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Foreign Exchange Implications of CBDCs and Their Integration via Bridge Coins

Alexis Derviz*

Abstract

When several central banks decide to introduce CBDCs, interoperability requirements create demand for a common payment infrastructure and a joint digital accounting unit (bridge coin). Many attributes of the latter resemble those of private digital currencies. At the same time, the CBDC-embracing authorities actively contribute to elevating digital wallets to the position of a household technology. Private agents discover ways to make domestic and foreign payments in the (digital) currency of their choice irrespective of the CBDC-issuing authorities' intentions. In such a world, will fiat currencies and the central banks that issue them be sidetracked by the bridge coin, or are old and new forms of international transactions able to coexist? What changes await the traditional FX market? These questions are addressed in a two-country, two-good, two-currency DSGE model with a global digital currency (digicoin). Under a certain structure of FX transaction costs, all three partial FX markets coexist and the use of fiat currency in foreign trade is unlikely to be eliminated completely as long as the bridge coin operator is unable to become a global banker as well.

Abstrakt

Když se několik centrálních bank rozhodne zavést CBDC, požadavky na interoperabilitu vytvoří poptávku po společné platební infrastruktuře a společné digitální zúčtovací jednotce (bridge coin). Mnoho atributů této zúčtovací jednotky připomíná soukromé digitální měny. Zároveň měnové autority, které se vydají cestou CBDC, aktivně přispívají k tomu, že z digitálních peněženek se stane technologie běžná v domácnostech. Soukromé subjekty objevují způsoby, kterými mohou provádět domácí i přeshraniční platby v (digitální) měně podle svého uvážení bez ohledu na záměry emitentů CBDC. Budou v takovém světě fiat měny a centrální banky, které je emitují, odsunuty na vedlejší kolej digitální zúčtovací jednotkou, nebo dokážou staré a nové formy mezinárodních transakcí koexistovat? Jaké změny čekají tradiční devizový trh? Na tyto otázky odpovídám pomocí DSGE modelu dvou zemí se dvěma druhy zboží, dvěma měnami a globální digitální měnou (digicoin). Při určité struktuře devizových transakčních nákladů všechny tři dílčí devizové trhy koexistují a není pravděpodobné, že by používání fiat měny v mezinárodním obchodu zcela vymizelo, za předpokladu, že se operátor digitální zúčtovací jednotky nedokáže zároveň stát globální bankou.

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Keywords: Bridge coin, cash in advance, CBDC, digital currency, FX market.

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1. Introduction

Issuing digital currencies is very often justified by international transaction facilitation. This is the case with both private cryptocurrencies, whose issuers promise freedom from conventional FX frictions, and central bank digital currencies (CBDCs). In the latter case, central banks that experiment with them, although primarily driven by the fear of loss of monetary sovereignty in favor of private digital token alternatives, list the removal of excessive cross-border payment costs among their main pursued goals.

Next, when more than one central bank decides to introduce a CBDC, these means of payment must be made interoperable (otherwise, the CBDC-use case becomes very weak). However, when a CBDC seeks interoperability with similar products in other countries, a common unit of account is a natural next step (BIS, 2021). This unit of account cannot exist as a mere abstraction; it requires common standards and protocols and the corresponding digital infrastructure. The central banks concerned may try to create this infrastructure as a common project inside their community (with all the difficulties associated with cooperation in CBDC matters among independent monetary authorities; see BIS, 2021, section 2) or commission it to a willing private entity. There are already several happy to oblige (Ripple may be the best known; see Qiu, 2019), and the tokens they attach to the accounting unit are sometimes called bridge coins. The latter gain attributes resembling many already existing private digital currencies.

In the meantime, the mass adoption of digital means of payment and units of account, be they offered by a state authority or a corporate, may itself become a strong incentive for economic agents to move into the realm of decentralized means of exchange in which central bank monopoly is non-existent. This is a realm from which there is no proper way back.[1] Indeed, a state authority incentivizing private entities to use its CBDC is simultaneously pushing them en masse into familiarizing themselves with electronic wallets, encryption, DLT, and tokens, thereby elevating digital wallets to the position of a household technology. Even if central banks try to build a barrier between CBDC-carrying wallets and private crypto products, it will only result in two-track wallet operation. The highest barrier, namely, the psychological one standing between the public and the adoption of decentralized payment technology, they have just helped tear down themselves. It is then only a matter of time before private agents discover ways of using their digital wallets (either the same or similarly operated) and bridge coin balances to routinely access the digital asset universe at large. Besides purely investment activities, they will, from now on, be able to make domestic and foreign payments in the (digital) currency of their choice irrespective of the CBDC-issuing authorities' intentions.

If the outlined scenario has a non-negligible probability of materializing, one is bound to ask: are fiat currencies and the central banks that issue them at risk of eventually being sidetracked? In this new world, what happens with the demand for fiat monies in jurisdictions that have embraced the mutually interoperable CBDC policy? Will this demand die out? Or, conversely, will the well-known limits to DLT payment solutions (poor scalability, price and conversion cost volatility, risks of technical glitches and cyber fraud, etc.) reduce private digital currency use to an insignificant

[1] As an analogy, one can think of the recent explosive adoption of teleconference applications under the pressure of COVID-19 restrictions: the mass fear of the pandemic and the accompanying restrictions themselves are largely gone, but people still attend fewer meetings in person and use the opportunity to connect online much more often than they did in pre-covid times.

minority option? In this paper, I try to answer a somewhat narrower question, namely, about the chances of the old and new forms of international transactions coexisting. What will be different in the traditional FX market? To what extent is the aforementioned bridge coin adoption able to crowd out the exchange of fiat currencies and move international trade toward settlement by means of decentralized digital assets?[2] Will agents even remember that fiat currencies and the conventional forex opportunity exist if they can pay for imported goods with a digital token anywhere in the world without any FX risk?

To address these questions, I formally restate them in the simplified environment of a dynamic stochastic two-country endowment economy model with foreign trade. The innovative element is a global privately managed digital currency (digicoin) which can take on the function of bridge coin between two local fiat currencies cast in CBDC form. I examine three stages of "digicoinizing" payments for imports between the two countries: (1) when the CBDC is only used domestically (similar to the current experiments with e-CNY in China, the *Sand Dollar* in the Bahamas, the *Stimulus* token in Bermuda, etc.), whereas cross-border payments take place in the conventional fiat-to-fiat forex; (2) when the bridge coin can be used to exchange fiat currencies with the benefit of a globally transparent common price (but not for purchasing goods in the foreign country directly); (3) when foreign goods can be bought directly with the bridge coin. In order to concentrate on outcomes that are sustainable in the long term, I consider balanced-growth equilibria and construct a numerical procedure for a full-distribution solution of the model. In particular, I design a numerical method to approximate the ergodic set of state vectors in each of the long-term equilibria considered. The obtained relative positions of the ergodic sets in the state space are then examined as a form of comparative statics exercise to make qualitative inferences concerning the intensity of use of the fiat and the digital means of payment in each of the settings studied.

It turns out that, under a certain structure of (both traditional direct and the new bridge) FX frictions, all three channels of import financing can coexist, whereas a different, seemingly just as natural form of forex transaction costs would lead to the collapse of one or two of the three partial FX markets. That is, even a small change in transaction technologies may trigger a sudden switch from one means of exchange to another. At the same time, the use of fiat currency in foreign trade is unlikely to be completely eliminated. The main reason is that modern fiat money is not so much a means of cash-in-advance payment as it is a credit-supporting technology, a role that a DLT-based alternative is unable to take over (if it tries to, its operator necessarily assumes the obligations of a global commercial banker, at which instance the DeFi realm is abandoned and the digital asset enters the TradFi one). In short, a reasonable trade-off between fiat and digital is possible as long as the transaction costs in all three FX segments sufficiently resemble the frequently observable inventory-minimization pattern of today's markets. A surprising result concerning the said fiat-digital symbiosis is that, while the overall consumption level in the long term remains quite stable as the role of digicoin expands, the share of domestic consumption settled in fiat on average becomes higher instead of falling. The digicoin becomes more prominent in import spending and the bridge forex thrives, but local shopping relies on the fiat even more than before. The intra-period credit opportunity offered by the fiat money is thus important for the domestic market. Therefore, central bankers should not fear losing their jobs even in the case of unrestricted public access to the bridge coin for direct international payments. In addition, when the digicoin in the model becomes

[2] On the separate issue of the use of fiat vs. crypto as a store of value regardless of domestic or cross-border payment possibilities, see Derviz (2019).

globally acceptable in goods trade, it not only fails to crowd out the fiat forex with the said transaction cost structure, but, instead, makes it more stable.

1.1 Literature Review

The potential of cryptocurrencies to disrupt or transform the forex market has so far been barely exploited in the academic literature. At most, one finds non-technical online reviews for practitioners and interested laypeople (an example is Baldwin, 2021). The related, but separate, issue of crypto assets' ability to substitute for fiat means of exchange has been treated from the vantage point of pre-existing public vs. private means-of-exchange theories (Schilling and Uhlig, 2019a,b), but these contributions usually leave the open economy aspect of cryptocurrency adoption aside.

Together with the crystallizing CBDC concept, certain attempts have been made to embed this phenomenon in the existing theories of money and banking. One of the more advanced ones is Fernández-Villaverde et al. (2021), with their focus on CBDC-induced infraction by a central bank of the traditional maturity transformation domain of commercial banking. This paper does not explicitly discuss international aspects of public access to virtual central bank money. A tentative theory of CBDC competition with private digital tokens in providing privacy vs. efficiency in transaction services in a closed economy is proposed in Ahnert et al. (2022). Similar issues of the influence of CBDCs on financial intermediation and welfare depending on privacy, interest-bearing, and network effects are addressed in Agur et al. (2022). A game-theoretic approach to currency competition in the presence of a digital currency alternative is offered in Cong and Mayer (2022). Otherwise, the questions of CBDC interoperability and the role of CBDCs in cross-border payments have mainly been the topic of non-technical policy papers (BIS, 2021). Theoretical literature on international CBDC linkages is, at present, non-existent.

On the other hand, some work has been done on international fiat-digital money competition for liquidity services provision. A paper very similar in spirit to the present one is Benigno et al. (2022), in which the authors discuss competition between two fiat moneys and one digital currency in a dynamic two-country asset-pricing model. Actually, Benigno et al. (2022) start their analysis at the point where the present paper ends: their digital currency has a global status for all operations and renders the same liquidity services as the fiat local moneys, and the FX trades are frictionless. Not surprisingly, under these conditions, there is forced synchronization of monetary policies and the fiat exchange rate value is pinned down by the interest rate parity in the periods when all three currencies are used. As is routinely the case in such models, interest rate equalization and exchange rate parity are necessary to infer some knife-edge market-use properties: the slightest disturbance of these conditions means the closure of one of the FX markets and currency abandonment. On the contrary, the present paper does not impose any unrealistic indifference requirements on interest and exchange rates, trying instead to examine stable equilibrium currency utilization patterns under persistent, even though quantitatively moderate, frictions. In a sense, similar in spirit to our own work is the analysis of Ikeda (2022), which uses Benigno et al. (2022) as its point of departure. In that paper, exogenous frictions imposed on the use of the digital currency make the circulation of all available moneys, fiat and digital, more balanced, and also national monetary policy autonomy does not have to be lost completely.

The rest of the paper is organized as follows. Section 2 introduces the common model framework as well as the three submodels corresponding to the three gradually expanding roles of digicoin.

Section 3 constructs equilibria of the previously defined variants of the model and outlines the numerical solution method. Section 4 discusses the numerical results obtained with regard to digicoin vs. national fiat money utilization depending on the digicoin status. Section 5 concludes.

2. Model

2.1 Primitives

There are two structurally identical countries, indexed, respectively, by *h* (domestic) and *f* (foreign), with a representative agent in each. Time is discrete with an infinite horizon. Each country produces a single non-durable consumption good demanded globally. Each agent has a period utility being a function of a consumption index that is a Cobb-Douglas mix of the quantities of the two nationally produced goods. The utility is separable between periods and the time preference rate is $\beta<1$. The representative agent is actually a household of two agents H1 and H2 who act independently for the duration of each period. One member is responsible for intra-period goods storage and deliveries to the other member, who makes purchasing/resale decisions. This refers to both country *h*- and country *f*-produced goods. At the very end of the period, both household members consume jointly.

