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Abstract

We present the upgraded version of g3+, the Czech National Bank's core forecasting model, which became operational in April 2024 and summarizes its additional modifications over 2024. This paper outlines the innovative features of the model and the motivations behind their adoption. The enhancements also reflect the period from 2020 to 2022, which was marked by extraordinary events such as the Covid-19 pandemic and a significant surge in energy commodity prices. The upgraded g3+ now includes, among others, the endogenous decomposition of foreign economic activity into gap and trend components, a refined structure of foreign producer prices, and adjusted links between foreign and domestic economies. In addition, several model parameters have been recalibrated to reflect current and anticipated economic conditions. The introduction of these model changes and parameter adjustments lead to improved forecasting performance relative to the previous version of the model.

JEL Codes: C51, C53, E27, E37, F41.
Keywords: Conditional forecast, DSGE, energy, g3+ model, small open economy, two-country model.

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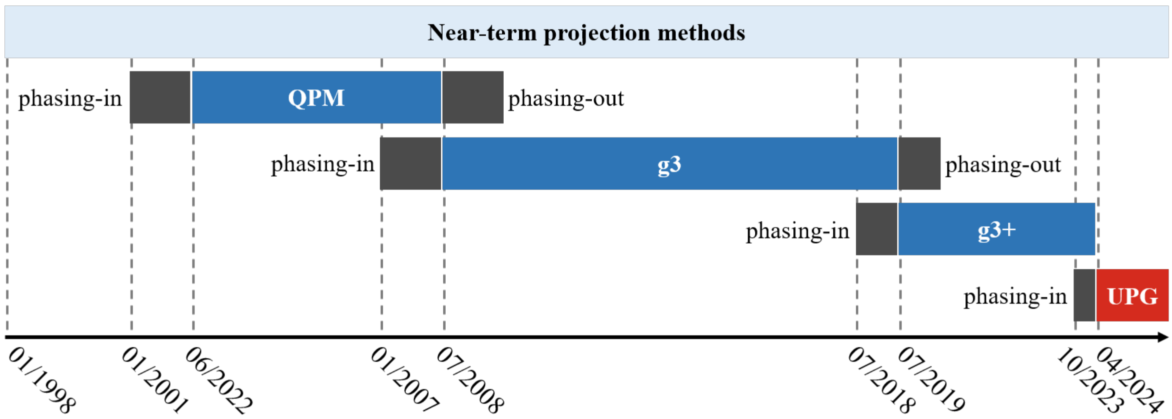
1. Introduction

The monetary policy framework of the Czech National Bank (CNB) incorporates the Forecasting and Policy Analysis System (FPAS) that includes a diverse set of analytical and forecasting tools. This system is centered around a core (primary) projection model, supported by various satellite models. Since the third quarter of 2019, the g3+ model has served as the central forecasting tool of the CNB (see Figure 1). To keep pace with recent economic developments and evolving forecasting practices, continuous refinement of the analytical apparatus is essential. We believe that this transparent approach increase credibility of the CNB in terms of economic modelling and forecasting.

Figure 1: Timeline of Implementation of the Core Forecasting Models at the CNB



Note: Dark blue bars indicate core forecasting models of the CNB. The abbreviation "QPM" represents the Quarterly Projection Model (see Coats et al. (2003)). The abbreviation "UPG" stands for the upgraded g3+ model.

Three main general motivations are being reflected in the modifications and improvements to the model. First, the extraordinary events of the 2020–2022 period highlighted the needs to improve our framework. In 2020 and 2021, numerous countries, including the Czech Republic, faced the COVID-19 pandemic associated with government measures limiting production capacities of economies. In 2022, a strong surge in energy commodity prices sparked significant inflationary pressures, that were particularly transferred through the producer prices. Second, our motivation to introduce novel features stems from insights gained from our forecasting and analytical practice since the introduction of the g3+ model in 2019 (referred as previous version). Third, these innovations align the framework more closely with current and future trends in the Czech economy.

Over the year 2024, we implemented enhancements in both components of the model – the foreign economy and the domestic economy block. The innovations related to *the foreign economy block*, which represents Czech trading partners in the euro area, focus on structural changes, adjustments in the observable data inputs and outlooks, and parametrization. Firstly, we introduce an endogenous decomposition of effective euro area economic activity into gap and trend components, while implementing a one-off shifter to account for temporary sharp supply changes. Secondly, we refine the composition of effective euro area producer prices by categorizing them into energy-related and core (non-energy) components, and add a time-varying weight for the components of foreign producer price inflation. Thirdly, we enhance the relationship between effective euro area consumer and producer prices, along with refining the real exchange rate USD/EUR gap equation. Fourthly, we redefine the composition of the effective euro area aggregate by including Austria as another important trading partner of the Czech Republic. Lastly, we recalibrate the parameters of foreign economy block and selected steady states.

The enhancements related to *the domestic economy block* encompass four primary areas. Firstly, we refine the link between foreign producer prices and domestic firms' costs, ensuring alignment with changes in the foreign economy block and facilitating the complete transmission of energy-related fluctuations to domestic firms' production costs. Additionally, we introduce a decomposition of the import price deflator into energy and non-energy components, thereby adding two new observable variables. Secondly, we incorporate constant elasticity of substitution production functions into the household consumption and export sectors, allowing for limited substitution between production factors. Thirdly, we introduce a time-varying elasticity governing external demand as a multiple of foreign economic activity. Furthermore, we adjust the links between foreign demand and domestic exports and investment. Fourthly, we recalibrate selected parameters and steady states within the domestic block.

Overall, this upgrade of the g3+ model represents a further advancement in structural modeling and forecasting practice at the CNB. The newly incorporated model features offer a closer alignment with observed data, resulting in a more accurate depiction of the economic environment. Additionally, these innovations support a better understanding of recent extraordinary events and partially replace expert judgments made in the past. Consequently, the innovations enhance the model's structure, providing a more robust foundation for economically sound and consistently interpreted analysis of economic events. Moreover, the upgraded framework demonstrates improved stochastic and forecasting properties when compared to its previous version.

The paper is structured as follows. Section 2 provides an overview of the g3+ model, outlining its characteristics and key components. This is followed by an exploration of innovations in the foreign and domestic economy blocks of the upgraded model in Sections 3 and 4, respectively. Model reparametrization is discussed in Section 5 and model properties are examined in Section 6. The paper concludes in Section 7.

2. g3+ Model in a Nutshell

The g3+ model is a structural two-agent New Keynesian (TANK) dynamic stochastic general equilibrium (DSGE) small open economy (SOE) model used for regular forecasting and policy analysis. Rooted in the New Keynesian (NK) tradition, it incorporates nominal rigidities and real frictions. The SOE component establishes the connection between the Czech economy and its primary trading partner, the effective euro area (EA).

The g3+ model, detailed by Brázdik et al. (2020), represents a significant advancement over its predecessor, the g3 model, which was introduced by Andrle et al. (2009). The g3+ is a business-cycle model implementing inflation-forecast-targeting monetary policy while relying on forward-looking model-consistent expectations in the vein of limited rational expectations (LIRE).[1] The crucial aspect of the model revolves around the endogenous nature of monetary policy described by the central bank's reaction function (monetary policy rule). Our reaction function setup provides a nominal interest rate trajectory aiming to achieve the inflation target within the monetary policy horizon.[2]

[1] Section 8 in Brázdik et al. (2020) describes the LIRE concept and its setup in detail.

[2] The monetary policy horizon denotes the time horizon during which monetary policy decisions exert their most significant impact. This horizon accounts for the typical transmission lag, resulting in a span of approximately 4–6 quarters ahead.

The CNB forecasts are conditioned on number of assumptions concerning both foreign economic developments (such as prices, economic activity, or interest rates) and several domestic factors (including fiscal policy, administered prices, and the impacts of taxes). The core model serves as a unifying framework, maintaining coherence among macroeconomic variables while allowing for expert judgments.

In what follows, we present a general description of foreign economy and domestic economy blocks of the g3+ model (including the upgrades made in this research paper) to introduce the model framework to the reader. The individual upgrades of the model compared to its previous version are then presented in Sections 3, 4 and 5.

2.1 Foreign Economy Block

The foreign economy block, reflecting the structure of Czech exports, is a small semi-structural model of major trading partners within the euro area. The role of the effective euro area economy block is to structurally describe the behavior of the foreign economy counterpart to the domestic economy. The rest of the world (RoW) is represented by an exchange rate link to the United States economy.[3]

The foreign economy block comprises core behavioral equations that allow for the interpretation of business cycle developments of major trading partners. The remaining equations include various definitions, identities and autoregressive processes. This block is based on the basic structure of NK open economy models and articulates the main economic linkages (see Figure 2):

- Economic activity is described by *the IS curve*, which establishes a connection between the output gap (representative of demand) and the real monetary condition index. This index encompasses real interest rate and real exchange rate gaps.

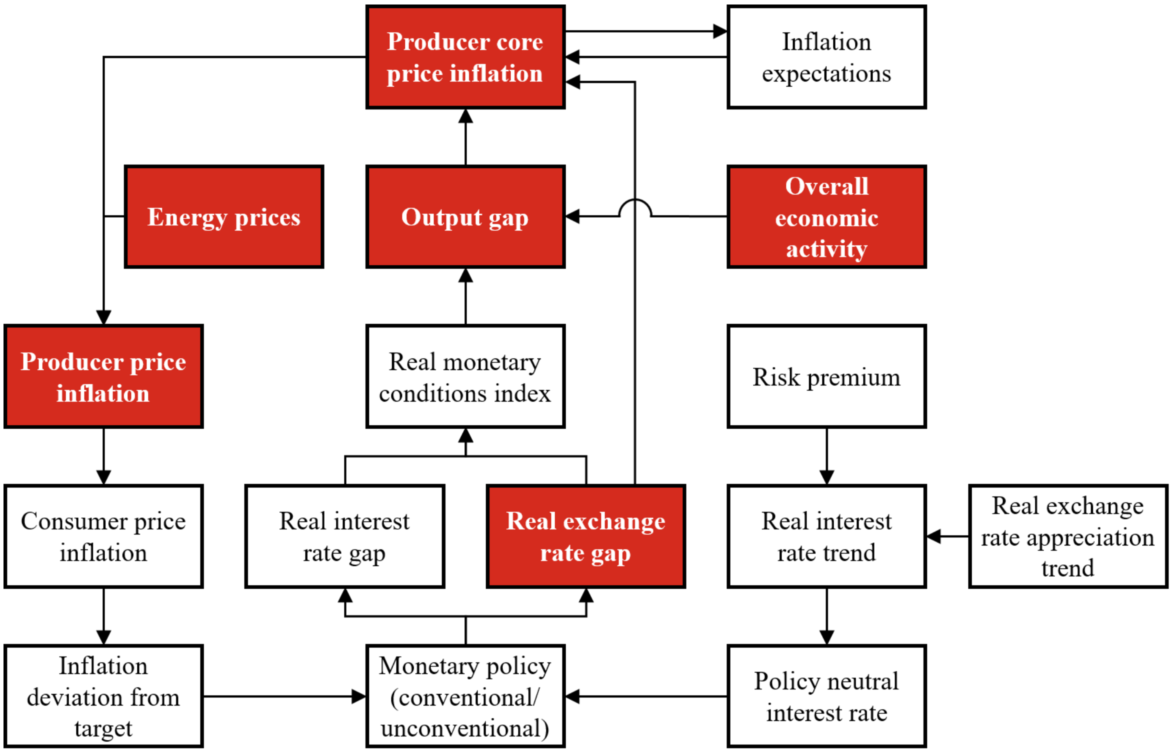
- *Producer price inflation* consists of two components – core and energy. The core component follows an open-economy version of the Phillips curve, driven by inflation expectations, inflationary pressures from the real economy, and the real exchange rate gap. The energy component is described by a simple autoregressive process.

- *Consumer price inflation* characterizes dynamics of prices across consumption goods and is linked to producer price inflation via a simple relation.

- The monetary authority follows *a Taylor-type reaction function*, aimed at achieving the inflation target over the medium term. This rule includes expected deviations of consumer price inflation from the inflation target and the output gap along with the interest rate smoothing. The model framework explicitly considers foreign unconventional monetary policy, incorporating shadow interest rates[4] in addition to the standard (conventional) policy interest rate. The shadow policy rate helps us assess the true foreign monetary policy stance when conventional interest rates are constrained.

[3] The model features only a simple link to USD/EUR exchange rate. See Section 3 for reference.
[4] For more details see the stream of works related to Krippner (2013) and Krippner (2020).

Figure 2: g3+ Model – Foreign Block



Note: Red color emphasizes model blocks with structural changes made to the model introduced in this research paper.

2.2 Domestic Economy Block

The main domestic parts of the g3+ model describe the following sectors: households, firms producing intermediate and final goods, and monetary and fiscal authorities. The goods production sectors scheme and respective price rigidities are depicted in Figure 3.

- *Households* are composed of two types, formulating the two-agent nature of the model. The first type of households, referred to as Ricardians, engage in the consumption of final goods within an internal habit formation paradigm. They rent capital, supply labor to intermediate producers, and earn wages based on contracts outlined in Calvo (1983). Ricardians participate in the trading of domestic nominal government bonds but lack access to foreign financial markets. The second type of households, referred to as hand-to-mouth, rule-of-thumb, or non-Ricardian households, entirely consume their labor income each period and do not have access to financial markets. As such, they cannot smooth their consumption. This distinction between the two household types introduces deviations from the permanent income hypothesis, as observed in studies such as Campbell and Mankiw (1989) and Mankiw (2000).

- *The intermediate production sector* includes three types of firms. The first one produces domestic intermediate goods by using capital and labor inputs while utilizing the Cobb-Douglas production technology. Importing firms comprise two types – energy and core (non-energy) importers. Imported intermediate production relies solely on imports, without any domestic inputs. Importing firms operate under the assumption of local currency, resulting in the stickiness of imported goods in the domestic currency. All firms use Calvo pricing schemes when setting their prices, resulting in the existence of NK Phillips curves for each intermediate good.

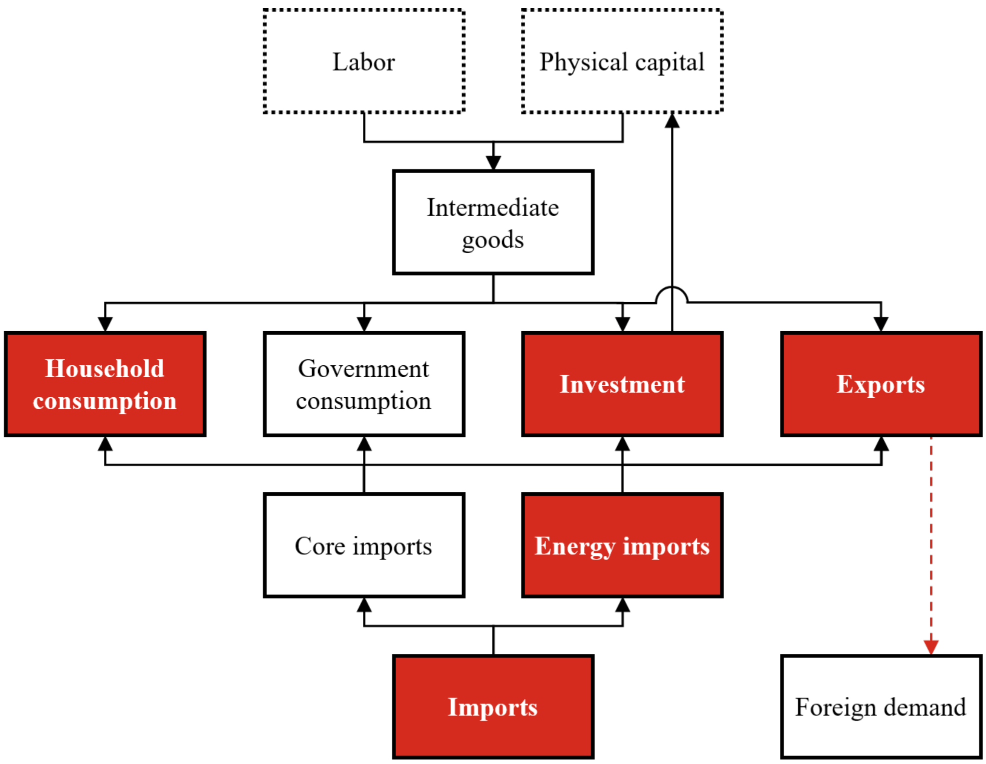
- *Final goods producers* bundle consumption, public-spending, investment, and export goods. All goods are produced using imported and domestically produced intermediate goods, while using specific technologies. Similar to the intermediate production sector, each final goods sector adopts Calvo price setting, giving rise to NK Phillips curves for every final good. Price stickiness of export goods is modelled under the foreign-currency assumption.

- *The monetary authority (central bank)* behaves in line with a monetary policy rule featuring interest rate smoothing. This rule establishes connection between nominal interest rates and deviations of expected year-over-year monetary policy-relevant inflation[5] from the inflation target.

- *The fiscal authority (government)* is characterized by the period budget constraint. The government engages in the consumption of non-productive public-spending goods.

- The profit-optimizing behaviour of *forex dealers*, who have access to international financial markets, implies the uncovered interest rate parity condition.

Figure 3: g3+ Model – Domestic Production Sectors and Price Rigidities



Note: Red color emphasizes model blocks/linkages with structural changes/enhancements made to the model introduced in this research paper. Dotted boxes represent production inputs for intermediate goods. Solid arrows depict rigidities in domestic currency, while dashed ones indicate rigidities in foreign currency.

[5] Monetary policy-relevant inflation is inflation to which central bank reacts. It is defined as headline inflation adjusted for first-round effects of changes to indirect taxes. More details are available here: https://www.cnb.cz/en/faq/What-is-monetary-policy-relevant-inflation.

3. Changes in the Euro Area Economy Block

The upgrades to the foreign economy block are primarily motivated by global economic developments caused by COVID-19 pandemic and the outbreak of war in Ukraine, which resulted in sharp, temporary supply-side disruptions and significant increases in energy prices. At the same time, we have refined the internal linkages within the model to more accurately reflect the evolving economic environment. Furthermore, we have revised the methodology for constructing external assumptions in the forecasting process and re-parameterized the model accordingly. A detailed description of the foreign block equations is provided in Appendix A. In this section, we focus on the key features of its updated specification.[6]

3.1 New Approach to Identify EA Business Cycle Position

The economic consequences of the COVID-19 pandemic in 2020 and 2021 were partly driven by government-imposed restrictions that directly impacted production capacity. Measures such as mandatory shutdowns of businesses, mobility limitations, and health-related disruptions led to abrupt and temporary supply-side constraints, causing a significant, though not permanent, decline in potential output. These unusual dynamics posed particular challenges for the g3+ structural model, which was originally designed to capture more gradual and persistent changes in economic fundamentals.

Although the model was able to reproduce the observed shifts in the euro area's potential output in 2020 and 2021, it required extensive expert judgment to supplement the model's internal structure. While these expert interventions made it possible to capture the immediate economic effects of the pandemic, they came at the cost of increased complexity in the forecasting process. These limitations highlighted the need for structural enhancements to the model—particularly the introduction of mechanisms capable of accommodating sudden and sizable shifts in potential output without relying heavily on external inputs.

3.1.1 Short-Term Sharp Changes to Supply

Semi-structural NK models typically employ a simple autoregressive (AR) process to capture the growth rate of potential GDP. Such approach performs sufficiently well most of the time; however, it falls short in replicating sudden and sharp changes. This failure originates from the mismatch between the short-term effects of such shocks and the growth rate described by a persistent process. Such model structure results in impacting potential GDP for several periods (see the lines labeled as "Persistent growth shock" in Figure 4 for illustration). To improve our framework and to address sudden and sharp changes to potential GDP (as depicted by the lines labeled as "Level shock" in the same figure), we introduce short-term (sharp) supply changes into the trend-cycle decomposition of foreign economic activity.

