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Vojtěch Molnár *

Abstract

The paper compares price level targeting and inflation targeting regimes in a New Keynesian model with bounded rationality. Economic agents form their expectations using heuristics—they choose between a few simple rules based on their past forecasting performance. In the paper, two main specifications of the price level targeting model are examined—the agents form expectations either about the price level or about inflation, which is ex ante not equivalent because of the sequential nature of the model. In addition, several formulations of the forecasting rules are considered. Both regimes are assessed by performing a loss function comparison. According to the results, price level targeting is slightly preferable in case where expectations are created about the price level under the baseline calibration. It is, however, sensitive to some model parameters and there is a risk of instability. Furthermore, when expectations are created about inflation, price level targeting loses credibility over time and leads to divergence of the economy. On the other hand, inflation targeting model functions in a stable manner. Therefore, while the potential benefits of price level targeting have been confirmed under certain assumptions, the results suggest that inflation targeting constitutes a more robust choice for monetary policy.

Abstrakt

Článek srovnává režimy cílování cenové hladiny a cílování inflace v novém keynesiánském modelu s omezenou racionalitou. Ekonomické subjekty vytvářejí svá očekávání pomocí heuristického přístupu—volí mezi několika jednoduchými pravidly v závislosti na jejich předchozí predikční schopnosti. V tomto článku jsou zkoumány dvě hlavní specifikace modelu cílování cenové hladiny—ekonomické subjekty formují svá očekávání buď ohledně cenové hladiny, nebo ohledně inflace, což vzhledem k sekvenční povaze modelu není ex ante ekvivalentní. V úvahu je vzato rovněž několik různých formulací predikčních pravidel. Oba režimy jsou vyhodnoceny prostřednictvím porovnání ztrátové funkce. Výsledky ukazují, že cílování cenové hladiny je mírně výhodnější v případě, kdy si ekonomické subjekty při výchozí kalibraci vytvářejí očekávání ohledně cenové hladiny. Je však citlivé na některé parametry modelu a obsahuje riziko nestability. Navíc v případě tvorby očekávání ohledně inflace cílování cenové hladiny časem ztrácí kredibilitu a vede k divergenci ekonomiky. Naopak model cílování inflace funguje stabilním způsobem. Přestože za určitých předpokladů byly potenciální výhody cílování cenové hladiny potvrzeny, výsledky naznačují, že cílování inflace představuje robustnější volbu pro měnovou politiku.

JEL Codes: E31, E37, E52, E58, E70.
Keywords: Bounded rationality, heuristics, inflation targeting, monetary policy, price level targeting.

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1. Introduction

In the last three decades, inflation targeting (IT) has become state-of-the-art in monetary policy-making and has been officially adopted by 41 central banks to date (IMF, 2020). In addition, other central banks use many aspects of IT despite not being formally inflation targeters. Nevertheless, there has been ongoing debate and research on alternative policy frameworks, reinforced in particular by the Great Recession and the subsequent era of nominal interest rates close to their zero lower bound. One significant stream of research on alternative frameworks is focused on price level targeting (PLT).

This paper contributes by examining whether price level targeting could be a viable monetary policy regime alternative to inflation targeting. In particular, its key contribution is the analysis of price level targeting in a model with imperfect rationality, i.e. the paper relaxes the rather strong assumption of rational expectations, which is often considered too restrictive in the context of PLT. The rational expectations hypothesis is replaced by the heuristic approach, as proposed for example by De Grauwe (2012). Economic agents form expectations by choosing between a few simple rules based on their past forecasting performance; in other aspects, the model corresponds to a standard 3-equation New Keynesian DSGE model. The model used by De Grauwe (2012) is extended here to include the price level targeting regime and its corresponding expectations formation rules.

Price level targeting is a regime in which the targeted path of the price level can still increase over time (consistent with a small but positive inflation rate), but contrary to inflation targeting, the price level targeting regime compensates for past deviations of inflation from its steady state rate. The difference seems small, but economic literature in general suggests that it could have a significant positive impact on the economy due to the stabilizing role of inflation expectations.

The crucial aspect of the PLT framework is that lower inflation now leads to higher inflation expectations for the future and vice versa. As a result, the dynamics of inflation expectations mitigate inflation shocks and decrease macroeconomic volatility. This mechanism works well in theoretical models with rational expectations and a fully credible PLT regime. These are, however, fairly strong assumptions. If the public did not understand the regime correctly or did not trust the central bank sufficiently, the inflation expectations would not adjust in the desired way and the key mechanism of the regime would no longer hold.

Moreover, there is very limited historical experience with price level targeting, so it is not possible to compare it with inflation targeting empirically. In fact, a lack of confidence in the strong assumptions among policymakers is probably the main reason the discussion remains only theoretical and no central bank uses pure PLT in practice. Therefore, probably the only option is to try to relax the assumptions within the framework of the theoretical models and to compare IT and PLT in a different setting. This is the goal of this paper.

The applied method of modelling expectations formation inherently contains endogenous credibility of a monetary policy framework and therefore helps us to examine the crucial aspects of PLT—its credibility and understandability among the public. Furthermore, the heuristic approach involves a greater departure from rational expectations than other models of bounded rationality, such as adaptive learning models. Heuristic methodology thus enables us to assess the robustness of the performance of price level targeting in a framework which is more removed from perfect rationality, so the monetary policy regime is subject to greater scrutiny. Furthermore, the choice of methodology is motivated and supported by some empirical evidence about expectations formation (see section 3 for details).