In the notation, unless detrimental to clarity, either the country origin of a variable is identified by a superscript, or the country *f*-variable is accompanied by an asterisk (in which case, the country *h*-counterpart has none).

Unless one considers simple special cases (subsection 4.1), there are two means of exchange in each country: the domestic fiat money $\mathscr{M}$ ($\mathscr{M}^*$ in country *f*) and the digital currency $\mathscr{Z}$ common to both countries. There is an FX market in which the fiat national currencies are traded. As regards the digital currency, in one, more elementary version of the model it is a purely domestic CBDC that can buy domestic goods but is not accepted in the other country. Although not particularly realistic, this version will serve for reference and comparison purposes. Whenever one allows for the two CBDCs to be interoperable, it is assumed that they become integral parts of a richer global palette of digital assets (crypto or otherwise) that agents, regardless of their country of residence, can hold in arbitrary quantities. For simplicity, this palette will be assumed to be fronted by one global token that I call digicoin. There will be two FX market segments involving $\mathscr{Z}$, one for each country, in which the digicoin is traded for the respective fiat national currency. If the digicoin is used for a two-step conversion from domestic fiat money to digicoin to foreign fiat money, or back, it is also called bridge coin. Moreover, in the last variant of the model considered, agents will be allowed to pay for both local goods with the digicoin directly.

In the sequel, I will define variables describing the operations involving $\mathscr{Z}$ in full generality even though individual versions of the model put restrictions on what agents are allowed to do with the digicoin. When a restriction is in place, the corresponding variable must be thought of as exogenously set equal to zero.

2.2 Prices

There are no explicit quantitative supply constraints: all variability and uncertainty in the production process is assumed to manifest itself in the exogenous price movements. In addition, the retail sides of goods distribution under fiat and digital settlement will be assumed to involve separate risk factors. As a result, there will be four separate supply-side risk circuits, one pertaining to the traditional and one to the digital distribution mode, in each of the two countries.

The focus of this model is on transactions, not investment and production. Accordingly, it would be beside the point to treat individual supply-side uncertainties explicitly. Instead, a wide range of such uncertainties is captured by summary statistics called prices. As mentioned earlier, there are four such random processes: P – domestic prices in fiat money, P^* – foreign prices in fiat money, P^z – domestic prices in digicoin, P^{*z} – foreign prices in digicoin. Price movements in the two distribution circuits, fiat and digital, are assumed mutually independent. Besides that, national prices within each circuit are assumed identically distributed pairwise mutually independent within each period and across periods.[3] The principal difference between the circuits will be the amplitude of price fluctuations: the fiat price volatility will be higher than the digital one. As a justification, one can think of higher transparency of digitally settled exchange of goods making abuse of market imperfections harder. Another reason may be more efficient supply-demand matching in the digital payment circuit, such as considered in Ahnert et al. (2022). Lower volatility should add some attractiveness to using digicoins in view of their otherwise significant limitations in terms of acceptance and cash-in-advance requirements. Numerically, this additional advantage leads to a more balanced $\mathcal{M}/\mathcal{Z}$-split in agents' wallets and facilitates graphical inspection of quantitative results. Otherwise, this assumption is not central and can be dropped.

We will also use three endogenous price variables: S – the price of one unit of $\mathcal{M}^*$ in $\mathcal{M}$-terms (the nominal exchange rate), Ξ – the price of $\mathcal{M}$ in $\mathcal{Z}$-terms, and Ξ^* – the price of $\mathcal{M}^*$ in $\mathcal{Z}$-terms.

2.3 Domestic and Cross-Border Goods Exchange, Supply of Means of Payment

The origination and circulation of consumer goods in the economy is modeled by splitting the representative household into two members. One, which we call H1, is responsible for setting the overall consumption levels of both domestic and foreign goods as well as deciding how much of each domestic consumption good will be financed by conventional (A^c) and how much by digital (A^d) spending. H1 also makes decisions about FX trades. Although the household as a whole typically carries over some cash balances from the previous period, in the fiat money spending case, they are insufficient to cover both A^c and import spending: the transaction technology requires higher cash balances as an input than what the household prefers to save between periods. Therefore, one resorts to intra-period $\mathcal{M}$-borrowing. (In the case of $\mathcal{Z}$, the cash-in-advance requirement is more constraining, since borrowing is impossible – see later.) The intra-period $\mathcal{M}$-loans to H1 constitute the primary money supply.

In principle, although overall consumption of each good ($A^c/P+A^d/P^z$ for the domestic good and A^f/P^* for the foreign one) is restricted to be positive by the functional form of the preferences, H1 may choose to set one of the quantities A^c, A^d to be negative (thereby partially subsidizing purchases

[3] Since the model will be solved numerically, the price distribution must be defined as a discrete approximation of the preferred theoretical density. It turns out that increasing the selected grid size has minimal qualitative consequences for the outcome (such as the positioning of ergodic sets). For the basic outline of the sought ergodic sets of the model, it is sufficient to use a very crude approximation with just three grid points (i.e., inflation in a given price circuit is either low, average, or high). The probability of average inflation is 0.5; the other two realizations have probabilities of 0.25 each. This choice leads to a substantial reduction in calculation time without jeopardizing the nature of the results. Also note that the implemented distributions refer to the de-trended and orthogonalized price residual series relevant in the balanced-growth equilibrium, justifying the absence of such effects as intertemporal and inter-circuit correlation.

in one money type with sales in another). In fact, some of the long-term equilibrium outcomes render negative A^c in multiple states of nature. It is better to think of such a possibility not as a transaction in physical goods but rather as an order in the forward market (the forward position can be both long and short). This is in line with the assumed mechanism of production and distribution of the local good.

Both the primary production and the distribution ("packaging and delivery") sectors are controlled by the other household member, H2. H2 receives orders (A^c,A^d) from the domestic H1 and similar export (from his perspective, import from the other country perspective) orders (I^{*c},I^{*d}) from the corresponding agent H*1 in the other country. The orders set in motion the production and distribution process. H2 not only processes orders, but also generates value added at a rate G>1 (for simplicity, we assume this rate to be constant, associating all the uncertainty and time variability with prices). Both generating value-added in the distribution sector and fulfilling consumption orders require transaction balances. This means that H2, in addition to cash-in-advance payments from H1 and H*1 (equal to $A^h = A^c + A^d \cdot P/P^z$ and SA^{*h}, respectively), must receive value added-covering payments $(G\text{-}1)(A^h + sA^{*h})$, needed as a transaction-technology input in the distribution sector. Let us assume that this additional cash is obtained from the financial sector (i.e., ultimately from the central bank or the digicoin issuer) in exchange for intra-period promissory notes or similar short-term non-tradable claims issued by H2, to be settled at the end of the period. If the A^c-type order is positive, H2 receives cash balances (there is a secondary money supply); if it is negative, he buys promissory notes back with cash obtained in connection with other order types, usually exports. Since G>1, H2 must repay more than what H1 redeemed, and there is a secondary monetary contraction (money withdrawal). The same holds for A^d (by construction, as will be seen later, import order A^{*h} is always positive). Since, on balance, $A^h + SA^{*h}$ must always be positive, the net flow of promissory notes at the start of the period is always from H2 to the cash provider, i.e., the aggregate secondary money supply is positive.

2.4 Budget Constraints

Let M_t be the household's fiat money holdings at the end of period t. Also, let Y^{ch} be the domestic good sold to H1 and paid for in $\mathscr{M}$ (superscript c stands for "conventional", as opposed to "digital" – see later), Y^{cf} the domestic good exported (ordered by H*1 and paid for with $\mathscr{M}$), M^z the quantity of $\mathscr{M}$ exchanged for $\mathscr{Z}$ by H1, and Q^x the quantity of $\mathscr{M}$* acquired by H1 in the fiat forex – all during the given period. Subscript t indicates the time period in which the transactions are made. Then

$$M_t = \mathrm{G}P_t\left(Y_t^{ch} + Y_t^{cf}\right) + FX_t^{tr} - (1+i)(A_t^c + S_tQ_t^x + M_t^z + H_t^x + H_t^z - M_{t-1}). \quad (1)$$

Equation (1) states that H1 decides upon three categories of the target $\mathscr{M}$-expenditure for the current period and then borrows the part of their sum that exceeds the money carried over from the previous period. This is how the primary monetary expansion takes place. H1 then exchanges goods for money with H2, whereas the latter also generates the $\mathscr{M}$-denominated value added (the secondary monetary expansion).

Here, H^x and H^z are the transaction costs paid by H1 in the two FX segments that involve $\mathscr{M}$. Households finance these costs together with the rest of the within-period spending or, put differently, pay gross amounts SQ^x+H^x and M^z+H^z in $\mathscr{M}$-units to effectuate net trades SQ^x and M^z. FX^{tr} is the revenue of the intermediary who facilitates FX transactions on the home country side (there is a similar one on the foreign country side). It is treated by the households as a lump-sum exogenous transfer. On the contrary, H1 knows the dependence of H^x and H^z on her choice of variables and takes it into account in her decision-making.

For simplicity, let $FX^{tr}=(1+i)(H^x+H^z)$, i.e., the entity collecting transaction fees within a country is assumed to be owned by the local households and to eventually turn the earned revenue-cum-interest over to them. At the same time, this intermediary is a part of the financial sector and earns the same competitive interest rate i on the funds lent to H1 for the purpose of participating in forex trades as the rest of the sector. Consequently, transaction costs do not formally appear in the monetary aggregate calculations, but shape the optimization problems and the equilibrium.

Households do not hold foreign fiat currency savings, spending all of the purchased currency on consumption of foreign goods within the same period. This is so because we assume additional costs of intra-period foreign currency storage, making this option suboptimal compared to saving in the home currency. Of course, if taken literally, this assumption is counterfactual. On the other hand, FX saving is, in essence, an investment activity from which we abstract in this paper. One can think of all investment (including in foreign currency) as productive activity taking place in the financial sector that we do not model explicitly. Its revenues are lumped together with all the other endowments managed by H2.

The budget constraint of the h-country household for $\mathcal{Z}$-balances is

$$Z_t = (1+\rho_z)Z_{t-1} + GP_t^z\left(Y_t^{dh} + Y_t^{df}\right) - A_t^d - ZC_t^* + \Xi_t M_t^z - \Xi_t^* M_t^{*z}, \qquad (2)$$

ρ_z being the rate of return on the digicoin.[4] ZC^* is the foreign good paid for by H1 in $\mathcal{Z}$ directly (in the variant of the model in which it is allowed; the corresponding value for agent H*1 in country f would be denoted Z^*C). Similarly to the fiat money case, Y^{dh} (Y^{df}) denotes the h-country good paid for in $\mathcal{Z}$ by agent H1 (H*1; the latter is non-zero only when direct import settlement in $\mathcal{Z}$ is accepted), M^z is the digicoin quantity purchased with $\mathcal{M}$, and M^{*z} is the $\mathcal{M}^*$-quantity purchased with digicoins, both by H1 (time subscripts obvious).

2.5 Cash-in-Advance Constraints for the Two Means of Exchange

So far, the operations agents conducted with both types of cash, $\mathcal{M}$ and $\mathcal{Z}$, may have seemed the same. The property that makes them different is the form of the cash-in-advance constraint. The assumption will be that intra-period credit is possible in fiat money but impossible in digicoin. Specifically, member H1 of the representative household can borrow $\mathcal{M}$ from its central bank in excess of what has been saved in the previous period and use the full amount to either order domestic consumption goods from H2, convert $\mathcal{M}$ to $\mathcal{M}^*$ in the fiat forex, or trade $\mathcal{M}$ for $\mathcal{Z}$ in the fiat/digicoin forex. The loan is repaid by the household as a whole at the end of the period, together with interest, the rate i of which is assumed deterministic and constant over time for simplicity. Repayment is possible thanks to the proceeds of the sale of domestic goods (to both domestic agent H1 and importer H*1 from the other country) received by H2 during the period. Since the value added per unit of production, G-1, is deterministic and positive, the households are identical, and all markets

[4] In theory, the sign of ρ_z is not restricted. In the calculations, we use a positive value lower than the $\mathcal{Z}$ stock growth rate, keeping in mind that our digicoin stands for many digital assets, some of which are capable of earning positive, even though highly uncertain, yields (for example, thanks to so-called yield farming).

clear in every period, there is no credit risk in aggregate: if one begins period t-1 with a positive M_{t-1}, the end-of-period money holdings will also be positive.