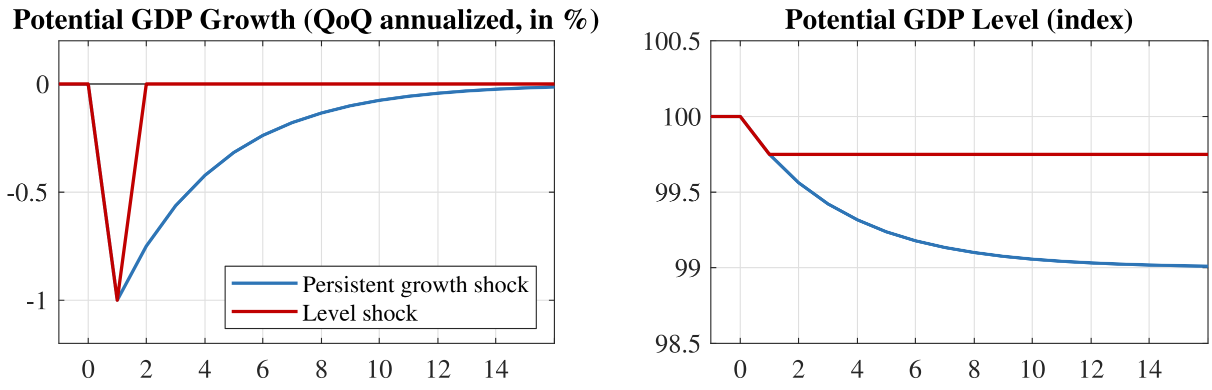
The dynamics of GDP is described by movements in potential output, serving as a proxy for the supply side, and changes in the output gap, acting as a proxy for the business cycle. Thus, quarter-over-quarter foreign GDP growth Δy_t^* is decomposed as follows:

$$\Delta y_t^* \quad = \quad (\hat{y}_t^* - \hat{y}_{t-1}^*) + \Delta y_t^{*trend}, \tag{1}$$

where Δy_t^{*trend} represents quarter-over-quarter foreign potential output growth and $\hat{y}_t^*$ is the foreign output gap.

[6] Unless stated otherwise, all variables presented in this section are expressed in logarithms.

Figure 4: Illustrating Growth and Level Shocks to Potential GDP



Note: The horizontal axis displays quarters.
Source: Authors' calculations.

The trend component of the effective euro area GDP is newly assumed to be an aggregate of two components. The first, which we call *the fundamental trend*, represents the persistent part of the supply side. The second component is the *a trend shifter*, a component that captures sudden distortions in potential GDP growth. The trend shifter is designed to capture sharp, temporary deviations from the fundamental trend, such as those caused by global supply shocks, pandemics, or war-related disruptions.

Accordingly, the change in foreign potential output growth, Δy_t^{*trend}, is decomposed as follows:

$$\Delta y_t^{*trend} = \Delta y_t^{*ftrend} + \Delta y_t^{*trendshift}, \tag{2}$$

where fundamental trend growth $\Delta y_t^{*ftrend}$ represents long-term movements, while the trend shifter $\Delta y_t^{*trendshift}$ captures short-lived deviations.

The quarter-over-quarter growth rate of the fundamental trend follows an AR(1) process of the following functional form:

$$\Delta y_t^{*ftrend} = \rho_{y*ftrend}\,\Delta y_{t-1}^{*ftrend} + (1-\rho_{y*ftrend})\,\Delta y^* + \varepsilon_t^{y*ftrend}, \tag{3}$$

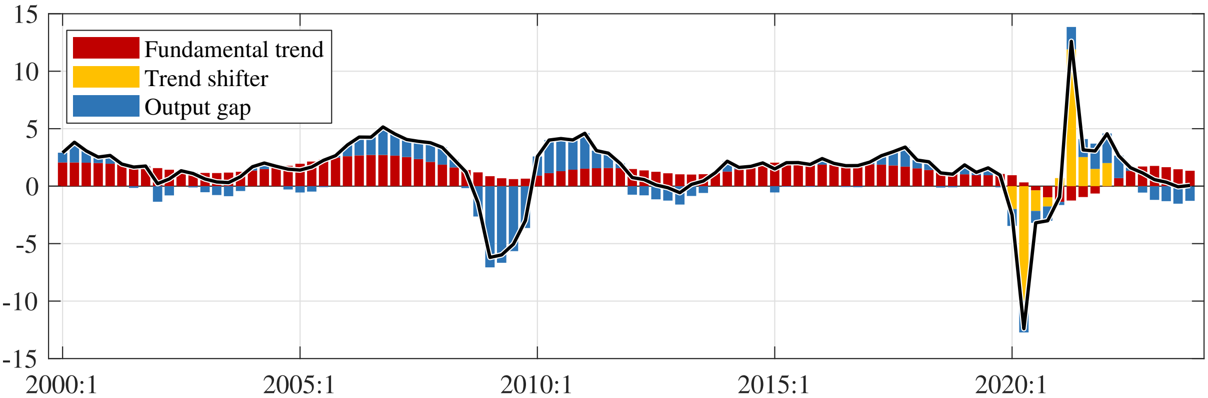
where Δy^* is the long-run equilibrium growth rate of foreign GDP and $\varepsilon_t^{y*ftrend}$ is a shock. The trend shifter is described by a one-off shock, $\varepsilon_t^{y*trendshift}$, that is:

$$\Delta y_t^{*trendshift} = \varepsilon_t^{y*trendshift}. \tag{4}$$

While the trend shifter can be estimated endogenously, its trajectory may be influenced by expert judgment in extreme cases, such as sudden supply disruptions. During forecasting rounds, these adjustments are informed by high-frequency indicators and real-time assessments.

To illustrate the flexibility of the implemented approach, we present the decomposition of the effective EA GDP growth into its components in Figure 5. Historically, growth was predominantly

Figure 5: Effective Euro Area GDP Growth Decomposition (YoY, in %, contributions in pp)



Source: Authors' calculations.

driven by its potential, with occasional instances where overheated demand played a significant role, notably during the aftermath of the European debt crisis in 2009. The COVID-19 period from 2021 to 2022 is primarily attributed to fluctuations in potential GDP, reflecting temporary business closures and constrained production capacities. Additionally, this period showcases the impact of our expert judgments through the trend shifter. The most recent figures are then interpreted in the context of subdued demand.

In addition to the changes to the potential output, we have also modified *the dynamic IS curve* that characterizes the foreign output gap. This equation incorporates output persistence, reflecting mechanisms such as consumption habits, investment inertia, and other real rigidities, the influence of real monetary conditions and external demand responses to changes in international price competitiveness. Compared to the previous version of the model, we have updated the timing of the real exchange rate gap into concurrent terms $\hat{z}_t$, which improves the alignment with standard open-economy theory and enhances the interpretability of the model's narrative. Consequently, the foreign output gap, $\hat{y}^*_t$, is described by the following equation:

$$\hat{y}^*_t \quad = \quad a_{y^*}\, \hat{y}^*_{t-1} - a^{r^*}_{y^*}\, \hat{r}^*_{t-1} + a^{z}_{y^*}\, \hat{z}_t + \varepsilon^{\hat{y}^*}_t\,, \tag{5}$$

where $\hat{r}^*_{t-1}$ is the lagged real interest rate gap, capturing the effects of foreign monetary policy, $\hat{z}_t$ stands for the USD/EUR real exchange rate gap, which accounts for the exchange rate channel in an open economy, $\varepsilon^{\hat{y}^*}_t$ is a foreign demand shock, and $a^{\prime}_{y^*}s$ are parameters characterizing elasticities.

3.1.2 Endogenous Decomposition of Euro Area Economic Activity

In the Brázdik et al. (2020)'s version of g3+ model, the position of the foreign economy within the business cycle is identified during forecasting process by tools external to the model structure. Specifically, we applied the Hodrick and Prescott (1997) filter, supplemented by expert judgment during regular forecasting rounds. As a result, the dynamics of the effective euro area block were conditioned on an exogenously observed decomposition of foreign economic activity. The model forecast treated the foreign output gap as an observed input rather than an internally estimated variable.

In the upgraded version of g3+, the position of the foreign economy in the business cycle is now assessed endogenously within the model structure. This new approach ensures structural consistency and improves the interpretability of the foreign output gap. The estimation now integrates additional macroeconomic indicators, such as inflation and the foreign monetary policy stance, resulting in a more comprehensive and economically grounded narrative.[7]

3.2 Revision of Euro Area Producer Prices Composition

In 2022, prices of energy commodities experienced a significant surge, leading to a notable increase in both producer and consumer price inflation. In the previous version of the g3+ model, foreign producer prices were already differentiated into energy-related and core components. Before the energy crisis unfolded, our attention was primarily focused on the price of oil as the only driver of the energy component. In response to the surge in commodity prices, specifically electricity and natural gas, in 2022, we adjusted the energy component at the data level to reflect this misalignment in our forecasts. In this model update, we address this issue fully by implementing a systematic change to the model structure itself to accommodate and reflect changes in the current and future economic environment. These include, among others, a gradual transition from conventional to cleaner energy sources, such as natural gas or electricity, leading to their increased significance in production costs.

3.2.1 Adjusting Components of the Producer Price Index

The previous version of the g3+ model decomposed foreign producer price index (PPI) inflation into three components: (a) the core component, capturing industries relevant to the Czech Republic's international competitiveness, (b) the oil component, reflecting industries linked to oil-related activities, and (c) the non-oil energy component, encompassing the remainder of energy-related industries (refer to Panel A in Figure 6).

Crucially, the previous model explicitly incorporated foreign oil price dynamics by including oil prices as an observed variable. This approach reflected the underlying assumption that oil prices were the dominant driver of overall energy prices and energy-related producer costs. Historically, this simplification was justified, as global oil prices tended to co-move with other energy prices, and in many cases, prices of non-oil energy commodities were directly indexed to oil. Consequently, oil price fluctuations served as the main channel through which energy-related shocks were transmitted from the foreign PPI block to the domestic economy.

However, the outbreak of war in 2022 and the resulting market disruptions weakened this historically stable relationship between oil and other energy commodities. In response to this, the model was revised to incorporate the observation of a broader set of energy prices, allowing it to better capture the evolving structure of energy markets and their impact on domestic producer costs.

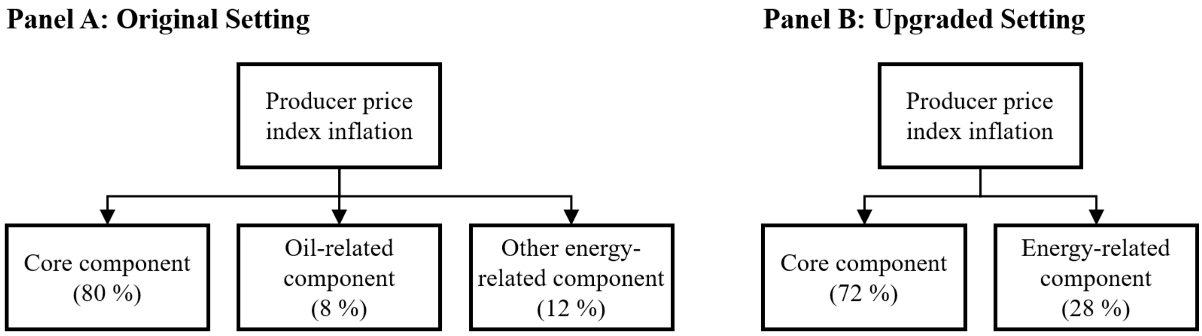
In the upgraded model, we streamline the decomposition of effective euro area PPI inflation into two parts: (a) the core component, and (b) the energy component, encompassing all relevant energy-related commodities, including electricity and gas (refer to Panel B in Figure 6).

Along this new decomposition, the weight of energies in the total PPI inflation has increased from 20% to 28%.[8] The 28% weight of the energy component was derived from Eurostat MIGs data, reflecting the average contribution of energy-related items to foreign PPI growth across key euro area economies. This weighting ensures alignment with sectoral price developments and improves

[7] A decomposition of the output gap into structural shocks is provided in Section 6.

[8] The increase in the energy share is also influenced by a methodological change in identifying the energy component. Consequently, the higher share is not due to a mere reorganization of the components.

Figure 6: Components of the Foreign Producer Price Index Inflation



Note: The numbers indicate the weight in the total index.

the model's responsiveness to energy shocks. Furthermore, all forms of energy now exert a direct impact on the production process within the Czech economy, as outlined in Section 4.

To describe core PPI inflation $\tilde{\pi}_t^{*core}$ dynamics, we employ an open-economy *Phillips curve*. It captures the relationship between PPI inflation, demand side pressures (proxied by foreign output gap $\hat{y}_t^*$) and external cost pressures (proxied by USD/EUR real exchange rate $\hat{z}_t$). Also, the Phillips curve includes the forward- and backward-looking elements $\tilde{\pi}_{t+1}^{*core}$ and $\tilde{\pi}_{t-1}^{*core}$, capturing the firms' learning, expectations and price rigidities. A key innovation relative to the original model is the inclusion of the contemporaneous real exchange rate gap, improving the responsiveness of the Phillips curve to imported inflation pressures. The Phillips curve for foreign producers core inflation $\tilde{\pi}_t^{*core}$ is given by:

$$\tilde{\pi}_t^{*core} \quad = \quad a_{\pi^{*core}}\, \tilde{\pi}_{t-1}^{*core} + (1 - a_{\pi^{*core}})\, \tilde{\pi}_{t+1}^{*core} + a_{\pi^{*core}}^{y^*}\, \hat{y}_t^* + a_{\pi^{*core}}^{z}\, \hat{z}_t + \varepsilon_t^{\tilde{\pi}^{*core}}, \tag{6}$$

where $\varepsilon_t^{\tilde{\pi}^{*core}}$ is a cost-push shock and $a'_{\pi^{*core}}s$ are elasticity parameters.

In contrast to the core component, the foreign energy PPI inflation $\tilde{\pi}_t^{*ener}$ is treated as an exogenous process with no structural link to foreign business cycle conditions. This component reflects global energy market developments and is modeled as an AR(1) process:

$$\tilde{\pi}_t^{*ener} \quad = \quad \rho_{\pi^{*ener}}\, \tilde{\pi}_{t-1}^{*ener} + (1 - \rho_{\pi^{*ener}})\, \tilde{\pi}^{*ener} + \varepsilon_t^{\tilde{\pi}^{*ener}}, \tag{7}$$

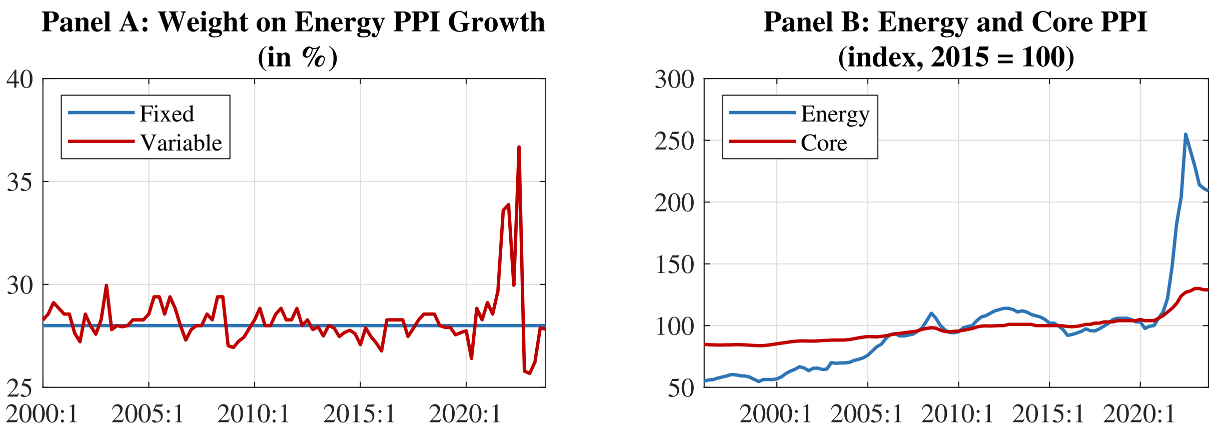
where $\tilde{\pi}^{*ener}$ denotes the steady-state energy inflation rate and $\varepsilon_t^{\tilde{\pi}^{*ener}}$ is an energy cost-push shock.

Overall, the changes to the PPI inflation structure significantly enhance the model's ability to replicate observed price dynamics in energy markets that have been recent driver of cost-side inflation. By broadening the scope of foreign energy prices definition and embedding them into the production cost structure, the upgraded model ensures an improved depiction of external inflationary pressures.

3.2.2 Introducing Time-Varying Energy Weight

As shown in Figure 6, the foreign PPI components contribute to the total index with specific weights. In the revised structure of foreign producer price inflation, 28% of the weight is assigned to energies, while the core component encompasses the remaining 72%. The reliability of fixed weights diminishes notably, particularly when significant relative fluctuations occur within the components of PPI inflation. When one component experiences a more pronounced growth rate than the other, it automatically exerts a greater influence on the overall PPI growth. Ignoring this effect could lead to an imprecise decomposition of foreign PPI dynamics within the modeling framework, yielding misleading implications due to the diverse nature of PPI components and their role within production.[9]

Figure 7: Effective EA PPI Components



Source: Eurostat; authors' calculations.

To account for this effect, we incorporate a time-varying weight for the components of foreign PPI inflation into the model structure. This adjustment allows us to capture the transient significant differences in the growth rates of the core and energy components and their varying impact on the overall PPI dynamics (refer to Panel A in Figure 7). Formally, foreign PPI inflation is defined as:

$$\tilde{\pi}^*_t = \rho_t^{*\frac{ener}{ppi}} \left(\tilde{\pi}_t^{*ener} - \tilde{\pi}_t^{*rp}\right) + \left(1-\rho_t^{*\frac{ener}{ppi}}\right)\tilde{\pi}_t^{*core} + \varepsilon_t^{\tilde{\pi}^*}, \tag{8}$$

where $\tilde{\pi}^*_t$ is foreign PPI inflation, $\tilde{\pi}_t^{*rp}$ represents the relative inflation accounting for differences in long-term dynamics between the components, $\rho_t^{*\frac{ener}{ppi}}$ stands for the time-varying weight, and $\varepsilon_t^{\tilde{\pi}^*}$ is a residual term addressing discrepancies.

The time-varying weight on the energy component is expressed in a multiplicative form, combining the fixed (steady-state) weight and the time-varying component, $\hat{\rho}_t^{*\frac{ener}{ppi}}$. This relationship is defined as:

$$\rho_t^{*\frac{ener}{ppi}} = \rho_{*\frac{ener}{ppi}} \hat{\rho}_t^{*\frac{ener}{ppi}}, \tag{9}$$

where the time-varying component is derived from a difference (gap) between the energy and core components of foreign PPI inflation:

$$\hat{\rho}_t^{*\frac{ener}{ppi}} = \tilde{\pi}_t^{*ener} - \tilde{\pi}_t^{*core} + \varepsilon_t^{\hat{\rho}^{*\frac{ener}{ppi}}}. \tag{10}$$

[9] For a more in-depth discussion on the role of energy and core components of the foreign PPI, refer to Section 3 and Appendix E in Brázdik et al. (2020).

