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Michal Franta*

Abstract

The paper demonstrates the benefits of multiple-output quantile regression for macroeconomic analysis. The domestic financial cycle, which is characterized by the co-movement of credit and property prices, is a natural subject of such methodology. More precisely, I examine the tails of the joint distribution of US house price growth and household credit growth since the late 1970s to shed some light on the evolution of systemic risk and its links to various economic and financial factors. The analysis finds that the crucial indicators include the banking sector's exposure to household credit, household leverage, house price misalignment and financial market volatility. This contrasts with the negligible role of real-economy factors. In addition, it is shown that the multiple-output quantile regression framework is a useful tool for forecasting and tracking systemic risk over time. The sustainable growth of house prices and credit can be distinguished from their growth accompanied by the rise in systemic risk to guide policymakers on an appropriate response.

Abstrakt

Tento článek ukazuje výhody kvantilové regrese s více výstupy pro makroekonomickou analýzu. Přirozeným předmětem zkoumání touto metodou je domácí finanční cyklus, který je charakterizován společným vývojem úvěrů a cen nemovitostí. Konkrétně zkoumám okraje sdruženého rozdělení růstu cen rezidenčních nemovitostí a růstu úvěrů domácnostem v USA od konce 70. let 20. století s cílem osvětlit vývoj systémového rizika a jeho vazby na různé ekonomické a finanční faktory. Analýza ukazuje, že mezi klíčové indikátory patří expozice bankovního sektoru vůči úvěrům domácnostem, zadluženost domácností, nesladěnost cen nemovitostí a volatilita na finančních trzích. Oproti tomu faktory z reálné ekonomiky hrají zanedbatelnou roli. Dále ukazuji, že kvantilová regrese s více výstupy je užitečným nástrojem pro prognózování a sledování systémového rizika v čase. Udržitelný růst cen bydlení a úvěrů můžeme odlišit od jejich růstu doprovázeného nárůstem systémového rizika a poskytnout tak tvůrcům hospodářských politik vodítko k odpovídající reakci.

JEL Codes: C32, E44, G10.

Keywords: Domestic financial cycle, multiple-output quantile regression, systemic risk.

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1. Introduction

The financial cycle is a broad concept which describes medium-term regularities in financial developments characterized by boom-bust cycles. One of the possible embodiments of the concept is the domestic financial cycle (DFC).[1] In the expansion phase, credit grows together with house prices. Economic agents – households and banks – change their attitude towards risk reflecting their optimistic economic outlook. Funding conditions are easy and demand for housing strong. Consequently, vulnerabilities in the form of the overvaluation of houses and exposure to risky credit increase in a self-reinforcing interaction – systemic risk builds up. At some point the economy enters the contraction phase, accompanied by credit defaults, a fall in house prices and, in many cases, a banking crisis. Systemic risk materializes. After some time, the expansion phase starts again.

In this paper, I examine the US domestic financial cycle since the late 1970s. To that end, I employ multiple-output quantile regression which is especially suited to this purpose for two reasons. First, by focusing on quantile regression in general, the tails of conditional distributions can be examined, thus shedding some light on the risk of serious disruption captured by the "left" tail of the distribution. For variables describing the financial system, the risk of such serious disruption defines the systemic risk.

Second, the DFC is an inherently multidimensional phenomenon. It is characterized by the medium-term co-movement of credit and house prices, which together constitute the most parsimonious set of variables capturing the cycle (Borio, 2014). Multivariate quantiles allow us to understand the joint behavior of the two variables in extreme situations. By focusing on the "south-west" tail of the joint distribution of credit and house prices, it is possible to track the evolution of systemic risk and look into the relationships between systemic risk and financial and macroeconomic variables.

To examine the DFC, I employ two basic approaches to multiple-output quantile regression – direct and directional. They complement each other, thus demonstrating the broad applicability of the tool. Note that the use of such methodology is rare in macroeconomics. While a few applications with some aspects of the directional approach can be found, the direct approach has yet to be explored.[2]

Based on the direct approach, the estimation results show the crucial role of banking sector exposure to household credit, household leverage, house price misalignment, and financial market volatility in the evolution of systemic risk. This contrasts with the negligible role played by real-economy factors. The risk/potential of the opposite situation, i.e. the pronounced simultaneous acceleration of both house prices and household credit, is characterized by its dominant link to past house prices and credit dynamics. The "north-east" tail of the joint conditional distribution of house price growth and household credit growth follows house price cycle and credit cycle, while systemic risk can diverge from the two cycles suggesting more complex and potentially nonlinear relationships related to the evolution of systemic risk.

[1] The concept of the domestic financial cycle was introduced in Minsky (1982) and Kindleberger (2000).

[2] Some aspects of the directional approach to the multivariate quantiles can be found in Polanski and Stoja (2017), who introduce multidimensional value-at-risk as a straightforward extension of standard univariate value-at-risk. Next, Montes-Rojas (2017) elaborates a vector directional quantile model, which is based on a system of univariate directional quantiles.

Macro-financial and real-economy indicators are linked to systemic risk in terms of their respective predictive power at the one-year horizon. Apart from that, causal inference is carried out for monetary policy shocks in order to contribute to the lean vs. clean debate. The result implies a warning that looking at conditional mean effects can mislead the argument. I found that systemic risk remains constant or increases after monetary policy tightening; there is no evidence of systemic risk reduction. The lean vs. clean debate should take into account the possibility that the distribution tails of asset volumes and asset prices can shift differently from conditional means after policy intervention.

The directional approach provides a measure which tracks the evolution of systemic risk over time and identifies periods in which it materializes and builds up. The measure matches past US banking crises and allows us to distinguish between periods of sustainable house price and credit growth and periods in which the growth of house prices and credit are associated with a rise in systemic risk. Finally, it is shown that the directional approach can be useful for systemic risk forecasting.

The application of multivariate quantile regression presented in this paper adds to the stream of literature which employs quantile regression to examine the effect of financial and macroeconomic indicators on the distribution tails of macroeconomic variables. Giglio et al. (2016) estimate the effect of systemic risk and market distress measures on real activity outcomes distribution. Adrian et al. (2019) extends the scope from quantiles to the whole distribution of GDP growth by fitting the estimated quantiles with the skewed *t*-distribution. The Growth-at-Risk (GaR) approach introduced in Adrian et al. (2019) has been implemented for various macroeconomic variables since then. Close to the application presented in this paper is the study by Deghi et al. (2020), which focuses on the left tail of house price distribution. This paper extends the examination to the second dimension of household credit.

Some studies dealing with the effects on distribution tails extend the usual specification of quantile regression and add quantiles to the set of independent variables (see Engle and Manganelli, 2004, in the univariate setting and White et al., 2015, for the multivariate extension). Such models primarily deal with distribution tail dependencies and are useful when cross-sectional aspects are of the main interest. In this paper, the time series dimension is the focus of analysis.

This paper interprets a particular tail of household credit growth and house price growth conditional distribution as a systemic risk measure. Therefore, studies which discuss various measures of systemic risk constitute another set of related papers. A recent review by Ellis et al. (2022) identifies 60 methodologies aimed at systemic risk measurement and finds that the measures usually draw on data for individual financial institutions. The approach in this paper aims at the stability of the aggregate system. Another example of the aggregate view is the GaR approach, which uses GaR as a way of quantifying systemic risk.[3] However, the GaR approach deals with the real-economy consequences of general financial disruptions rather than the accompanying phenomena (slump in credit and property prices) of a narrowly defined domestic financial cycle.

The structure of the paper is as follows: Before introducing multiple-output quantile regression in Section 3, Section 2 looks at multivariate quantiles and the intuition behind them. This is because

[3] See, for instance, Krygier and Vasi (2022) for the use of GaR as a measure of systemic risk in the Sveriges Riksbank.

the concept is not so common in macroeconomics. Section 4 describes the application of multiple-output quantile regression on the US domestic financial cycle and Section 5 concludes.

2. Multivariate Quantiles

The modeling framework employed in this paper draws on multiple-output quantile regression, but the intuition can be discussed for multivariate quantiles, i.e. cases where there are no covariates/predictors to explain the observed data. I have done this in the following two subsections.

Several approaches to multivariate quantiles and multiple-output quantile regression have been developed. For a survey of possible methods see Hallin and Šiman (2017). The basic distinction is whether the approach works directly in the multi-dimensional space (direct approach) or whether the approach reduces the problem to the well-known univariate distribution problem for a given direction (directional approach).

In this paper, I employ both approaches as they complement each other in their application on the DFC. The direct approach allows a more detailed analysis of distribution tails, but the size of the model is limited due to the computational complexity. The directional approach is less computationally intensive and the related econometrics is already well established. On the other hand, there are some arbitrary assumptions behind the approach that could in general weaken its applicability.

2.1 Direct Approach

The direct approach to multivariate quantiles is represented by "geometric quantiles" (Chaudhuri, 1996). They are based on the multivariate extension of the standard check function known from univariate quantile regression (Koenker and Basset, 1978), which is expressed as follows:

$$\rho_\tau(z) := (1-\tau)|z|\mathbb{I}_{z<0} + \tau|z|\mathbb{I}_{z\geq 0} = \frac{1}{2}(|z| + uz) =: \frac{1}{2}\Phi_u(z), \tag{1}$$

where $u := 2\tau - 1$ and $\mathbb{I}_{(\cdot)}$ represents an indicator function. The multivariate version of the check function is defined in an analogous way:

$$\boldsymbol{\Phi}_{\mathbf{u}}(\mathbf{z}) := \|\mathbf{z}\| + \mathbf{u}'\mathbf{z}, \tag{2}$$