Overall, the analysis suggests that inflation targeting is more robust and is thus a safer choice of monetary policy regime than price level targeting in the presence of model uncertainty. The theoretical advantages of price level targeting are confirmed in certain model specifications, but they are sensitive to particular assumptions about expectations formation—the PLT regime is unstable for other specifications. Conversely, inflation targeting performs reasonably well in all the cases examined.

The paper is structured as follows: section 2 presents the key characteristics of price level targeting and the related literature review. Section 3 discusses the macroeconomic model with expectations created using heuristics. Section 4 presents the results of the analysis, while section 5 assesses their implications. Section 6 concludes. The appendices contain some supplementary mathematical derivations and calibration together with a robustness analysis.

2. Price Level Targeting

In theory, price level targeting could offer several benefits when compared to inflation targeting. As Cournede and Moccero (2011) or Popescu (2012) discuss, if the regime is credible and well understood by the public, then any shock to prices in one direction causes inflation expectations to move in the opposite direction. The real interest rate then adjusts in the desired direction even in the case of a stable nominal interest rate. This mechanism not only leads to a less volatile nominal interest rate, but more importantly, it can also anchor inflation closer to its long-term desired rate and decrease output volatility (see subsection 2.1 for a more detailed discussion on volatility under PLT). In addition, there is also a lower probability of hitting the lower bound on nominal interest rates.

Both Cournede and Moccero (2011) and Popescu (2012) also mention that price level targeting could decrease the probability or severity of asset price bubbles. Nominal rates below the neutral level for a sustained period of time can contribute to the creation of asset price bubbles and—as discussed above—PLT requires a less aggressive adjustment of the rates. In addition, PLT leads to lower price level uncertainty in the long term, which makes intertemporal planning easier. On the contrary, under IT, the price level is not stationary and its variance increases up to infinity in the very long term (Minford and Peel, 2003).

The benefits of PLT hinge on the anchoring of expectations about future prices. The mechanism functions very well when all the economic agents are forward looking and rational, and when the monetary policy regime is fully credible. But such assumptions are hardly realistic. In practice, central bank communication would be difficult, as the short-term targeted inflation rate changes over time in reaction to past shocks to the price level, while communicating the targeted price level (e.g. the level of the consumer price index) would be cumbersome as this is a much more abstract variable. Moreover, where central bank credibility is imperfect, people could no longer form inflation expectations in the direction opposite to the shock and the key advantage of price level targeting would no longer be valid. Finally, PLT is prone to the problem of dynamic inconsistency (Bohm et al., 2012).

These challenges show why the regime remains a rather theoretical concept and why central banks are reluctant to use it in practice. Swedish monetary policy in the 1930s is the only example from history widely considered as price level targeting (see e.g. Berg and Jonung (1999) and Straumann and Woitek (2009)). Bohm et al. (2012) suggest that the deflationary policy that was part of the currency reform in Czechoslovakia after WWI could be also considered as PLT. Nevertheless, neither experience says much about the effectiveness of the PLT framework in general, as both were short-lived, adopted during turbulent times and had very specific characteristics.[1]

On the other hand, after its monetary policy framework revision in August 2020, the Fed effectively adopted average inflation targeting, which allows the temporary overshooting of the inflation target after a period of inflation below the target. The policy can be considered as a compromise between IT and PLT.[2] While PLT has not been adopted in the pure sense, this shift in Fed policy confirms that it is to the forefront of current monetary policy discussions and corroborates the need for further research in this area.

2.1 Literature Review

Much of the research regarding price level targeting is influenced by the seminal paper of Svensson (1999a). The model presented in his paper contains a neoclassical Phillips curve and assumes that a central bank has complete control over the inflation rate and discretionarily chooses the rate in each period in order to minimize loss function. The loss function has two main specifications—one with the deviation of the inflation rate from its target (corresponding to IT) and the other with the deviation of the price level from its targeted path (corresponding to PLT); deviation of the output gap from its (non-negative) target is present in both versions. If the output gap is at least moderately persistent, PLT leads to lower inflation variability than IT. In particular, if the desired output gap is zero, output gap persistence above 0.5 is sufficient for such a result. Moreover, even if society prefers inflation stabilization as opposed to price level stabilization (i.e. society's loss function is specified in terms of inflation deviation from the target), assigning the goal of price level stabilization to the central bank leads to results closer to optimal monetary policy than assigning an inflation target.

While Svensson uses a backward-looking model, Vestin (2006) confirms his key result in a standard forward-looking New Keynesian model. He assumes that a central bank can operate only in a discretionary environment, but an optimal commitment solution can be replicated under PLT in a scenario in which there is no inflation persistence. This is a rather strong result. In addition, PLT leads to a better inflation-output volatility trade-off than IT even with inflation persistence. Cover and Pecorino (2005) also complements Svensson's analysis and shows that his results are valid under a broader set of assumptions (even without output gap persistence).

Some further support for PLT is provided e.g. by Berentsen and Waller (2011) in a real business cycle setting; by Ball et al. (2005) in an inattention framework; by Cateau (2017) in the presence of model uncertainty; and by Eggertsson and Woodford (2003) in the presence of the zero lower bound.

[1] The price level targeting element is also part of the "foolproof way of escaping from a liquidity trap" presented by Svensson (2000), which served as the theoretical underpinning of the exchange rate commitment in the Czech Republic in 2013-2017. As Franta et al. (2014) discuss, the Czech National Bank in fact foresaw that the commitment would lead to an increase in inflation above the 2% target, followed by convergence to the target from above. This was expected to compensate for the previous period of low inflation and bring the average inflation rate close to the target; the policy therefore implicitly included price level targeting characteristics. On the other hand, the policy was based on a nominal exchange rate peg which had no commitment to a future price level. Furthermore, there was no intention of abandoning the inflation targeting framework. Such an approach hence represented both the potential benefits of PLT as well as the reluctance of policymakers to adopt the regime fully due to the perceived cognitive limitations of the public.