In this equation, $\varepsilon_t^{\hat{\rho}^{*\frac{ener}{ppi}}}$ denotes a structural shock to the time-varying energy weight. The time-varying component $\hat{\rho}_t^{*\frac{ener}{ppi}}$ is normalized to one in the steady state, ensuring that the long-run share of energy in PPI inflation remains consistent with historical averages. When the energy component grows faster (slower) than the core one, the gap widens (narrows), leading the weight $\rho_t^{*\frac{ener}{ppi}}$ to rise above (drop below) its steady-state level. This specification captures the short-term fluctuations in the relative contribution of energy prices to overall PPI inflation and allows for abrupt changes during episodes of high volatility in energy markets. It also provides a simple and tractable way to endogenously adjust the influence of energy shocks in the model without imposing structural breaks or hard-coded weight changes.

As illustrated in Panel B of Figure 7, both PPI components display distinct trend growths, in particular prior the mid-2010s. These differing long-term patterns highlight the need for a modeling approach that can account for component-specific deviations from common growth trend without imposing permanently divergent trends from observed data.

To address this, we maintain the modeling of foreign relative producer price inflation, which represents energy inflation relative to core inflation. This formulation allows the model to accommodate medium-term divergences while preserving the assumption that, in the long run, the steady-state growth rates of both components are equal. By doing so, the model captures historical variability in relative price movements without introducing structural non-stationarities or long-run imbalances between energy and core producer prices.

3.3 Additional Changes in the Euro Area Economy Block

In addition to the two primary sets of enhancements, we make adjustments to the following two links within the foreign economy block. The first adjustment pertains to the connection between consumer price index inflation and producer price index inflation. The second adjustment addresses the real exchange rate gap equation.

3.3.1 CPI-PPI Relation

Given that the foreign monetary policy operates under the inflation targeting regime focused on consumer price inflation, the previous version of the model employed a straightforward equation based on the multiple of inflation gaps[10], linking producer price and consumer price inflations (refer to Panel A in Figure 8), as both gaps exhibit similar dynamics but with different amplitudes.

In the upgraded version of g3+, we enhance this relationship by introducing a persistence component $\rho_{\pi^{*cpi}}(\tilde{\pi}_{t-1}^{*cpi} - \tilde{\pi}^{*cpi})$, which adds an additional layer of inertia to the CPI inflation gap. The added persistence term reflects the observed inertia in consumer price responses to producer prices, which may arise from contract rigidities, regulatory delays, and the role of distribution margins. This change improves alignment of the model with real-world inflation pass-through dynamics.

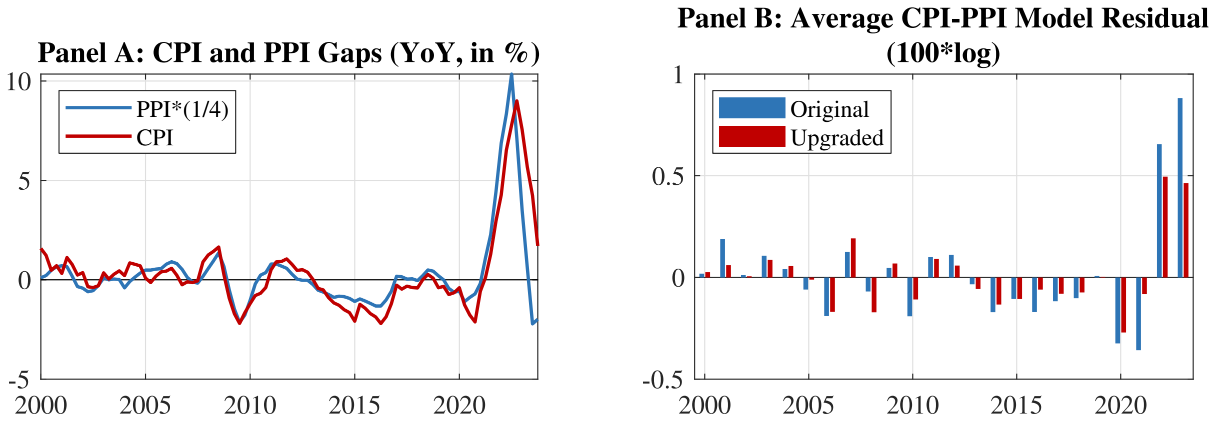
The upgraded CPI-PPI link is thus specified as follows:

$$\tilde{\pi}_t^{*cpi} - \tilde{\pi}^{*cpi} = \rho_{\pi^{*cpi}}(\tilde{\pi}_{t-1}^{*cpi} - \tilde{\pi}^{*cpi}) + (1-\rho_{\pi^{*cpi}})a_{\pi^{*cpi-ppi}}(\tilde{\pi}_t^{*} - \tilde{\pi}^{*}) + \varepsilon_t^{\tilde{\pi}^{*cpi}}, \quad (11)$$

where $\tilde{\pi}_t^{*cpi}$ and $\tilde{\pi}^{*cpi}$ is foreign CPI inflation and its steady state, respectively, $\tilde{\pi}^{*}$ is the steady state of foreign PPI inflation, $\rho_{\pi^{*cpi}}$ is the persistence parameter, and $a_{\pi^{*cpi-ppi}}$ is the elasticity of

[10] Inflation gaps are given by the difference between inflation dynamics and its steady state value.

Figure 8: Effective EA CPI and PPI



Note: CPI and PPI gaps in Panel A are differences between the time series and the steady state of 2%.
Source: Eurostat; authors' calculations.

CPI inflation gap to PPI inflation gap $(\tilde{\pi}^{*}_{t} - \tilde{\pi}^{*})$. The residual term $\varepsilon^{\tilde{\pi}^{*cpi}}_{t}$ accounts for unexplained discrepancies in the CPI-PPI inflation relationship.

This innovation improves the connection between CPI and PPI inflations, leading to a reduction in the identified residual (refer to Panel B in Figure 8). Implicitly, it also accounts for price-vertical rigidities between intermediate and consumer goods, providing a more nuanced reflection of the underlying dynamics.

3.3.2 Real Exchange Rate USD/EUR Gap

The foreign block captures the effective EA, which is assumed to be an open economy. Its counterpart, the RoW, is represented by the United States. The dynamics of the nominal US dollar-euro exchange rate is established by a version of an uncovered interest rate parity (UIP) condition. Despite its simplicity, this framework allows for capturing the persistence of capital mobility between the effective EA and the RoW to some extent.

In the previous version of the model, the real exchange rate gap was influenced by the change in the nominal exchange rate, the deviation of effective EA PPI inflation from its steady state, and the inclusion of the country risk premium. While this was operationally and model-wise appealing, the incorporation of the country risk premium lacked theoretical justification. The premium did not impact the equilibrium exchange rate or the natural rate of interest, essentially representing an exchange rate shock with persistence. In the upgraded version of g3+, we exclude the country risk premium and introduce the RoW inflation target wedge instead. The equation for the real exchange rate gap $\hat{z}_t$ is reformulated as follows:

$$\hat{z}_t - \hat{z}_{t-1} \quad = \quad -wedge^{\bar{\pi}^{RoW}}_{t} - \Delta usdeur_t - \tilde{\pi}^{*}_{t}, \qquad (12)$$

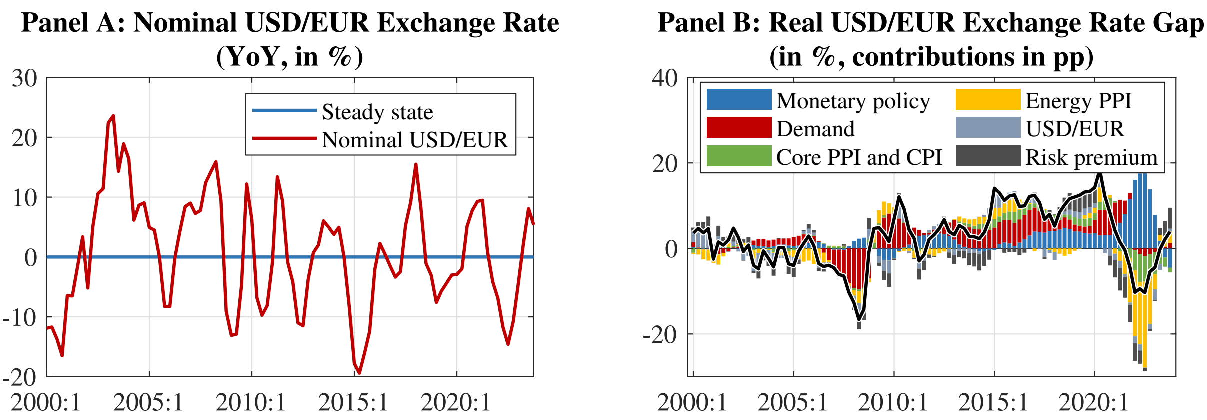
where $\Delta usdeur_t$ is the change in the nominal USD/EUR[11], $wedge^{\bar{\pi}^{RoW}}_{t}$ is the RoW inflation target wedge modelled as an AR(1) process. It represents a structural wedge that ensures the long-run consistency of the real exchange rate dynamics by allowing for persistent differences in inflation

[11] A positive change in $\Delta usdeur_t$ means appreciation of EUR against USD.

targets across regions. The long term equilibrium for $\Delta usdeur_t$ is assumed to be -2%, mirroring the 2% steady-state of effective EA PPI inflation, with a zero change in the nominal US dollar-euro exchange rate in equilibrium (refer to Panel A in Figure 9).

Panel B in Figure 9 illustrates the breakdown of the real USD/EUR exchange rate gap into structural shocks. The decomposition underscores the influence of real economic activity prior to the Global Financial Crisis and during the European Sovereign Debt Crisis, with demand playing a significant role in overvaluation initially and later in undervaluation. Additionally, subdued inflation pressures around 2015 further contributed to the undervaluation of the real exchange rate. The framework also identifies substantial positive contributions stemming from the loose monetary policy of the ECB post-2015. Conversely, the period after 2021 is characterized by a notable increase in prices, which contributed to the appreciation of the real exchange rate.

Figure 9: Nominal USD/EUR Exchange Rate and Its Real Gap



Note: Positive values in Panel B resemble to real undervaluation of EUR against USD.
Source: Bloomberg; authors' calculations.

3.4 Re-Defining the Effective Euro Area

In addition to structural model changes, adjustments have been made to the preparation of effective EA data. Previously, data for CPI and PPI indices, and GDP were prepared for five countries (the so called EA5): Germany, Slovakia, France, Italy, and Spain. Using the weights based on individual countries' shares on Czech exports, we composed effective EA indicators. We used data for all EA countries in export effective terms to define historical data, while the EA5 outlooks approximating economic developments in the whole EA served as assumptions for forecasting purposes.[12]

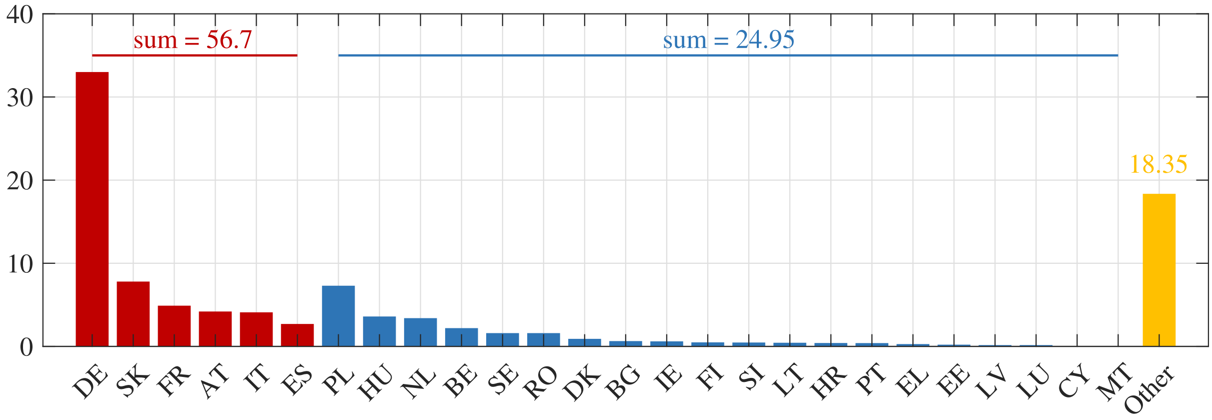
With the upgraded version of the model, our monitoring scope has expanded by including Austria, forming an EA6 aggregate. As illustrated in Figure 10, the EA6 aggregate now accounts for approximately 57% of Czech exports.[13] Furthermore, this upgrade adopts a consistent approach based on EA6 indicators in handling historical data and conditioning foreign economy outlooks. We ground

[12] The EA17 aggregate omitted Luxembourg and Malta (due to limited data availability) and Croatia, which was not a member of the EA until 2023.

[13] Poland and Hungary are not included in our indicators, despite having higher shares in Czech exports than four countries in the EA6 aggregate. This omission is intentional, as Poland and Hungary are not members of the EA. Their inclusion would compromise the coherence of the monitored aggregate, as their monetary policies are not described by the policy instruments of the ECB. Nevertheless, we can apply experts judgements should we need to account for external developments outside the EA that severely impact the Czech international trade.

this assumption on observation that overall PPI and GDP dynamics exhibit similar trajectories in effective terms for both the EA6 and the broader EA17 composition (refer to Figure 11).

Figure 10: Trading Partners of the Czech Republic (share, in %)



Note: Shares are based on Czech Republic exports in 2023. Red and blue bars indicate EA6 and other EU countries, respectively. DE - Germany, SK - Slovakia, FR - France, AT - Austria, IT - Italy, ES - Spain, PL - Poland, HU - Hungary, NL - Netherlands, BE - Belgium, SE - Sweden, RO - Romania, DK - Denmark, BG - Bulgaria, IE - Ireland, FI - Finland, SI - Slovenia, LT - Lithuania, HR - Croatia, PT - Portugal, EL - Greece, EE - Estonia, LV - Latvia, LU - Luxembourg, CY - Cyprus, MT - Malta.
Source: The United Nations COMTRADE database.

On the top of the change of the effective EA aggregate, the introduction of structural improvements leads to a modification in the set of observed variables of the foreign economy block. As we identify the position of the effective EA in the business cycle endogenously within the foreign economy block, we reduce the number of observed variables related to economic activity solely to GDP. Additionally, we substitute Brent oil prices, previously used as the underlying data series for estimating the energy component of the PPI, by directly introducing effective EA energy PPI. The energy component encompasses a broader range of commodities beyond oil, including electricity and gas, reflecting a more comprehensive representation of the energy sector. The breakdown of the overall PPI into its components is derived from detailed data collected by Eurostat in accordance with the Main Industrial Groupings (MIGs) classification.[14] The new EA6 aggregate includes the most economically relevant euro area economies for Czech trade with stable structure as shown in Appendix A.5, balancing statistical representativeness and dampening sensitivity to short-lived trade shocks. The exclusion of smaller or less integrated members reduces noise and improves predictive power for Czech external demand and foreign inflation.

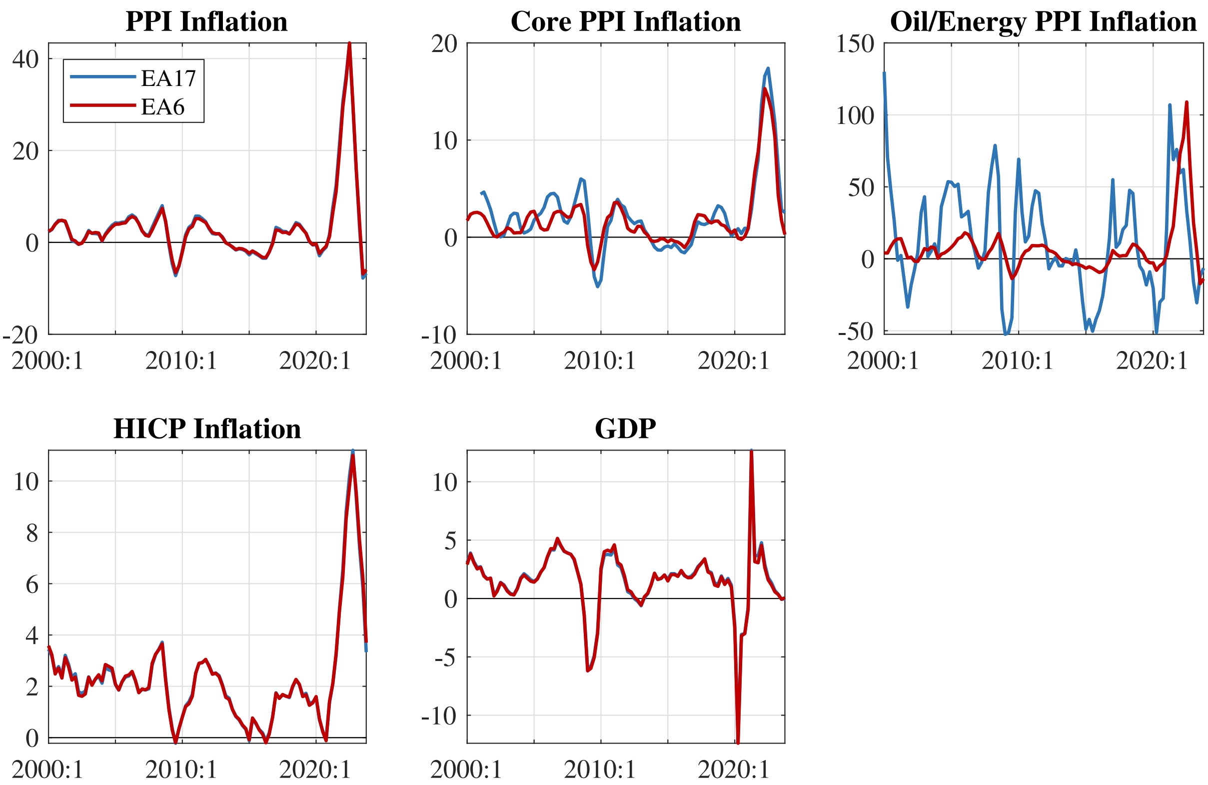
Changing the composition of observed variables and redefining the effective EA translate into differences in observed variables compared to the previous version of g3+. These changes concern real GDP growth, producer prices and their breakdown, and HICP inflation. The other foreign assumptions remain unchanged. As illustrated in Figure 11, the most significant distinctions emerge in the components of effective EA PPI, while the overall PPI dynamics closely resemble the one used in the previous version of g3+. Both core and energy PPI inflations exhibit reduced volatility compared to their previous definitions.[15] Regarding core PPI, the disparities stem from its altered

[14] Eurostat's MIGs are end-use categories based on the NACE rev.2 classification and are defined by European Commission (2007).
[15] In this context, Brent oil price inflation serves as a direct counterpart to the energy component of effective EA PPI, given its analogous role in the model production vertical.

composition. Effective EA GDP growth shows only slight differences. This adjustment is associated with the shift to a narrower effective EA aggregate, resulting in an implicit increase in the weights of countries that have experienced slower growth rates in recent years, notably Italy and Germany.

Altogether, the foreign block now incorporates information from nine observed variables: GDP, CPI and PPI inflation along with PPI energy and core components (all expressed in terms of effective EA indicators), 3M Euribor, shadow 3M Euribor, EA nominal equilibrium interest rate, and the USD/EUR exchange rate. Data on GDP, HICP, and PPI inflations are sourced from the Eurostat database, while 3M Euribor and the USD/EUR exchange rate are obtained from the Bloomberg database. The shadow interest rate is derived based on internal calculations using data on market interest rates and various asset purchase programs of the ECB. The shadow rate incorporates the impact of these measures to reflect their equivalent in terms of a hypothetical interest rate. The EA nominal equilibrium interest rate reflects the communication of the ECB. Observed data cover all main macroeconomic indicators, starting from 1998Q1. Non-observed variables are retrieved based on model-based structural relations using the Kalman smoother as outlined by Kalman (1960).