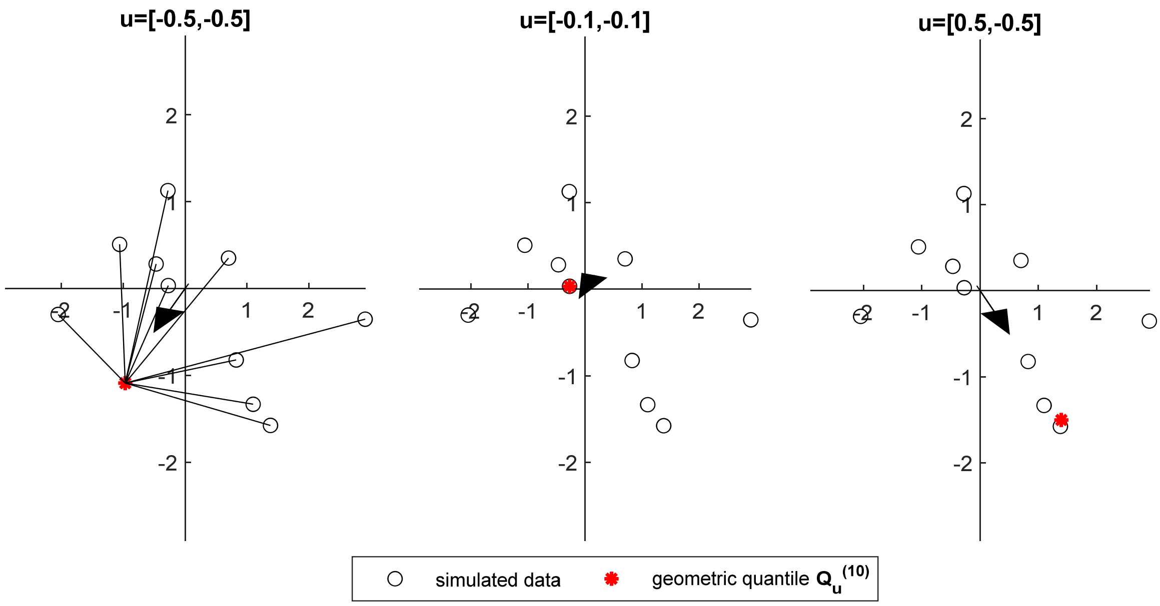
where $\mathbf{u}$ ranges over the open unit ball $B^{(d)} = \{\mathbf{u} | \mathbf{u} \in \mathbb{R}^d, \|\mathbf{u}\| < 1\}$ and the operator $\|\cdot\|$ denotes the Euclidean norm.
For the data set $\mathbf{Y} = \{\mathbf{Y}_1, \dots, \mathbf{Y}_n\}$, the $\mathbf{u}$-th geometric quantile is defined as

$$\mathbf{Q}_{\mathbf{u}}^{(n)} := \arg\min_{\mathbf{q}\in\mathbb{R}^d} \sum_{i=1}^{n} \boldsymbol{\Phi}_{\mathbf{u}}(\mathbf{Y}_i - \mathbf{q}). \tag{3}$$

The interpretation of the geometric quantile is that the average of the unit vectors with the direction given by the quantile and observations yields vector $-\mathbf{u}$, i.e. $\frac{1}{n}\sum_{i=1}^{n} \frac{\left(\mathbf{Y}_i - \mathbf{Q}_{\mathbf{u}}^{(n)}\right)}{\left\|\mathbf{Y}_i - \mathbf{Q}_{\mathbf{u}}^{(n)}\right\|} = -\mathbf{u}$, (Serfling, 2002). It follows that the index vector $\mathbf{u}$ of the geometric quantile allows us to distinguish between "extreme" and "central" observations in a multivariate data cloud. If $\|\mathbf{u}\|$ is close to one then the geometric quantile represents an extreme quantile, while $||\mathbf{u}||$ close to zero corresponds to a central quantile. In addition, the $\mathbf{u}$-th geometric quantile represents orientation information given by unit vector $\tilde{\mathbf{u}} = \mathbf{u}/||\mathbf{u}||$. For instance, $\tilde{\mathbf{u}} = \left[-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]$ represents realizations which are below the median in both dimensions.

To illustrate the intuition behind the geometric quantile, Figure 1 shows geometric quantiles for various index vectors based on a simulated data set of a two-dimensional data series containing 10 observations. First of all, the left panel of Figure 1 indicates all vectors defined by geometric quantile and observations. The average of the normalized vectors results in the opposite of the index vector. Next, the effect of the size of the index vector is illustrated in the left and right panels of Figure 1 with $||[-0.5, -0.5]|| = ||[0.5, -0.5]|| = 0.71$, which represent extreme quantiles, and in the middle panel of Figure 1 with $||[-0.1, -0.1]|| = 0.14$ representing central quantiles. Finally, the effect of orientation information is illustrated in the left and right panel of Figure 1 by comparing geometric quantiles in the "south-west" tail and "south-east" tail.

Figure 1: Three Geometric Quantiles Based on a Simulated Data Set of 10 Observations



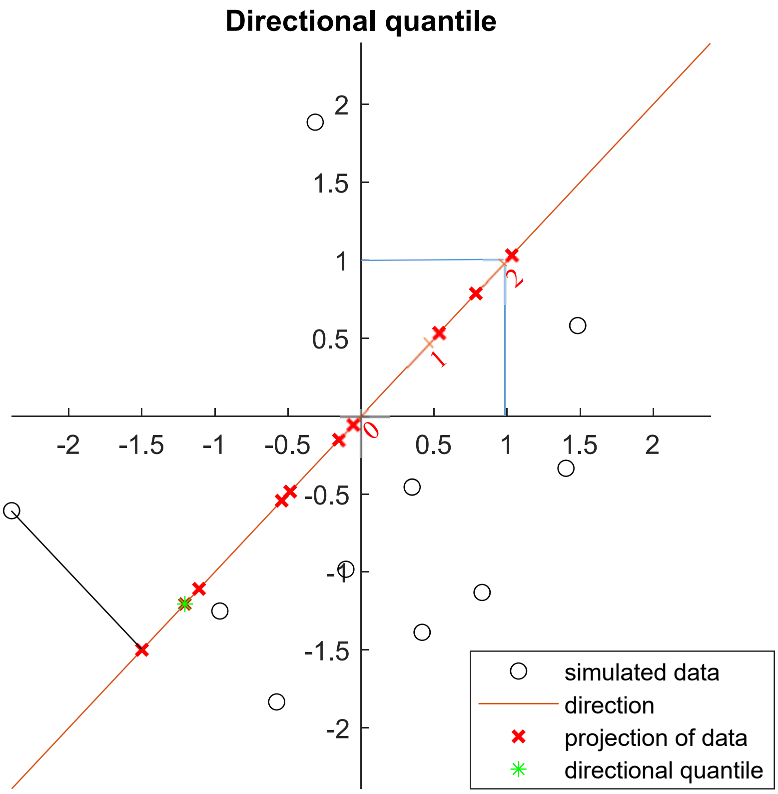
Note: The arrows indicate the index vector $\mathbf{u}$. The lines in the left panel indicate all vectors given by the geometric quantile and observations.

2.2 Directional Approach

The purpose of the directional approach is to project multidimensional observed data in a given direction obtaining a univariate distribution of data. A standard quantile can then be considered. A set of such quantiles in all directions yields various entities representing multivariate quantiles (quantile biplots, quantile regions, etc.) – see, for example, Kong and Mizera (2012).

To illustrate the approach, Figure 2 shows a simulated data set of 10 observations, $\mathbf{Y}$. The direction is given by a direction vector $\mathbf{w} = [1,1]$. The projections on the line with such direction are given by $\mathbf{w}'\mathbf{Y}$, i.e. $y_1 + y_2$. The directional quantile $\mathbf{q}_{\tau\mathbf{w}}$ (or projection quantile) is defined as a standard τ-th quantile of univariate distribution on the projection line, $q_{\tau\mathbf{u}}$, together with the direction vector $\mathbf{w}$, which is transformed into a unit vector $\widetilde{\mathbf{w}}$, so $\mathbf{q}_{\tau\mathbf{w}} := q_{\tau\mathbf{w}}\widetilde{\mathbf{w}}$.

Figure 2: Directional Quantile Based on a Simulated Data Set of 10 Observations; Direction $w = [1, 1]$ and $\tau = 0.20$



If appropriately chosen, the underlying variables together with the direction vector result in a projection, which possesses the interpretation of the domestic financial cycle measure. Consider, for instance, a three-dimensional vector of data consisting of medium-term cycles of credit, the credit-to-GDP ratio and property prices, and the direction vector $\mathbf{w} = [1/3, 1/3, 1/3]$. The projection, which is in fact the average of the three variables, is the domestic financial cycle indicator from Drehmann et al. (2012), which is viewed as a benchmark DFC indicator in the literature (Aldasoro et al., 2020). A univariate quantile of the DFC indicator from Drehmann et al. (2012) can be interpreted as a directional quantile of the distribution of the three constituent variables, and the tail of the indictor's distribution can be interpreted as the tail of the multivariate distribution for a given direction. So, the left tail of the projected (univariate) distribution corresponds to the "south-west" tail of the multivariate distribution.

3. Multiple-Output Quantile Regression

In this section, the multivariate quantiles from Section 2 are extended to multivariate quantile regression. The quantiles thus become conditional on the vector of observables $\boldsymbol{x}_t$. The two basic approaches – direct and directional – are again discussed. Finally, I look at the construction of confidence bands.

3.1 Direct Approach

In what follows, I closely follow Chakraborty (2003). The starting point is a multiple-output linear model:

$$\boldsymbol{y}_t = \boldsymbol{\beta}\boldsymbol{x}_t + \boldsymbol{e}_t, \qquad t = 1, \ldots, T \tag{4}$$

where $\boldsymbol{y}_t$ denotes *d*-dimensional response vectors and $\boldsymbol{x}_t$ represents the *k*-dimensional vectors of dependent variables. The error term $\boldsymbol{e}_t$ is assumed to be i.i.d. and independent of $\boldsymbol{x}_t$.

The regression quantiles are then defined as

$$\widehat{\boldsymbol{\beta}}_T(\mathbf{u}) = \arg\min_{\boldsymbol{\beta}\in\mathbb{R}^{d\times k}} \sum_{t=1}^{T}[\|\boldsymbol{y}_t - \boldsymbol{\beta}\boldsymbol{x}_t\| + \mathbf{u}^T(\boldsymbol{y}_t - \boldsymbol{\beta}\boldsymbol{x}_t)], \tag{5}$$

where $\mathbf{u}$ is an element of the open unit ball $B^{(d)} = \{\mathbf{u}|\mathbf{u} \in \mathbb{R}^d, \|\mathbf{u}\| < 1\}$. The first component in the sum (5) captures the size of residuals given the observed data $\{\boldsymbol{y}_{t,}\boldsymbol{x}_t\}$ and the parameter $\boldsymbol{\beta}$. When left alone in the objective function, the minimization of the Euclidean distance of the observations to the hyperplane $\boldsymbol{y}_t - \boldsymbol{\beta}\boldsymbol{x}_t$, the geometric median regression, is obtained. When looking for non-trivial geometric regression quantiles, a new component appears in the objective function – the residuals are weighted coordinate-wise by the index vector components. The more extreme the quantile, the higher the weight of residuals in comparison to their norm.