The cash in advance constraint for the fiat money side of the economy consists in adding to the period utility a smooth, strictly increasing, strictly concave function w of real money balances M_t/P_t, approaching minus infinity at zero and zero at infinity. Function w represents both a penalty for fiat cash balances getting imprudently small and a dwindling reward for keeping them at an unnecessarily high level. The negative infinity limit of w at zero forces M to be positive at all times.[5] However, the constraint is imposed on the end-of-period balances, whereas within the period, running a negative cash balance is allowed thanks to intra-period credit. The constraint on the digicoin side is much stricter. Essentially, everything purchased for $\mathcal{Z}$ within a period must be paid for by the digicoins available at the start of the period.

Formally, define ZI_t as $(1+\rho_z)Z_{t-1} - A_t^d - ZC_t^*$ (I meaning immediate). Budget constraint (2) can be rewritten as

$$Z_t = ZI_t + GP_t^z\left(Y_t^{dh} + Y_t^{df}\right) + \Xi_t M_t^z - \Xi_t^* M_t^{*z}. \tag{3}$$

The cash-in-advance constraint on $\mathcal{Z}$-balances is imposed by adding to the household period utility a term v qualitatively similar to w, but made a function of real immediate balances ZI/P^z. In other words, as opposed to fiat money balances, it is not enough to expect an end-of-period income that will fully cover current $\mathcal{Z}$-expenditure; one must have sufficient funds to cover the current period consumption already at the end of the previous period.[6]

We know that credit is an either non-existent or rudimentary feature in currently circulating cryptocurrencies.[7] So, if the assets represented in $\mathcal{Z}$ mainly include DLT-based instruments, the assumption seems warranted. Of course, as long as the digicoin remains on the level of a national CBDC that simply provides a digital representation of the fiat money within the country, disbursed loans can, in principle, be transferred to the borrower's CBDC account. It is entirely up to the issuing central bank to impose or not to impose any restrictions on the transfer. However, once the CBDC is made externally convertible, the rules of fiat-CBDC conversion have to be agreed upon with other participating central banks as well as with the supranational bridge coin provider if any such is involved. Put differently, one needs a globally consensual monetary policy for the digicoin. Accordingly, a CBDC-issuing central bank willing to maintain unrestricted monetary sovereignty

[5] Accordingly, the constraint we impose (both for the fiat balances here and for the digicoin, to be introduced shortly) is a smoothed-out one, making it formally look like a money-in-the-utility specification. Since, qualitatively, both forms of constraints on cash balances work in the same way (Feenstra, 1986), we use the term cash-in-advance in this broadened sense.

[6] Note that $\mathcal{Z}/\mathcal{M}$ forex transactions are excluded from the cash-in-advance constraint. This assumption, beside simplifying the calculations, turns out to be warranted by the small magnitudes of open $\mathcal{Z}$-forex positions in equilibrium. Also, trade in $\mathcal{Z}$ forex is intermediated by specialized entities (see the discussion following equation (1) above) and is intended to secure the desired $\mathcal{M}/\mathcal{Z}$ -fund mix for the period to follow, not the current one, thereby separating this trade from the current consumption spending.

[7] Credit is present in most stablecoins, which are centrally managed instruments, making proper functioning of any obligations resulting from a stablecoin-denominated loan the responsibility of the coin operator. At the same time, money creation that results from lending always involves the fiat money leg of the stablecoin, i.e., the redemption promise. This means that the stablecoin operator takes on obligations in a fiat currency (or currencies) similar to those of the conventional commercial banker who lends in fiat. In other words, credit in stablecoins is effectively credit in fiat with an extra component of credit risk (the coin issuer's solvency) and does not violate the spirit of the assumption discussed.

has no choice but to exclude its CBDC from domestic credit aggregates and only allow it as a fallback means of exchange, fully in line with our assumption.

2.6 Preferences

To ensure the existence of balanced-growth equilibria, the utility in the model will contain an element tying it to recently attained consumption levels. If, as already mentioned

$$\Omega(C_t, C_t^*) = (C_t)^\gamma (C_t^*)^{1-\gamma}$$

is the household consumption index in period *t*, we will denote by $\bar{\Omega}_{t-1}$ the average of this index across all states of nature in the previous period. The comparison utility in period *t* used here will be defined as a power utility with elasticity parameter σ of a Cobb-Douglas mix of the individual and the past aggregate consumption indices, plus the two cash-in-advance terms defined in subsection 2.5:

$$u(M_t, ZI_t, C_t, C_t^*) = \frac{1}{1-\frac{1}{\sigma}}\left((\bar{\Omega}_{t-1})^{1-\rho}\left(\Omega(C_t, C_t^*)\right)^\rho\right)^{1-\frac{1}{\sigma}} + w\left(\frac{M_t}{P_t}\right) + v\left(\frac{ZI_t}{P_t^z}\right)$$
$$= uc(C_t, C_t^*) + w\left(\frac{M_t}{P_t}\right) + v\left(\frac{ZI_t}{P_t^z}\right), \qquad (4)$$

with ρ being a positive parameter below, but close to, unity which determines the weight of the past aggregate consumption level in the current felicity. The expectations of these utilities in all future periods discounted by the appropriate power of the time preference rate β are then summed to form the usual total expected utility of future transaction means holdings and consumption streams.

2.7 Transaction Costs in the Three FX Segments

As mentioned at the outset, the whole point of introducing an internationally interoperable CBDC is to reduce the current FX trade frictions. Therefore, a model aspiring to provide joint treatment of CBDC and forex must offer a formal representation of these frictions. We shall do so by proposing a particular quantification of FX transaction costs.

Let us start with the fiat $(\mathscr{M}/\mathscr{M}^*)$ segment. The total transaction costs associated with trading FX volume Q^x are often defined as

$$H^x(Q^x) = \varepsilon^x Q^x + \varphi(Q^x), \qquad (5)$$

where the component $\varepsilon^x Q^x$ is the usual flat fee and the second, non-linear component, φ, is a strictly increasing, strictly convex function of volume with $\varphi(0)=0$. This latter term expresses the growing additional costs of transacting quantities exceeding the usual lot size. To cover both purchases and sales in the same expression, definition (5) can be extended to negative values of Q^x, in which case the flat fee is $\varepsilon^x|Q^x|$, and function φ is usually assumed symmetric around zero. (In our present set-up, including negative values of Q^x is not necessary, since the roles of *f*-currency buyers and sellers are separated: H1 buys $\mathscr{M}^*$, H*1 buys $\mathscr{M}$.)

Specification (5) can be interpreted as a crude formal capture of the pricing behavior of major forex dealer banks. The latter, prima facie, set the mid-quote in order to earn as much as possible on the anticipated customer order flow (Evans and Lyons, 2002). However, they also incur additional costs when they need to process oversized trades, and those costs are routinely charged to the order originators in the form of less attractive quotes for extra lots. Customers tend to accept the charges, given that even if they try to use an alternative, order-based trade venue (an electronic or voice broker), the cost curve as a function of order size they face there is qualitatively the same, further encumbered by the uncertainty of order execution terms inherent in brokered trades.

On the other hand, the forex structure has evolved during the last two decades. What used to be a two-tier market of customers at the mercy of dealers now contains a third tier of online intermediaries offering customers favorable quote search across dealers and trade venues as well as other sophisticated order execution amenities (technical analysis, algo trading of various degrees of complexity and other AI-tools, full or partial outsourcing of FX desk tasks, etc.; see Moore et al., 2016, or Schrimpf and Sushko, 2019). The online brokers comprising the new tier have different order flow preferences compared to traditional dealers: their objective is to process every potential sale/purchase regardless of size for which they are able to find a buyer/seller, with as small an open position on their hands at the end of the trading day as possible. Although most of them are prepared to accept some degree of risk warehousing (that is, holding a non-zero currency position for a period during which a persistent price shift is likely), the desired outcome remains so-called straight-through processing (STP), where every incoming order is matched by an opposite one (or more) that clears the order warehouse quickly. Such a broker, by means of trade fee adjustment, is likely to encourage orders contributing to approaching the anticipated aggregate flow value and to discourage those that lead the ultimate aggregate astray from that target.

To reflect the said STP proliferation, we modify the non-linear part of the transaction cost specification. Namely, the transaction costs in a market with STP-preferring (warehousing-averse) liquidity providers can be defined as

$$H^x(Q^x) = \varepsilon^x Q^x + \tau^x \phi\left(\frac{Q^x}{\tau^x} - 1\right), \tag{6}$$

with ϕ being, again, a strictly convex function with zero at the origin and τ^x being the volume of currency conversions preferred by the intermediary. This latter value reflects both the liquidity providers' expected transaction volume in the market in the given period and their preferred level of involvement (that would, in the case of them having perfect foresight, guarantee the STP ideal). This is the cost function faced by the *h*-country household (more exactly, its member H1). In this paper, we will work with quadratic function

$$\phi(q) = \frac{a^x}{2} q^2.$$

2.8 Household Optimization Problems

The state variables of the optimization problem are transaction means holdings *M* and *Z*, the evolution of which is given by finance constraints (1) and (2). The decision variables of the household in country *h* are: consumption levels *C* and *C** as well as the proportion thereof paid for in digicoin (when accepted), $\mathscr{M}^*$-purchases Q^x with $\mathscr{M}$, $\mathscr{M}$-sales M^z for $\mathscr{Z}$, and $\mathscr{M}^*$-purchases M^{*z} with $\mathscr{Z}$. Quantities M^z and M^{*z} are non-zero when the digicoin acts as a bridge currency. Equivalently, instead of *C* and its digicoin portion, one can set control variables to be A^c and A^d (recall subsection 2.3). As regards *C**, in view of the assumption of no $\mathscr{M}^*$-storage (subsection 2.4), its level is pinned down by the choice of Q^x, M^{*z}, and ZC^* (when non-zero). It may also be convenient to treat M and Z themselves as controls instead of A^c and A^d, since there is a one-to-one correspondence between (M,Z) and (A^c,A^d) for any fixed pair of past state variable values and the remaining controls. Under this last choice of control variables, the first-order optimality conditions consist of two dynamic (Euler) equations that follow from optimizing M and Z and the remaining system of intra-period algebraic equations implied by optimality of the other controls. These remaining controls are: fiat forex purchases Q^x (in all variants of the model), digicoin purchases M^x and sales M^{*z} (when $\mathcal{Z}$ acts as a bridge coin), and direct import orders in digicoin ZC^* (when $\mathcal{Z}$ is a globally accepted means of payment).

The Euler equations of the problem have the form well-known from the usual money-in-the-utility models. Let us denote by uc'_h the partial derivative of the comparison utility component uc of expression (4) with respect to domestic consumption. Then, the existence of an internal M-optimum for period t implies the first-order condition

$$-\frac{1}{P_t}uc'_h(C_t,C_t^*)+\frac{1}{P_t}w'\left(\frac{M_t}{P_t}\right)+\beta E_t\left[\frac{1}{P_{t+1}}uc'_h(C_{t+1},C_{t+1}^*)\right]=0 \tag{7}$$

and the existence of an internal Z-optimum the condition

$$-\frac{1}{P_t^z}uc'_h(C_t,C_t^*)+\frac{1}{P_t^z}v'\left(\frac{ZI_t}{P_t^z}\right)+\beta E_t\left[\frac{1}{P_{t+1}^z}uc'_h(C_{t+1},C_{t+1}^*)\right]=0. \tag{8}$$

As usual, E_t denotes expectations conditional on the information available in period t. The uncertainty in the model is associated with the four price processes defined in subsection 2.2 and the price information is assumed symmetric. Therefore, operator E_t is common to the households in both countries.

One should recall that the exact relation of C to (M,Z) and C^* to Q^x (as well as the remaining control variables, if any) depends on the selected role of digicoin, i.e., on the version of the model. The most general relationship follows from the accounting identities

$$C=\frac{A^c}{P}+\frac{A^d}{P^z},\; C^*=\frac{SQ^x}{P^*}+\frac{M^{*z}}{P^*}+\frac{ZC^*}{P^{*z}}=\frac{1}{P^*}I^c+\frac{1}{P^{*z}}I^d, \tag{9}$$

in which one should set equal to zero those components that correspond to switched-off digicoin functions. (For instance, both M^{*z} and ZC^* are zero if $\mathcal{Z}$ exists only as a local CBDC.) I^c and I^d are nominal import orders for the f-country good placed by H1 (cf. I^{*c}, I^{*d} mentioned in subsection 2.3). The relation of (A^c,A^d) to (M,Z) is then given by (1) and (2).