[2] Technically, average inflation targeting with the average computed over an infinite time horizon corresponds to price level targeting.

While there is clearly substantial support for price level targeting in rational expectations models, several papers relax some of the strong assumptions of such models. For example, Masson and Shukayev (2011) introduce an escape clause which allows the price level target to be reset after large shocks. It turns out that this possibility endangers the credibility of the policy regime, may lead to multiple equilibria, and may result in PLT performing worse than IT.

Honkapohja and Mitra (2020) replace rational expectations with adaptive learning; the full credibility of PLT is replaced by only partial credibility, which evolves endogenously over time. Their results are mixed—as long as PLT has at least some initial credibility, the regime outperforms IT during a liquidity trap; on the other hand, IT is superior when the zero lower bound constraint is not binding. The implications of imperfect knowledge and adaptive learning are also explored by Eusepi and Preston (2018) and Gaspar et al. (2007) who find that price level targeting can constitute an appropriate policy regime even without rational expectations.

The transition from IT to PLT is examined by Cateau et al. (2009), who use a large-scale macroeconomic model used in the Bank of Canada (ToTEM). They model the transition between regimes as a Markov switching process with an endogenous state of either low or high policy regime credibility. Low credibility corresponds to the positive probability of switching back to IT, while high credibility represents rational expectations consistent with PLT. It turns out that the potential welfare gain from a switch to PLT can be quite substantial, and it would become negative only if the economy was in the low credibility state for at least 13 years.

Yetman (2005) introduces consumers who use rule-of-thumb forecasting into an otherwise standard macroeconomic model with rational expectations and shows that even a small proportion of rule-of-thumb consumers reverses the optimality of price level targeting. While the performance of both IT and PLT diminishes with increasing number of these consumers, PLT is less robust and deteriorates faster. Such a conclusion is actually in line with the results of this paper presented in section 4, even though the particular details of both the methodology and the results of this work differ from those in Yetman's paper.

This paper uses a heuristic approach to model deviation from rational expectations. To the author's best knowledge, the only previous work using this methodology on the issue of price level targeting is the paper by Ho et al. (2019). The authors use an open-economy version of the model to examine different monetary policy regimes, and they conclude that PLT leads to greater economic stability and social welfare than IT. While the general approach applied in that paper is similar to the methodology used here, these works differ in several ways. Ho et al. (2019) examine only one very specific case in which agents forecast the future price level under PLT (not future inflation), while the targeted price level remains constant. The analysis conducted here will examine the implications of a broader set of assumptions—people making forecasts about future inflation will be explored as well, and other versions of forecasting rules (not present elsewhere in the literature) will be considered. The model used here also differs slightly in the exact formulation of policy rules; it allows for a positive inflation target and a positive natural real interest rate. Most importantly, it turns out that the general implications of the analysis presented here will be quite different from the results of Ho and his co-authors.

More detailed surveys of the literature on price level targeting are provided by Ambler (2009), who generally confirms the superiority of PLT in rational expectations models; and by Hatcher and Minford (2016), who focus on developments since Ambler's survey and discuss papers on optimal monetary policy, the zero lower bound, the transition from IT to PLT, and financial frictions. Overall, the authors also favor PLT, although this conclusion is not unequivocal.

To summarize, models with rational expectations provide strong support for price level targeting in various model settings. There have been a few attempts to model expectations differently, which have led to mixed results—PLT is still superior to IT in some cases (although at least a certain level of credibility is crucial for such result). According to other studies, however, it ceases to perform well without rational expectations.

3. Model

While rational expectations models generally favor price level targeting, they are usually considered too unrealistic in this context. This motivates the use of an approach with bounded rationality. In particular, rational expectations will be replaced by heuristics, where economic agents form their expectations based on a few simple rules, as proposed by De Grauwe (2012). The method is also motivated by some empirical evidence (e.g. by Branch (2004) from survey data and by Hommes (2011) based on a laboratory experiment) suggesting that agents actually form expectations by switching between a few simple rules. Furthermore, a significant stream of literature, including Mankiw et al. (2003) and Andrade et al. (2016), shows significant and time-varying heterogeneity in inflation expectations (and, in the case of the latter, also in expectations of other macroeconomic variables) across economic agents, which can be captured using the heuristic approach.

The heuristic approach is not the only possible way of modelling imperfect rationality. One common branch of bounded rationality models uses adaptive learning. Although the details depend on a particular setup, the departure from rational expectations is generally still quite small in the adaptive learning framework, as economic agents are assumed to know the full structure of the economic model—they just do not know the values of the model parameters. They instead estimate those using simple econometric techniques, usually least squares. They re-estimate the parameters in each time period and learn about their true values over time. Such an approach generally converges to rational expectations, but transition dynamics may have consequences for the business cycle. By contrast, heuristic expectations represent a larger departure from rational expectations with no convergence towards them. The methodology of this paper thus scrutinizes the performance of price level targeting to a greater extent than adaptive learning models.[3]

Apart from expectations formation, the applied model corresponds to a standard small New Keynesian DSGE model. In particular, a dynamic IS curve (or aggregate demand equation) based on the utility maximization of individual households looks as follows:

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1-a_1) y_{t-1} + a_2 (i_t - \widetilde{E}_t \pi_{t+1} - \bar{r}) + \varepsilon_t, \tag{1}$$

where y_t is the output gap in period t, i_t is the nominal interest rate, $\bar{r}$ is the steady state real interest rate (assumed to be constant over time for simplicity), π_t is the inflation rate, and ε_t is a white noise disturbance term (no autocorrelation in the error term is used in the baseline model). The tilde above the expectations operator E captures the fact that expectations are not rational. The lagged output gap accounts for habit formation in consumption. The a_1 parameter is inversely related to the degree of habit formation ($0 < a_1 < 1$) and $a_2 < 0$.