Geometric regression quantiles defined in this way possess several undesirable properties. For example, they are not equivariant under coordinate-wise scale transformations. It means that they do not change correspondingly if a component of $\boldsymbol{y}_t$ is rescaled. To resolve this, Chakraborty (2003) introduced the adaptive transformation retransformation (TR) procedure, which makes the regression quantiles affine equivariant, i.e. not dependent on the choice of coordinate system. The purpose of the TR procedure is to change the coordinate system according to the observed data in order to render it affine equivariant. The new coordinate system is indexed by $\hat{\alpha}$ consisting of time indices used to set up the new coordinate system.

The change in the coordinate system leads to the transformed response vectors $\boldsymbol{z}_t^{(\hat{\alpha})}$ and transformed index vector $\mathbf{v}(\hat{\boldsymbol{\alpha}})$. Let $\hat{\boldsymbol{\Gamma}}_T^{(\hat{\alpha})}(\mathbf{u})$ be the $\mathbf{v}(\hat{\alpha})$-th regression quantile obtained from the transformed response vectors regressed on $\boldsymbol{x}_t$:

$$\hat{\boldsymbol{\Gamma}}_T^{(\hat{\alpha})}(\mathbf{u}) = \arg\min_{\boldsymbol{\Gamma}\in\mathbb{R}^{d\times k}} \sum_{t\notin\hat{\alpha}} \left[\left\|\boldsymbol{z}_t^{(\hat{\alpha})} - \boldsymbol{\Gamma}\boldsymbol{x}_t\right\| + \mathbf{v}(\hat{\alpha})^{\mathrm{T}}\left(\boldsymbol{z}_t^{(\hat{\alpha})} - \boldsymbol{\Gamma}\boldsymbol{x}_t\right)\right]. \tag{6}$$

Then the multivariate TR regression quantile $\widehat{\boldsymbol{\beta}}_T^{(\widehat{\alpha})}(\mathbf{u})$ is a retransformation of $\hat{\boldsymbol{\Gamma}}_T^{(\widehat{\alpha})}(\mathbf{u})$ back to the original coordinate system. Details of the minimization procedure and the TR procedure can be found in Appendix A.

3.2 Directional Approach

Several possible algorithms are used to implement the directional approach to multiple-output quantile regression – see, for example, Paindaveine and Šiman (2011). Regardless of the specific approach, an estimation procedure synthesizes results for a given direction in all directions.

In the application discussed in this paper, however, a specific direction implies an interpretation of the projected data as a measure of the financial cycle. We thus stick to the specific direction and do not employ the full estimation procedure. In such case, the estimation approach is restricted to the standard univariate quantile regression optimization procedure with the left-hand-side variable constituted by the data projected from the response space onto the given direction. So, the linear relationship between the τ-th quantile of the projected data $\mathbf{w}\mathbf{y}_t$ and predictors $\boldsymbol{x}_t$ is found to be a minimizer of the following optimization problem:

$$\hat{\Gamma}_T(\tau) = \arg\min_{\boldsymbol{\beta}\in\mathbb{R}} \sum_{t\in T} \left[\tau * \mathbb{I}_{\mathbf{w}\mathbf{y}_t \geq \boldsymbol{\beta}\boldsymbol{x}_t} |\mathbf{w}\mathbf{y}_t - \boldsymbol{\beta}\boldsymbol{x}_t| + (1-\tau) * \mathbb{I}_{\mathbf{w}\mathbf{y}_t < \boldsymbol{\beta}\boldsymbol{x}_t} |\mathbf{w}\mathbf{y}_t - \boldsymbol{\beta}\boldsymbol{x}_t|\right], \quad (7)$$

where $\mathbb{I}_{(\cdot)}$ denotes the indicator function, $\boldsymbol{y}_t$ denotes $d \times 1$ vectors and $\mathbf{w}$ is a $1 \times d$ unit vector of weights representing a direction in $\mathbb{R}^d$.

3.3 Bootstrapping of Confidence Intervals

A moving block bootstrap is employed to construct the confidence intervals of the estimated regression quantiles. The procedure follows Fitzenberger (1997), who demonstrates its usefulness for standard (single-response) quantile regression. The bootstrap samples of the length of the original sample are constructed from blocks of length $\lfloor \sqrt[3]{T} \rfloor$ in order to retain the autocorrelation structure of the data. We generate 5,000 bootstrap samples in order to obtain 90% and 80% confidence bands.

4. US Domestic Financial Cycle

In this section, we employ multiple-output quantile regression to examine the US domestic financial cycle. The direct approach allows us to estimate the statistical relationships between various macroeconomic and financial indicators and systemic risk, while the directional approach allows us to identify periods in which systemic risk related to the banking sector builds up and materializes. In addition, the directional approach can serve as a tool for systemic risk forecasting.

Systemic risk is represented by the "south-west" tail of the bivariate conditional distribution of house price growth and household credit growth. House prices and private credit (i.e. household and corporate credit) constitute the most parsimonious set of variables able to represent the DFC. This means that the two variables are sufficient to capture the relationship between the financial cycle, business cycle and financial crises (Borio, 2014). Moreover, Büyükkarabacak and Valev

(2010) show that the empirical link between credit and banking crises is more straightforward for household credit than for credit to firms. Therefore, we focus on household credit instead of general private credit.

Quarterly data are used to estimate multiple-output quantile regression, mostly covering 1977 Q1–2022 Q1. House prices and household credit are from the BIS database. Other series are downloaded from the Fred database. A detailed description of data, some descriptive statistics and a discussion on the detrending of financial variables can be found in Appendix B. Finally, a comparison of the multiple-output quantile regression approach and separate univariate quantile regression is presented in Appendix C to demonstrate the possible pitfalls if the analysis draws on univariate models/distributions only.

4.1 Predictors of Systemic Risk

I start by examining the effect of various indicators on the geometric quantile of house price growth and household credit growth distribution four quarters ahead drawing on the following model:

$$\begin{bmatrix} \Delta_4 hp_{t+4} \\ \Delta_4 hc_{t+4} \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} \Delta_4 hp_{t-1} \\ \Delta_4 hc_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{13} \\ \beta_{23} \end{bmatrix} z_{t-1} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \tag{8}$$

where $\Delta_4 hp_{t+4}$ stands for the growth rate of real house prices between periods t and $t + 4$ (i.e. the year-on-year growth rate computed as the difference of the logs of real house prices multiplied by 100), $\Delta_4 hc_{t+4}$ for the growth rate of real household credit four quarters ahead and z_{t-1} represents a lagged real-economy or financial indicator. A separate model specification for each indicator z_{t-1} is necessary due to the computational complexity of the estimation procedure.[4]

The indicators z_{t-1} are chosen drawing on two structural models. Presenting details of the models is beyond the scope of the paper, and thus only a very brief description follows. The first model is from Deghi et al. (2020), who present a New Keynesian type-model similar to Guerrieri and Iacoviello (2017). The model includes households/lenders and households/borrowers, who face occasionally binding collateral constraint. It is able to replicate housing crises and suggests several possible predictors of house price downside risk: household leverage, financial conditions, house price misalignment, and real GDP growth as a proxy for household income.

The second model is introduced in Liu and Fan (2021). It is a DSGE model which, in addition to households/borrowers and banks, contains firms. The banking sector endogenously decides about the portion of credit allocated to households and to firms. The model is able to replicate the stylized facts of the run-up to the global financial crisis (GFC): a rapid rise in house prices, an increase in the mortgage-to-GDP ratio, a fall in the real mortgage rate, and an increase in the ratio of household credit to commercial and industrial credit.

Based on the two structural models, the set of indicators applied in turn in (8) consists of the Chicago Fed's National Financial Condition Index (NFCI) as the most general measure of financial sector conditions. Then more specific financial indicators are employed. Non-financial leverage is represented by the credit ratio (household credit/credit to non-financial corporations) and by household leverage (household credit/GDP). The price aspect is captured by house price

[4] The adaptive transformation retransformation procedure is the most time-consuming part of the estimation procedure.

misalignment (real house prices/real GDP per capita). Finally, following the literature on the effects of financial market volatility on risk-taking and financial crises (e.g. Danielsson et al., 2018), the implied stock market volatility index VIX is considered. The real-economy indicators are represented by real GDP growth and the real mortgage rate.

While we consider various geometric quantiles in what follows, we focus mainly on the $[-0.5, -0.5]$-th geometric quantile because it serves as a measure of systemic risk. The direction of the quantile implies that it describes adverse scenarios of simultaneous very low growth in household credit and house prices, i.e. the circumstances of systemic risk materialization within the DFC. Furthermore, the distance of the quantile from the origin ($\|[-0.5, -0.5]\| = 0.71$) suggests that the quantile represents an extreme situation.[5] The robustness check for a less extreme quantile with the same direction can be found in Appendix D.

Finally, note that the geometric quantile is a vector with two components: a house price component and household credit component. Therefore, the regression quantiles based on (8) relate either to the first or second component and can be interpreted as being related either to vulnerability stemming from the elevated house valuation process or vulnerability related to excessive borrowing by households.

Table 1: Estimation Results for [-0.5,-0.5]-th Geometric Quantile of House Prices and Household Credit

Equation:	Prices	Credit	Prices	Credit	Prices	Credit	Prices	Credit	Prices	Credit
Constant	-1.48	-2.24**	-9.33**	-3.61**	-1.29	-1.80**	-1.02	-1.48**	-2.89**	-2.39**
Lagged house price growth	0.79**	0.11	1.11**	0.28**	0.59**	0.02	0.47**	-0.12	0.74**	0.10
Lagged household credit growth	-0.10	0.52**	-0.73	0.50**	-0.01	0.56**	0.19	0.74**	0.30**	0.72**
NFCI	-1.43*	-1.93*								
VIX			0.48**	0.09						
Credit ratio					-16.00**	-7.36**				
Household leverage							-0.76**	-0.46**		
House price misalignment									-1.34**	-0.30

Notes: **denotes statistical significance at 5%; * denotes statistical significance at 10%

The estimation results of the quantile regression (8) for different indicators z_{t-1} are presented in Table 1 and Table 2. The main finding is the striking difference between the role of financial indicators (Table 1) and real-economy indicators (Table 2). While financial indicators relate significantly to changes in systemic risk (components of the $[-0.5, -0.5]$-th geometric quantile), the role of real-economy indicators is negligible.