Still, in all considered variants of the model, the fiat forex transaction volume Q^x is present. For each set of fixed remaining states/controls, there exists an internal optimal value of Q^x characterized by the within-period first-order condition (subscript f denotes partial derivative w.r.t. foreign consumption)

$$\frac{1}{P_t^*}uc'_f(C_t,C_t^*)-\frac{S_t}{P_t}uc'_h(C_t,C_t^*)\left(1+\varepsilon^x+a^x\left(\frac{Q_t^x}{\tau^x}-1\right)\right)=0, \tag{10}$$

which is easily derived by combining (1), (9), and the intertemporal utility definition. An analogous equation holds for the foreign country household. Let us assume that both the constants ε^x and a^x and the target transaction volume of the FX intermediary are the same in country f. More exactly, τ^x is denominated in $\mathcal{M}^*$ and so, by symmetry, the corresponding value for country f intermediaries should be denominated in $\mathscr{M}$ and equal to $S\tau^x$. Then, since our definition (6) of H^x is homogeneous of degree one in (Q^x, τ^x), we have the identity

$$H^{*x}(SQ^x, S\tau^x) = SH^x(Q^x, \tau^x),$$

allowing for a perfectly symmetric definition.

It is convenient to introduce a separate variable for the marginal rate of substitution between foreign and domestic consumption:

$$MRS_h^f(t) = \frac{uc'_f(C_t, C_t^*)}{uc'_h(C_t, C_t^*)}$$

and an analogous variable $MRS *_f^h (t)$ for the foreign household. Equation (10) can then be restated as

$$S_t\left(1 + \varepsilon^x + a^x\left(\frac{Q_t^x}{\tau^x} - 1\right)\right) = \frac{P_t}{P_t^*} MRS_h^f(t), \tag{11}$$

and a similar argument applied to the f-country agent renders

$$\frac{1 + \varepsilon^x + a^x\left(\frac{Q_t^x}{\tau^x} - 1\right)}{S_t} = \frac{P_t^*}{P_t} MRS *_f^h (t). \tag{12}$$

These two equations, although not explicitly providing $\mathscr{M}^*$ demand and supply schedules in the fiat forex (the MRS terms are themselves Q^x- and S-dependent), will be useful in the later analysis of special cases.

Next, let us consider the cases where the bridge forex (exchanging $\mathscr{M}$ for $\mathscr{Z}$ and the latter for $\mathscr{M}^*$) is open. We will assume a friction structure formally analogous to the fiat forex, although with different parameters generating lower transaction costs. Specifically, assume that all transaction costs are paid in the fiat currency that enters the trade and let the intermediaries in the $\mathscr{M}/\mathscr{Z}$-segment charge their fees according to

$$H^z(M^z) = \varepsilon M^z + \frac{a}{2} S\tau\left(\frac{M^z}{S\tau} - 1\right)^2. \tag{13}$$

In the same way as in (6), τ is the targeted transaction volume (expressed in $\mathscr{M}^*$). In (13), the flat fee ε is assumed to be lower than ε^x in (6) and the quadratic term coefficient a will also be smaller than a^x, which should create an extra incentive to use the bridge forex.[8] For reasons of symmetry, the transaction cost function H^{*z} in the $\mathscr{M}^*/\mathscr{Z}$-segment will be the same, including the parameter values.

Combining (1), (2), (9), and the utility definitions for both countries, one arrives at the pair of intra-period first-order conditions at the optimal level of M^z, one from the country h- and the other from the country f-perspective:

[8] Although not conceptually pivotal, this proves to be useful in the calculations and in the visualization of the results because the transaction volume magnitudes in the three forex segments become closer to each other.

$$\frac{P_t}{P_t^z}\Xi_t - 1 - \varepsilon - a\left(\frac{M_t^z}{S\tau} - 1\right) = 0, \frac{P_t^{*z}}{P_t} MRS *_f^h (t)\left(1 - \varepsilon - a\left(\frac{M_t^z}{S\tau} - 1\right) - \Xi_t\right) = 0. \quad (14)$$

A similar equation pair can be derived for the $\mathcal{M}^*/\mathcal{Z}$-segment:

$$\frac{P_t^z}{P_t^*} MRS_h^f(t)\left(1 - \varepsilon - a\left(\frac{M_t^{*z}}{\tau} - 1\right) - \Xi_t^*\right) = 0, \frac{P_t^*}{P_t^{*z}}\Xi_t^* - 1 - \varepsilon - a\left(\frac{M_t^{*z}}{\tau} - 1\right) = 0. \quad (15)$$

We have stated the six equations (11), (12), (14), and (15) for six variables (Q^x, S, M^z, Ξ, M^{*z}, and Ξ^*) in the maximum possible generality independently of the particular digicoin status. Their exact specifications differ for different versions of the model because the chosen role of digicoin has implications for the MRS values. Individual variants of the model, including the properties of the solutions of equations (11)–(15), will be discussed in section 4.

3. Equilibrium

3.1 Policy Functions, Market Clearing, and Balanced Growth

We will be looking for an equilibrium of the described model in Markov feedback form. It is a vector of optimal policies for the domestic agent and another identically structured vector for the foreign agent that depend on the current prices and the last-period values of the four cash variables (M, M^*, Z, and Z^*). As explained in subsection 2.8, there is more than one possibility for the choice of controls to be expressed by the optimal policy functions. For definiteness, we take the policies of the country h agent to be the rules for domestic fiat-financed consumption expenditure, A^c, digicoin-financed consumption expenditure, A^d, the fiat forex transaction volume (in $\mathcal{M}^*$), Q^x, and, possibly, further quantities pertaining to the different versions of the model. Specifically, there are no more controls when the digicoin is a purely local CBDC. When the digicoin is traded against $\mathcal{M}$ and $\mathcal{M}^*$, there are policies M^z (sales of $\mathcal{M}$ for $\mathcal{Z}$) and M^{*z} (purchases of $\mathcal{M}^*$ for $\mathcal{Z}$). Finally, when $\mathcal{Z}$ is a global means of exchange, there is one more policy component ZC^* – direct $\mathcal{Z}$-spending on foreign goods. Equilibrium requires that the individually preferable policies are mutually consistent and all markets clear. For the three forex segments, this consistency is captured by equation pairs (11), (12) and (14), (15), in which the price and the transacted quantity variables are common. For output/endowment quantities appearing in (1) and (2), market clearing in the two consumption goods segments means that, in country h, $PY^{ch}=A^c$, $PY^{cf}=I^{*c}=SQ^x+M^z$, $P^zY^{dh}=A^d$, $P^zY^{df}=Z^*C$, and the same is true on the f-country side: $P^*Y^{*ch}=I^c= Q^x+M^{*z}$, $P^*Y^{*cf}=A^{*c}$, $P^{*z}Y^{*dh}=ZC^*$, $P^{*z}Y^{*df}=A^{*d}$. Note, in line with the discussion in subsection 2.3, that Y^{ch} (Y^{*cf}) is allowed to be negative, whereas the export volume sold for fiat, Y^{cf} (Y^{*ch}), is always positive and, naturally, the total orders for the local good settled in the fiat currency $Y^c=Y^{ch}+Y^{cf}$ ($Y^c=Y^{*ch}+Y^{*cf}$) are positive as well. The same holds for goods purchased with digicoins.

The model allows for a balanced-growth equilibrium. That is, the equilibrium states and controls of all agents grow asymptotically at mutually consistent constant rates. Those rates in the present model are those of the means of exchange variables M, M^*, Z, and Z^* (the same rate G_m for all four; this is also the growth rate of the remaining controls as defined at the start of subsection 2.8) and the price variables (again, the same rate G_p for all four). Real fundamentals (output and consumption levels) then grow at the common rate equal to G_m/G_p. As usual in growth models with money, the time preference rate must be sufficiently low so that agents do not have an incentive to hoard cash or save excessively: $1+i<G_m<1/\beta$. Also, the gross output growth rate G must be higher than the gross interest rate $G_i= 1+i$ and the gross digicoin return $1+\rho_z$.

After ensuring that the exogenous trends are consistent with a balanced-growth equilibrium, one can deflate the states and controls, restating the transition and the Euler equations in stationary terms. It is also natural to assume the same growth trend G_m for target transaction volumes τ^x and τ. Since all prices grow at the same average rate, price ratios are stationary, and so are exchange rates S, Ξ, and Ξ^*. This fact easily follows from (14) and (15).

To allow for a straightforward derivation of a balanced-growth equilibrium, the model has been designed as homogeneous of degree one in states and controls. However, such homogeneity implies indeterminacy of the equilibrium consumption (or any other equivalent variable) scale. The indeterminacy has been removed by pinning down aggregate local consumption in both countries in the state of the world corresponding to the mid-values of all the price variables in such a way that, in this state, the selected consumption levels are subjectively optimal.

3.2 Symmetry and Ergodicity

By assuming perfect symmetry of the two countries, one can reduce the number of policy functions sought. Put $N=(M,Z)$, $N^*=(M^*,Z^*)$, $p=(P,P^*,P^z,P^{*z})$. For such p, p^{sw} means (P^*,P,P^{*z},P^z). The pair of (vector-valued) state transition equations for countries h and f under policies J and J^* will be symbolically written as

$$N= \mathcal{T}(p,N_0,N^*_0,J,J^*),\ N^*= \mathcal{T}^*(p,N_0,N^*_0,J,J^*).$$

Also, let $\boldsymbol{J}(p,N_0,N^*_0)$ be the equilibrium policy vector in country h and $\boldsymbol{J}^*(p, N_0,N^*_0)$ the corresponding policy vector in country f. State transition equations under equilibrium policies will be shortened to

$$N=\boldsymbol{T}(p,N_0,N^*_0),\ N^*=\boldsymbol{T}^*(p,N_0,N^*_0).$$

Symmetry means that

$$\mathcal{T}^*(p,N_0,N^*_0,J,J^*)= \mathcal{T}(p^{sw},N^*_0,N_0,J^*,J),$$

$$\boldsymbol{J}^*(p, N_0,N^*_0)= \boldsymbol{J}(p^{sw},N^*_0,N_0),$$

$$\boldsymbol{T}^*(p,N_0,N^*_0)= \boldsymbol{T}(p^{sw},N^*_0,N_0).$$

Even after exploiting the symmetry assumption, we have to deal with vector-valued optimal policies that are functions of eight variables. One way to proceed would be to find an appropriate parsimonious representation of eight-argument functions on the defined state space (for instance, via neural networks) and then apply a suitable numerical procedure to solve both the Euler equations (7), (8) and the intra-period algebraic maximization equations (11), (12), (14), (15). Although available (see, for example, Andreasen, 2018, Mendoza, 2020, Derviz, 2020), with a growing state-space dimension such methods produce solutions in a form that would be difficult to visualize or attach intuitive meanings to. One would still need to propose a way to inspect the formal numerical results from an economically relevant angle, which is a separate non-trivial task. A much more practical approach, to be presented in this paper, is to select a few characteristics of the solution that persist in the long run and look at the corresponding summary statistics.

When restated in de-trended variable terms, the system of difference equations (1), (2), (7), (8) for the four state variables (after the solutions to the intra-period optimality conditions have been substituted into them) must possess an ergodic set. An approximate idea of the position of this set in the state space can be obtained by means of the following heuristic observation. Recall the equivalence of controls and next-period states (subsection 2.8). If, in addition, these state variables have been deflated by their growth rates to become stationary, their optimality must be closely related to state preservation. Intuitively, mapping $\boldsymbol{T}$ is a result of applying optimal policies. However, state vectors obtained from optimal policies are themselves optimal, so that $\boldsymbol{T}$ should conserve them for every given p. Accordingly, for each p, one should expect an optimal state vector to also be a fixed point of map $\boldsymbol{T}(p, .)$.

More exactly, take an arbitrary pair $Q_0=(N_0,N^*_0)$. Define a sequence of state space points iteratively by the rule $Q_{k+1}=(\boldsymbol{T}(p,Q_k), \boldsymbol{T^*}(p,Q_k))$, $k=0,1,2,\ldots$. State transition equations in de-trended variables are mean-reverting, which means that the sequence $\{Q_k\}_{k\geq 0}$ must possess at least one limit point. The latter has two properties important for the analysis at hand: it must be a fixed point of $\boldsymbol{T}(p, .)$ and belong to the ergodic set of the dynamic system defined by our model specification. We shall call the collection of such limit points spanned by a representative set of p-values *the ergodic set anchor points*, or simply (ergodic) anchor points.