[3] Adaptive learning models are summarized in a comprehensive way by Evans and Honkapohja (2001) and more recently surveyed by Milani (2012) and Eusepi and Preston (2018)). Another option for modelling bounded rationality is, for example, behavioral New Keynesian model containing a cognitive discounting parameter directly in the IS curve and the Phillips curve introduced by Gabaix (2020). A further overview of the topic can be found in Woodford (2013).

The New Keynesian Phillips curve (aggregate supply equation) based on the profit maximization of individual firms operating in monopolistic competition and a Calvo price setting environment then has the form:

$$\pi_t = b_1 \widetilde{E}_t \pi_{t+1} + (1-b_1)\pi_{t-1} + b_2 y_t + \eta_t, \tag{2}$$

where η_t is a white noise disturbance term. The equation also includes lagged inflation, which captures the indexation of prices. The b_1 parameter captures the relative weight of forward and backward looking terms and is a function of the underlying Calvo parameter ($0 < b_1 < 1$).

The model is closed by the central bank interest rate rule. Central bank targeting inflation uses the Taylor rule with interest rate smoothing:

$$i_t = c_3 \left[\pi^* + \bar{r} + c_1^{IT}(\pi_t - \pi^*) + c_2 y_t\right] + (1-c_3)i_{t-1} + u_t^{IT}, \tag{3}$$

and the rule corresponding to price level targeting has the form:[4]

$$i_t = c_3 \left[\pi^* + \bar{r} + c_1^{PLT}(p_t - \overline{p}_t) + c_2 y_t\right] + (1-c_3)i_{t-1} + u_t^{PLT}, \tag{4}$$

where p_t is the log of the price level in period t, while $\overline{p}_t$ is the targeted price level in that period. π^* is the inflation target in the case of IT and an increase in the targeted price level under PLT (it is assumed to be same in both policy regimes). The price level target then develops according to:[5]

$$\overline{p}_t = \overline{p}_{t-1} + \pi^{*q}. \tag{5}$$

See Appendix A.1 for an explicit solution of the two versions of the model for the given expectations about the output gap and inflation (to be defined below).

Furthermore, the quadratic loss function of the standard form (in line with e.g. Clarida et al. (1999) and Walsh (2003)) will be defined to compare the two monetary policy regimes as follows:

$$L = \sum_{t=1}^{T} \left[(\pi_t - \pi^*)^2 + \lambda y_t^2\right]. \tag{6}$$

The function thus punishes deviations of inflation from the target and of output from its potential, with larger deviations having higher importance (given the quadratic nature of the function). The λ parameter captures the relative weight of output gap stability and T is the number of periods over which the comparison is performed. Note that the loss function is the same for both policy regimes—even loss under PLT is defined in terms of inflation volatility, which is the ultimate goal; PLT is approached as just a tool which may deliver more stable inflation (and more stable output gap).

Let us turn now to the description of expectations formation based on De Grauwe (2012). It is worth noting, however, that De Grauwe does not consider price level targeting at all—this paper thus extends his model for use in this monetary policy framework.

[4] This type of rule is called Wicksellian (see Woodford (2003) and Giannoni (2014) for a discussion on this). Furthermore, note that the interest rate smoothing parameter may not necessarily be the same in both policy rules, but using different values in each of the rules does not change the key results of the analysis.

[5] Superscript q makes the distinction that the relationship holds for quarterly inflation, while model equations contain annualized inflation. The log-linearized relationship is simply $\pi = 4 * \pi^q$.

In particular, economic agents are assumed to choose between two simple rules for forecasts of inflation and separately of the output gap. For both variables, there is a fundamentalist rule (corresponding to the variable at its equilibrium level) and an extrapolative rule (when the agents use adaptive expectations). Let us first focus on output gap forecasts. Formally, fundamentalist and extrapolative rules for the output gap under both policy regimes have the form:

$$\widetilde{E}_t^f y_{t+1} = 0, \tag{7}$$

$$\widetilde{E}_t^e y_{t+1} = y_{t-1}. \tag{8}$$

While economic agents are not assumed to have rational expectations, they do not choose the forecasting rules completely randomly—they take the past performance of the rules into account; we are talking about bounded rationality here. The subsequent formalization of switching between the rules is based on discrete choice theory as described e.g. by Anderson et al. (1992) and Brock and Hommes (1997). The agents evaluate the past forecasting performances of both rules by calculating the utilities (defined as the negative weighted mean squared forecast error of given rule) as follows:

$$U_{f,t}^y = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{t-k-2}^f y_{t-k-1} \right]^2, \tag{9}$$

$$U_{e,t}^y = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{t-k-2}^e y_{t-k-1} \right]^2, \tag{10}$$

where $U_{f,t}^y$ and $U_{e,t}^y$ are the utilities from fundamentalist and extrapolative rules in output gap forecasting, respectively (calculated in period t); and ω_k are geometrically declining weights—errors made in the distant past have a lower weight than recent errors—which captures the tendency to forget. When we define $\omega_k = (1-\rho)\rho^k$ with $0 < \rho < 1$, we can rewrite equations 9 and 10 as follows (This is shown explicitly in Appendix A.2):

$$U_{f,t}^y = \rho U_{f,t-1}^y - (1-\rho) \left[y_{t-1} - \widetilde{E}_{t-2}^f y_{t-1} \right]^2, \tag{11}$$

$$U_{e,t}^y = \rho U_{e,t-1}^y - (1-\rho) \left[y_{t-1} - \widetilde{E}_{t-2}^e y_{t-1} \right]^2. \tag{12}$$

The coefficient ρ then captures people's memory. $\rho = 0$ means no memory, while as ρ increases (up to 1), the importance of more distant errors grows as well.