[5] The fact that the $[-0.5, -0.5]$-th geometric quantile describes extreme realizations from the joint distribution can be viewed also from the perspective of standard univariate quantiles. Using the single-response relation $u = 2\tau - 1$ mentioned in (1), the $[-0.5, -0.5]$-th geometric quantile corresponds to the 0.15-th quantile of the two underlying variables. If the two variables are independent, then the probability of observing the realization of both variables simultaneously to be less or equal to 0.15 is $(0.15)^2 = 0.0225$, which is below the quantile order of 0.05 usually employed in the Macro-at-Risk literature. The independence of the two variables represents the lower bound of joint probability; correlated variables result in higher probabilities.

Table 2: Estimation Results for [-0.5,-0.5]-th Geometric Quantile of House Prices and Household Credit

Equation:	Prices	Credit	Prices	Credit
Constant	-1.20	-2.60**	-0.13	-3.36*
Lagged house price growth	0.67**	0.08	0.72**	0.04
Lagged household credit growth	0.05	0.54**	-0.01	0.57**
GDP growth	-0.07	0.23		
Real mortgage rate			-0.22	0.19

Notes: **denotes statistical significance at 5%; * denotes statistical significance at 10%

In summary, by focusing on macro-financial indicators more precisely, the severity of an extreme simultaneous drop in house price growth and household credit growth is explained by lagged house price growth for the house price component of the quantile and by lagged household credit growth for the household credit component. The potential drop is more severe (systemic risk is higher) in circumstances of lower stock market volatility, a more leveraged banking sector and household sector, and more overvalued houses.

Focusing on the indicators, which affect both components of the $[-0.5, -0.5]$-th geometric quantile, and are thus directly interpretable in terms of the direction of their effect on systemic risk, it turns out that the leverage indicators exhibit the most profound effect.[6] The ratio of household credit to credit to firms is negatively associated with the conditional geometric quantile four quarters ahead. A one-standard-deviation increase in the ratio accounts for a drop in the $[-0.5, -0.5]$-th quantile of 2.40 percentage points (pp) for the house price component and of 1.11 pp for the household credit component – see Table 3 for the effects of indicators of one standard deviation in size. The increase in the credit ratio means that household credit increases more relative to corporate credit. While the effect of additional corporate credit on the economy is usually more persistent through investment and its positive effect on the economy's potential, the increase in household credit can smooth household consumption, but is still without any significant long-lasting positive effect (Büyükkarabacak and Valev, 2010). Therefore, more profound growth in household credit (compared to corporate credit) can be unsustainable for the economy and systemic risk rises.

[6] Note that for the two leverage indicators, the estimates of the effect on the geometric quantile can be the subject of omitted variable bias. The two indicators are strongly correlated in the sample (0.87), and thus including a single indicator in the model provides biased estimates analogically to the OLS regression. For a single-response quantile regression, the omitted variable bias is elaborated in Angrist et al. (2006). The bias is not expected for other indicators because they do not exhibit a strong correlation between each other.

Table 3: One-Standard-Deviation Effect on [-0.5,-0.5]-th Geometric Quantile of House Prices and Household Credit

NFCI	[-1.27*,-1.71*]
VIX	[1.40**,0.27]
Credit ratio	[-2.40**,-1.11**]
Household leverage	[-2.67**,-1.61**]
House price misalignment	[-3.94**,-0.89]

Notes: **denotes statistical significance of the underlying coefficient at 5%; * denotes statistical significance of the underlying coefficient at 10%

In a similar vein, a one-standard-deviation increase in the ratio of household credit to GDP accounts for a decrease in the geometric quantile components of 2.67 pp and 1.61 pp respectively, and a house prices to GDP one-standard-deviation rise predicts a drop of 3.94 pp in the house price component of the geometric quantile. The increase in household credit to GDP can be viewed as the increase in debt with respect to household income. At some point, an income shock results in binding collateral constraint, and house prices, along with real GDP, enter into a vicious circle of falling aggregate demand, fire sales and a fall in income and house prices. That is why household leverage and house price misalignment account for a rise in systemic risk.

Finally, the effect of the stock market volatility index – although statistically significant in terms of house prices only – is in accordance with Minsky's famous "stability is destabilizing" (Minsky, 1977). Lower volatility is associated with an increase in the house price component of systemic risk. Low volatility periods affect the risk-taking behavior of economic agents resulting in vulnerability stemming from the overvaluation of houses.

While the above-mentioned financial indicators concern a slow accumulation of vulnerabilities in the economy, the NFCI reflects immediate stress on financial markets related rather to the actual materialization of systemic risk. As such, its effect on systemic risk is statistically weaker than that of the other indicators.

The estimated predictive power of various financial indicators of systemic risk is in contrast with the "north-east" tail, which captures the pronounced simultaneous growth of both house prices and household credit. The considered indicators are mostly statistically insignificant for the $[0.5,0.5]$-th geometric quantile (see Table D1 and D2 in Appendix D). Only lagged house price growth and credit growth predict their respective component of the $[0.5,0.5]$-th geometric quantile. Moreover, in comparison to the $[-0.5,-0.5]$-th geometric quantile, the lagged household credit growth coefficient increases, suggesting an increase in the persistence of the respective tail component. The tail predominantly follows house price cycle and credit cycle dynamics.

4.2 Monetary Policy and Systemic Risk

In addition to the indicators suggested by structural models, we estimate the quantile regression (8) with the monetary policy shocks in order to contribute to the "lean vs. clean" debate, which is historically related to the DFC concept (Aldasoro et al., 2020). The point of the debate, formulated in terms of house prices and systemic risk, is whether the monetary authority should respond to the build-up of systemic risk (a house price bubble) or just intervene when systemic risk materializes.

The narratively-identified monetary policy shocks in Romer and Romer (2004), updated up to 2007 by Wieland and Yang (2020), are employed to estimate the causal effect of monetary policy on systemic risk. The Romer and Romer (2004) approach, however, exhibits a different effect of a monetary policy shock on economic activity in comparison to other standard approaches (Coibion, 2012). Therefore, the set of monetary policy shocks estimated using the DSGE model by Smets and Wouters (2007) is also considered. The series of shocks is taken from Coibion (2012) and covers the period to 2004. Note that the shock measures enter the model specification (8) not lagged.

Table 4: Estimation Results for the Specification with MP Shocks

Geometric quantile:	[-0.5,-0.5]		[0,0]		OLS	
Equation:	House prices	Credit	House prices	Credit	House prices	Credit
Romer and Romer shocks						
Constant	-2.99**	0.26	-0.55	2.09**	-0.95**	2.13**
Lagged house price growth	0.71**	-0.19	0.71**	-0.07	0.80**	-0.06**
Lagged credit growth	0.26	0.49**	0.48**	0.67**	0.23**	0.61**
MP shock	-0.26	-0.34	-0.33	-0.42	-0.57**	-0.33**
Smets and Wouters shocks						
Constant	-1.51**	1.29*	-0.15	2.42**	0.06**	3.28**
Lagged house price growth	0.55**	-0.30**	0.68**	-0.12	0.64**	-0.26**
Lagged credit growth	0.50**	0.66**	0.54**	0.68**	0.52**	0.71**
MP shock	-0.55*	-0.31	-0.48**	-0.35	-0.53**	-0.36**

Notes: **denotes statistical significance at 5%; * denotes statistical significance at 10%

Table 4 shows that the estimated relationship between monetary policy and systemic risk does not support the "lean" approach because the effect of monetary policy shocks on systemic risk is in the opposite direction if any. The impact of a 100 b.p. unexpected monetary policy tightening on a house price component of $[-0.5, -0.5]$-th geometric quantile is negative for the shocks in Smets and Wouters (2007), i.e. it leads to an increase in systemic risk. The effect of the shocks in Romer and Romer (2004) is not statistically significant.[7] Similar effects are found for the geometric median. Interestingly, the equation-by-equation OLS estimate suggests effects similar to the effects on geometric quantiles. However, they are always statistically significant.

Looking separately at the conditional means of household credit growth and house price growth without any knowledge of the effect on tails would incorrectly support the use of the leaning strategy by concluding that tightening monetary policy slows the growth in both house prices and credit. However, through the lens of multiple-output quantile regression, it turns out that the whole conditional joint distribution of house price growth and household credit growth shifts in the south-west direction within a horizon of one year (Smets and Wouters shock) or the shift of the tail is not statistically significant (Romer and Romer shocks). A pre-emptive monetary policy tightening can thus unintentionally increase systemic risk.

[7] Note that the employed monetary policy shocks are estimated quantities and should be treated accordingly leading to less precise estimates. The confidence bands for the shocks are not available, however, and we thus stick to the shocks considered as directly observed quantities.