Figure 11: Effective Euro Area Assumptions (YoY, in %)



Note: "Oil/Energy PPI Inflation panel" displays inflations in Brent oil price (in USD) and the energy component of effective EA PPI.
Source: Eurostat; authors' calculations.

4. Innovations in Domestic Economy Block

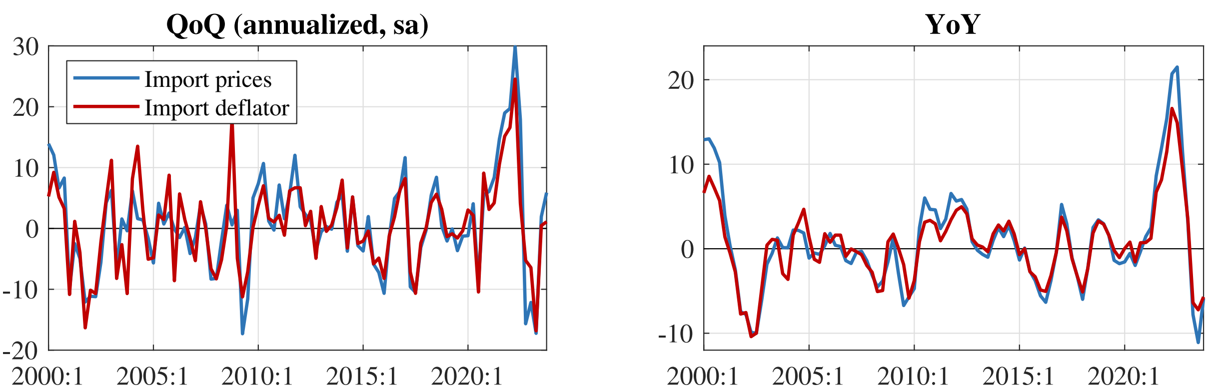
The enhancements applied to the foreign economy block of the g3+ model necessitate corresponding adjustments in the domestic economy counterpart. The revised structural interpretation of domestic imports involves a shift in emphasis from oil imports to overall energy imports, mirroring the change decomposition of the foreign PPI structure. This also motivates the change of the domestic production side to focus on energy as a general production factor. Furthermore, an analysis of Czech export activity data uncovers a downward trend in its growth rate over the past decade, prompting a further round of innovations. In addition, we reevaluate the link between foreign demand and domestic investment, as our forecasting practice indicated room for improvement in this domain.

4.1 Adjusting Links between Foreign and Domestic Prices

The domestic economy of the g3+ model operates on the premise that the import price deflator is interconnected with foreign producer prices. Therefore, any alterations in the composition of the foreign PPI must be accurately mirrored in the composition of the domestic import price deflator. However, its decomposition is limited by data constraints. Unfortunately, our data sources lack components of the import price deflator that correspond to the industrial classification as seen in the Czech PPI.

To address this challenge, we rely on domestic import price indices as a foundational source for the decomposition of the import price deflator. Our chosen approach is supported by historical data demonstrating a robust co-movement between the import price deflator and the overall import price index, as depicted in Figure 12. This close co-movement serves as a basis for navigating the constraints posed by the lack of detailed industrial classification data within the deflator.

Figure 12: Import Prices vs Import Price Deflator (in %)



Source: Czech Statistical Office.

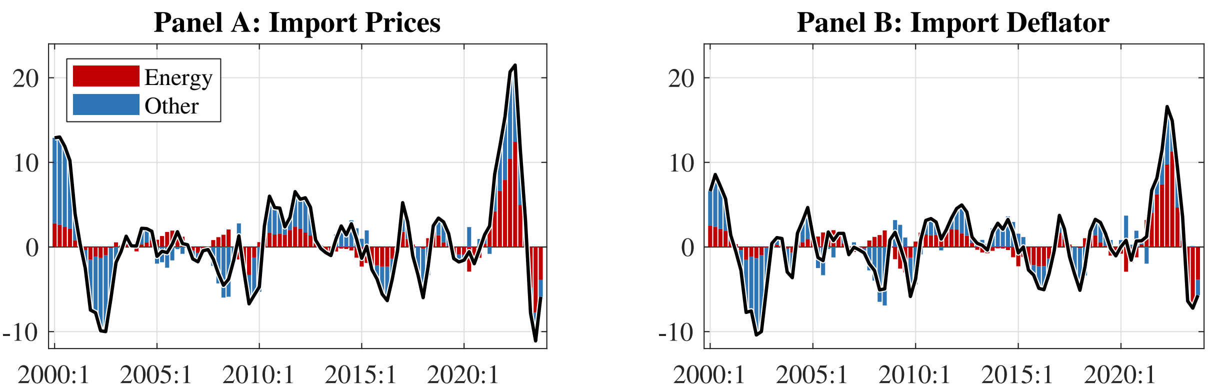
To construct an approximate breakdown of the import price deflator into core and energy components, we make use of detailed data on import prices. We base this breakdown on the observation that the structure of import prices adheres closely to the Standard International Trade Classification (SITC), which serves as proxy to the NACE classification employed in the decomposition of the foreign PPI into core and energy components. Moreover, we have access to the weights associated with import prices. With the support of existing weights and assuming the identical structure of import prices and the import price deflator, we decompose the change in the import price deflator, $\pi_t^N = \frac{p_t^N}{p_{t-1}^N}$, into changes in energy and core (other) components, $\pi_t^{Nenergy}$ and π_t^{Ncore}, as follows:

$$\pi_t^N = \left(\pi_t^{Ncore}\right)^{1-\rho_t^{\frac{pNener}{pN}}} \left(\pi_t^{Nenergy}\right)^{\rho_t^{\frac{pNener}{pN}}}, \tag{13}$$

where the time-varying weight of the energy component $\rho_t^{\frac{pNener}{pN}}$ is derived from the import prices data.[16]

Utilizing the methodology built upon the aforementioned assumptions, the applied approach facilitates the breakdown of the growth rate of the import price deflator, illustrated in Figure 13. Panel A displays both the raw import prices data and their decomposition, while Panel B showcases the decomposition of the import price deflator. Moreover, with access to the corresponding weights, we have the capability to construct the price level indices.

Figure 13: Decomposition of Import Prices and Import Price Deflator (YoY, in %, contributions in pp)



Source: Czech Statistical Office; authors' calculations.

Employing this approximative breakdown of import prices enables us to determine price changes for energy and core import goods. This decomposition serves as a reference point for identifying the price pressures stemming from import goods when manufacturing final goods (refer to Section 4.2). The new distinction is also evident in the pricing equations of Czech importers. *The New Keynesian Phillips curves*, which account for nominal stickiness and characterize the price-setting behavior of energy and core importers, take the following form:

$$\log\left(\frac{\pi_t^{Nl}}{\pi_{t-1}^{Nl}}\right) = \beta \log\left(\frac{\pi_{t+1}^{Nl}}{\pi_t^{Nl}}\right) + \frac{(1-\xi_{Nl})(1-\beta\xi_{Nl})}{\xi_{Nl}} \log(rmc_t^{Nl}\Theta^{Nl}) + \varepsilon_t^{\pi^{Nl}}, \quad (14)$$

where index $l \in \{energy, core\}$ stands for either the energy or core imports sector, β is the discount factor, π_t^{Nl} represents core/energy import price deflator, ξ_{Nl} is the Calvo parameter, Θ^{Nl} is the markup, rmc_t^{Nl} represents real marginal costs, and $\varepsilon_t^{\pi^{Nl}}$ is a cost-push shock.

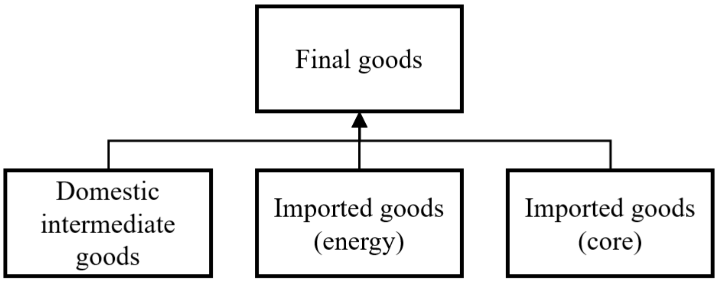
4.2 Refining Production of Final Goods

The g3+ model assumes three production inputs utilized in manufacturing final goods designated for household and government consumption, investment activities, and exports. These inputs comprise

[16] This weight is derived from the import price components within the category labeled "Mineral fuels, lubricants, and related materials."

intermediate domestic goods and energy and core imports (see Figure 14). Technically, the assumed production structure translates into the composition of production functions for final goods. The previous version of g3+ assumed that all final goods, denoted as $m_t \in C,G,J,X$ (representing household and government consumption, investment, and exports), are produced using a Leontief production function[17] with domestic intermediate input y_t^m and the imported energy and core contents, $n_t^{energy,m}$ and $n_t^{core,m}$. The use of Leontief production functions implies that the factors of production are employed in fixed (pre-determined) proportions, without the flexibility for substitution between them.

Figure 14: Final Goods and Inputs



In the upgraded g3+ model, we assume the possibility of substitution between production factors for the production of goods for household consumption and exports. This becomes particularly important in exceptional situations, such as the recent sharp increase in energy prices, when other production inputs were substituted for overly expensive energy commodities, albeit to a limited extent (see, for example, Hassler et al. (2021)). To model the substitutability, we introduce the constant elasticity of substitution (CES) production function for final goods for household consumption and exports, $o_t \in \{C,X\}$, of the form[18]:

$$o_t = \left[(\omega^o \phi^o)^{\frac{1}{\eta^o}} (o_t^{ener,o})^{\frac{\eta^o-1}{\eta^o}} + (\omega^o(1-\phi^o))^{\frac{1}{\eta^o}} (o_t^{core,o})^{\frac{\eta^o-1}{\eta^o}} + (1-\omega^o)^{\frac{1}{\eta^o}} (y_t^o)^{\frac{\eta^o}{\eta^o-1}}\right]^{\frac{\eta^o}{\eta^o-1}}, \quad (15)$$

where η^o is the constant elasticity of substitution parameter, ω^o denotes import intensity of producing good o_t and ϕ^o denotes energy share on the imported production inputs. CES production is implemented for household consumption and exports goods, while the Leontief production is retained for investment and government goods due to limited substitution flexibility in those areas.[19]

[17] The Leontief production function is a special case of the constant elasticity of substitution function.

[18] Lower case typeface in this section denotes the stationary version of the variable considered if not stated otherwise. For more details on stationarizing the model, refer to Section 2 in Brázdik et al. (2020).

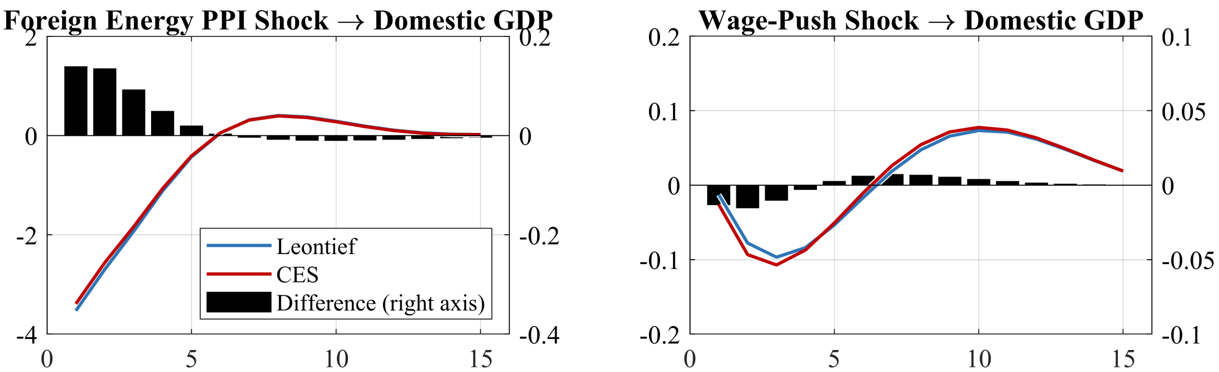
[19] The choice of these two sectors is driven by the significant rigidity present in the government sector, which is bound by various processes and regulations. Additionally, the investment sector is left out due to the uniqueness and difficulty of substituting imported investment goods and fixed capital structure.

Denoting $Q_t^o = \left[\omega^o \phi^o (p_t^{Nenergy})^{1-\eta^o} + (1-\omega^o)\phi^o (p_t^{Ncore})^{1-\eta^o} + (1-\omega^o)\right]^{\frac{1}{1-\eta^o}}$, with $p_t^{Nenergy}$ and p_t^{Ncore} being the prices of energy and core production inputs[20], the implied demands for factors are:

$$\begin{aligned}
n_t^{energy,o} &= \phi^o \omega^o \left(\frac{p_t^{Nenergy}}{Q_t^o}\right)^{-\eta^o} o_t \\
n_t^{core,o} &= (1-\phi^o)\omega^o \left(\frac{p_t^{Ncore}}{Q_t^o}\right)^{-\eta^o} o_t; \\
y_t^o &= (1-\omega^o)\left(\frac{1}{Q_t^o}\right)^{-\eta^o} o_t, \qquad (16)
\end{aligned}$$

where $n_t^{energy,o}$ is the amount of imported energy good, $n_t^{core,o}$ is the amount of imported core good, and y_t^o is the amount of domestic intermediate goods used for production.

Figure 15: Impulse Responses of Domestic GDP to Production Costs (QoQ annualized, in %)



Note: Impulse responses show reactions to 50 pp and 1 pp quarter-over-quarter annualized shocks (on impact) in foreign energy PPI inflation and domestic wage growth. The horizontal axis displays quarters.
Source: Authors' calculations.

The described innovation is illustrated in Figure 15 where impulse responses to the energy component of foreign PPI inflation and domestic wage growth are presented. Both graphs showcase the model's reactions with Leontief production functions and CES production functions for household consumption and exports. An increase in foreign PPI inflation, driven by an increase in energy prices, leads to a smaller contraction in domestic output under the CES production specification. This is because the CES structure allows substitution of less costly domestic inputs for expensive energy imports, which is not possible under the Leontief case. A rise in domestic wages leads to weaker GDP dynamics under the Leontief specification. However, in the CES case, more expensive domestic labor input is offset by imported inputs, thereby mitigating the effect of wage increase.

4.3 Revisiting Foreign Demand

The analysis of export elasticity in relation to foreign economic activity indicates a downward trend. To incorporate this trend into the model's framework, we opt to substitute the constant elasticity of foreign demand for domestic exports with a time-varying elasticity. This approach allows us to more

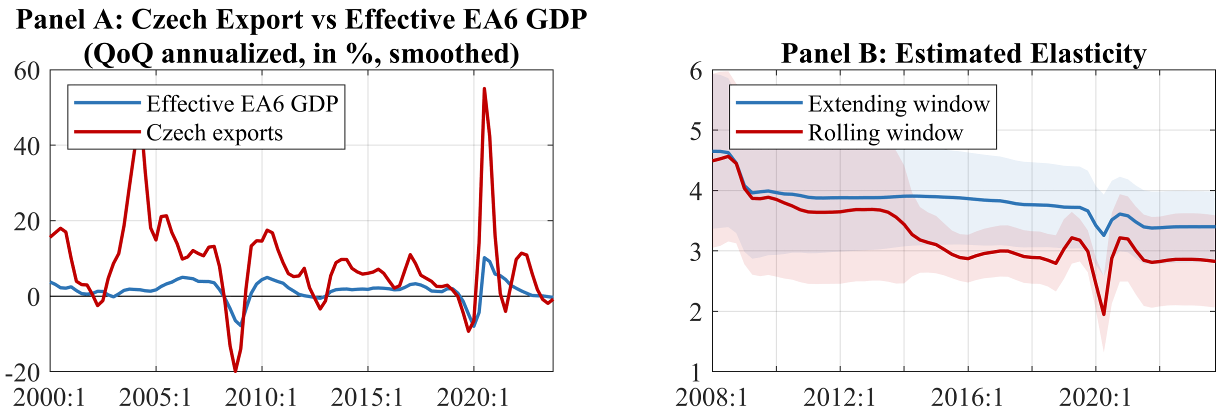
[20] The price of the domestic intermediate product is assumed to be nummeraire, such that $p_t^y \equiv 1$.

accurately reflect the Czech transition path experience while aligning with our current medium-term perspective. The assumption of a non-constant elasticity also requires adjustments to the connection between the foreign and domestic economies. Furthermore, we revisit the link between the domestic investment cycle and foreign economic activity.

4.3.1 Time-Varying Elasticity of Foreign Demand

As pointed by Andrle et al. (2009), foreign demand for Czech exports can be expressed as a multiple of effective euro area GDP. The original g3 model, the predecessor of g3+, as well as g3+ itself, were built on the assumption of elasticity equal to four. In other words, foreign demand was well approximated by effective euro area GDP multiplied by a factor of four.

Figure 16: Illustrating Foreign Demand



Note: Panel A presents QoQ annualized dynamics smoothed by the HP filter with $\lambda = 1$. Shaded areas in Panel B depict 95% confidence intervals.
Source: Czech Statistical Office; Eurostat; authors' calculations.

To validate this assumption, we run extending-window and rolling-window regressions covering the period from 1997Q1 to 2023Q4, using a simple regression model given by:

$$x_t^{qoq} = \beta_t y_t^{*,qoq} + \nu_t \tag{17}$$

where $x_{sm,t}^{qoq}$ and $y_{sm,t}^{*,qoq}$ represent quarter-over-quarter growths in Czech exports and effective EA GDP, respectively, and ν_t is an error term. Both time series are additionaly smoothed using the HP filter with $\lambda = 1$, which removes low frequency noise as displayed in Panel A of Figure 16. The extending window starts with 40 observations and incrementally extends its length by one quarter in each iteration, while the rolling-window has a span of 40 quarters.

We implement both approaches to see how the elasticity evolves over individual decades, and asses the elasticity while utilizing the whole sample in the end.

The estimation results are displayed in Panel B of Figure 16. The highest elasticity values, nearing five, are observed in the early 2010s when the Czech Republic joined the European Union, gaining access to the European market with fewer constraints. Following this period, the rolling-window estimates show a gradual decline in elasticity, settling around four by 2014. From that point onwards, the rolling-window estimate remains relatively stable at around three, except for the disruption caused by the COVID-19 pandemic in the 2020s,which significantly affected global trade flows. The gradual decline in elasticity observed since the late 2010s may also reflect the slowing pace of real economic convergence between the Czech Republic and its euro area counterparts. Due to its cumulative nature, the extending-window estimate exhibits a more gradual descent from earlier high values. As a result, it shows elasticity to be approximately 0.5 percentage points higher at the end of the sample period compared to the rolling-window estimate.

In light of the available evidence, we revisit the "rule of four" in the upgraded model and set the long-term value for elasticity to three. Additionally, to better capture historical dynamics, we allow this elasticity to vary over time. Specifically, we adopt the same approach used for the time-varying weight of the energy component in foreign PPI growth (see Equation (9)).

Accordingly, the time-varying foreign demand elasticity is specified as:

$$\eta_t^{n*} = \eta^{n*}\hat{\eta}_t^{n*} \tag{18}$$

where η^{n*} denotes the long-run steady-state elasticity of Czech exports demand to effective foreign GDP, and $\hat{\eta}_t^{n*}$ captures temporary deviations from this steady-state. These deviations reflect changes in the responsiveness of demand for Czech exports to foreign economic activity, such as, changes in the structure of foreign GDP and its connection or global value chain integration.