The utilities represent the deterministic components in the choice between the two rules, but there are also stochastic elements $\varepsilon_{f,t}$ and $\varepsilon_{e,t}$. The resulting probability of choosing the fundamentalist rule is

$$\alpha_{f,t} = P \left[U_{f,t}^y + \varepsilon_{f,t} > U_{e,t}^y + \varepsilon_{e,t} \right]. \tag{13}$$

Such specification is based on the idea that the choice between the rules is influenced by both the actual performance of the rules and by the current mood of the individual decision makers, which is captured by the stochastic components. Furthermore, $\varepsilon_{f,t}$ and $\varepsilon_{e,t}$ are assumed to be logistically distributed, which leads to the probability of selecting the fundamentalist rule as follows:

$$\alpha_{f,t} = \frac{\exp(\gamma U^y_{f,t})}{\exp(\gamma U^y_{f,t}) + \exp(\gamma U^y_{e,t})}, \tag{14}$$

and to the probability of choosing the extrapolative rule:

$$\alpha_{e,t} = P\left[U^y_{e,t} + \varepsilon_{e,t} > U^y_{f,t} + \varepsilon_{f,t}\right] = \frac{\exp(\gamma U^y_{e,t})}{\exp(\gamma U^y_{f,t}) + \exp(\gamma U^y_{e,t})} = 1 - \alpha_{f,t}, \tag{15}$$

where the γ parameter measures the intensity of choice or, in other words, the willingness to learn from past errors; and it is given by the variance of $\varepsilon_{f,t}$ and $\varepsilon_{e,t}$. When the variance is zero, $\gamma = \infty$ and the choice is purely deterministic. When the variance goes to infinity, $\gamma = 0$ and the choice becomes random—the probability of choice of each rule is 0.5.

The market expectations are then given by

$$\widetilde{E}_t y_{t+1} = \alpha_{f,t}\widetilde{E}^f_t y_{t+1} + \alpha_{e,t}\widetilde{E}^e_t y_{t+1} = \alpha_{f,t}0 + \alpha_{e,t} y_{t-1}. \tag{16}$$

The same approach is applied for inflation forecasting. The extrapolative rule under IT is defined by

$$\widetilde{E}^{e,IT}_t \pi_{t+1} = \pi_{t-1}, \tag{17}$$

and the fundamentalist rule by

$$\widetilde{E}^{f,IT}_t \pi_{t+1} = \pi^*. \tag{18}$$

The situation is, however, more complicated for PLT. The model is constructed in a way that an agent at time t forms expectations about $t+1$ based on data from $t-1$. As $t-1$ and $t+1$ are not adjacent periods, it actually matters whether the agent makes expectations about future prices or inflation. In the baseline model, let us assume that expectations are formed about the price level; the other case will be described and examined in subsection 4.2. The model will be solved for p_t using the relationship $\widetilde{E}_t \pi^q_{t+1} = \widetilde{E}_t p_{t+1} - p_t$. Agents therefore form expectations $\widetilde{E}_t p_{t+1}$, while inflation expectations $\widetilde{E}_t \pi^q_{t+1}$ are created only ex post, after p_t is determined. The fundamentalist rule in this case is thus

$$\widetilde{E}^{f,PLT}_t p_{t+1} = \overline{p}_{t+1}, \tag{19}$$

and the extrapolative rule in the baseline specification is

$$\widetilde{E}^{e,PLT}_t p_{t+1} = p_{t-1} + 2 * \pi^q_{t-1}. \tag{20}$$

As $\pi^* = 2\%$, even extrapolative agents have to address increasing trend in prices. Forecasting the future price level to be equal to the past price level does not make sense. This rule is thus of a purely extrapolative nature—agents simply assume that the economy will develop in the same way as it has done in the past. The rationale of such a rule corresponds to the logic behind the IT extrapolative rule in equation 17.

The forecast performances of both rules are represented by their respective utilities computed from weighted mean squared forecast errors just as for the output gap. This leads to probabilities of choosing fundamentalist and extrapolative rules in inflation/price level forecasting (for both regimes) as follows:

$$\beta_{f,t} = \frac{\exp(\gamma U_{f,t}^{\{\pi,p\}})}{\exp(\gamma U_{f,t}^{\{\pi,p\}}) + \exp(\gamma U_{e,t}^{\{\pi,p\}})}, \tag{21}$$

$$\beta_{e,t} = \frac{\exp(\gamma U_{e,t}^{\{\pi,p\}})}{\exp(\gamma U_{f,t}^{\{\pi,p\}}) + \exp(\gamma U_{e,t}^{\{\pi,p\}})}. \tag{22}$$

Finally, the inflation market forecast under IT is given by

$$\widetilde{E}_t^{IT}\pi_{t+1} = \beta_{f,t}\widetilde{E}_t^{f,IT}\pi_{t+1} + \beta_{e,t}\widetilde{E}_t^{e,IT}\pi_{t+1} = \beta_{f,t}\pi^* + \beta_{e,t}\pi_{t-1}, \tag{23}$$

and under PLT by

$$\widetilde{E}_t^{PLT}p_{t+1} = \beta_{f,t}\widetilde{E}_t^{f,PLT}p_{t+1} + \beta_{e,t}\widetilde{E}_t^{e,PLT}p_{t+1} = \beta_{f,t}\overline{p}_{t+1} + \beta_{e,t}\widetilde{E}_t^{e,PLT}p_{t+1}, \tag{24}$$

where $\widetilde{E}_t^{e,PLT}p_{t+1}$ will be specified later. Furthermore, the $\beta_{f,t}$ parameter can also be interpreted as the credibility of a given monetary policy regime.