Our symbolic notation $\boldsymbol{T}$ should not obfuscate the fact that the ergodic anchor points cannot be calculated by simply searching for fixed points (e.g., by iteration). The difficulty is that the definition of $\boldsymbol{T}$ involves optimality conditions (Euler equations) that still have to be analyzed explicitly. Nevertheless, due to the linearity of the state-transition equations in the policy variables at the anchor points (even though not elsewhere!), the policies that conserve the state variables (M and Z) are known linear combinations of the latter. After substituting them into the Euler equations, one obtains an equation system for the states that can be solved much more easily than the original (state transition plus Euler) dynamic system. The solution provides an anchor point characterization for every chosen p.

If one manages to calculate the anchor points for a sufficiently representative set of price patterns, one gains a fair idea of the ergodic set as a whole. Qualitatively, in the long run the dynamic system under consideration will be randomly jumping inside some neighborhood of the ergodic set depending on the realizations of random vector p. At each jump, the dynamic system will be following the general direction of the anchor point associated with the current p-value. Naturally, the system will never place itself exactly at any of the anchor points, all we know is that it oscillates inside the set spanned by them. Therefore, the anchor point properties should not be interpreted literally as predictions of the model behavior, only as indicators of likely tendencies in that behavior, particularly to make comparisons of different versions of the model studied in parallel.

In the next section, I specify individual sub-models that differ in their digicoin status. We will inspect both some general properties of their equilibria and some quantitative features of their ergodic set anchor points.

4. Expanding the Opportunities of Digicoin

4.1 Prequel: Fiat-Only and Digicoin-Only Economies

I start the discussion with a pair of relatively simple cases. One is an economy without any digicoins, where only two fiat national currencies exist. The other is the exact opposite: there is just one common digicoin available for transactions in both countries. Obviously, in the first case, only random prices P, P^* in the conventional distribution circuit apply, whereas in the second, the only relevant price risks are P^z and P^{*z}.

Let us start with the $\mathscr{M}/\mathscr{M}^*$-only model. Apparently, $A^c = PY^{ch} = PC$. (To keep track of the gradually growing sophistication of the model versions discussed, we still write A^c for domestic goods orders by H1 and Y^c for domestic wholesale production overseen by H2, even though there is no A^d or Y^d here, in the same way as there is no A^c or Y^c in the $\mathscr{Z}$-only model.) Further, $P^*C^* = Q^x$. The only FX market in this model is the one for fiat currencies, and we shall now examine the special cases of (11) and (12) characterizing it.

Regardless of the version of the model considered, under the utility function definition (4), MRS has a fairly simple form

$$MRS_h^f = \frac{1-\gamma}{\gamma}\frac{C}{C^*}. \tag{16}$$

Therefore, (11) becomes

$$S_t\left(1 + \varepsilon^x + a^x\left(\frac{Q_t^x}{\tau^x} - 1\right)\right) = \frac{1-\gamma}{\gamma}\frac{A_t^c}{Q_t^x}. \tag{17}$$

Analogously, (12) is reduced to

$$1 + \varepsilon^x + a^x\left(\frac{Q_t^x}{\tau^x} - 1\right) = \frac{1-\gamma}{\gamma}\frac{A_t^{*c}}{Q_t^x}. \tag{18}$$

One immediate consequence is the equilibrium nominal exchange rate as the ratio of the nominal domestic consumption spending levels in the two countries:

$$S_t = \frac{A_t^c}{A_t^{*c}}. \tag{19}$$

This is true in all periods regardless of the FX transaction volume Q^x. The latter, as follows from (18), can be considered a function of A^{*c} and τ^x.

Now let us look at the model behavior at the anchor points. Since, in the balanced-growth equilibrium, M^*, A^{*c}, τ^x, and Q^x grow asymptotically at rate G_m, it is useful to deflate the (special case of) transition equation (1) for country f . To keep the notation simple, we will use the same symbols for normalized variables. After taking into account market clearing, the evolution of the $\mathscr{M}^*$-stock can be written as

$$M_t^* = \frac{G_i}{G_m}M_{t-1}^* + (G - G_i)(A_t^* + Q_t^x). \tag{20}$$

Put δ_m=1-G_i/G_m. In view of the assumptions made in section 3, δ_m is a small positive number. Next, denote by λ_m the ratio of δ_m to G-G_i. Recall that at the ergodic set anchor points, optimal policies conserve the money stocks. Therefore, in the new notation, (20) is reduced to the following relation between the money stock and the policies at every anchor point:

$$\lambda_m M^* = A^{*c} + Q^x. \qquad (21)$$

Together with (18), (21) demonstrates that, at the anchor points, given the target FX transaction volume τ^x, the money stock is in a one-to-one correspondence with the nominal domestic absorption. The exact level of either of them for given price pattern p follows from the Euler equation. The fundamental values (money stocks and absorption levels) for country h follow by symmetry.

This version of the model, similarly to a couple of others to be discussed below, allows for multiple equilibria associated with different choices of τ^x. To narrow down the ambiguity, one can pick some ex ante-plausible assumption with regard to τ^x and then try an empirical check. One such assumption is suggested by equation (18). Put

$$y^x = \varepsilon^x + a^x\left(\frac{Q_t^x}{\tau^x} - 1\right)$$

and rearrange (18) in the form

$$\frac{Q^x}{A^{*c}} = \frac{1-\gamma}{\gamma}\frac{1}{1+y^x}$$

(time superscripts dropped for simplicity). Essentially, this equation says that the ratio of imports to domestic absorption must be equal to the ratio of foreign to domestic goods shares in the consumption index times a positive number close to unity, dictated by the FX market frictions. The quantity on the left-hand side is observable and can be derived from the usual macroeconomic statistics, while the right-hand side pins down the ratio of Q^x to τ^x. The benchmark choice seems to be y^x=ε^x, corresponding to the case of perfect foresight of the forex intermediary, when $Q^x = \tau^x$. Other choices compatible with data should have quantitatively similar implications. Therefore, in this paper, we show the ergodic set anchor points in the (M,M^*) space computed under the said perfect foresight assumption.

Figure A1 shows nine ergodic anchor points in the state space, corresponding to three consumer price inflation realizations in each country: low, medium, and high. The ergodic set itself should be close to the convex hull of these points (not exactly the hull itself, because the trajectories of state-transition equations are not perfectly linear, even though they have low curvature in absolute terms). Another important illustration of the model behavior is the collection of exchange rate values at the anchor points. Figure A2 compares the exchange rate given by (19) with the purchasing power parity-suggested diagonal (solid line). Apparently, the exchange rate is more volatile (has a stronger reaction to changes in the inflation differential) than what PPP would dictate, for reasons related to the forex frictions.

Next, consider a two-country world with a common digital currency. This time, market clearing means $P^zC=A^d=P^zY^{dh}$ and $P^{*z}C^*=ZC^*$. There are no FX markets. The evolution of the Ƶ-balances of the country h household follows the simplified law of motion

$$Z_t = (1+\rho_z)Z_{t-1} + (G-1)A_t^d - ZC_t^* + GZ^*C_t. \tag{22}$$

ZI_t is still given by $(1+\rho_z)Z_{t-1} - A_t^d - ZC_t^*$. Whereas A^d-optimization results in the usual Euler equation already stated in (8), the optimal choice of imports ZC^* implies the intra-period first-order condition

$$\frac{P_t^z}{P_t^{*z}} MRS_h^f(t) = 1. \tag{23}$$

In view of (16), the optimal import expenditure is exactly the fixed proportion of A^d given by

$$ZC_t^* = \frac{1-\gamma}{\gamma} A_t^d, \tag{24}$$

and an analogous formula holds for country f imports. Indeed, in this two-country world, there is no national currency exchange and hence no FX frictions either. This allows one to rewrite (22) in terms of domestic absorption volumes in the two countries. Next, one should conduct variable detrending with factor G_m in the same way as in the previous fiat money model. Analogously to the fiat money case, put $\delta_z=1-(1+\rho_z)/G_m$ and $\lambda_z=\delta_z/(G-1)$. Again, similarly to the case of (21), keeping the same notation for de-trended states and controls as the original ones for the sake of notational simplicity, one can derive the following relation between the Ƶ-stock at the anchor points and the local absorption policies in both countries that preserve those stocks:

$$\lambda_z Z = -\left(\frac{1}{G-1}\frac{1-\gamma}{\gamma} - 1\right) A^d + \frac{G}{G-1}\frac{1-\gamma}{\gamma} A^{*d}. \tag{25}$$

The Euler equations (8) for both countries, together with (25) and its analogue for country f, render the levels of Ƶ-stocks in countries h and f at the ergodic anchor points for all price patterns.

Figure A3 shows these anchor points on the same graph, together with the previously calculated anchor points of the pure fiat money case. The most salient difference between the two is the magnitude of typical transaction balances: agents need more means of exchange when intra-period credit is impossible. This is a feature that will reappear in subsequent versions of the model. On the contrary, consumption patterns do not exhibit such dramatic differences (Figure A4). If anything, the use of common digicoin seems to dampen somewhat the cross-country imbalances related to asynchronous inflation.[9]

4.2 Digicoin as a CBDC for Local Use

This version of the model essentially leaves only one reason for using digicoin: exploitation of current price differences in the two distribution circuits within a country, as well as hedging against

[9] To avoid a simplistic interpretation of this and the other graphs, one should keep in mind that the anchor points shown there, as well as the ergodic sets spanning them, are just attractors of the corresponding dynamic system, i.e., although the state vectors are driven toward them by the equilibrium laws of motion, the system is positioned exactly in them with zero probability. Each new shock moves the system to a new position from which the subsequent movement is directed toward a different attractor defined by the current price shock realizations.

future such differences. Prima facie, the formal treatment of the model does not differ much from the fiat money case of the previous subsection except for the additional component in domestic consumption covered by the domestic CBDC. Starting with the evolution of the $\mathcal{M}$-holdings of the agent in country h , after de-trending with the same money growth rate G_m as before, we observe (keeping the same state and control names for the deflated variables) that it can be reduced to

$$M_t = (1 - \delta_m)M_{t-1} + (G - G_i)(A_t^c + S_t Q_t^x),$$

which is formally the same as (20), but for country h instead of f. The analogue of (21) at the ergodic anchor points,

$$\lambda_m M = A^c + SQ^x, \tag{26}$$

is also valid. Further, since the digicoin in this version is not involved in foreign trade transactions, the $\mathcal{M}^*$-value of country h exports (and country f imports) is still equal to Q^x. Equations similar to (17) and (18) hold for the (fiat) FX market, but with domestic goods consumption spending settled in fiat, A^c, replaced by overall domestic spending, $A=A^c+PA^d/P^z$. For this reason, the nominal exchange rate expression analogous to (19) holds as well:

$$S = \frac{A}{A^*}\,. \tag{27}$$

The evolution of the $\mathcal{Z}$-holdings of country h households follows an even simpler rule than (22):

$$Z_t = (1 + \rho_z)Z_{t-1} + (G - 1)A_t^d. \tag{28}$$

At the ergodic anchor points, (28) is reduced to

$$\lambda_z Z = A^d. \tag{29}$$

Purely optically, as opposed to rule (25) in the previous subsection, digitally settled consumption is now linearly related to Z without any formal dependence on the other country.

In spite of all the superficial similarity with the fiat-only case, the equilibria of the two models are quite different in many respects. First of all, four price risks must now be taken into account instead of two. Second, no matter how rudimentary the role of $\mathcal{Z}$ seems to be, the presence of an additional consumption financing source has far-reaching quantitative implications.

As before, calculating $\mathcal{M}$- and $\mathcal{Z}$-holdings at the ergodic anchor points requires substituting (26), (27), and (29) into the Euler equations and solving them numerically. Setting the FX target volume parameter τ^x proceeds as before, based on the same perfect foresight assumption.