The time-varying component $\hat{\eta}_t^{n*}$ is modeled as a log-linear autoregressive process:

$$\log(\hat{\eta}_t^{n*}) = \rho_{\hat{\eta}^{n*}} \log(\hat{\eta}_{t-1}^{n*}) + (1-\rho_{\hat{\eta}^{n*}})\log(\hat{\eta}^{n*}) + \varepsilon_t^{\hat{\eta}^{n*}} \tag{19}$$

with the steady-state level $\hat{\eta}^{n*}$ normalized to 1, and $\varepsilon_t^{\hat{\eta}^{n*}}$ is a structural shock that captures unexpected shifts in foreign demand sensitivity.

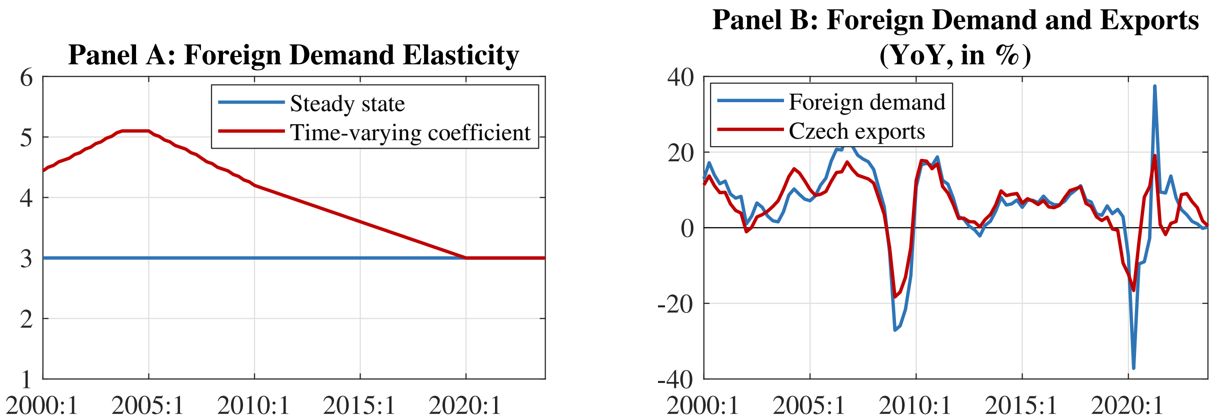
This shock plays an important role in the model: it allows for the endogenous adjustment of export demand elasticity in response to unobserved or evolving economic conditions, without requiring ad hoc re-specifications of the model structure. As such, this formulation improves the model's ability to match historical export patterns, while ensuring that long-run properties and theoretical consistency remain intact.

The time-varying elasticity component is obtained via model-consistent filtering, using data until 2023Q4. Nevertheless, to avoid frequent coefficient changes with updated data and considering the robust evidence from the provided estimates, we opt to fix the coefficient within the historical range, as displayed in Panel A of Figure 17. Also, we assume the elasticity to remain stable throughout the forecast horizon, pinned to its steady state. The cyclical and convergence disparities between the Czech and effective EA economies and the impacts of the COVID-19 pandemic continue to be captured by selected model technologies (see Brázdik et al. (2020)). Panel B of Figure 17 shows filtered year-over-year growth rates of foreign demand and Czech exports to illustrate the precision of the new model feature.

4.3.2 Updating Link between Domestic Export and Foreign Demand

The link between domestic export and foreign demand (as a key factor determining it) in the g3+ model originates from the demand function for domestic exports based on CES aggregation. In the previous version of the g3+ model, the link incorporated fixed multiples adhering to the "rule of four", which defined foreign demand (refer to Section 3.4 in Brázdik et al. (2020) for detailed information). With the introduction of time-varying elasticity of foreign demand in the upgraded model, it becomes necessary to adjust the relationship between domestic exports and foreign demand accordingly.

Figure 17: Illustrating Foreign Demand in g3+



Note: Panel B displays model time series retrieved via the Kalman smoother. Foreign demand resembles effective EA GDP growth multiplied by the time-varying coefficient given by Equation (18) and displayed in Panel A.
Source: Authors' calculations.

The re-defined demand function based on CES aggregation takes the form:

$$X_t \quad = \quad \left(\frac{P_t^X}{P_t^{*core}}\right)^{-\theta_x} (Y_t^*)^{\eta_t^{n*}}, \tag{20}$$

where P_t^X and P_t^{*core} is the price of exports and the foreign core PPI price level, respectively, θ_x is the price elasticity, and Y_t^* represent foreign output. The modification here applies to the introduction of the time-varying elasticity η_t^{n*} instead of the fixed multiple equal to four.

The novelty then translates into the stationary version of Equation (20) as follows[21]:

$$x_t \quad = \quad \left(\frac{p_t^X}{p_t^{*core}}\right)^{-\theta_x} (\hat{y}_t^*)^{\eta_t^{n*}} \frac{(Y_t^{*trend})^{\eta_t^{n*}}}{tech_t^x}, \tag{21}$$

where $tech_t^x$ is the export-specific technology.[22] Also, the domestic export-foreign economy link is closed via an updated definition of aggregated difference between the foreign and domestic trends, $wedge_t^{trends}$,[23] such that:

$$wedge_t^{trends} \quad \equiv \quad \frac{(Y_t^{*trend})^{\eta_t^{n*}}}{tech_t^x} \tag{22}$$

where the structural export wedge, $wedge_t^{trends}$, that captures the deviation between trend of foreign demand growth, Y_t^{*trend}, and the growth of domestic export capacity is modelled as a simple AR(1) process. Economically, this wedge reflects the non-neutral effects of structural changes such as shifting foreign trade patterns, global competitiveness, or evolving sectoral specialization. By explicitly modeling this wedge, the framework allows the export block to decouple long-run foreign demand dynamics from domestic export potential, capturing medium-term misalignments between global demand and the national capacity to supply.

[21] The division of foreign output between trend and gap components stems from the decomposition of foreign GDP present in the model and their individual treatment when stationarizing the model.
[22] The export-specific technology captures Bamol-Bowen and Balassa-Samuelson effects.
[23] See Appendix C in Brázdik et al. (2020) for a detailed explanation.

4.3.3 Adjusting a Connection between Domestic Investment Activity and Foreign Demand

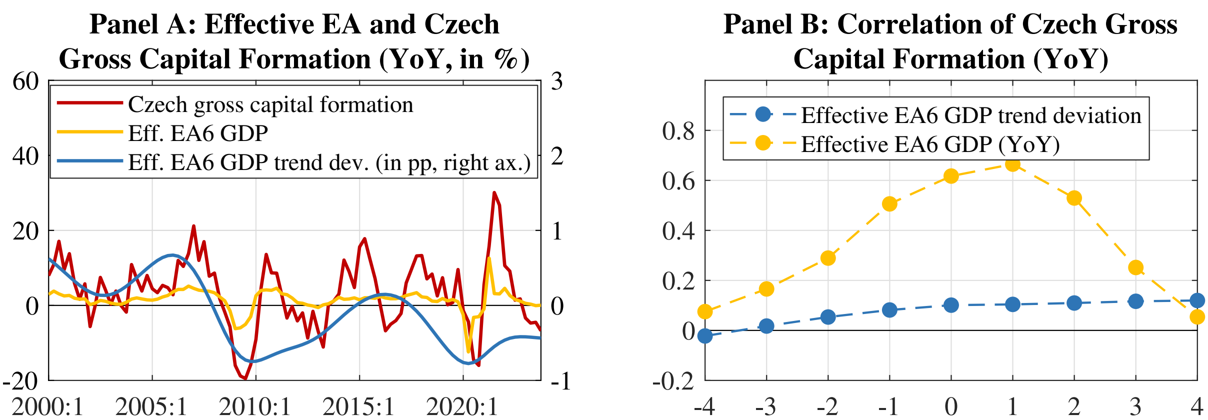
As highlighted by Gürtler (2018), there exists a robust connection between effective euro area activity and Czech investment activity, driven by the Czech Republic's intensive participation in global value chains and the consequent demand for new production capacities. To effectively model this phenomenon, the g3+ model features a dedicated link in the investment growth equation, labeled as an investment growth add-on[24], $\Delta aJnp_t$, such that:

$$\Delta j_t \quad = \quad \frac{j_t}{j_{t-1}} \Delta tech_t^j \Delta aJnp_t \tag{23}$$

where j_t denotes the stationary level of gross capital formation with Δj_t being its growth rate, and $\Delta tech_t^j$ is a collection of technologies used to impose stationarity of the model.

In the previous version of the g3+ model, the investment growth add-on was based on the deviations of long-term component of foreign demand growth from its steady-state growth. Our forecasting practice motivates us to enhance the model's ability to capture the connection between external demand and domestic investment. The correlation analysis indicates a notable synchronized movement between effective EA6 GDP growth and domestic investment growth (refer to Figure 18).[25]

Figure 18: Illustrating Foreign Demand-Domestic Investment Link



Note: Effective EA6 GDP trend deviation stands for the deviation of the YoY trend growth from the steady-state growth of 1.6%. The trend is retrieved using the HP filter with $\lambda = 1,600$. Correlation coefficients depicted in Panel B are computed on the sample covering 1997Q1–2023Q4. The horizontal axis in Panel B displays lags/leads.
Source: Authors' calculations.

In the upgraded version of g3+, we integrate short-term effects originating from the effective euro area business cycle evolution, represented by output gap change $\frac{\hat{y}_t^*}{\hat{y}_{t-1}^*}$, on top of the long-term effects represented by the deviation of potential output from its long term equilibrium growth $\frac{\Delta y_t^{*trend}}{\Delta y^{*trend}}$. This improvement is also grounded in historical simulations of the model, demonstrating superior predictive capabilities compared to those observed in a model lacking such structure. Formally, the

[24] Refer to Brázdik et al. (2020) for more details.
[25] Even though the correlation analysis points towards a one-lag-relation, we keep the timing in the concurrent terms, as such setting delivers forecasting superiority of the model.

investment growth add-on is defined as:

$$\Delta aJnp_t \quad = \quad \left(\frac{\hat{y}^*_t}{\hat{y}^*_{t-1}} \frac{\Delta y^{*trend}_t}{\Delta y^{*trend}} \right)^{\eta_{aJnp}\eta^{n*}_t} \exp\{\varepsilon^{aJnp}_t\} \tag{24}$$

where η_{aJnp} is the coefficient that, together with the time-varying elasticity of foreign demand η^{n*}_t, defines the elasticity governing the link between foreign demand and domestic investment, and ε^{aJnp}_t is a non-productive investment shock allowing for deviations in the specified relation.

5. Re-Parametrization

The new foreign block structure and new data inputs in terms of the effective EA result in the re-parametrization of the foreign block. The changes in the foreign block steady-state values necessitate reciprocal changes in the setup of the domestic part of the model. Also, we need to establish values for new parameters related to re-worked structural relations. The changes to the domestic steady-state values and parameters are consistent with the observations of past economic developments and also the expected future evolution of the Czech economy.

5.1 Foreign Economy Block

5.1.1 Steady States

The primary changes in the foreign block relate to the inclusion of new variables and the removal of others. As a result, it is necessary to define steady-state values for the newly introduced variables to ensure consistency within the model. The energy component of the effective euro area PPI is assumed to grow at an annual rate of 2%, consistent with the steady-state growth of both the core PPI component and the aggregate PPI index (considering constant weights). This assumption ensures a balanced long-run contribution of each component to overall producer price dynamics.

In addition, the steady-state value of the rest-of-the-world (RoW) inflation target wedge is set at –2%. This wedge captures the permanent deviation between the steady-state inflation rates of the EA economy and the rest of the world. It is calibrated to ensure consistency with the long-run equilibrium condition embedded in the real exchange rate dynamics (see Equation (12)). Specifically, it helps preserve the stationarity of the real exchange rate by aligning inflation differentials with corresponding productivity or preference shifts between regions.

All other steady-state values related to the foreign sector remain unchanged from the previous version of the model. A full summary of the steady-state assumptions is provided in Table 1.

5.1.2 Parameters

The next set of modifications includes the dynamic parameters of the model. Leveraging the relatively small size of the foreign block, we opt for estimating the model using the Bayesian techniques. The data set spans the period from 2000 to 2019 (covering 80 quarters) and includes nine observed variables, as listed in Section 3.4. We exclude the period from 2020 onwards in the estimation due to the occurrence of several extraordinary events, such as government-imposed lockdowns in response to the COVID-19 pandemic, the Russian war aggression against Ukraine, or subsequent surge in energy prices.

Table 1: Steady-State Assumptions in the Foreign Block

Variable		**Previous**	**Upgraded**
QoQ (annualized), %			
GDP	Δy^*	1.6	1.6
GDP Potential	Δy^{*trend}	1.6	1.6
CPI	$\bar{\pi}^{*cpi}$	2.0	2.0
PPI	$\bar{\pi}$	2.0	2.0
Core PPI	$\bar{\pi}^{*core}$	2.0	2.0
Oil/Energy PPI	$\bar{\pi}^{*ener}$	n/a	2.0
Oil Price	$\bar{\pi}^{brent,usd}$	4.0	n/a
USD/EUR	$\Delta usdeur$	0.0	0.0
RoW Inflation Target Wedge	$wedge_t^{\bar{\pi}^{RoW}}$	n/a	−2.0
Level, %			
Nominal Interest Rate	i^*	2.5	2.5
Natural Rate	r^*	0.5	0.5
Risk Premium	$prem^{usdeur}$	0.0	0.0

We revise all parameters of the model with support of estimation except for the elasticity of the policy rates to inflation and the persistence parameter in the IS curve (a_{y^*}), which we calibrate.[26] Table 2 provides an overview of the parameters in the foreign block in comparison to the initial model configuration. We offer the summary of the remaining parameters in Table A.1 in Appendix A.

Table 2: Deep Parameters in the Foreign Block

Parameter		**Previous**	**Upgraded**	**Parameter**		**Previous**	**Upgraded**
IS Curve				**Monetary Policy Rule**			
Lag	a_{y^*}	0.600	0.750	Lag	a_{i^*}	0.700	0.791
Real IR Gap	$a_{y^*}^{r^*}$	0.150	0.113	Inflation	$a_{i^*}^{\pi^*}$	2.000	2.000
Real ER Gap	$a_{y^*}^{z}$	0.025	0.017	Output Gap	$a_{i^*}^{y^*}$	0.250	0.198
PC (Core PPI)				**USD/EUR UIP**			
Lag	$a_{\pi^{*core}}$	0.450	0.187	Lead	ρ_{usdeur}	0.700	0.334
Output Gap	$a_{\pi^{*core}}^{y^*}$	0.070	0.108				
Real ER Gap	$a_{\pi^{*core}}^{z}$	0.010	0.010				

Significant differences emerge primarily in terms of persistence parameters (denoted as lags). Both the IS curve and the monetary policy rule demonstrate heightened persistence in comparison to the model's prior version. Conversely, the Phillips Curve (PC) for effective EA core PPI inflation displays a notably reduced level of persistence. Additionally, the forward-looking aspect of the USD/EUR UIP condition has been suppressed, indicating that the cross exchange rate is more responsive to current economic developments rather than anticipated ones.

In summary, the estimation has resulted in a more accurate fit of the model, surpassing the previous version that relied mainly on calibration. Learning from the new estimation have proven beneficial.

[26] We initially estimated the foreign block parameters, and based on the evaluation of model properties we adjusted these parameters to discipline the responses of domestic variables to foreign shocks. As such, the Bayesian estimation was indicative in certain instances.

New values of parameters notably support mitigation of unreasonable volatility and counteractive effects observed in historical decompositions provided by the previous version of the g3+ model. We illustrate these improvements in Section 6 by showing historical structural shock decompositions of selected foreign variables from their steady states.

5.2 Domestic Economy Block

5.2.1 Steady States

The Czech economy has faced multiple structural breaks over the past decade due to major shocks like the aftermath of the Global Financial Crisis, the COVID-19 pandemic, and other events. Additionally, Czech economy is facing many challenges mainly related to international trade and its extensive integration into global value chains. Since 2016, industrial stagnation, a shift towards services, weakening productivity growth, and limitations in infrastructure, innovation, and institutional quality have contributed to this slowdown. Moreover, it appears that Czech economy growth based on high-performing industry is showing signs of strain, therefore its long-term potential growth is lowered from 3% to 2.5%, as Livorová et al. (2024) explain. Simultaneously, the long-term characteristics of real activity were further adjusted, as summarized in Table 3

Table 3: Steady-State Assumptions in the Domestic Part

Variable		Previous	Upgraded
QoQ (annualized), %			
GDP	Δy	3.0	2.5
Household Consumption	Δc	3.0	2.5
Investment	Δj	3.0	2.5
Government Consumption	Δg	3.0	2.5
Imports	Δn	6.4	4.8
Exports	Δx	6.4	4.8
CZK/EUR	Δs	−1.5	−1.0
Level, %			
Nominal Interest Rate	i	3.0	3.0
Inflation Target	π^{tar}	2.0	2.0

The adjustment of foreign demand elasticity, representing the relationship between foreign demand and effective EA GDP growth as its multiple, necessitated a re-calibration of the growth rates for domestic exports and imports. These revised rates now stand at 4.8%, compared to the previously assumed value of 6.4%.

The steady-state appreciation of the Czech koruna has been revised downward from 1.5% to 1.0%. This adjustment reflects a comprehensive reassessment of medium-term economic conditions, taking into account external competitiveness and the relative economic performance of key trading partners. The revision aligns with observed exchange rate trends and is intended to provide a more accurate characterization of the long-term exchange rate trajectory. This modification ensures consistency with the recalibration of other model parameters and enhances the overall coherence of the upgraded forecasting framework.

5.2.2 Parameters

The revised composition of the production block that is accounting for general energy imports instead of oil-specific components necessitates recalibrating the energy shares in imported content across sectors. Based on the OECD (2021) Input-Output Tables, we calibrate the import energy shares, ϕ^m for $m \in \{c, x, g, j\}$, as follows: 10% for household consumption, 7% for exports, 7% for government consumption, and 2% for investment. These adjustments are summarized in Table 4.

Table 4: Deep Parameters of the Domestic Block

Parameter		**Previous**	**Upgraded**
Calvo Imports Oil/Energy	$\xi_{Nenergy}$	0.15	0.25
Elasticity of Substitution Consumption	η^c	0.00	0.05
Elasticity of Substitution Exports	η^x	0.00	0.05
Foreign Demand Elasticity	η^{n*}	4.00	3.00
Oil/Energy Share on Imported Content, %			
Household Consumption	ϕ^c	4	10
Exports	ϕ^x	4	7
Government Consumption	ϕ^g	1	7
Investment	ϕ^j	1	2

Note: We report oil-related values for initial calibration, as the previous model worked with oil components instead of general energy ones.

Also, the new production component prompts a calibration of the price stickiness parameter specific to energy imports, denoted as $\xi_{Nenergy}$.[27] In the previous version of g3+, which operated with oil-related imports, the corresponding Calvo parameter was set at 0.15. However, considering the broader range of energies now involved, including those with higher rigidities such as electricity prices, the Calvo parameter is adjusted to a higher value of 0.25. The updated Calvo parameter value reflects recent micro evidence suggesting higher frequency of price adjustments during inflationary periods (e.g., Klein et al. (2024)). While micro-level PPI data are not available for Czech firms, this calibration aligns with international evidence.

Additionally, the shift to CES production functions in the household consumption and export sectors requires setting substitution elasticities between production inputs, η^c and η^x. We set these at 0.05, reflecting the limited substitutability observed in empirical studies such as Hassler et al. (2021) and further supported by Atalay (2017). Finally, the long-run elasticity of foreign demand for Czech exports is recalibrated from 4.0 to 3.0 (see Section 4.3).