The rationale for heuristic rules is provided e.g. by Hommes and Lustenhouwer (2019), who derive heuristic switching rules from micro-foundations with individuals having bounded rationality. Furthermore, some empirical support for this way of inflation expectations formation by Branch (2004) or Hommes (2011) has already been mentioned.

Finally, let us define the supplementary variable AS_t, which stands for *animal spirits*:

$$AS_t = \begin{cases} \alpha_{e,t} - \alpha_{f,t} & \text{if } y_{t-1} > 0, \\ -\alpha_{e,t} + \alpha_{f,t} & \text{if } y_{t-1} < 0. \end{cases} \tag{25}$$

The variable AS_t hence ranges between -1 and 1 and is equal to 0 when the fraction of the agents using each rule is the same, i.e. 0.5. When $y_{t-1} > 0$, the extrapolative rule is optimistic, while the fundamentalist rule is pessimistic. The more agents that use the extrapolative rule (the higher $\alpha_{e,t}$), the higher the fraction of optimists and the higher AS_t. The opposite situation holds for $y_{t-1} < 0$. The variable AS_t hence captures the degree of optimism and pessimism. Values close to 1 indicate a substantial wave of optimism, while values close to -1 mean strong pessimism. Values around 0 suggest a neutral state of the economy.

For the calibration of the model parameters, see Appendix B.1. Let us now briefly discuss the main characteristics of heuristic models as described by De Grauwe (2012) and in subsequent research.

In particular, the model leads to self-fulfilling waves of optimism and pessimism, which arise when extrapolative rules prevail for some time after a shock. De Grauwe (2012) calls the waves *animal spirits* (and equation 25 is a formalization of this term), which refers to the famous concept coined by Keynes (1936), and more recently emphasized by Akerlof and Shiller (2009). Note, however, that Keynes used the term in the somewhat positive sense of spontaneous optimism, while Akerlof and Shiller (2009) and De Grauwe (2012) consider *animal spirits* to be behavioral tendencies which drive the economy out of its equilibrium.

Furthermore, the model generates fat tails in output gap distribution. Periods of large bubbles and recessions are hence more probable than in the mainstream models.

It should be noted that the overall superiority of the heuristic model compared to the mainstream models has not been established so far, as few studies have attempted to fit data and compare the heuristic and rational expectations models. One such analysis was conducted by Liu and Minford (2014), who actually reject the behavioral model on US data, while the rational expectations model passes the test. On the other hand, Jang and Sacht (2016) prefer the heuristic approach for the euro area using the method of moments, while they admit that this model has a problem dealing with the structural break in US data at the beginning of Great Moderation. Furthermore, Kukacka et al. (2018) use the simulated maximum likelihood estimation for the euro area and the US and they also favor the behavioral model, even though not unequivocally.

There is hence no clear consensus regarding a comparison of the models. The heuristic approach surely has certain relevance and it can serve as a tool to look into PLT issues from a different perspective. At the same time, it should not be considered as clearly superior to the mainstream models and the results should thus be treated with some caution.

4. Results

This section presents the results obtained from simulating the model described in section 3. The agents in the price level targeting regime are assumed to form expectations about the price level in the first subsection and about inflation in the second one. The agents in the inflation targeting regime always form expectations about inflation (with the exception of a robustness analysis, as discussed later).

In each case, simulations will be conducted 1000 times (unless stated otherwise) for 1000 time periods for both monetary policy regimes, and the loss function described by equation 6 will be computed for each simulation of each regime. The models of both regimes are simulated under the same set of random disturbances. The graphs presented here depict one random draw from the 1000 simulations.

4.1 Expectations about the Price Level

Simulating the model with rules 19 and 20 for forecasting price level under PLT leads to mixed results. Price level targeting is generally more stable and clearly superior to inflation targeting. However, it is, at the same time, susceptible to model divergence.[6]

The divergence of the PLT model occurs in 18.9% of cases. This number captures simulations in which the software actually provided a *not available* result. Nevertheless, Table 1 reveals that while most of the remaining simulations lead to a value of loss function closely distributed around the median, there are several extremely large observations. The mean value is actually higher than the 0.97 quantile, and the steep increase in loss value occurs around this quantile. Clearly, the

[6] To analyze with greater precision the probability of divergence within a given time span, the model has been simulated 100,000 times in this part of analysis.

extreme observations also represent model divergence—the divergence is not so substantial that the software would provide a *not available* result, but economically it still captures a very large level of instability. Therefore, all simulations with a loss of higher than the 0.97 quantile will be considered as divergent (these account for an additional 2.4% of the original 100,000 simulations).

Table 1: PLT Loss Functions—Descriptive Statistics

Statistic	Value
0.05 quantile	0.036
Median	0.042
0.95 quantile	0.055
0.96 quantile	0.068
0.97 quantile	0.201
0.98 quantile	1.2
0.99 quantile	7.44
Maximum	186.6

Note: The numbers are based on 81.1% stable simulations.

With this adjustment in mind, let us look at the actual results. Table 2 shows that price level targeting leads to better results with 78.7% probability, i.e. in all cases where it behaves in a stable manner. But with 21.3% probability, the PLT model diverges, while IT still performs relatively well.