4.3 The Evolution of US Systemic Risk

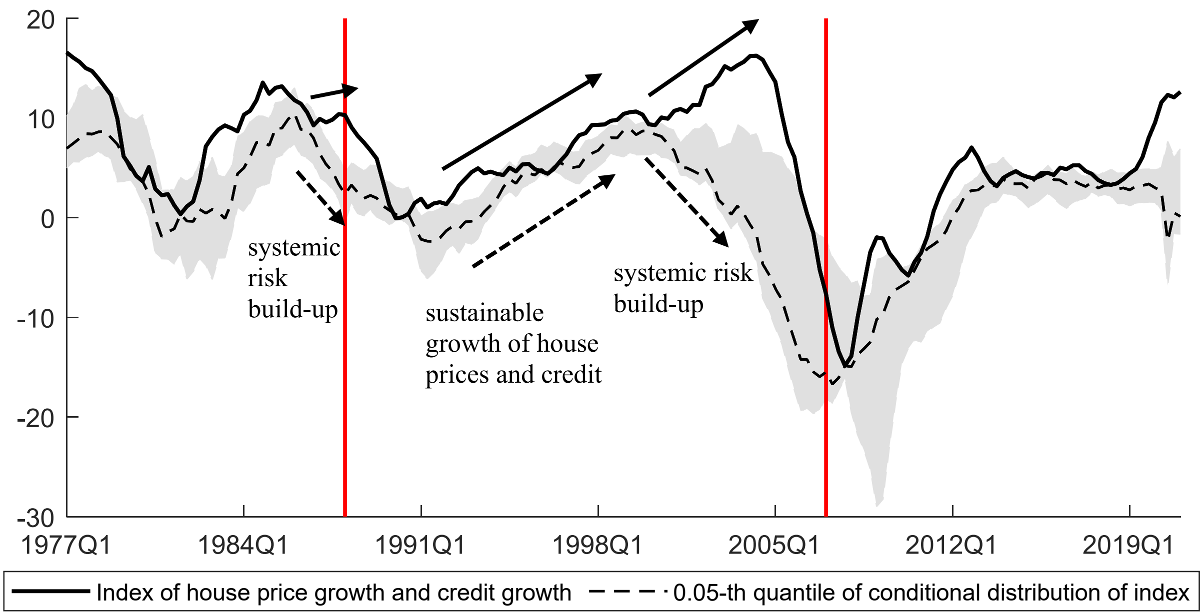
The estimates of conditional geometric quantiles based on (8) with different indicators in fact provide quantiles of different distributions because the distribution is conditional on the respective indicator. Observed data are, however, realizations from the distribution conditional on all relevant indicators. Therefore, to obtain a complete picture, the estimated quantiles should take into account all such indicators. This is the reason the directional approach to multivariate quantiles is employed.

We start with the following model with the index of house price growth and household credit growth on its left-hand side:

$$w_1 * \Delta_4 hp_{t+4} + w_2 * \Delta_4 hc_{t+4} = \beta_0 + \boldsymbol{\beta}_1 [\Delta_4 hp_{t-1}, \Delta_4 hc_{t-1}, \boldsymbol{z}_{t-1}]' + \varepsilon_t, \qquad (9)$$

where $\boldsymbol{z}_{t-1}$ is now a vector of all indicators from Subsection 4.1, which cover the whole sample. It includes the NFCI, the ratio of household credit to firms' credit, household leverage, house price misalignment, real output growth, and the real mortgage rate. The weights in the index $\mathbf{w} = [w_1, w_2] = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ are set in order to project the house price and household credit growth data cloud onto the south-west/north-east direction. Such direction is motivated by the financial cycle measure of Drehmann et al. (2012) – see Section 2.2 – and is ex-post empirically justified below. Note that systemic risk is now represented by a single variable and corresponds to the lower tail of the index distribution.

Figure 3: Index and 0.05-th Quantile of its Conditional Distribution

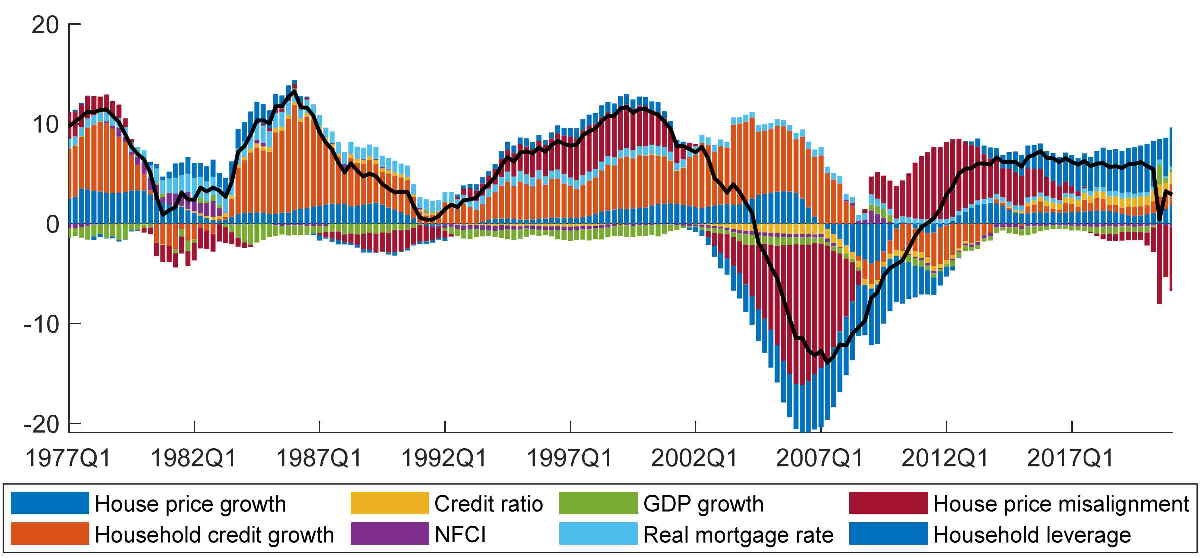


Note: The vertical lines indicate the year in which the systemic banking crisis started according to Laeven and Valencia (2012). The shaded area indicates a 90% confidence band for the predicted quantile. The arrows indicate examples of various combinations of systemic risk (dashed arrows) and the index (solid arrows) evolutions referred to in the text.

The evolution of systemic risk represented by the 0.05-th conditional quantile of the index distribution is reported in Figure 3. The fall in the estimated 0.05-th conditional quantile implies the rise in the severity of a potential fall in the index i.e. the build-up of systemic risk – see two examples of such situation indicated by the left and right arrows in Figure 3. The middle set of arrows indicates the situation involving an increase in the index and a simultaneous increase in quantile i.e. a decrease in systemic risk. This situation can be observed throughout the 1990s and suggests that the observed growth in credit and house prices does not indicate systemic risk build-up and represents sustainable growth in house prices and credit.

In addition, Figure 3 demonstrates the empirical relevance of the choice of weights $\mathbf{w}$ and of the directional approach in general by relating both the index and the 0.05-th conditional quantile of its distribution to observed events. The materialization of systemic risk is indicated by the index approaching and reaching the predicted 0.05-th conditional quantile. Such materialization happens either due to an extreme economic or financial shock or the appearance of nonlinear effects (e.g. a financial accelerator). Ultimately, the materialization can result in systemic banking crises. Figure 3 provides several examples of systemic risk materialization. First, the materialization of systemic risk is observed during the early 1980s as a consequence of the 1979 oil crisis and the tight monetary policy pursued by Paul Volcker. The second period of systemic risk materialization can be observed in the late 1980s during the savings and loan crisis. The preceding rapid increase in systemic risk can be observed starting in 1986 (see arrows in Figure 3) coinciding with the deregulation of the US financial sector. The materialization had started by 1988 which, according to Leaven and Valencia (2012), is the year the banking crisis started. Finally, systemic risk materialization can be observed around the time the housing bubble burst in 2007 with preceding systemic risk build-up starting around 1999, which coincides with the repealing of the Glass-Steagall Act of 1933.

Figure 4: The Decomposition of the 0.05-th Quantile of the Conditional Distribution of the Index into the Contribution of Right-Hand-Side Variables in (9)



Note: The intercept is not presented in the decomposition; the presented quantile is shifted by the value of the intercept to correspond to the sum of the independent variables' contribution.

To interpret changes in systemic risk, a decomposition of the predicted 0.05-th conditional quantile of the index distribution into the contribution of right-hand-side variables in (9) is presented in Figure 4. It turns out that systemic risk build-up preceding the GFC relates mainly to the overvaluation of houses and the increase in household leverage. The abovementioned sustainable growth of house prices and credit in the 1990s is characterized by the opposite contribution of the two factors.

4.4 Out-of-Sample Performance of the Directional Approach

The directional approach can be applied in real time to inform policymakers on the outlook for systemic risk and the phase of the financial cycle. To assess the ability of the directional approach in the forecasting of systemic risk, the quantile R^2 is computed for various specifications (see Table D5 and the accompanying text for the computational procedure and results). It turns out that the best forecasting performance at the one-year horizon is obtained for the specification with lagged house price growth, lagged household credit growth, credit ratio, GDP growth, real mortgage rate, and house price misalignment. Based on this specification, I estimate the model in (9) on windows starting with the 1977 Q1–2001 Q4 window and adding one quarter at a time until the last window covers the whole benchmark period from 1977 Q1–2021 Q1.

Figure 5: Observed Index of House Price Growth and Household Credit Growth, in-Sample Estimate of the 0.05-th Quantile of Conditional Index Distribution and the Pseudo out-of-Sample Forecast of the 0.05-th Quantile of Conditional Index Distribution

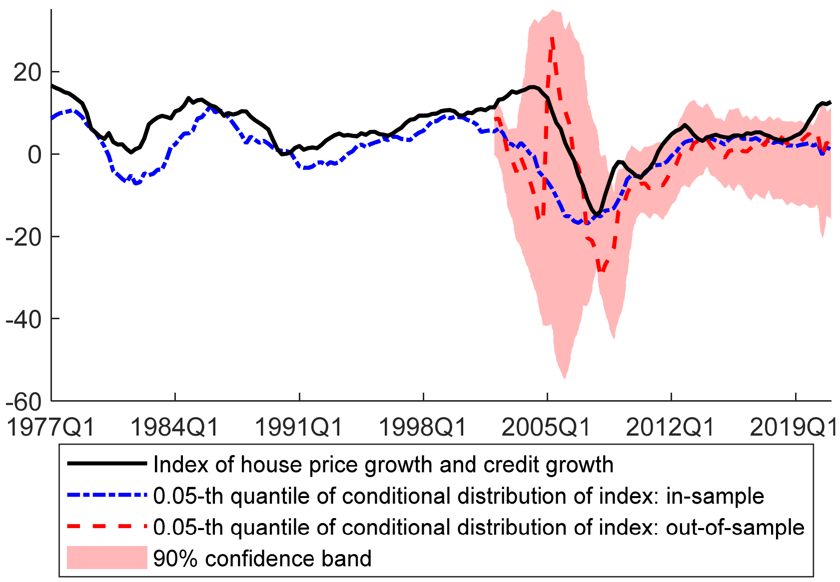


Figure 5 shows the comparison of the 0.05-th quantile of conditional distribution of the index based on the consecutive estimation windows and full sample. It points out two observations. First, the uncertainty related to out-of-sample prediction is so high for some periods that the prediction is essentially useless (e.g. the period from 2005–2010, i.e. a period of systemic risk materialization). On the other hand, the systemic risk forecast provides an important signal for some periods, which can be viewed as statistically significant (e.g. a recent period with the 0.05-th conditional quantile diverging from the observed index suggesting an increase in systemic risk).

