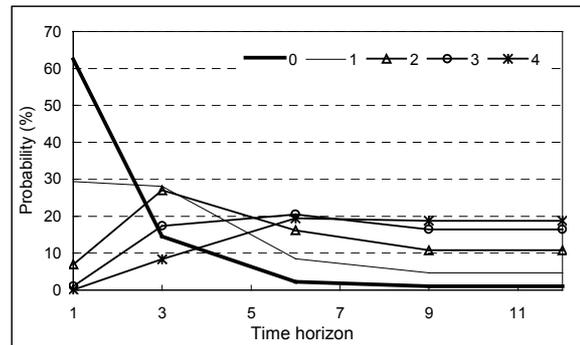
29 November 2001 ($\Delta = -0.25$)



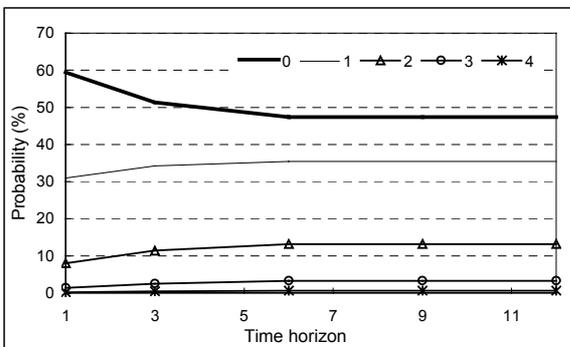
19 December 2001 ($\Delta = -0.25$)



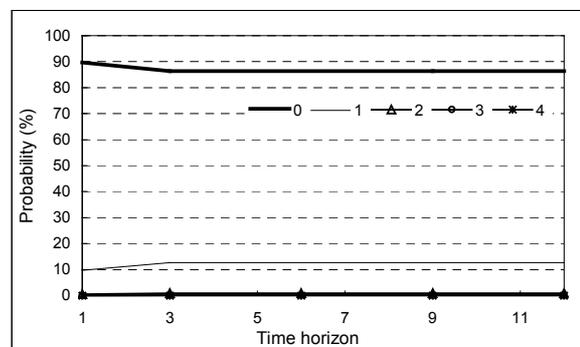
21 December 2001 ($\Delta = -0.25$)



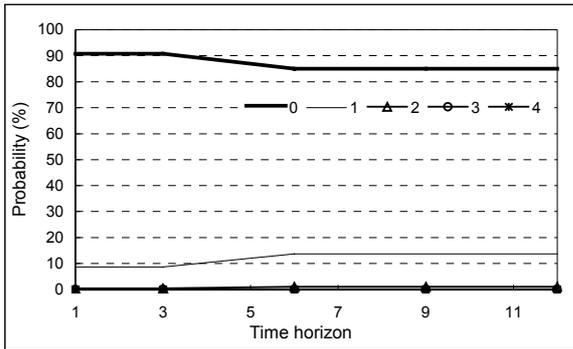
30 January 2002 ($\Delta = -0.25$)



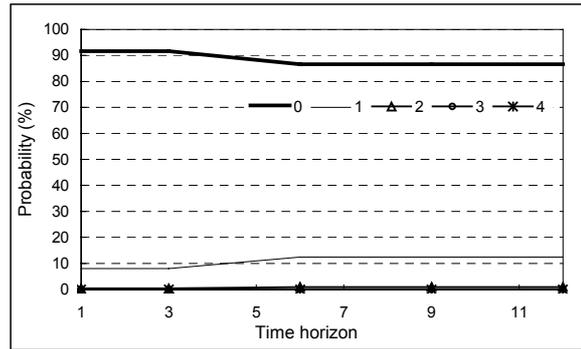
1 February 2002 ($\Delta = -0.25$)



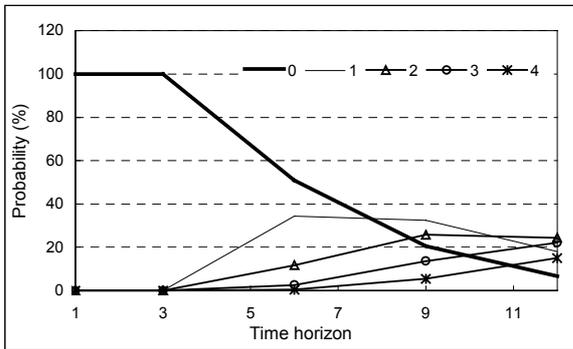
27 February 2002 ($\Delta = -0.25$)



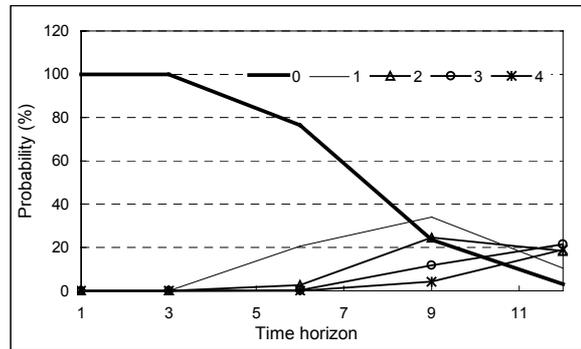
29 February 2002 ($\Delta = -0.25$)



27 March 2002 ($\Delta = +0.25$)



29 March 2002 ($\Delta = +0.25$)



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