6. Model Properties

An important component of the model development involves examining both stochastic and deterministic properties of the model, as well as its forecasting performance. This section presents the outcomes of the following analyses:

(i) *Impulse response functions*: This part focuses on understanding how the model responds to shocks by tracing the dynamic effects over time. Impulse response functions offer valuable information on the short- and long-term impacts of shocks on the economic system.

[27] Input-output tables 2021 edition covers data from 2018.

(ii) *Forecast error variance decomposition*: The results of this analysis shed light on the contributions of different shocks to forecast errors, providing insights into the relative importance of various factors in the forecasting process.

(iii) *Model-consistent identification of structural shocks*: Here, the goal is to present identification of structural shocks consistent with the model specifications. This helps in attributing observed variations to specific shocks and enhances the interpretability of the model's dynamics.

(iv) *Recursive filtering and forecasting*: This analysis involves evaluating the model's forecasting performance through recursive filtering and forecasting, while using exogenous assumptions in each iteration.

6.1 Impulse Response Functions

We analyze the dynamic properties of the current g3+ model in comparison to its initial version, highlighting differences resulting from the upgrades. The selected foreign shocks, such as foreign demand, foreign prices, and foreign monetary policy, are highly relevant to the model changes discussed in the previous sections. Additionally, we present responses to four domestic shocks in Appendix B – exchange rate, monetary policy, demand, and wage push shocks – to demonstrate that the upgraded model maintains similar characteristics to its predecessor in terms of fluctuations caused by domestic factors. We direct readers to Brázdik et al. (2020) for a discussion on other impulse responses.

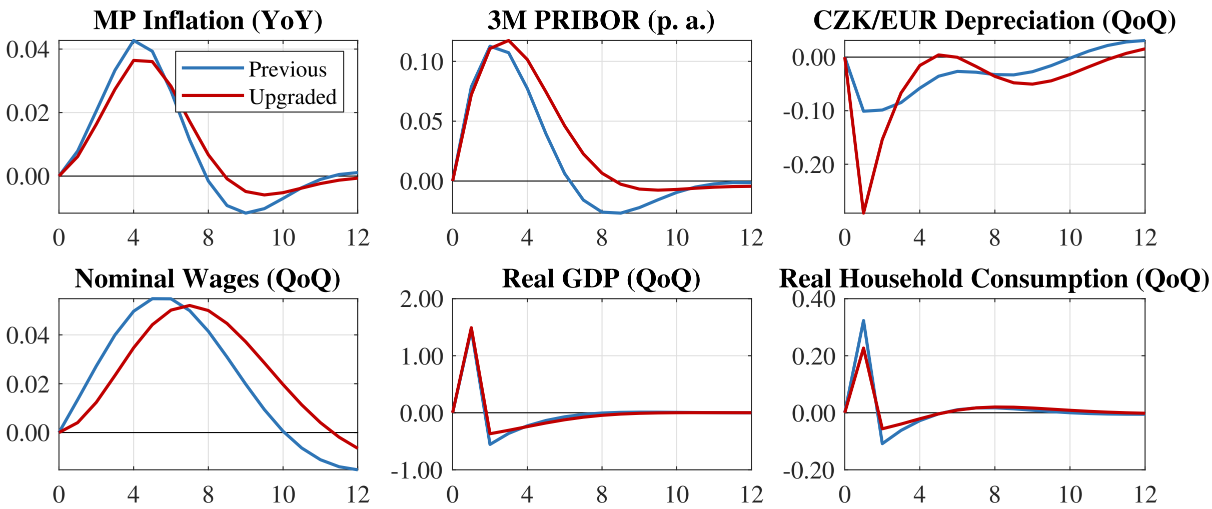
The figures presented below show the impulse responses of the variables to a one percent deviation surprise shocks. The responses are presented in percentage deviations from the steady state.

6.1.1 Foreign Demand Shock

The positive shock to foreign demand (see Figure 19), indicating an overheated foreign economy, results in heightened foreign inflation pressures – in terms of overall CPI and core PPI prices – and the consequent tightening of foreign monetary policy. The surge in foreign demand leads to an increased demand for domestic exports, thereby stimulating domestic economic activity and fostering higher labor demand and wages. Domestic real household consumption experiences an upswing due to higher real income of households. Responding to increased inflation pressures from a tighter labor market, the domestic interest rate goes up. The appreciation of CZK/EUR mirrors the enhanced net foreign asset position (NFA), outweighing the impact of the decline in the domestic-foreign interest rate differential.

Compared to the previous version of the model, we observe subtle alterations, particularly in terms of responses duration. The foreign IS equation, characterized by increased persistence, results in more widespread and prolonged effects. Simultaneously, foreign core PPI inflation exhibits greater sensitivity to influences originating from foreign economic activity, leading to more substantial impacts compared to the earlier version. Conversely, monetary policy shows less responsiveness to changes in the output gap. These effects collectively contribute to a more pronounced impact on the domestic economy. The CZK appreciation against the EUR is more evident initially, as the competitiveness of Czech exporters improves further due to a more significant increase in foreign core PPI inflation and an enhanced NFA position. The more enduring effects from foreign demand manifest in broader impacts on domestic wages and monetary policy inflation. Consequently, the response of domestic monetary policy also exhibits increased persistence.

Figure 19: Foreign Demand Shock



Source: Authors' calculations.

6.1.2 Foreign Monetary Policy Shock

The positive shock to foreign policy rates (refer to Figure 20) is linked to reduced foreign demand and subdued foreign inflation. Concurrently, a strict foreign monetary policy triggers the depreciation of the Czech koruna through the UIP condition. The koruna's depreciation contributes to an increase in imported goods inflation, prompting a rise in the domestic interest rate. The competitiveness of Czech exporters suffers due to the decline in foreign core prices, as Czech exporters compete on foreign non-energy goods and services markets. Domestic consumption and investment decrease in response to the diminished foreign demand. An increase in nominal wage dynamics in subsequent quarters reflects a compensation of heightened inflation.

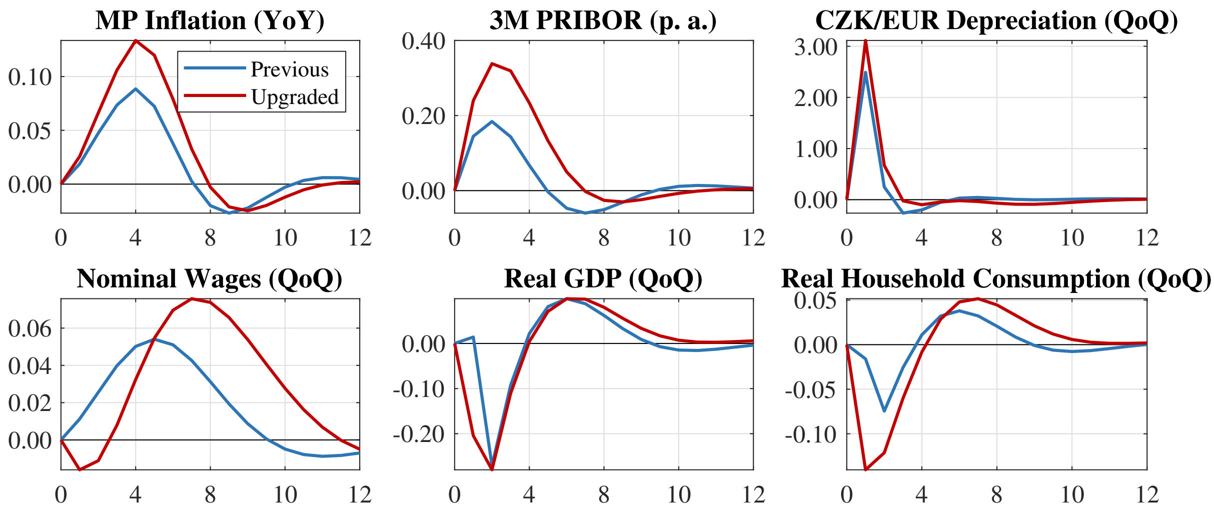
In the upgraded model, the response of the foreign policy rate exhibits increased persistence, thus it is associated with a more significant decrease in foreign core PPI, leading to a more pronounced depreciation of the CZK/EUR exchange rate. Additionally, wages experience a more substantial increase. These combined effects result in higher domestic inflation and interest rates. An initial drop of nominal wage dynamics, compared to its increase on impact in the previous version of g3+, is associated with faster decline in domestic economic activity.

6.1.3 Foreign Core PPI Inflation

The shock to foreign core producer price inflation (refer to Figure 21) results in a tightening of foreign monetary policy to counteract inflation pressures and realign inflation with the target. The higher foreign real interest rates contribute to a decline in foreign GDP. Improved price competitiveness for domestic exporters leads to increased exports and imports (the latter due to the high import intensity of exports). The Czech koruna appreciates through the NFA channel. Growth in import prices, coupled with higher demand for Czech exports, pushes domestic inflation upward.

In the innovated model, the effects are less pronounced. Foreign core PPI inflation displays significantly less persistence, resulting in more subdued impacts on the foreign economy. Additionally, domestic real household consumption increases initially, as nominal wages do not decrease at first but rise in response to heightened domestic economic activity.

Figure 20: Foreign Monetary Policy Shock



Source: Authors' calculations.

6.1.4 Foreign Energy PPI Inflation

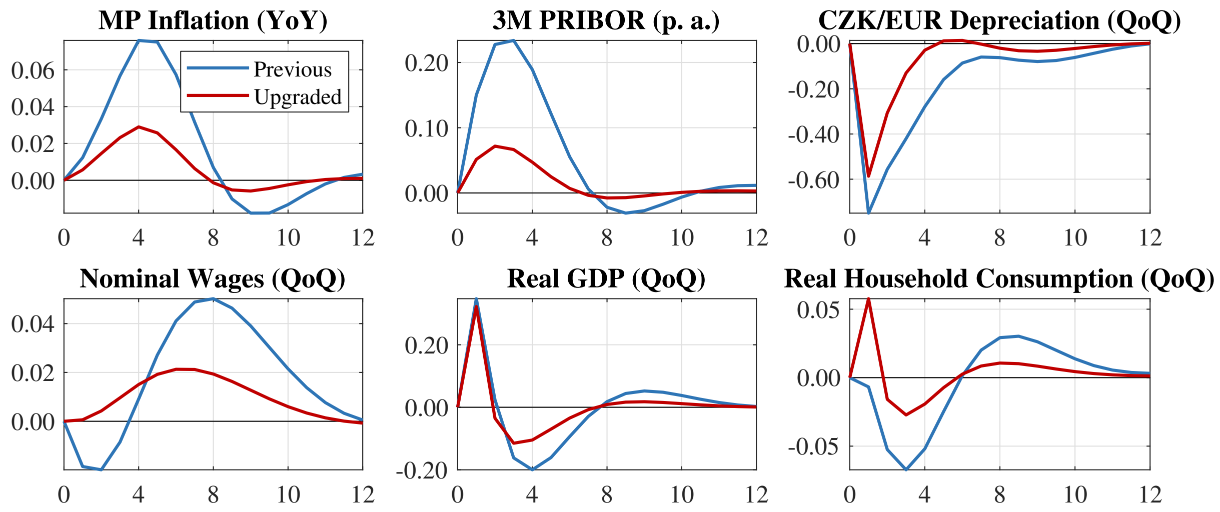
The shock to foreign energy PPI inflation (refer to Figure 22) leads to an increase in foreign price pressures, prompting the central bank to respond by raising interest rates. Foreign core producer price inflation decreases due to lower foreign demand. The decline in foreign economic activity is mainly caused by tighter foreign monetary conditions. Elevated energy import prices affect domestic producers' costs, leading to a rise in domestic inflation. The decrease in foreign core prices worsens the price competitiveness of Czech exporters. The combination of lower foreign core prices and subdued foreign demand results in a decrease in Czech exports, followed by a decline in imports as indicated by import intensities. The nominal exchange rate depreciates through the NFA channel. The diminished domestic production restrains labor demand and wage growth. Nevertheless, the increase in domestic prices, coupled with the depreciated currency, leads to higher domestic interest rates.

In contrast to the previous model version, the role of foreign energy PPI inflation is more significant. This is because only the oil component of foreign PPI influenced domestic developments, while other energies aggregated in the "other energy" component had no direct effect on the domestic economy. Consequently, the effect in the new framework is three to four times more stronger, given that the energy component of the PPI constitutes almost 30% of the overall PPI. This is in stark contrast to the previous model, where the weight of the oil-related component was only 8%.

6.2 Forecast Error Variance Decomposition

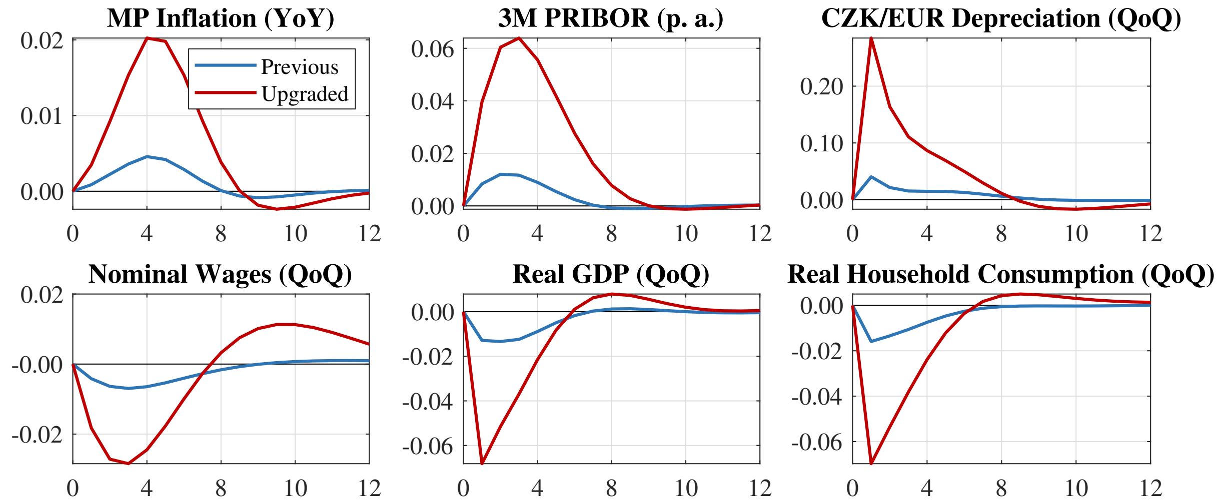
Figure 23 presents the forecast error variance decompositions (alternatively, variance decompositions) of the variables most significantly affected by model changes introduced in previous two sections, on top of three main policy-relevant variables. Namely, we focus on MP inflation, CZK/EUR depreciation, 3M Pribor and quarter-over-quarter growths in investment, export and import. The variance decompositions are presented for both the previous (Panel A) and the current model (Panel B), allowing for a direct comparison.

Figure 21: Foreign Core PPI Inflation Shock



Source: Authors' calculations.

Figure 22: Foreign Energy PPI Inflation Shock

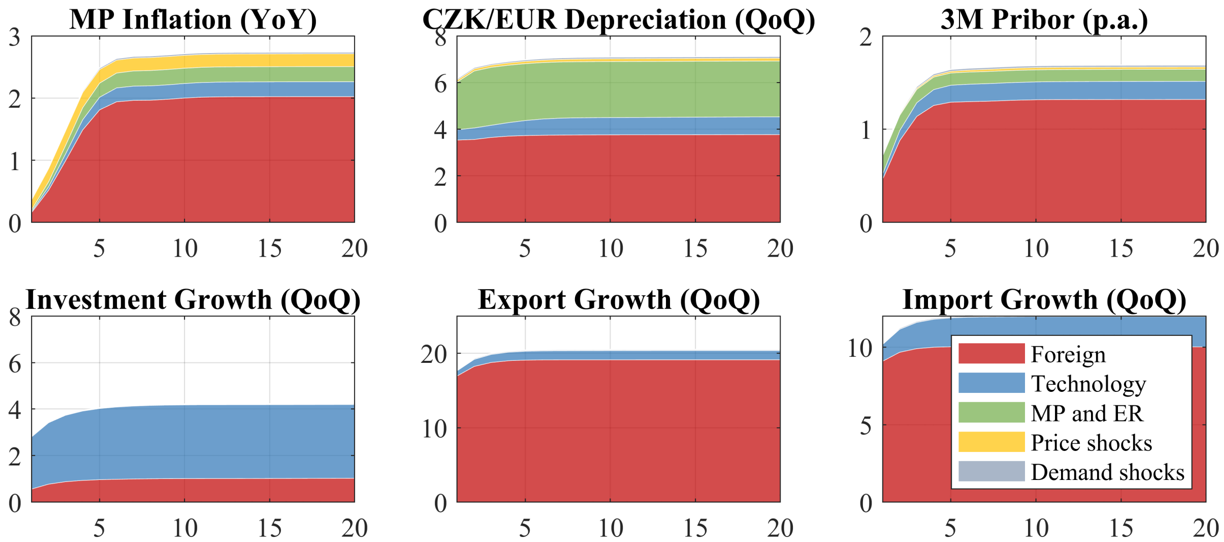


Note: Impulse responses to the brent oil price shock are depicted in the case of the initial model.
Source: Authors' calculations.

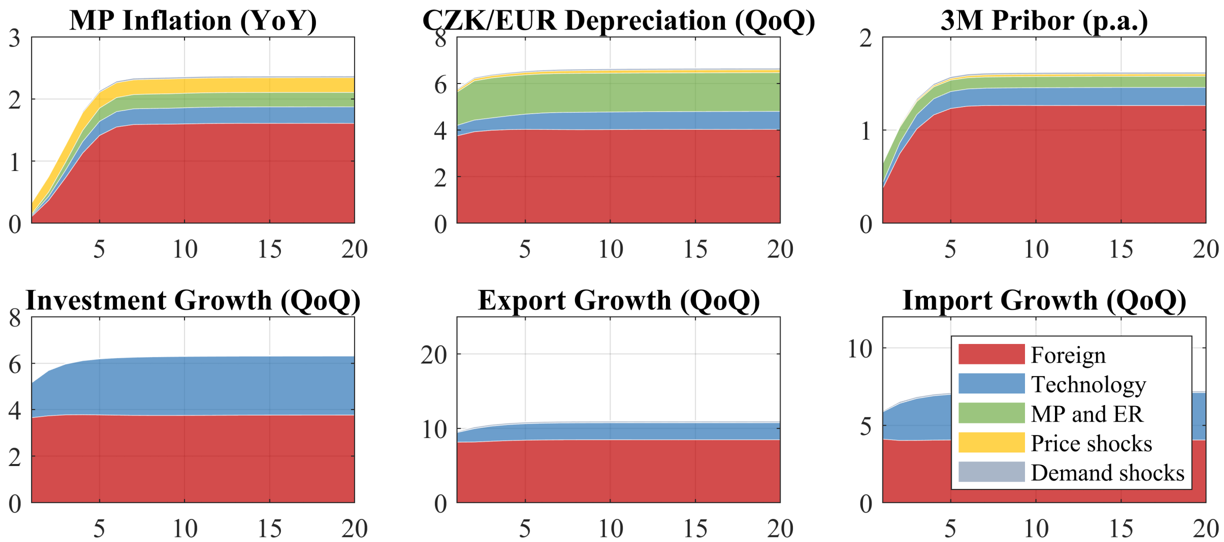
Concerning MP inflation and variables related to monetary conditions, our upgrades lead to similar variations with fairly similar contributions in individual categories in the upgraded model. The most affected variables are those associated with international trade, as one of the major model novelties relate to the adjustment in foreign demand elasticity. The reduction in its value from four to three results in substantially reduced fluctuations in export growth. Also, the newly defined energy component of foreign PPI is less volatile than oil prices, thus contributing to lower volatility of exports and imports. Given the close relationship between exports and imports, driven by high-import intensity of exports, the decreased variation in exports directly translates into the one of imports. In the case of import growth, the upgraded model indicates slightly higher contributions from technology shocks.