Figure A5 shows the split between local goods orders denominated in the fiat currency and the local CBDC. As already mentioned in subsections 2.3 and 3.1, the short positions in goods ordered for fiat and the long positions in those ordered in digicoin are agent H1's perfectly legitimate choices. In this version of the model, this combination of trade signs occurs in all states of nature. The short positions in $\mathcal{M}$ are more than compensated by long ones in $\mathcal{Z}$, so total consumption is positive. The overall local goods orders in fiat also remain positive, since the negative A^c values are always lower in absolute terms than the positive nominal exports. The values of the latter at the ergodic anchor points are shown in Figure A6 for all three models discussed so far. On average, foreign goods orders (which are either exactly or approximately proportional to overall domestic spending) do not show any visible shift in either direction when one goes from one model to another. Instead, one sees substantial growth in variability. The dispersion of the foreign trade patterns is lowest for the digicoin economy and grows somewhat when one goes over to the fiat money case, whereas the ergodic set expands multiple times when a pair of domestic CBDCs is injected into the two-country fiat money world.

4.3 Digicoin as an FX-Bridge

This set-up, although not the most general formally (the ability of digicoin to exchange directly for imports is switched off), is the most demanding technically, as will become clear in subsection 4.4. The evolution of the $\mathcal{M}$-stock follows (1) verbatim. It is convenient to restate it after taking into account market clearing and subsequent de-trending, while, in the same way as in subsections 4.1 and 4.2, keeping the original variable names for notational simplicity:

$$M_t = (1 - \delta_m)M_{t-1} + (G - G_i)(A_t^c + S_t Q_t^x + M_t^z). \tag{30}$$

Indeed, since agent H*1 from country *f* spends the whole quantity of $\mathcal{M}$ acquired within the current period on imports of *h* goods, the import order size in $\mathcal{M}$ terms is exactly SQ^x+M^z (recall from the discussion in subsection 3.1 that M^z is the quantity of $\mathcal{M}$ both sold for $\mathcal{Z}$ by H1 and purchased with $\mathcal{Z}$ by H*1). The source of H*1's $\mathcal{M}$-balances is twofold: the fiat cash forex and the $\mathcal{M}/\mathcal{Z}$-forex.

An analogous procedure for deriving from (2) the equilibrium de-trended evolution law for *Z* leads to

$$Z_t = (1 - \delta_z)Z_{t-1} + (G - 1)A_t^d + \Xi_t M_t^z - \Xi_t^* M_t^{*z}. \tag{31}$$

The three forex segments present in this set-up are characterized by equations (11), (12), (14), and (15), with the MRS expressions corresponding to the available consumption financing regimes. Taking into account (16), one obtains:

$$\frac{P}{P^*} MRS_h^f = \frac{1-\gamma}{\gamma} \frac{A^c + \frac{P}{P^z} A^d}{Q^x + M^{*z}},\ \frac{P^*}{P} MRS *_f^h = \frac{1-\gamma}{\gamma} \frac{A^{*c} + \frac{P^*}{P^{*z}} A^{*d}}{SQ^x + M^z}.$$

This can be regarded as a system of six equations for three prices, *S*, Ξ, and Ξ*, and three quantities, Q^x, M^z, and M^{*z}. The system can be solved numerically for a wide range of values of four parameters, of which two (τ^x and τ) characterize forex trade and the other two are composite policy functions that provide the values of total local goods expenditure in fiat money units: $A=A^c+PA^d/P^z$ and $A^*=A^{*c}+P^*A^{*d}/P^{*z}$. They are functions of price patterns and cash holdings, knowledge of which requires computation of the full dynamic equilibrium. There is no closed-form solution for the nominal exchange rate similar to (19) or (27) in this case as long as one considers general initial states.

In the same way as in the previously discussed two models that involved transaction costs of FX trades, there is an equilibrium multiplicity associated with target transaction volumes τ^x and τ set by intermediaries. (Recall that we understand these variables as a kind of expected long-term average of the envisaged transactions that the intermediaries try to rationally assess.) In the present case, this indeterminacy can be decomposed along two dimensions. One, as before, corresponds to the ratio of overall long-term imports $S\tau^x+\tau$ to domestic consumption spending. This one we are able to remove by positing the same empirically plausible imported/local consumption ratio as in the previous cases. The other refers to a continuum of possibilities for splitting long-term import spending between the fiat and the bridge forex channels. Fortunately, in the hypothetical long-term equilibrium, in which perfect foresight implies $Q^x=\tau^x$, $M^z=S\tau$, and $M^{*z}=\tau$, the equation system (11), (12), (14), (15) degenerates to one in which prices (i.e., exchange rates) are simply given by the usual laws of one price

$$S=\frac{PP^{*z}}{P^*P^z},\ \Xi=\frac{P^z}{P},\ \Xi^*=\frac{P^{*z}}{P^*} \tag{32}$$

independently of the absorption policies. Under these exchange rates, market clearing implies that monetary aggregates are independent of the share of the target transaction volume τ in total imports. Therefore, as long as we investigate ergodic anchor points, the exact split of import financing between forex segments does not matter. We can choose an arbitrary value of the ratio $\tau/(S\tau^x+\tau)$, e.g., 0.5.

This property does not hold in general, not even on the ergodic set outside the anchor points, which means that the choice of targeted order volumes by the FX intermediaries influences the exact adjustment dynamics. In this regard, arbitrariness in the choice of τ^x and τ gives rise to multiple equilibrium *trajectories*, but does not affect the uniqueness of the current *direction* of evolution of the system. The latter is unique for every price pattern realization.

Equations (30) valid at the anchor points to some extent facilitate the calculation of the position of the ergodic set. Absorption policies that preserve the money quantities for a fixed price pattern are also easily calculated linear functions of (M,M^*,Z,Z^*) that invert (30), (31), and the two corresponding equations for country f, under the restriction of constant moneys. This inversion is, as mentioned above, independent of the chosen equilibrium $\tau/(S\tau^x+\tau)$ ratio. Nonetheless, calculating the money values at the anchor points that satisfy the Euler equations is a separate numerical exercise which we do not discuss here for space reasons. The outcomes of these calculations will be discussed in the next subsection jointly with the ones for the last version of the model.

4.4 Fully Global Digicoins

The state-transition equations in the universally acceptable digicoin case are exactly as written down in (1) and (2). The Euler equations are also formally the same as in (7) and (8). Compared to the model of subsection 4.3, the representative agent in each country has one more decision variable, the quantity of imports ordered directly in $\mathcal{Z}$. Optimization along this dimension is intra-period and the (necessary and sufficient) F.O.C. looks like (23) for country h, or

$$\frac{P_t^z}{P_t^{*z}} MRS *_h^f(t)=1 \tag{33}$$

for country f. In other words, direct goods arbitrage in the digital payment circuit results in equalization of the MRS values in the two countries, which must both coincide with the relative (digicoin) price of the foreign good, a textbook international trade theory result.

However, there is only one outcome in the three forex markets compatible with (23), (33):

$$\varepsilon^x + a^x\left(\frac{Q_t^x}{\tau^x} - 1\right) = 0, \varepsilon + a\left(\frac{M_t^z}{S\tau} - 1\right) = 0, \varepsilon + a\left(\frac{M_t^{*z}}{\tau} - 1\right) = 0 \tag{34}$$

for trade volumes and (32) for exchange rates. Note that in the global retail digicoin case, the laws of one price (32) hold at all times and in all states, not just at ergodic anchor points. At the same time, these exchange rates support transaction volumes that are always lower than the targets of intermediaries: Q^x=(1-ε^x/a^x)τ^x, M^z=(1-ε/a)$S\tau$, M^{*z}=(1-ε/a)τ. Also note that, were the transaction costs given by (5) instead of (6), no analogue of (34) would be possible, there would be no solution to the system of the type (11)–(15), and one of the FX channels, either the direct fiat or the bridge one, would close down, depending on the relative burden of transaction costs.

With the intra-period optimization conditions resolved, we can go over to the dynamic part of the equilibrium determination. Compared to the bridge FX model of subsection 4.3, there is an additional equilibrium indeterminacy dimension with implications for the system's behavior both at and away from the ergodic anchor points. The overall digicoin utilization is a choice, hence endogenous, variable (M^z for country *h* and M^{*z} for country *f*, in the notation of subsection 2.4). However, the absence of conventional-against-digital-arbitrage in the goods markets in the present case makes the agents indifferent between direct import order placement in $\mathcal{Z}$ and conversion to $\mathcal{M}^*$ with subsequent placement of fiat-denominated orders. The pure laws of one price (32) are universally valid when the digicoin is usable without restrictions (in the present case), but invalid away from ergodic anchor points when direct digicoin payments abroad are impossible (the models of subsections 4.2 and 4.3). In the present case, there are three possibilities for financing imports: through the fiat forex, through the bridge forex, and direct digicoin payments. Whereas the ambiguity in the relative weights of $S\tau^x$ and τ does not affect the money-conserving absorption policies for the same reasons as in the bridge coin model, the relative weight of direct $\mathcal{Z}$-payments for imports has a (moderate) quantitative effect. Since, in this exercise, we do not explicitly model any technological or political economy factors that decide the extent of direct digital currency use in foreign trade, there will be one ad hoc assumption that fixes the said digicoin-denominated order share at 30 per cent, the rest being split equally between the fiat and the bridge forex segments.

With this restriction on direct digicoin participation in mind, we proceed to examining the model behavior at the ergodic anchor points. Numerical results are shown on the same graphs for the models of subsections 4.2 and 4.3 and the present one.

Figure A7 depicts the anchor points for (*M*,*Z*) pairs in the state space. Apparently, the bridge coin case generates the most dispersion, i.e., the largest ergodic set, followed by the global digicoin case. Not surprisingly, in the local CBDC case, digicoin holdings are less sensitive to shocks on the fiat side of the economy, mainly adjusting to the current $\mathcal{Z}$-price pattern. Also understandable is the highest sensitivity to the fiat price patterns in the bridge coin model: the bridge FX trades are used there to hedge the fiat foreign price risks, whereas in the global digicoin model, agents have an additional opportunity to relocate consumption orders directly to the digicoin payment circuit, which requires a smaller $\mathcal{Z}$-holdings adjustment.

Consumption orders themselves are the subject of Figure A8, which is an extension of Figure A5 to the two further models with expanding digicoin significance. One recognizes the purely short domestic goods orders in the fiat payment circuit of the local CBDC case, contrasted with a more balanced division between short and long orders, depending on the price pattern, of the other two models with greater "digital freedom." Note the general direction of the order split evolution in line with the growing role of digicoin: whereas with digicoin granted the bridge FX function, the weight of short fiat orders is still substantial, in the global digicoin case short and long orders seem to be represented almost equally. At the same time, the magnitude of digital orders diminishes slightly as the opportunities of digicoin expand.

4.5 General Results and Common Messages

Comparing the long-term (ergodic) behavior of the various models defined in the previous four subsections, one notices that the presence of "functions-gaining" digicoins does not mean a simple interpolation between the pure fiat and the pure digicoin worlds. Only the main distinctive feature of the digicoin, that is, the absence of intra-period credit, is prominent in all the cases considered. The ergodic levels of $\mathcal{Z}$ needed to support consumption under the cash-in-advance constraint markedly exceed those of $\mathcal{M}$. In other aspects, the parallel existence of the two means of exchange causes many qualitative differences compared to one-currency economies.

For example, the ergodic sets are larger in the presence of the digicoin, with the largest, against a naive expectation, being the one of the bridge coin model. Apparently, once a second means of exchange is introduced, partial restrictions on its cross-border utilization create a less stable environment than complete liberalization. The stable benefits of frictionless credit in fiat guarantee that this medium is demanded even in states of nature that favor a temporary surge in digicoin use (e.g., for digital payment circuit price advantage reasons).

The same factor (fiat credit availability) is mirrored in the domestic absorption patterns as well. In Figure A8, it is evident that, at least in the neighborhood of the ergodic set, digital local consumption orders are uniformly higher when digicoin is restricted to the domestic CBDC function than when these central bank liabilities become interoperable and enter global markets. Clearly, as long as there are no other uses for the digicoin, it will be spent on domestic goods in full, to the detriment of the domestic fiat money. The bridge coin model entails more moderation with regard to digital vs. fiat utilization in the local markets, since the bridge forex attracts a certain portion of $\mathcal{Z}$-balances formerly used at home. The global digicoin case means a further shift back toward fiat orders, at least on a part of the ergodic set.