The second observation from Figure 5 is the crucial role of the GFC in the forecasting of systemic risk. The historical highs and lows of the index are observed around the GFC. Such observations presumably have a significant effect on quantile regression (9). If, for example, the GFC represents a unique regime distinct from the past, the systemic risk forecast cannot be too informative. However, for recurrent events – regardless of whether they represent regime change – the systemic risk forecast is a useful indicator for policymakers. For example, the fall in the 0.05-th quantile from 2002–2005 (a period of systemic risk build-up) warned policymakers about the heightened systemic risk in real-time.

5. Conclusion

In this paper, I examine the potential of multiple-output quantile regression in applied macroeconomics. In contrast to the univariate case, where the ordering of numbers makes the definition of the distribution quantile straightforward, the definition in the multivariate setting is not straightforward and several approaches have emerged. I discuss both basic concepts – the direct approach and directional approach – and employ them to explain their pros and cons.

Multiple-output quantile regression is applied to the US domestic financial cycle. There are essentially two reasons for such a choice. First, the domestic financial cycle is an inherently two-dimensional phenomenon and as such is of a multivariate nature but is still simple enough to allow for the estimation. Second, systemic risk and its evolution is an important issue for the policymakers who design macroprudential policies to deal with it.

I show that the structure of banking sector assets, household leverage, the overvaluation of house prices, and financial market volatility are crucial factors in understanding the evolution of systemic risk. The approach also identifies periods of systemic risk materialization and build-up. In addition, it turns out that past house prices and household credit are important indicators in understanding the evolution of tails, especially the tail representing the potential for rapid simultaneous growth in both house prices and household credit.

The approach is more general and can be applied in Macro-at-Risk literature when the subject of risk is represented by more variables and nonlinear dynamics different from the dynamics at conditional means is expected. Various spirals, feedback effects and regime changes come to mind in this respect.

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Appendix A: Estimation Procedure of Geometric Regression Quantiles

This appendix describes the estimation procedure behind the direct approach to multiple-output quantile regression closely following Chakraborty (2003). Since this procedure is not common in economics, it is described explicitly here. Note that in contrast to Chakraborty (2003), who works with a cross section, the procedure is applied to time series in this paper and the notation is adjusted accordingly.

Step 1: Obtain a sample variance covariance matrix of the residuals obtained from ordinary least squares applied to the equation $\boldsymbol{y}_t = \boldsymbol{\beta}\boldsymbol{x}_t + \boldsymbol{e}_t$, for $t = 1, \ldots, T$ and denote it as $\hat{\Sigma}$.

Step 2: Define S_T to be a set of all subsets of $k + d$ indices from the set $\{1, \ldots, T\}$. For each $\alpha = \{t_1, \ldots, t_k, s_1, \ldots, s_d\} \in S_T$ define $\mathbf{W}(\alpha)$ as $k \times k$ matrix with columns $\boldsymbol{x}_{t_1}, \ldots, \boldsymbol{x}_{t_k}$ and $\mathbf{Z}(\alpha)$ as $d \times k$ matrix with columns $\boldsymbol{y}_{t_1}, \ldots, \boldsymbol{y}_{t_k}$. Next, let $\mathbf{E}(\alpha)$ be $d \times d$ matrix consisting of columns $\boldsymbol{y}_{s_1} - \mathbf{Z}(\alpha)\{\mathbf{W}(\alpha)\}^{-1}\boldsymbol{x}_{s_1}, \ldots, \boldsymbol{y}_{s_d} - \mathbf{Z}(\alpha)\{\mathbf{W}(\alpha)\}^{-1}\boldsymbol{x}_{s_d}$.

Step 3: Compute the matrix $\mathbf{V}(\alpha) = \{\mathbf{E}(\alpha)\}^{\mathrm{T}}\hat{\Sigma}^{-1}\mathbf{E}(\alpha)$ and the quantity

$$t(\alpha) = \frac{[\mathrm{trace}\{\mathbf{V}(\alpha)\}]/d}{[\det\{\mathbf{V}(\alpha)\}]^{1/d}}.$$

Step 4: Minimize $t(\alpha)$ with respect to $\alpha \in S_T$ and denote such α as $\hat{\alpha}$. The practical implementation of the minimization task is such that searching for optimal α is stopped whenever $t(\alpha)$ is sufficiently close to 1.[8]

Step 5: Transform all response vectors $\boldsymbol{y}_t$ to $\boldsymbol{z}_t^{(\hat{\alpha})} \equiv \{\mathbf{E}(\hat{\alpha})\}^{-1}\boldsymbol{y}_t$ for all $t \notin \hat{\alpha}$ and transform the index vector $\mathbf{u}$ as follows:

$$\begin{aligned} \mathbf{v}(\hat{\alpha}) &= \frac{\{\mathbf{E}(\hat{\alpha})\}^{-1}\boldsymbol{u}}{\|\{\mathbf{E}(\hat{\alpha})\}^{-1}\boldsymbol{u}\|}\|\mathbf{u}\| \qquad &&\text{if } \mathbf{u} \neq \mathbf{0} \\ \mathbf{v}(\hat{\alpha}) &= \mathbf{0} &&\text{if } \mathbf{u} = \mathbf{0}. \end{aligned}$$

Given the transformation defined above, the multiple-output quantile regression problem becomes as follows:

$$\hat{\mathbf{\Gamma}}_T^{(\hat{\alpha})}(\mathbf{u}) = \arg\min_{\mathbf{\Gamma}\in\mathbb{R}^{d\times k}} \sum_{t\notin\hat{\alpha}} \left[\left\|\boldsymbol{z}_t^{(\hat{\alpha})} - \mathbf{\Gamma}\boldsymbol{x}_t\right\| + \mathbf{v}(\hat{\alpha})^{\mathrm{T}}\left(\boldsymbol{z}_t^{(\hat{\alpha})} - \mathbf{\Gamma}\boldsymbol{x}_t\right)\right]. \qquad \text{(A1)}$$

The existence of the unique minimizer of (A1) follows from the convexity of the Euclidean norm and linearity of the second term in (A1).

Step 6: To find a minimizer of the problem (A1), we employ the simplex search method implemented in MATLAB command *fminsearch*. As a check, we also employ Newton's method to solve the following system of $d * k$ equations representing the first order necessary conditions of an unconstrained minimization problem:

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[8] In this specific exercise the distance from 1 is set to be at least 10^{-9}.

$$\sum_{t\notin\hat{\alpha}}\left[\frac{\boldsymbol{z}_t^{(\hat{\alpha})}-\boldsymbol{\Gamma}\boldsymbol{x}_t}{\left\|\boldsymbol{z}_t^{(\hat{\alpha})}-\boldsymbol{\Gamma}\boldsymbol{x}_t\right\|}+\mathbf{v}(\hat{\alpha})\right]\otimes\boldsymbol{x}_t^{\mathrm{T}}=\mathbf{0}. \tag{A2}$$

Before employing Newton's iterative procedure, one should check the non-degeneracy condition. For each subset of size k, $\{t_1,\dots,t_k\}\subset S_n\setminus\hat{\alpha}$ form matrix $\boldsymbol{W}$ with columns $\boldsymbol{x}_{t_1},\dots,\boldsymbol{x}_{t_k}$ and matrix $\boldsymbol{Z}$ with columns $\boldsymbol{z}_{t_1}^{(\hat{\alpha})},\dots,\boldsymbol{z}_{t_k}^{(\hat{\alpha})}$ and check the following condition:

$$\left\|\sum_{t\notin\hat{\alpha}\cup\{t_1,\dots,t_k\}}\left[\frac{\boldsymbol{z}_t^{(\hat{\alpha})}-\mathbf{Z}\mathbf{W}^{-1}\boldsymbol{x}_t}{\left\|\boldsymbol{z}_t^{(\hat{\alpha})}-\mathbf{Z}\mathbf{W}^{-1}\boldsymbol{x}_t\right\|}+\mathbf{v}(\hat{\alpha})\right]\otimes\boldsymbol{x}_t\right\|\le(1+\|\mathbf{u}\|)\sum_{t\in\{t_1,\dots,t_k\}}\|\boldsymbol{x}_t\|. \tag{A3}$$

If the condition is satisfied for some $\{t_1,\dots,t_k\}$ then set $\widehat{\boldsymbol{\beta}}_T^{(\hat{\alpha})}(\mathbf{u})=\{\mathbf{E}(\hat{\alpha})\}\boldsymbol{Z}\{\boldsymbol{W}\}^{-1}$. Otherwise proceed to the application of Newton's algorithm.

The non-degeneracy procedure avoids situations where the minimized function is not continuous. However, in the application presented in this paper, the initial values chosen are presumably close to the solution and the condition is not checked.

The solution of (A2) can be found iteratively. The initial iteration $\hat{\boldsymbol{\Gamma}}_T^1(\mathbf{u})$ is computed by regressing $\boldsymbol{z}_t^{(\hat{\alpha})}$ on $\boldsymbol{x}_t$ using coordinate-wise single-output QR with order $0.5*(\mathrm{v}_i+1)$, for $i=1,2$, to ensure that the initial step is sufficiently close to the minimizer and thus the procedure converges. Let $\hat{\boldsymbol{\Gamma}}_T^1(\mathbf{u}),\dots,\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})$ be the successive approximations of $\hat{\boldsymbol{\Gamma}}_T(\mathbf{u})$ obtained in consecutive iterations. To compute the next iteration $\hat{\boldsymbol{\Gamma}}_T^{n+1}(\mathbf{u})$ define:

$$\boldsymbol{\delta}^{\boldsymbol{n}}\equiv\sum_{t\notin\hat{\alpha}}\left[\frac{\boldsymbol{z}_t^{(\hat{\alpha})}-\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})\boldsymbol{x}_t}{\left\|\boldsymbol{z}_t^{(\hat{\alpha})}-\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})\boldsymbol{x}_t\right\|}+\mathbf{v}(\hat{\alpha})\right]\otimes\boldsymbol{x}_t$$

and

$$\boldsymbol{\Phi}^{\boldsymbol{n}}\equiv\sum_{t\notin\hat{\alpha}}\left\{\frac{-1}{\left\|\boldsymbol{z}_t^{(\hat{\alpha})}-\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})\boldsymbol{x}_t\right\|}\left[\mathbf{I}_d-\frac{\left\{\boldsymbol{z}_t^{(\hat{\alpha})}-\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})\boldsymbol{x}_t\right\}\left\{\boldsymbol{z}_t^{(\hat{\alpha})}-\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})\boldsymbol{x}_t\right\}^{\mathrm{T}}}{\left\|\boldsymbol{z}_t^{(\hat{\alpha})}-\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})\boldsymbol{x}_t\right\|^2}\right]\right\}\otimes\boldsymbol{x}_t\,\boldsymbol{x}_t^{\mathbf{T}},$$

where $\mathbf{I}_d$ is d-dimensional identity matrix. The next iteration is then

$$\hat{\boldsymbol{\Gamma}}_T^{n+1}(\mathbf{u})=\hat{\boldsymbol{\Gamma}}_T^n(\mathbf{u})-(\boldsymbol{\Phi}^{\boldsymbol{n}})^{-\mathbf{1}}\boldsymbol{\delta}^{\boldsymbol{n}}.$$

The iterative procedure is stopped when two consecutive iterations are sufficiently close.[9]

Step 7: The **u**-th geometric regression quantile is $\widehat{\boldsymbol{\beta}}_T^{(\hat{\alpha})}(\mathbf{u})=\{\mathbf{E}(\hat{\alpha})\}\hat{\boldsymbol{\Gamma}}_T(\mathbf{u})$.