Table 2: Comparison of the Regimes

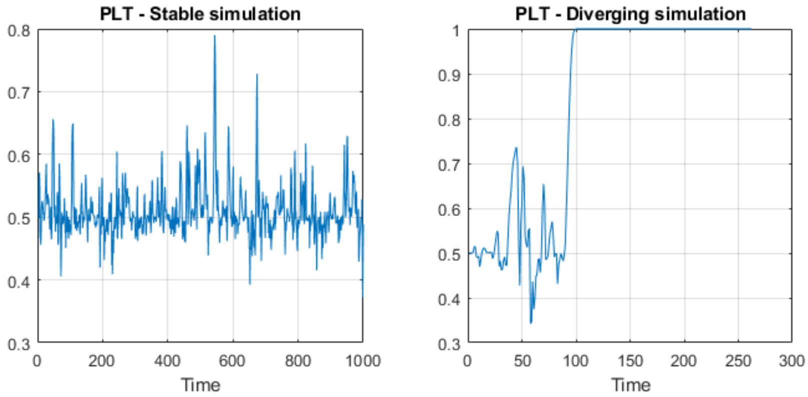
	IT	PLT
Divergence	0	21.3%
Mean loss (all simulations)	0.41	n/a
Mean loss (stable simulations)	0.39	0.04
Higher loss	78.7%	0%
Standard deviation of inflation (all simulations)	0.018	n/a
Standard deviation of inflation (stable simulations)	0.018	0.005
Standard deviation of the output gap (all simulations)	0.014	n/a
Standard deviation of the output gap (stable simulations)	0.014	0.009

Note: The mean of inflation and that of the output gap are not shown here, as when there are over 1000 simulations they just correspond directly to their equilibrium values in both policy regimes.

The mechanism of divergence is such that if a sufficiently large shock causes the economy to deviate greatly from its steady state, the fundamentalist rule becomes very imprecise and the extrapolative rule prevails. Once this happens, the situation is much more difficult to reverse under PLT than under IT. In addition, PLT performs very poorly under the extrapolative rule. Section 5 discusses the whole process in greater detail. Figure 1 shows the typical development of fraction of extrapolative agents under PLT in a stable simulation and in a diverging simulation. The main result is not sensitive to the value of the c_1^{PLT} parameter —the model diverges with a probability of 15-25% for all reasonable values of c_1^{PLT}.

Note that 1000 periods means 250 years, so a probability of divergence of 21.3% within this time span is not that high. For example, PLT outperforms IT with about 96% probability in a period

Figure 1: Fraction of Extrapolative Agents—PLT



spanning 50 years, while it has a very poor outcome in the remaining cases. The risk of divergence is therefore neither very high nor completely negligible.

Figures 2 and 3 depict several key variables for both regimes in a stable simulation. The *animal spirits* variable around the extreme values of -1 and 1 in the case of IT suggests strong waves of optimism and pessimism, while the PLT regime results in a much lower number of extreme *AS* values. Note also the difference in scales for other variables, which are better anchored around the steady state values for PLT. The persistence of the variables is higher under IT than under PLT, especially in the case of inflation persistence (the first lag autocorrelation of 0.95 versus 0.60). Figure 4 depicts the autocorrelation function for additional lags and confirms that PLT substantially decreases inflation persistence due to its compensation for past inflation shocks; autocorrelation in fact turns negative in the fourth period. The shape of the autocorrelation function nicely illustrates the key mechanism of price level targeting—higher inflation now causing lower inflation in the future, and vice versa. Finally, Figure 5 shows the fraction of agents using the extrapolative rules for both the output gap and inflation and for both policy regimes.

In addition, one more benefit of price level targeting is not captured by the loss function—as Figure 6 shows, the development of the price level under PLT is very stable around the targeted path, while it fluctuates much more in the IT regime.

So far, the analysis compared models with just two forecasting rules in order to contrast the characteristics of the individual rules. It is, however, easily possible to extend the model to incorporate more rules. Let us introduce another rule for forecasting price level under PLT, which can be called a rule with partial credibility:

$$\widetilde{E}_t^{pc,PLT} p_{t+1} = p_{t-1} + 2 * \pi^{*q}. \tag{26}$$

This rule contains substantial credibility of the central bank, as inflation at times t and $t+1$ is assumed to be at its long-term target, but delivering the price level target is not expected. Therefore we have one purely extrapolative rule (in which agents do not trust the central bank and base their expectations on past values only), one fundamentalist rule (in which agents trust that the central bank will deliver its targeted price level, so they expect compensation for any potential past inflation deviation), and one partial credibility rule between the two (agents trust that the central bank will be able to stabilize inflation at the long-run target, but they do not expect a return on the targeted