Figure 23: Forecast Error Variance Decomposition

Panel A: Previous



Panel B: Upgraded



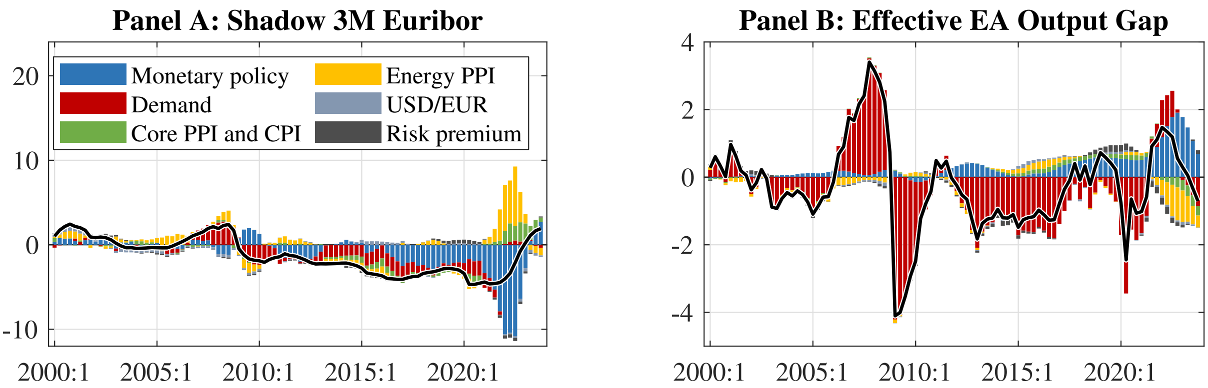
Note: The horizontal axis displays quarters.
Source: Authors' calculations.

In the previous version of the g3+ model, the variation in investment was primarily influenced by technology shocks, with the remaining impact attributed to foreign shocks. In its upgraded version, the composition shifts toward a predominant role of foreign shocks, supplemented by technology shocks instead. Additionally, there is a significant increase in the overall variation, which is in line with our intuition as investment proves to be the most volatile component of GDP. These changes in characteristics arise from a revised connection between domestic investment activity and foreign demand. The new link now considers not only the long-term effects from the foreign economy but also the short-term fluctuations described by changes in the foreign output gap.

6.3 Model-Consistent Identification of Structural Shocks

To demonstrate the model's capacity for narrative and structural interpretation, we begin by presenting historical decompositions of selected foreign variables in terms of structural shocks. Specifically, we focus on the decomposition of the foreign policy rate represented by the shadow 3M Euribor interest rate[28] and the effective euro area (EA) output gap. In addition, we present the decomposition of the domestic inflation rate relative to the model's steady-state values.

Figure 24: Structural Shocks Decomposition of Foreign Variables (in %, contributions in pp)



Note: The figure displays the decomposition as the deviation from the steady state. The solid black line is the sum of the contributions.
Source: Authors' calculations.

Figure 24 presents the decompositions of two foreign variables. In Panel A, we examine the shadow 3M Euribor interest rates, highlighting anti-inflationary pressures over the period from 2011 to 2021. These pressures are primarily attributed to subdued demand (illustrated by negative red bars) and core PPI and CPI inflation (shown by negative green bars). The model emphasizes the ECB's efforts to counteract these anti-inflationary pressures, maintaining a low shadow interest rate from 2015 onward, which significantly deviates from the level suggested by the policy rule. In contrast, the period from mid-2021 to the end of 2023 exhibits pronounced inflationary pressures, largely driven by energy-related price increases. Simultaneously, the model detects notable negative contributions from monetary policy shocks in 2022, reflecting the ECB's cautious stance prior to rate hikes amid rising inflationary pressures in the euro area.

Panel B of the Figure 24 presents the decomposition of the effective EA output gap into structural shocks.[29] The updated estimates reveal an overheating of the EA economy before the global financial crisis, followed by a recession triggered by demand shocks. The economy then operated below its potential, particularly between 2012 and 2018.[30] The decomposition suggests that ECB's monetary policy was more accommodative than the model policy rule prescribed, especially from 2015 onward. Initially, this divergence is attributed to the ECB's efforts to counteract deflationary pres-

[28] The shadow policy rate is an analytical concept used by the CNB to assess the overall stance of monetary policy, encompassing both conventional and unconventional measures. Unlike the standard policy rate, which reflects only the interest rate controlled by the central bank, the shadow interest rate accounts for the additional effects of non-standard monetary tools (asset purchase programmes), thereby offering a more comprehensive view of monetary policy conditions.

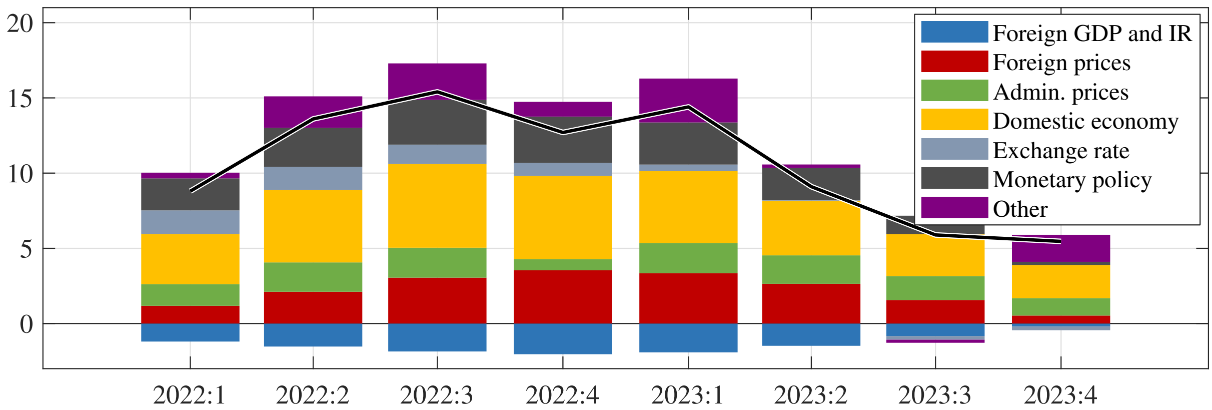
[29] The output gap includes expert assessments, particularly linked to government measures implemented during the COVID-19 pandemic.

[30] The increasingly negative effective EA output gap post-2012 aligns closely with estimates from institutions like the International Monetary Fund (IMF). For further details, refer to Beyer et al. (2023).

sures using unconventional monetary policy tools. The years following 2020 are characterized by a cautious monetary stance, preceding the initiation of a rate-hiking cycle. Additionally, increased post-COVID demand contributed to the euro area's economic overheating, while negative contributions from energy PPI emerged post-2021, indicating adverse impacts from elevated production costs.

Figure 25 provides a breakdown of domestic CPI inflation's deviation from the CNB's 2% target, detailing contributions from key groups of structural shocks. It provides insight into the nature of inflation surge in 2022 and its gradual unwinding in 2023.

Figure 25: Decomposition of Deviation of Inflation from the Target (in %, contributions in pp)



Note: The figure displays the decomposition as the deviation from the steady state. The solid black line is the sum of the contributions.
Source: Authors' calculations.

The surge in inflation during 2022 was predominantly driven by external cost-push shocks, particularly the steep increase in import prices (red bars), reflecting surge in foreign producer price linked to global energy and commodity market tensions. This surge in energy costs raised input costs for domestic firms, resulting in higher final prices for goods and services. These pressures were subsequently reversed to some extent in 2023 as global conditions stabilized.

Anti-inflationary forces stemmed primarily from weak foreign demand and tighter monetary policy abroad (blue bars), which eased price pressures by reducing external cost and demand impulses. The exchange rate contributed to disinflation through the appreciation of the Czech koruna (light grey bars), supported by improved regional sentiment, and both actual and verbal foreign exchange interventions by the CNB.

On the domestic side, inflation was driven above the target by several factors. The ECB's delayed and milder tightening compared to the CNB (dark grey bars) widened the interest rate differential, supporting koruna appreciation. The domestic economy's strong inflationary influence (illustrated by yellow bars) was largely driven by increase in profit margins of firms and an expansionary fiscal policy. In response to surging energy costs, firms increased prices beyond the actual cost increase, amplifying inflationary pressures. Portion of the price escalation was attributed to rising energy costs causing a significant increase in household energy bills as the administered prices increase (green bars) responded to global energy price surge. The high inflation environment eroded household purchasing power, resulting in lower real incomes and a gradual decline in consumer demand, which supported easing of inflationary pressures in 2023.

In response to sustained inflationary pressures and above-target inflation, domestic interest rates continued to rise until mid-2022, with the 2-week repo rate reaching 7%, where it remained until the end of 2023. The accommodative stance of domestic monetary policy during this period added to the inflationary pressures throughout 2022 and 2023.

6.4 Recursive Filtering and Forecasting

To assess the forecasting performance of the model, we employ the recursive filtering and forecasting approach, also known as in-sample simulations. This method involves filtering historical data, generating forecasts using external assumptions without expert judgments and evaluating them recursively against observed data. In addition to this, we produced a comprehensive forecast, referred to as a shadow forecast, and published it in the Annex of CNB (2024). This was done to evaluate the operational effectiveness and quality of the model in a real-time forecasting procedure.

In-sample simulations follow a recursive design. We set the starting period for the evaluation period (in this case, 2000Q1), and the procedure unfolds through the following steps:

(i) The initial state of the forecast is estimated by filtering[31], encompassing the historical period of the corresponding simulation (for example, 1996Q1–2000Q4).

(ii) Based on the initial state, a forecast conditional on foreign and domestic outlooks for eight quarters ahead is created[32], while true data outturns are used as conditioning outlooks.

(iii) The data sample used for filtering is extended by one observation (one quarter), and steps (i) to (iii) are reiterated until the final period of the evaluation is reached.

We apply the root mean square errors (RMSE) to evaluate the forecasting performance at round j as follows:

$$RMSE_j^M = \sum_{j=1}^{N} \sqrt{\frac{\sum_{i=1}^{H} (x_{j,i}^M - \bar{x}_{j,i})^2}{H}}, \tag{25}$$

where M stands for the model used (initial or upgraded g3+), $j = 1,\ldots,N$ represents the rounds of recursive filtering and forecasts, $i = 1,\ldots,H$ stands for the number of forecast quarters included in the computation of the statistics based on the selected horizon, and $x_{j,i}^M$ and $\bar{x}_{j,i}$ are the forecasted and the observed values of the respective variable. The RMSE for each model is computed as the sum of $RMSE_j^M$ over all considered periods.

Table 5 presents results for two distinct data samples with varying lengths. The first sample spans from 2000Q1 to 2019Q4, excluding the turbulent period 2020–2023 marked by extreme economic events such as the COVID pandemic. To show how the model performs over extended history, we also show results based on the period until 2023Q4.[33] Statistics are computed for two different forecasting horizons: the full forecast horizon covering six quarters ($i = 1,\ldots,6$ in Equation (25)) and the monetary policy (MP) horizon encompassing the 4th to 6th quarters of the forecast ($i = 4,5,6$

[31] The filtering step is done using the Kalman smoother.

[32] Domestic assumptions include: fiscal policy, administered prices inflation and tax impacts. Foreign assumptions encompass:effective EA real GDP and its decomposition into trend and gap, effective EA producer prices inflation and its components (energy-related and core), effective EA HICP inflation, nominal EURIBOR 3M interest rates and their shadow unconventional counterpart, and USD/EUR exchange rate.

[33] The results covering the period after 2020 should be treated cautiously as this period was characterised by many non-economic and non-standard events far exceeding the standard business cycle dynamics.

Table 5: Forecasting Performance – In-Sample Simulations

	Full Forecast Horizon		**MP Horizon**	
Variable	**Till 2019Q4**	**Till 2023Q3**	**Till 2019Q4**	**Till 2023Q3**
Exchange Rate CZK/EUR	1.01	0.95	1.00	0.97
YoY, %				
MP Inflation	1.07	1.02	0.99	0.97
Wages in Market Sectors	0.85	0.90	0.87	0.95
Real Consumption	0.95	0.98	1.00	1.02
Real Investment	0.82	0.82	0.79	0.86
Real Import	0.93	0.89	1.01	0.98
Real Export	0.75	0.73	0.85	0.81
Consumption Price Deflator	0.94	0.96	0.99	1.02
Investment Price Deflator	0.86	0.84	0.85	0.85
Import Price Deflator	0.95	0.87	0.98	0.90
Export Price Deflator	1.06	0.97	1.05	0.96

Note: The table reports the relative values of the RMSE statistics given by Equation (25) for the upgraded and initial version of model g3+. Numbers less (greater) than one in green (red) cells indicate an improvement (worsening) of the in-sample performance of the upgraded model in the respective variable.

in Equation (25)). For ease of interpretation, RMSE statistics for both the initial and upgraded models are expressed as ratios.

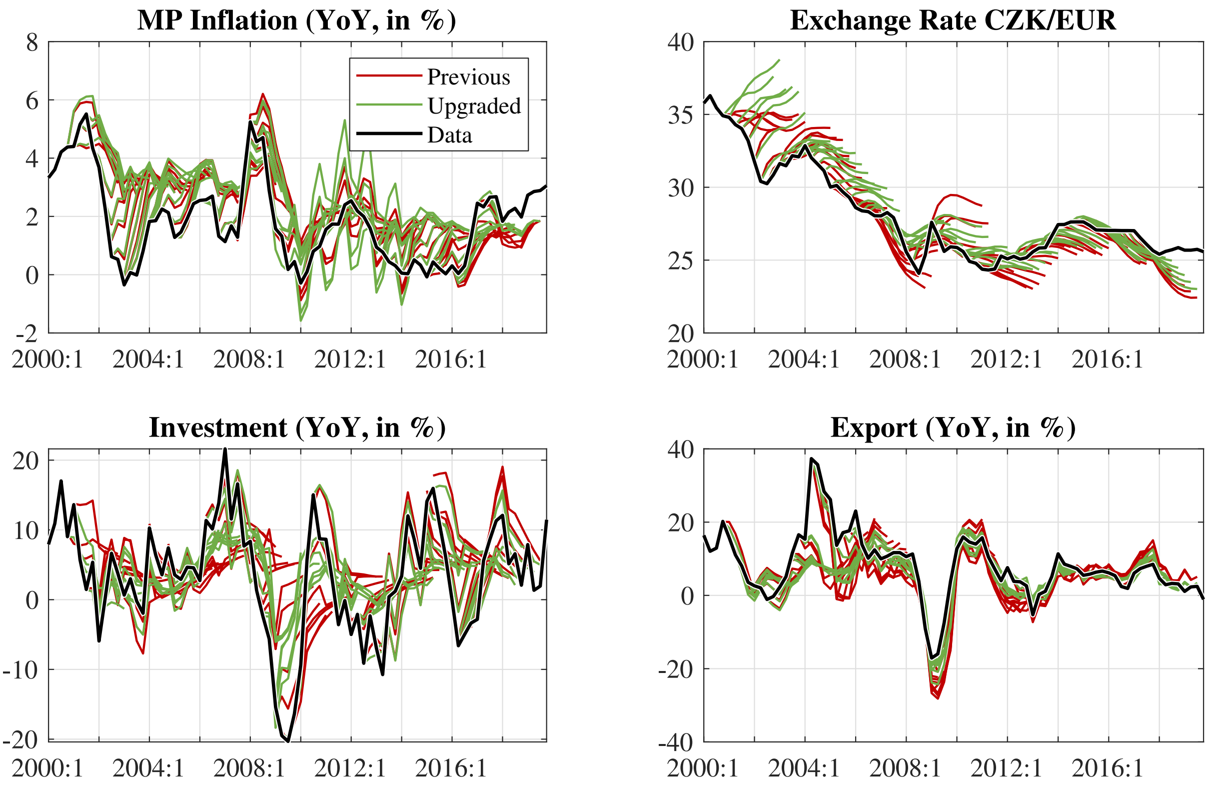
The resulting values of relative RMSE metrics indicate an enhancement in the updated model across the majority of listed variables, regardless of the chosen time horizon or period. This implies that the new features and calibration not only align better with typical economic cycles but also yield improvements during turbulent periods. The most significant improvement emerge in variables directly influenced by implemented upgrades, notably the updated domestic investment-foreign demand link and alterations to foreign demand itself. These changes result in notable advancements in forecasting performance of real investment and real export growth throughout the entire forecast horizon. Wages also exhibit considerable improvement, given their close connection to economic activity as such.

The upgraded model outperforms the original version in forecasting the exchange rate only when the 2020–2023 period is included in evaluation. This improvement stems from the recalibration of the equilibrium pace of Koruna appreciation, which enhances the model's ability to capture exchange rate dynamics in the most recent historical period (see Figure 26 for details). While forecasts of monetary policy-relevant inflation are less accurate when evaluated over the full forecast horizon, the upgraded model demonstrates a modest improvement within the monetary policy horizon. This is particularly important, as inflation developments over this shorter horizon play a key role in shaping timely and effective monetary policy decisions.

To illustrate the in-sample simulations, we plot recursive forecasts of selected variables (the MP conditions and the two variables attaining the most improvement – real investment and real export growth) in Figure 26.

Overall, the recursive filtering and forecasting exercise demonstrates the enhanced framework's superiority over the previous model, outperforming it by up to 20% in specific cases. While there is

Figure 26: Forecasting Performance – In-Sample Simulations



Source: Authors' calculations.

small deterioration of forecasting power in certain group of variables, it is only mild and constitutes a minority of instances.

7. Conclusion

The core forecasting model constitutes a crucial component of the Forecasting and Policy Analysis System, serving as a key element for CNB monetary policy. Given model's role in the system, it needs continuous attention and enhancements that align it with recent economic events and the experiences of professional forecasters and modelers. The events of the 2020–2023 period motivate us to introduce new features and thoroughly scrutinize existing relationships within the Czech National Bank's core forecasting model, g3+, with the aim to provide a more accurate description of economic developments and better understanding of the implications for setting monetary policy.

In this work, we summarize new features and revisions to the g3+ model introduced in 2024, also we provide motivation and discussions behind these modifications. Five significant upgrades were made to the foreign block, with three involving structural adjustments: the endogenous decomposition of foreign economic activity into gap and trend components, the division of foreign producer prices into energy-related and core components with a time-varying energy weight, and two minor adjustments: consumer price-producer price link and the USD/EUR real exchange rate gap equation. Additionally, the composition of the effective euro area aggregate was changed. Finally, the last area of upgrades involved the update of selected parameters of the foreign block.

The upgrades related to the domestic block of the model encompass four key areas. Innovations in the domestic domain were directly initiated by modifications of the foreign economy block, aligning with those adjustments. New structure of foreign economy allowed to modify domestic production by introducing a constant elasticity function for production of household consumption and export sector goods. Additional innovations concerned foreign demand and its impact on domestic exports and investment. Last but not least, selected dynamic parameters and long-term equilibrium values were re-calibrated.