[9] In this specific exercise the threshold for the distance of norms is set to 10^{-13}.

Appendix B: Data

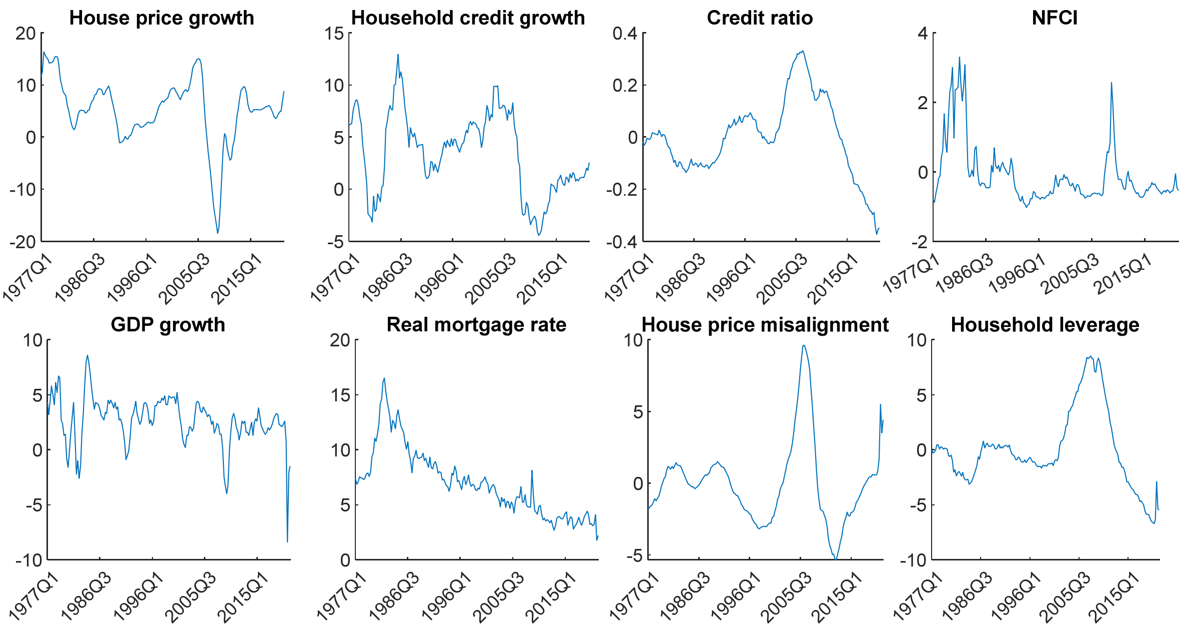
This appendix presents some descriptive statistics of the data entering the application of multiple-output quantile regression on the US domestic financial cycle. Table B1 describes all employed series and their sources.

Table B1: Data Sources

Variable	Source	Description	Period
Real house price index	BIS	Residential property prices, existing dwellings, per dwel., SA.	1977Q1-2022Q1
Household credit	BIS	Credit to households and NPISHs from all sectors at market value.	1977Q1-2022Q1
Credit to firms	BIS	Credit to non-financial corporations from all sectors at market value	1977Q1-2022Q1
Real GDP growth	Fred	Real gross domestic product, percent change from quarter one year ago, SA.	1977Q1-2022Q1
NFCI	Fred	Chicago Fed National Financial Conditions Index.	1977Q1-2022Q1
VIX	Fred	Stock market volatility index, not SA.	1990Q1-2022Q1
CPI	Fred	Consumer price index for all urban consumers: all items in US city average, SA.	1977Q1-2022Q1
Mortgage rate	Fred	30-year fixed rate mortgage average in the United States.	1977Q1-2022Q1
Monetary policy shocks			
Romer and Romer shocks		Wieland and Yang (2020) update of Romer and Romer (2004) monetary policy shocks.	1977Q1-2007Q4
Smets and Wouters shocks		Coibion (2012) estimate of monetary policy shocks using Smets and Wouters (2007).	1977Q1-2004Q4
Real GDP per capita	Fred	Real gross domestic product per capita, SA.	1977Q1-2022Q1
Real GDP	Fred	Real gross domestic product, SA.	1977Q1-2022Q1

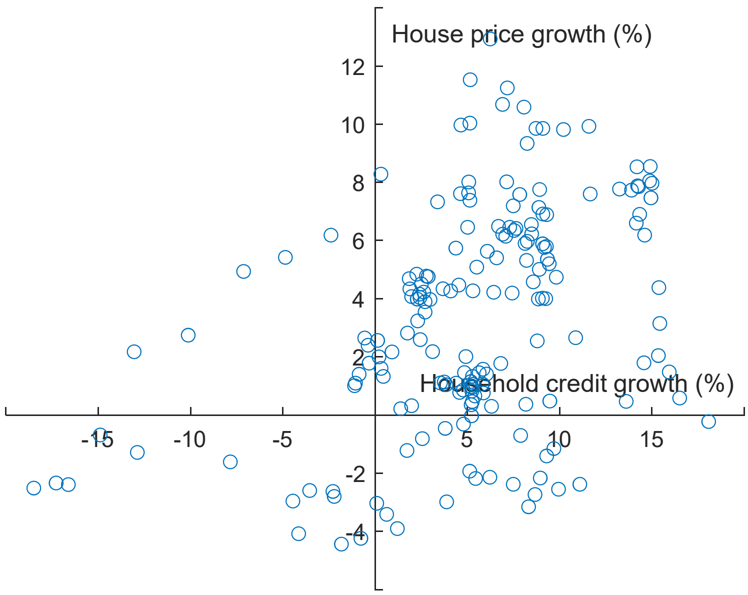
Note that household credit is deflated by the CPI and the real mortgage rate is computed as the nominal mortgage rate minus CPI inflation. House price misalignment is computed as the ratio of real house prices and real GDP. The ratio is multiplied by 10,000 in order to work with standard deviations of a similar magnitude. Next, household leverage is defined as the ratio of household credit to GDP and is also multiplied by the factor of 10,000.

Figure B1: Series Entering the Models



The series entering the models specified in (8) and (9) are reported in Figure B1. In addition, the credit ratio, real mortgage rate, house price misalignment and household leverage are linearly detrended. Linear detrending is chosen in order to avoid detrending approaches, which potentially cancel out the short frequencies that are relevant as signals for systemic risk build-up and materialization. As demonstrated by Schüler (2018), detrending financial series using HP or a band-pass filter can affect short-term fluctuations considerably.

Figure B2: Scatter Plot of Household Credit and House Prices



Finally, as we are primarily interested in the multiple-output quantiles of household credit and house prices, Figure B2 shows the scatter plot of the two variables. The paper's focus is on the observations in the south-west direction, which capture a simultaneous drop in house price growth and household credit growth.

Appendix C: Multiple-Output Quantile Regression vs. Univariate Quantile Regression

This appendix demonstrates that employing quantile regression instead of standard linear regression estimated by the OLS and employing multiple-output quantile regression instead of separate univariate quantile regression yield different results.

Figure C1: The Estimated Coefficients at the Ratio of Housing Credit to Credit to Firms for Various Geometric Quantiles of House Prices and Household Credit

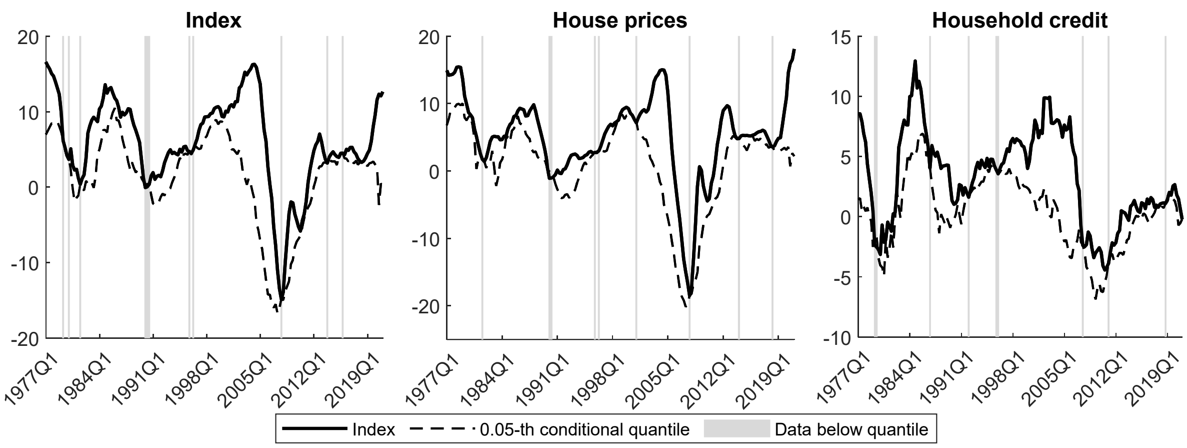
Figure C1 shows the estimated coefficients at the credit ratio for various geometric quantiles suggesting different estimates for different geometric quantiles. Importantly, the estimated coefficients at the credit ratio are different from the geometric median suggesting the importance of the use of the quantile approach for inference regarding the tails of house prices and household credit joint distribution.