Still, short domestic fiat money-funded consumption orders are frequent in all the cases considered, being most pronounced under heavy digicoin use restrictions. In those states of nature that induce such decisions, agents buy domestic consumption goods with $\mathcal{Z}$ while at the same time short-selling a part of them for $\mathcal{M}$. These sales are, of course, dwarfed by much larger volumes of exports. This behavioral pattern suggests an analogy (admittedly crude and superficial) with what was seen in the Chinese economy during its export boom years: most of industrial production was exported (for fiat yuans or dollars), whereas domestic consumption goods were paid for to a greater and greater extent out of private electronic wallets (WeChat, Alipay) – the precursors and catalysts of the CBDC launched by the People's Bank of China (e-CNY).

Finally, we look at the real exchange rate values at the anchor points, analogously to the outcomes shown in Figure A2. Figure A9 compares the actual exchange rate realizations with the PPP-implied values (depicted as a diagonal on the graph). Since the dependence of the exchange rate on prices at the anchor points is the same for the bridge coin and the global digicoin models, they cannot be distinguished on the graph. Qualitatively, the original observation made in the fiat-only case (Figure A2), namely, that forex frictions result in overreaction of the exchange rate to relative price movements compared to the PPP benchmark, is confirmed. In addition, the full solution of the local CBDC case allows us to assess the extent of this overreaction depending on the international role of the digicoin. It turns out that the highest overreaction takes place when the digicoin is purely domestic, whereas the two models in which the national digicoins are globally interoperable generate a more dampened response to relative price movements. In addition, one should expect more exchange rate volatility outside the ergodic anchor points in the bridge coin case than in the global retail digicoin model because, in the latter, goods market arbitrage restricts the exchange rate movements in all states of nature.

5. Conclusion

This paper introduces a family of national CBDC models that gradually gain functions related to international interoperability. Starting with a purely domestic CBDC, we proceed to examine a global digital currency freely exchangeable for fiat ones, thereby creating competition to traditional fiat forex, with the global digital currency (digicoin) taking on the function of bridge coin between two local fiat currencies. Finally, we allow the digital currency to be used in direct payments (for goods to be imported) anywhere in the world. All the models are embedded in a simplified environment of a two-country, two-good, two-currency, dynamic stochastic endowment economy. The full-distribution numerical solution of the model (an approximation of the ergodic equilibrium subspace of the state space) allows one to investigate the consequences of expanding bridge coin functionalities for demand for fiat moneys and the survival of fiat FX transactions in connection with foreign trade.

We analyze full-distribution solutions of these models by concentrating on the relative positions of their ergodic sets in the state space. For this purpose, we develop an algorithm for spanning an ergodic set with a subset of special "anchor points" and calculating state and policy variable values at these points. This procedure provides a parsimonious qualitative comparative method of analysis of the long-term use tendencies for fiat and digital payments depending on the functions entrusted to the digicoin.

It is shown that forex intermediaries, whose earnings in the model mimic the currently widespread online brokerage services with a preference for straight-through processing, are able to support all three coexisting FX segments (direct exchange between fiat currencies and two-stage exchange through the digicoin), regardless of the presence of direct digicoin purchases of foreign goods. Moreover, demand for fiat currencies in domestic markets happens to be revived, instead of killed off, by the expanding status of the digicoin. In addition, the digicoin "gain of function" process contributes to a higher degree of stability of both foreign trade flows and the exchange rate.

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Appendix

Figure A1: Money Stock Ergodic Anchor Points, Pure Fiat Money Model

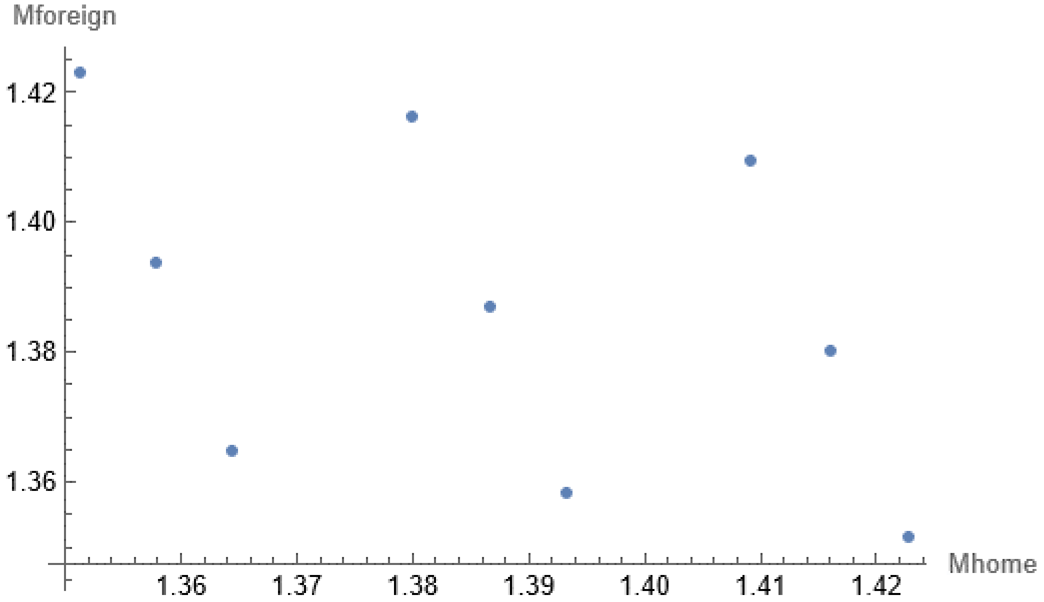


Figure A2: Actual and PPP-Implied Exchange Rate, Pure Fiat Money Model

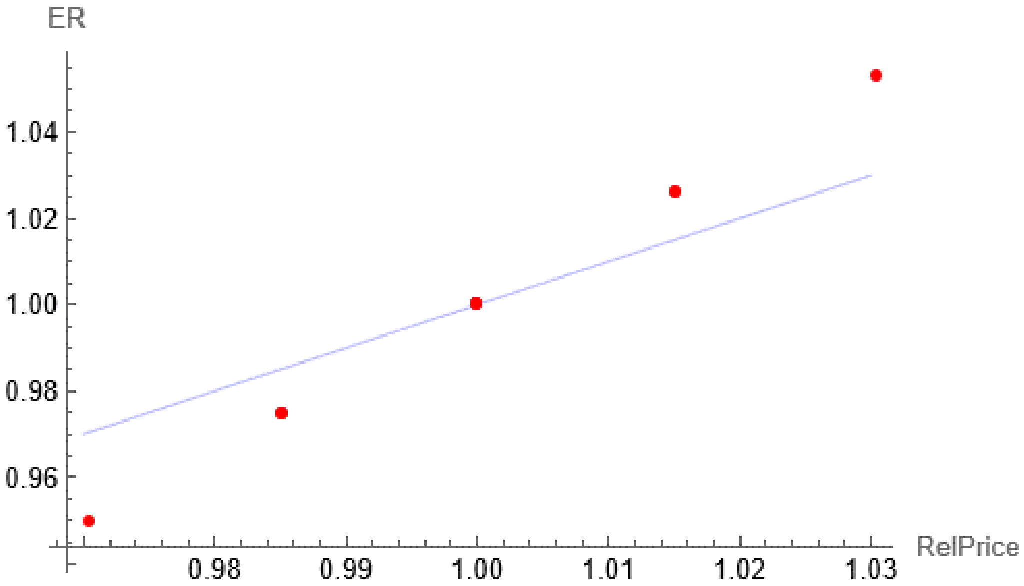


Figure A3: Ergodic Anchor Points for Cash Stocks in the Pure Fiat and the Pure Digicoin Economy

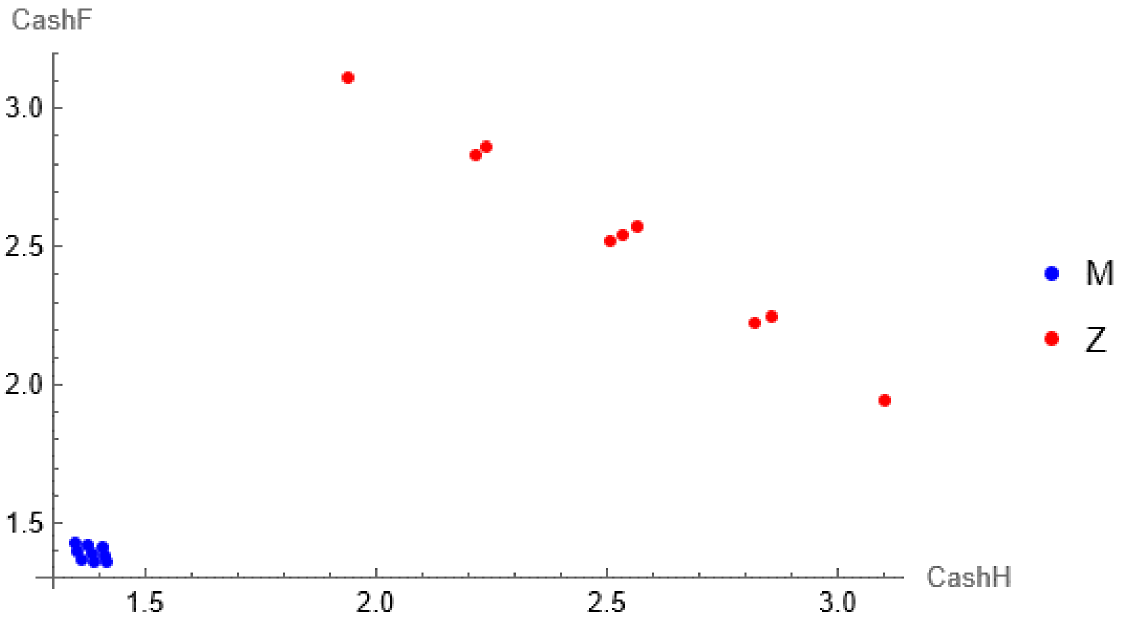


Figure A4: Local Goods Consumption in the Pure Fiat and the Pure Digicoin Economy

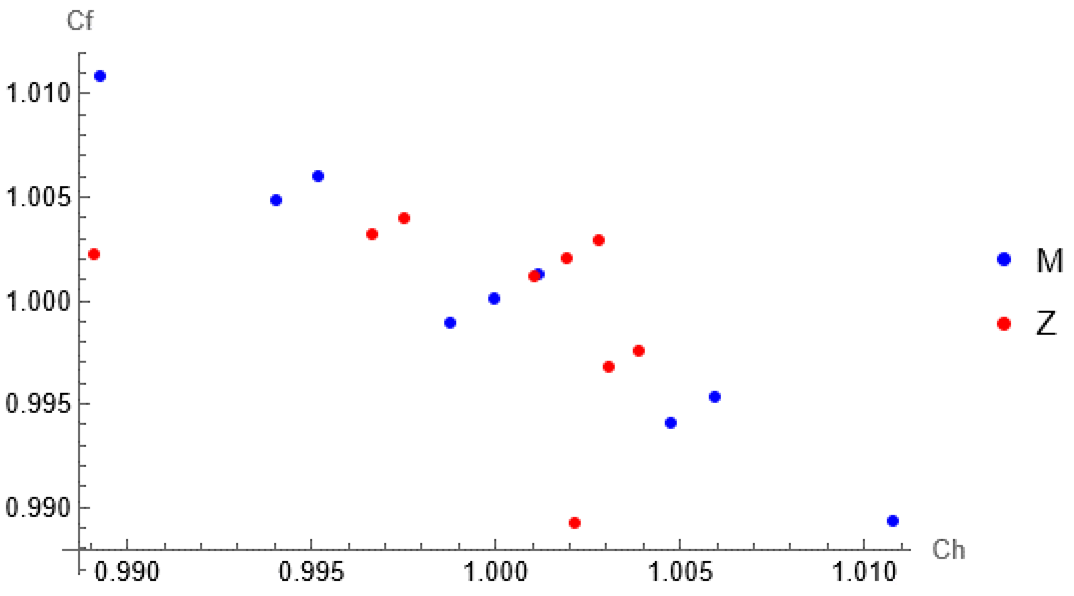


Figure A5: Domestic Consumption Orders in Fiat and Digital Currencies, Domestic CBDC Model