All these upgrades collectively enhance the richness of the model's structure. Pre-implementation tests conducted before the model was put into practice along with our forecasting routine in 2024, have demonstrated usefulness of the upgraded framework. It enabled us to better identify sources of economic fluctuations and provide internally consistent and economically sound interpretations of recent extraordinary events. The upgraded version of the model also demonstrates improvements in stochastic and forecasting properties compared to its previous version. These improvements extend beyond "standard" business cycle periods and also encompass the extraordinary years from 2020 to 2023.

At the CNB, model development is continuous process, and we acknowledge that the modeling framework will necessitate further enhancements in the future. Potential areas for improvement include a more detailed representation of global monetary policy, even capturing the actions of the Federal Reserve System and their spillover effects. Additionally, incorporating the term structure of interest rates into the model could offer deeper insights for policymakers, especially in periods of heightened market volatility or unconventional policy measures. While the model accounts for aggregate demand and supply shocks, it does not explicitly capture geopolitical or trade policy shocks such as tariffs or sanctions. These risks may manifest as shocks to foreign demand, inflation, or exchange rates, but a structural extension would be needed to explicitly account for such scenarios.

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Appendix A: Semi-Structural Model of the Effective Euro Area

The following equations sketch the design of the semi-structural model of the foreign economy represented by the effective euro area in the upgraded g3+ model.[34]

A.1 Behavioral Equations

The dynamic IS (DIS) curve:

$$\hat{y}^*_t = a_{y^*}\,\hat{y}^*_{t-1} - a^{r^*}_{y^*}\,\hat{r}^*_{t-1} + a^{z}_{y^*}\,\hat{z}_t + \varepsilon^{\hat{y}^*}_t. \tag{A.1}$$

The New Keynesian Phillips curve (NKPC) for the core PPI inflation:

$$\tilde{\pi}^{*core}_t = a_{\pi^{*core}}\,\tilde{\pi}^{*core}_{t-1} + (1-a_{\pi^{*core}})\,\tilde{\pi}^{*core}_{t+1} + a^{y^*}_{\pi^{*core}}\,\hat{y}^*_t + a^{z}_{\pi^{*core}}\,\hat{z}_t + \varepsilon^{\tilde{\pi}^{*core}}_t. \tag{A.2}$$

The inflation-targeting monetary policy (MP) rule:

$$i^*_t = a_{i^*}\,i^*_{t-1} + (1-a_{i^*})\left(i^{*eq}_t + a^{y^*}_{i^*}\,\hat{y}^*_t + a^{\pi^*}_{i^*}\left(\frac{\tilde{\pi}^{*cpi4}_{t+4}}{4} - \bar{\pi}^{*cpi}\right)\right) + \varepsilon^{i^*}_t. \tag{A.3}$$

The USD/EUR UIP condition:

$$0 = \rho_{usdeur}\,\Delta usdeur_{t+1} - (1-\rho_{usdeur})\,\Delta usdeur_t + (i^*_t - i^{*eq}_t - prem^{usdeur}_t) + \varepsilon^{\Delta usdeur}_t. \tag{A.4}$$

The link between CPI and PPI price inflations:

$$\tilde{\pi}^{*cpi}_t - \bar{\pi}^{*cpi} = \rho_{\pi^{*cpi}}(\tilde{\pi}^{*cpi}_{t-1} - \bar{\pi}^{*cpi}) + (1-\rho_{\pi^{*cpi}})a_{\pi^{*cpi-ppi}}(\tilde{\pi}^*_t - \bar{\pi}^*) + \varepsilon^{\tilde{\pi}^{*cpi}}_t, \tag{A.5}$$

A.2 Definitions and Aggregations

The structural definitions include, (except numerous definitions of year-over-year growth rates expressed from quarter-over-quarter ones):

$$\tilde{\pi}^*_t = \rho^{*\frac{ener}{ppi}}_t\left(\tilde{\pi}^{*ener}_t - \tilde{\pi}^{*rp}_t\right) + \left(1 - \rho^{*\frac{ener}{ppi}}_t\right)\tilde{\pi}^{*core}_t + \varepsilon^{\tilde{\pi}^*}_t, \tag{A.6}$$

$$\hat{\rho}^{*\frac{ener}{ppi}}_t = \frac{\tilde{\pi}^{*ener}_t}{\tilde{\pi}^{*core}_t}\varepsilon^{\hat{\rho}^{*\frac{ener}{ppi}}}_t, \tag{A.7}$$

$$\hat{z}_t - \hat{z}_{t-1} = -wedge^{\tilde{\pi}^{RoW}}_t - \Delta usdeur_t - \tilde{\pi}^*_t, \tag{A.8}$$

$$\hat{r}^*_t = i^*_t - \tilde{\pi}^{*cpi}_{t+1} - (i^{*eq}_t - \bar{\pi}^{*cpi}), \tag{A.9}$$

$$prem^{*shadow}_t = i^*_t - i^{*shadow}_t, \tag{A.10}$$

$$\Delta y^*_t = (\hat{y}^*_t - \hat{y}^*_{t-1}) + \Delta y^{*trend}_t, \tag{A.11}$$

$$\Delta y^{*trend}_t = \Delta y^{*ftrend}_t + \Delta y^{*trendshift}_t. \tag{A.12}$$

[34] All variables are expressed in logarithms. See Appendix A.4 for notation glossary and parametrization.

A.3 Unobserved Variables and Other Relations

For model parsimoniousness, some of unobserved variables and variables without fully micro-founded structural backup are modeled as AR(1) processes, to match their persistent behavior:

$$wedge_t^{\bar{\pi}^{RoW}} = \rho_{wedge^{RoW}} wedge_{t-1}^{\bar{\pi}^{RoW}} + (1-\rho_{wedge^{RoW}}) wedge^{\bar{\pi}^{RoW}} + \varepsilon_t^{wedge^{\bar{\pi}^{RoW}}}, \quad (A.13)$$

$$i_t^{*eq} = \rho_{i^{*eq}} i_{t-1}^{*eq} + (1-\rho_{i^{*eq}}) i^{*eq} + \varepsilon_t^{i^{*eq}}, \quad (A.14)$$

$$prem_t^{*shadow} = \rho_{prem^{*shadow}} prem_{t-1}^{*shadow} + \varepsilon_t^{prem^{*shadow}}, \quad (A.15)$$

$$prem_t^{usdeur} = \rho_{prem^{usdeur}} prem_{t-1}^{usdeur} + \varepsilon_t^{prem^{usdeur}}, \quad (A.16)$$

$$\tilde{\pi}_t^{*ener} = \rho_{\pi^{*ener}} \tilde{\pi}_{t-1}^{*ener} + (1-\rho_{\pi^{*ener}}) \bar{\pi}^{*ener} + \varepsilon_t^{\tilde{\pi}^{*ener}}, \quad (A.17)$$

$$\Delta y_t^{*ftrend} = \rho_{y^{*ftrend}} \Delta y_{t-1}^{*ftrend} + (1-\rho_{y^{*ftrend}}) \Delta y^{*} + \varepsilon_t^{y^{*ftrend}}. \quad (A.18)$$

A.4 Parameters and Variables

Table A.1: Rest of Parameters in the Foreign Block

Parameter		**Previous**	**Current**
Energy Share in PPI	$\rho_{*\frac{ener}{ppi}}$	n/a	0.28
AR Processes			
RoW Inflation Target Wedge	$\rho_{wedge^{RoW}}$	n/a	0.681
Equilibrium Interest Rate	$\rho_{i^{*eq}}$	0.800	0.800
Energy PPI Inflation	$\rho_{\pi^{*ener}}$	n/a	0.692
CPI-PPI Link	$\rho_{\pi^{*cpi}}$	n/a	0.300
Fundamental trend	$\rho_{y^{*ftrend}}$	0.700	0.751
Premium	$\rho_{prem^{*shadow}}$	0.700	0.724

Table A.2: Variables in the Foreign Block

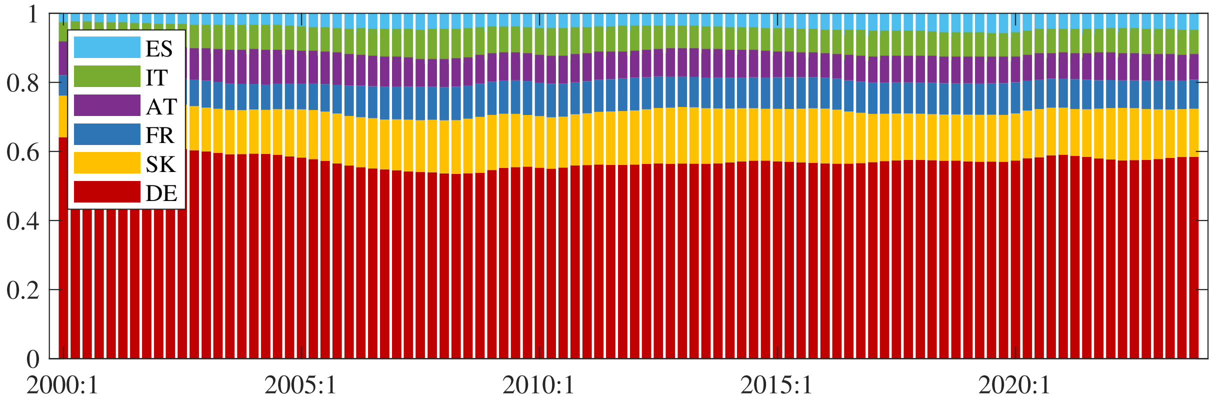
Variable	**Notation**	**Variable**	**Notation**
Economic activity			
Output	y_t^*	Potential output shift	$y_t^{*trendshift}$
Output gap	$\hat{y}_t^*$	Fundamental potential output	$y_t^{*ftrend}$
Potential output	y_t^{*trend}		
Prices			
PPI	$\tilde{\pi}_t^*$	Relative PPI inflation	$\tilde{\pi}_t^{*rp}$
PPI (core)	$\tilde{\pi}_t^{*core}$	CPI inflation	$\tilde{\pi}_t^{*cpi}$
PPI (energy)	$\tilde{\pi}_t^{*energy}$		
Monetary conditions			
Policy nominal interest rate	i_t^*	USD/EUR exchange rate	$usdeur_t$
Equilibrium nominal interest rate	i_t^{*eq}	USD/EUR real exchange rate	$\hat{z}_t$
Shadow nominal interest rate	$i_t^{*shadow}$	Risk premium	$prem_t^{usdeur}$
Real interest rate	r_t^*		
Rest of world link			
RoW inflation target wedge	$wedge_t^{\bar{\pi}^{RoW}}$		

A.5 Effective Euro Area Weights

Since the role of the foreign economy block is to represent the economies of the Czech Republic's main trading partners, its observed variables are constructed as weighted aggregates. Specifically, foreign GDP serves as a proxy for external economic activity, while foreign PPI and CPI inflation are used to capture price developments abroad. These indicators are weighted by the export shares of the Czech economy to each trading partner, which ensures that they reflect the structure and intensity of actual trade relationships. As a result, these composite indicators provide an effective measure of foreign demand and price pressures relevant to the Czech economy.

The export weights used in constructing these effective indicators are based on data from the Czech Statistical Office's International Trade database for cross-border goods movements. These weights are re-scaled to sum to unity. The underlying macroeconomic data for the foreign economies come from the Eurostat database, ensuring consistency and comparability across countries.

Figure A.1: Composition of Export Weighted Aggregate

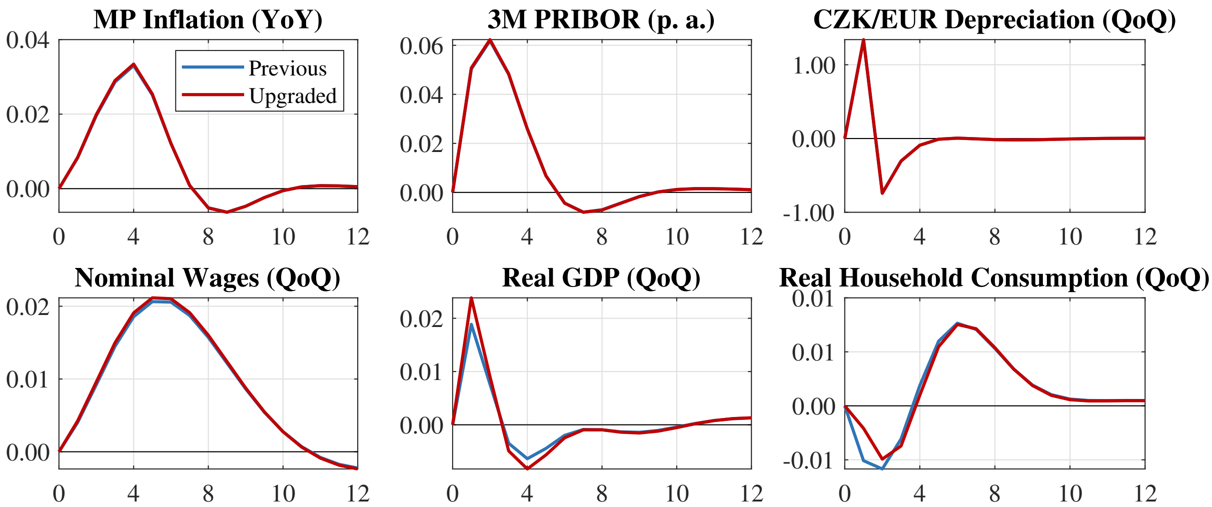


Source: Authors' calculations based on CZSO data.

Appendix B: Impulse Responses to Selected Domestic Shocks

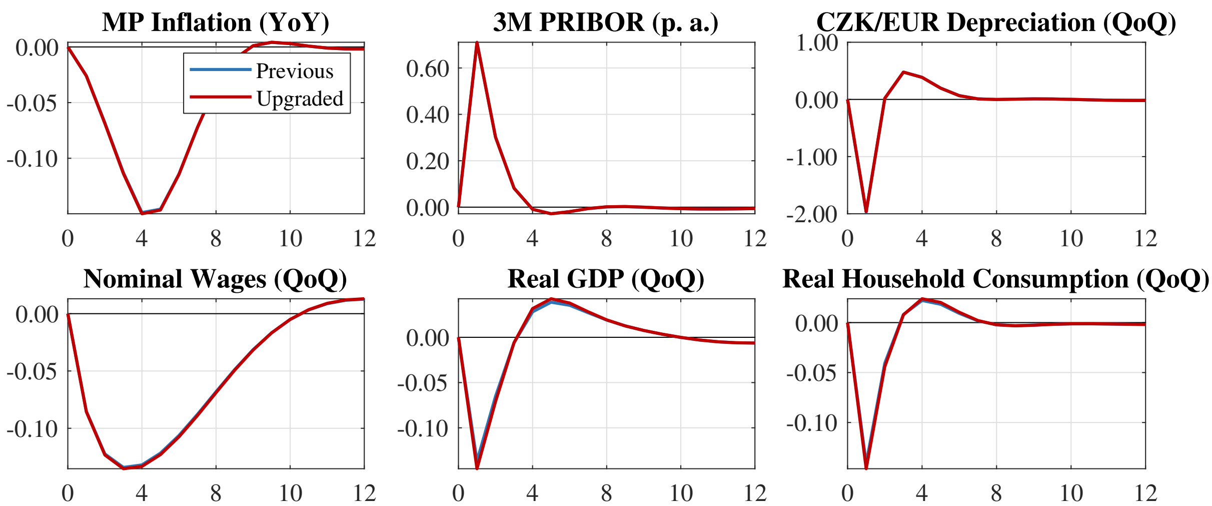
The figures shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state.

Figure B.1: Short-Term Exchange Rate Shock



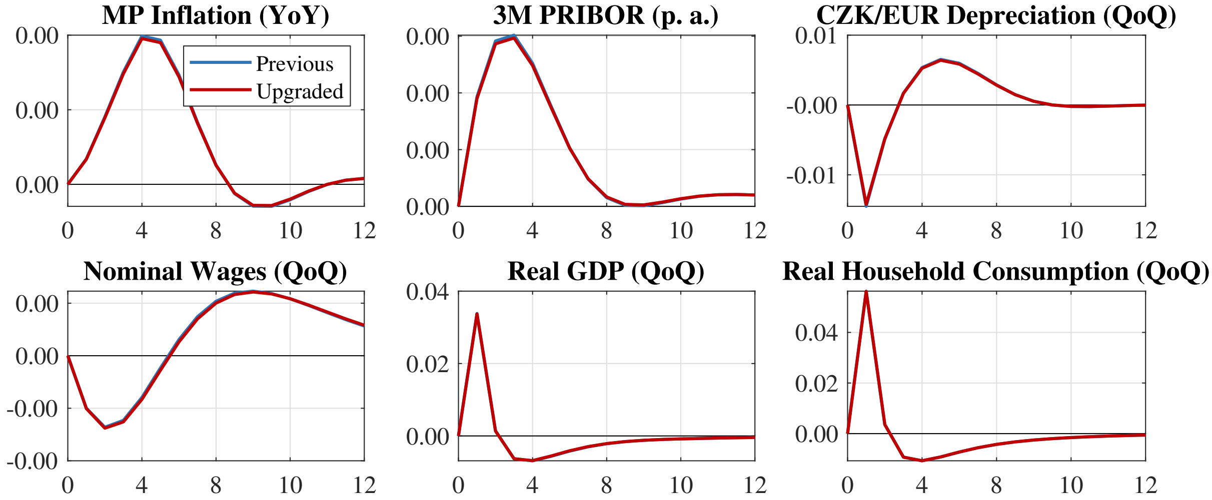
Source: Authors' calculations.

Figure B.2: Monetary Policy Shock



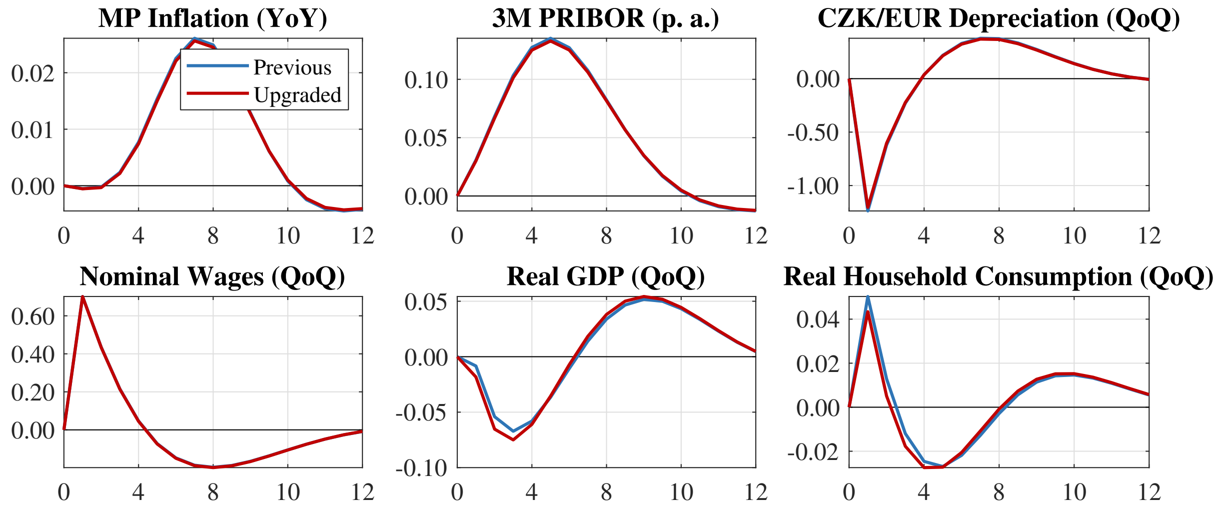
Source: Authors' calculations.

Figure B.3: Households' Demand Shock



Source: Authors' calculations.

Figure B.4: Wage Push Shock



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