Figure C2: The Estimated Coefficient for Univariate Quantile Regression (QR) and Multiple-Output Quantile Regression (MQR) for the Specification with the Credit Ratio

Note: (*) and (+) markers denote the estimated coefficient in quantile regression model (8) for index vectors $\mathbf{u} = [-0.5, -0.5]$, $[-0.3, -0.3]$, $[-0.1, -0.1]$, $[0.3,0.3]$, $[0.5,0.5]$. The index vectors are set to correspond to the quantile order according to $\tau = (u_i + 1)/2$.

Regarding the direct approach, Figure C2 suggests that separate univariate quantile regression estimates of the effect of the credit ratio on house price growth and household credit growth differ from the corresponding multivariate regression quantile components. For example, for the 0.75-th quantile of the house price growth conditional distribution, the unit change of the credit ratio is associated with a rise in the quantile by 3.75 pp, while the house price component of [0.5,0.5]-th falls by 4.67 pp.

Figure C3: The $\mathbf{0.05}$-th Conditional Quantile of the Index of House Price Growth and Household Credit Growth, House Prices and Household Credit and Implied Periods of Risk Materialization



Note: The shaded areas indicate the periods in which the observed data are below the estimated conditional quantile.

Similarly, the estimated 0.05-th quantile differs according to whether it is based on the directional approach (Figure C3, left panel) or whether univariate quantile regression is applied directly to house price growth (Figure C4, middle panel) and household credit growth (Figure C3, right panel) and then combined. Figure C3 shows that the periods in which the observed data fall below the corresponding 0.05-th conditional quantile (shaded areas) differ. In addition, the periods in which data are below the estimated quantiles for house prices and household credit at the same time are different from those periods constructed for their index.

Appendix D: Additional Results and Robustness Checks

This appendix presents several additional results and robustness checks, which are referred to in the main text.

The first set of additional results concerns the estimation of the multiple-response quantile regression model (8) for the [0.5,0.5]-th geometric quantile, which is the quantile in the opposite direction to the quantile representing systemic risk (Table D1 and D2).

Table D1: Estimation Results for [0.5,0.5]-th Geometric Quantile of House Prices and Household Credit

Equation:	Prices	Credit	Prices	Credit	Prices	Credit	Prices	Credit	Prices	Credit
Constant	4.19**	3.39**	-1.26	0.80	4.52**	3.21**	4.64**	3.28**	4.05**	3.53**
Lagged house price growth	0.62**	-0.01	0.71**	0.14**	0.53**	-0.02	0.52**	-0.04	0.56**	-0.02
Lagged household credit growth	0.09	0.91**	-0.26	0.87**	0.16	0.87**	0.14	0.87**	0.19*	0.83**
NFCI	0.31	1.32								
VIX			0.30**	0.05						
Credit ratio					-4.67	-0.93				
Household leverage							-0.10	-0.12		
House price misalignment									-0.47*	0.18

Notes: **denotes statistical significance at 5%; * denotes statistical significance at 10%

Table D2: Estimation Results for [0.5,0.5]-th Geometric Quantile of House Prices and Household Credit

Equation:	Prices	Credit	Prices	Credit
Constant	5.26**	3.19**	4.93**	0.33
Lagged house price growth	0.57**	-0.01	0.64**	0.08
Lagged household credit growth	0.25	0.88**	0.08	0.65**
GDP growth	-0.41	-0.02		
Real mortgage rate			-0.11	0.44*

Notes: **denotes statistical significance at 5%; * denotes statistical significance at 10%

In the main text, the $[-0.5, -0.5]$-th geometric quantile of house price growth and household credit growth serves as a measure of systemic risk. The quantile whose length is equal to 0.71 is extreme enough (see footnote 5). However, the tail characteristics are notoriously difficult to estimate precisely. Therefore, as a robustness check, Table D3 and D4 present the estimated regression quantiles from (8) for the $[-0.4, -0.4]$-th geometric quantile, whose length is equal to 0.57.

Table D3: Estimation Results for [-0.4,-0.4]-th Geometric Quantile of House Prices and Household Credit

Equation:	Prices	Credit	Prices	Credit	Prices	Credit	Prices	Credit	Prices	Credit
Constant	-0.35	-1.25*	-7.12**	-2.75*	-0.43	-1.04*	-0.25	-0.80	-1.53*	-1.36*
Lagged house price growth	0.73**	0.06	1.01**	0.25**	0.58**	0.03*	0.50**	-0.06	0.69**	0.09
Lagged household credit growth	0.03	0.62**	-0.65	0.54**	0.13	0.64**	0.21	0.74**	0.26*	0.71**
NFCI	-1.14	-1.76*								
VIX			0.42**	0.09						
Credit ratio					-14.47*	-5.54**				
Household leverage							-0.68**	-0.34**		
House price misalignment									-1.26**	-0.27

Notes: **denotes statistical significance at 5%; * denotes statistical significance at 10%

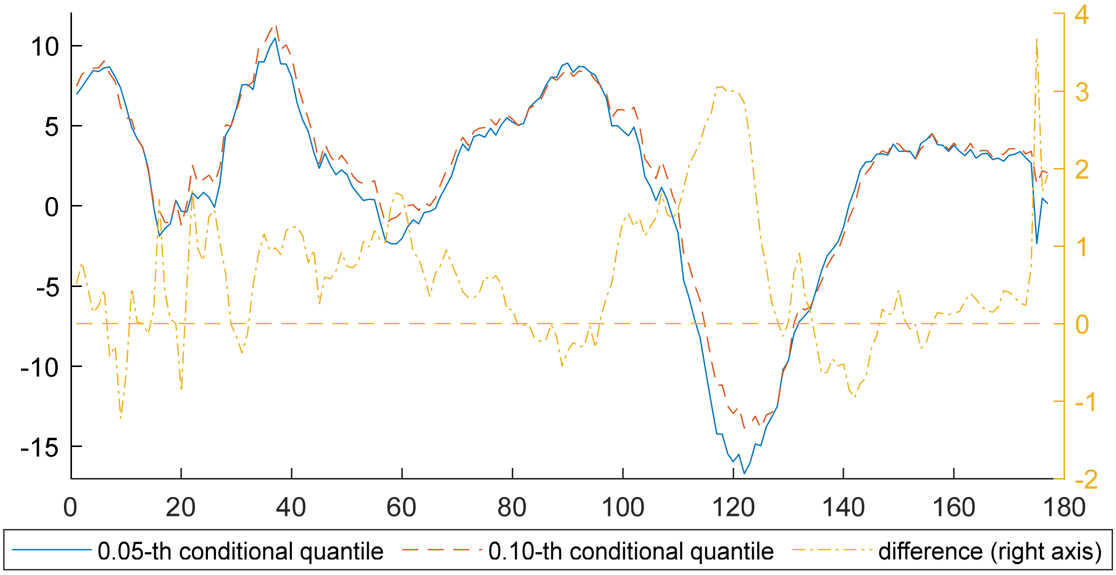
Table D4: Estimation Results for [-0.4,-0.4]-th Geometric Quantile of House Prices and Household Credit

Equation:	Prices	Credit	Prices	Credit
Constant	0.25	-1.42*	2.70	-0.66
Lagged house price growth	0.59**	0.04	0.63**	0.05
Lagged household credit growth	0.23	0.60**	0.17	0.65**
GDP growth	-0.12	0.26		
Real mortgage rate			-0.41	-0.03

Notes: **denotes statistical significance at 5%; * denotes statistical significance at 10%

A similar robustness check is carried out for the distribution of the index of house price growth and household credit growth. Figure D1 shows that the 0.05-th and 0.10-th quantiles of index distribution are close. There are some instances of quantile crossing observed – see the difference between the 0.10-th quantile and the 0.05-th quantile shown on the right axis. The issue of quantile crossing is not dealt with, as our interest lies in the behavior of distribution tails and the conclusions about systemic risk evolution would not change.

Figure D1: Conditional Quantiles of the Index Distribution and Their Difference



Finally, the assessment of the forecasting ability of the directional approach in Subsection 4.4 is based on the specification which is superior in the terms of its forecasting performance at the one-year horizon. The quantitative assessment of the out-of-sample forecasting ability of various specifications is based on the quantile R^2. The measure compares the loss using conditional information to that based on the unconditional historical quantile $\hat{q}_t^\tau$ estimated on the respective estimation window:

$$R^2 = 1 - \frac{\frac{1}{T}\sum[\rho_t(w_1 * \Delta_4 hp_{t+4} + w_2 * \Delta_4 hc_{t+4} - \hat{\beta}[1, \Delta hp_{t-1}, \Delta hc_{t-1}, \mathbf{z}_{t-1}]')]}{\frac{1}{T}\sum[\rho_t(w_1 * \Delta_4 hp_{t+4} + w_2 * \Delta_4 hc_{t+4} - \hat{q}_t^\tau)]}, \tag{D1}$$

where $\rho_t(x) = x(\tau - I_{x<0})$ is the quantile loss function and the sums are computed over T estimation windows given by time indices $1, \ldots, t$. More precisely, I estimate the quantile regression in (9) with the 1977 Q1–2001 Q4 window and adding one quarter at a time until the last window covers the whole benchmark period from 1977 Q1–2021 Q1.

Table D5: Adjusted R2 for Various Specifications of the Quantile Regression Model

Specification	Adjusted R2
Intercept, lagged house price growth, lagged household credit growth, credit ratio, NFCI, GDP growth, real mortgage rate, house price misalignment, household leverage.	-0.11
Intercept, lagged house price growth, lagged household credit growth, credit ratio, NFCI, GDP growth, real mortgage rate, house price misalignment.	0.24
Intercept, lagged house price growth, lagged household credit growth, credit ratio, GDP growth, real mortgage rate, house price misalignment.	0.34
Intercept, lagged house price growth, lagged household credit growth, credit ratio, real mortgage rate, house price misalignment.	0.32
Intercept, lagged house price growth, lagged household credit growth, credit ratio, house price misalignment.	0.19

The specification without NFCI and household leverage yields the highest quantile R^2 and is therefore used for the forecasting exercise.

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