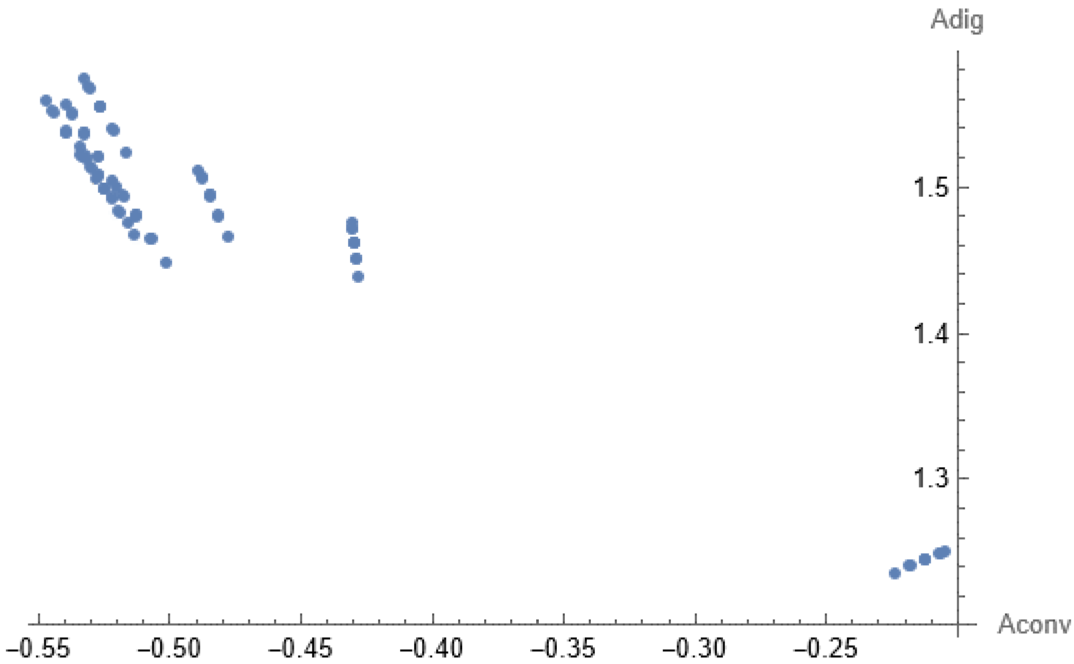


Figure A6: Export-Import Patterns in the Pure Fiat, Pure Digicoin, and Domestic CBDC Models

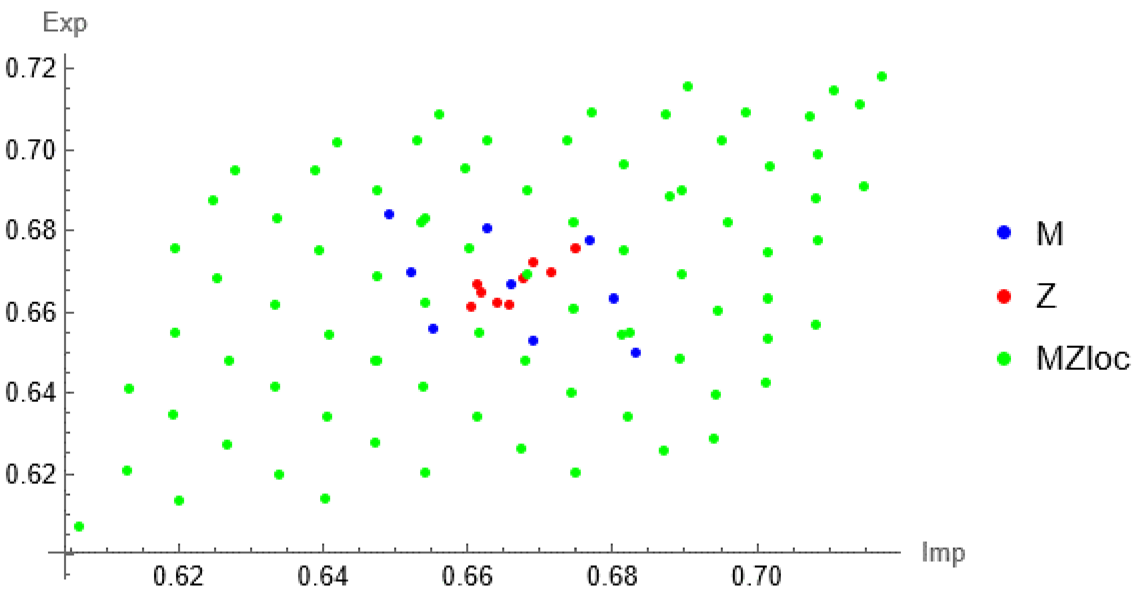


Figure A7: Ergodic Set Anchor Points in the Fiat/Digital Cash Space

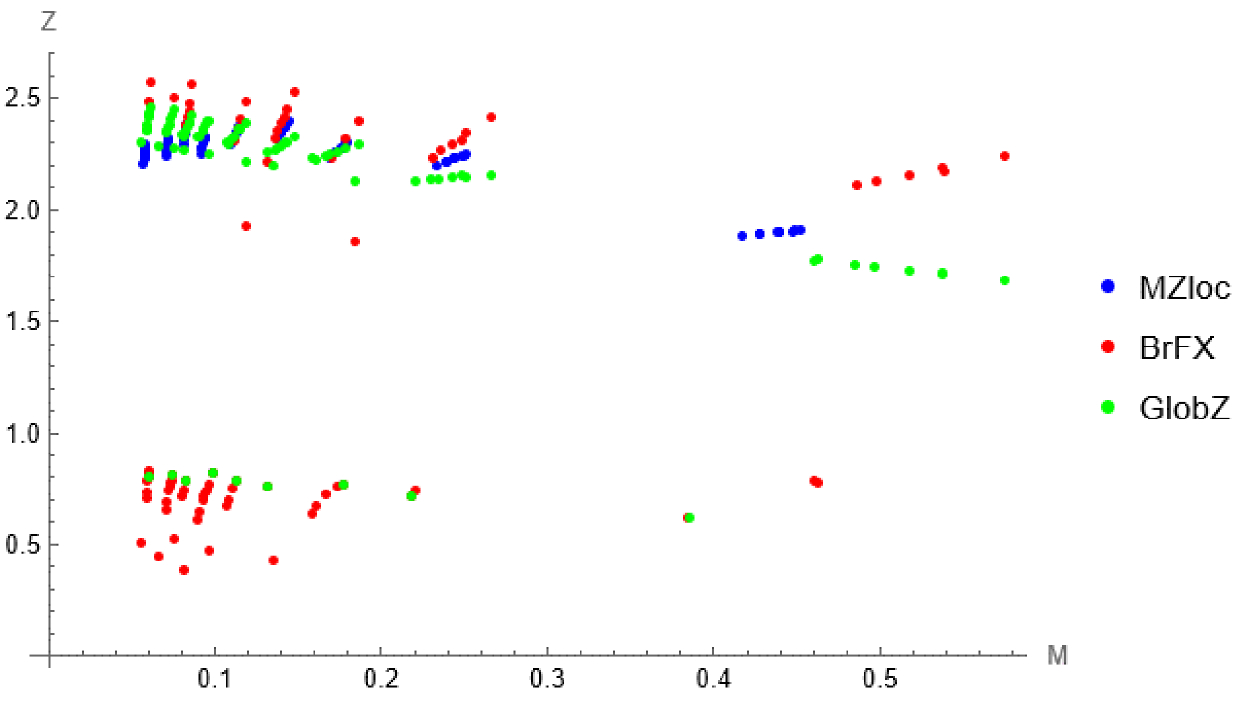


Figure A8: Domestic Consumption Orders in Fiat and Digital Currencies under Expanding Digicoin Functions

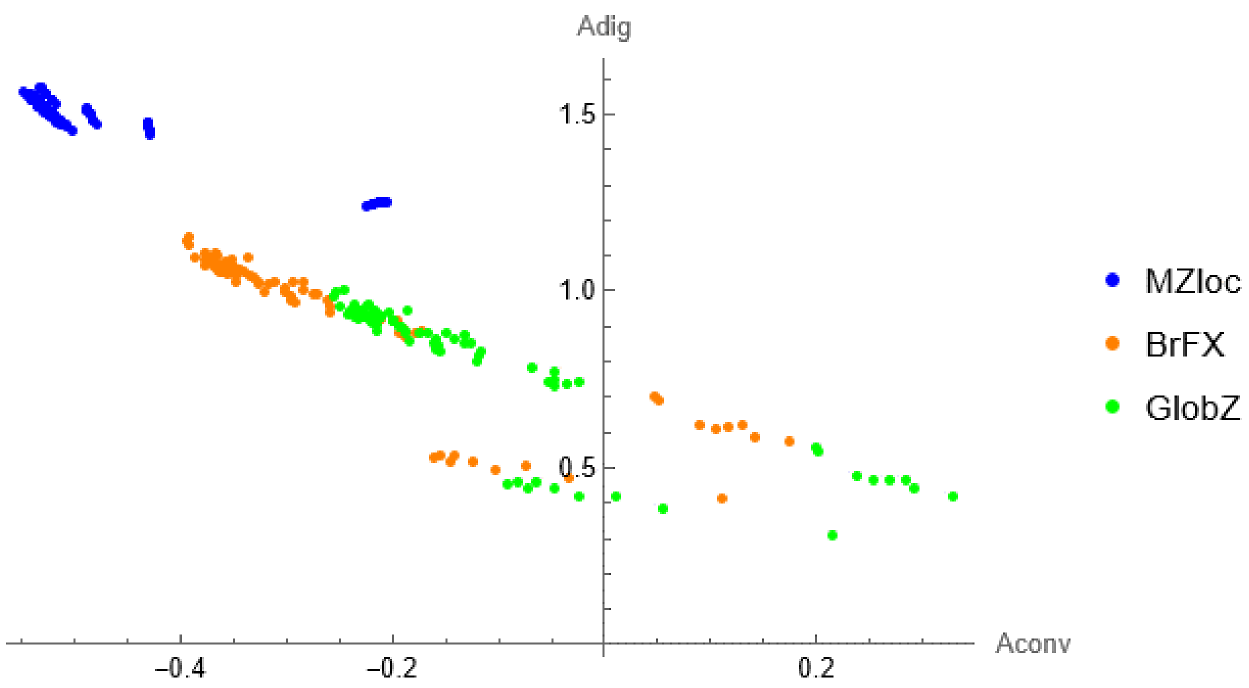
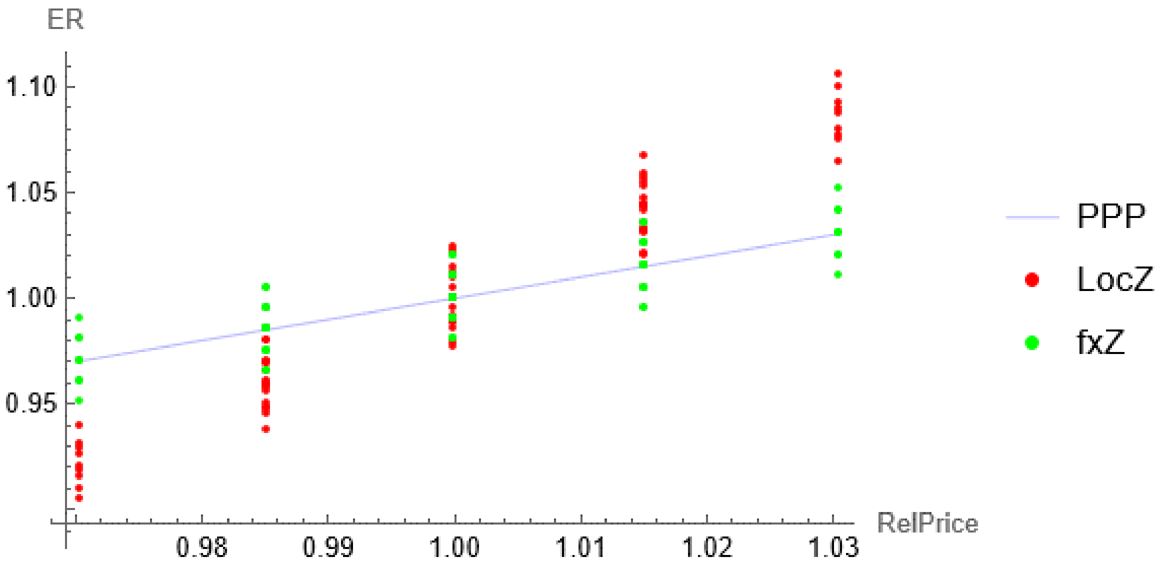


Figure A9: Actual and PPP-Implied Exchange Rate under Expanding Digicoin Functions



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