Figure 2: Variables under IT

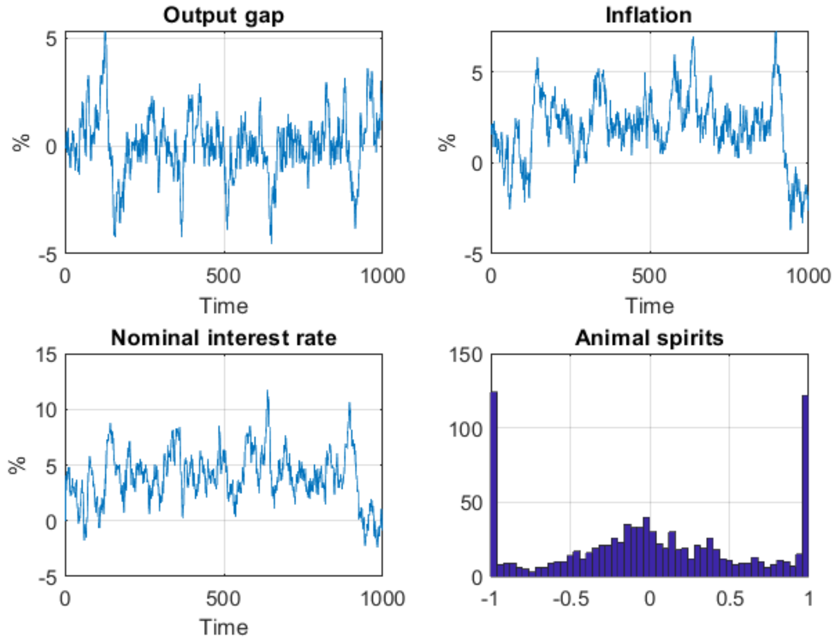


Figure 3: Variables under PLT

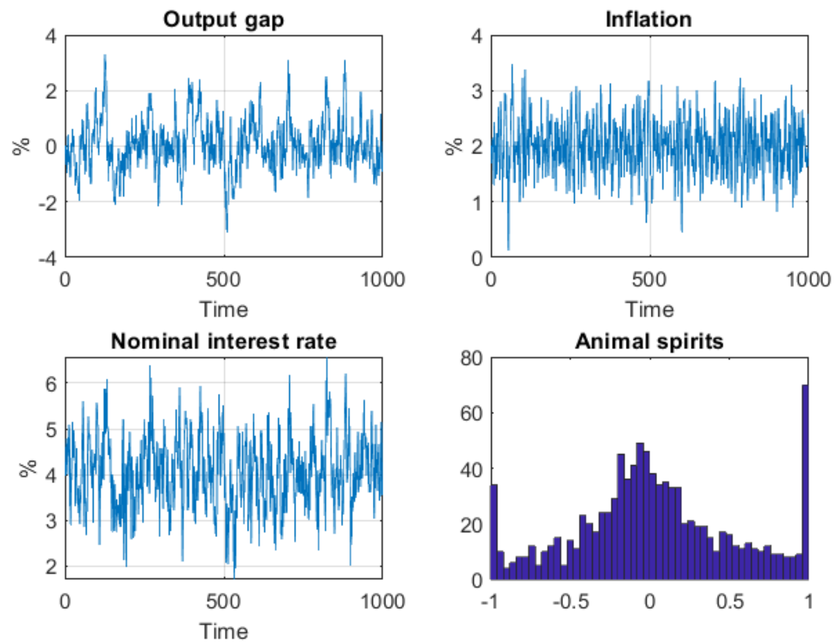


Figure 4: Inflation Autocorrelation Function

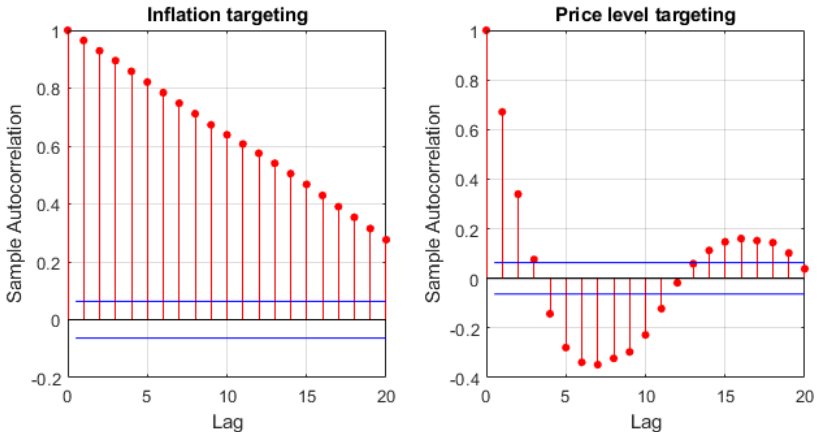


Figure 5: Fraction of Extrapolatives—Comparison of the Regimes

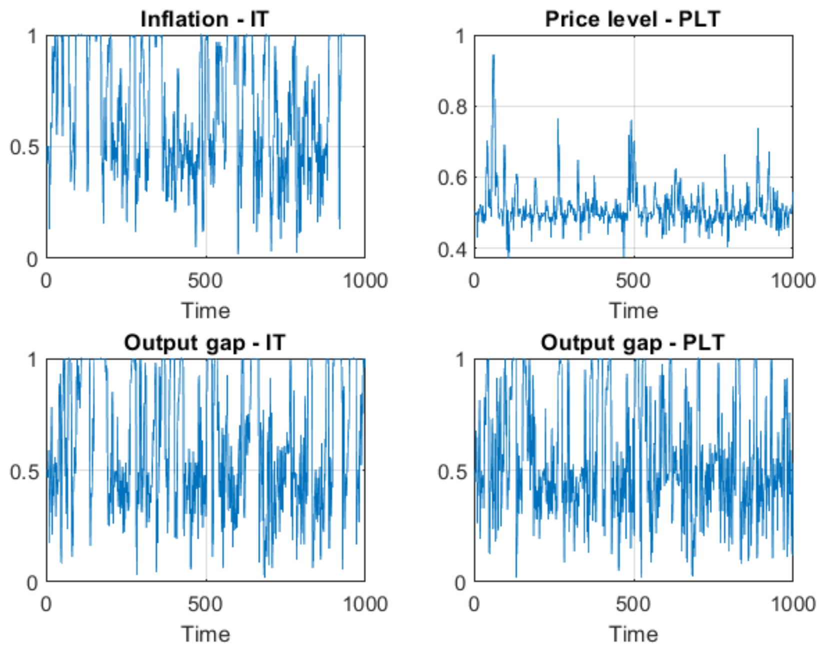
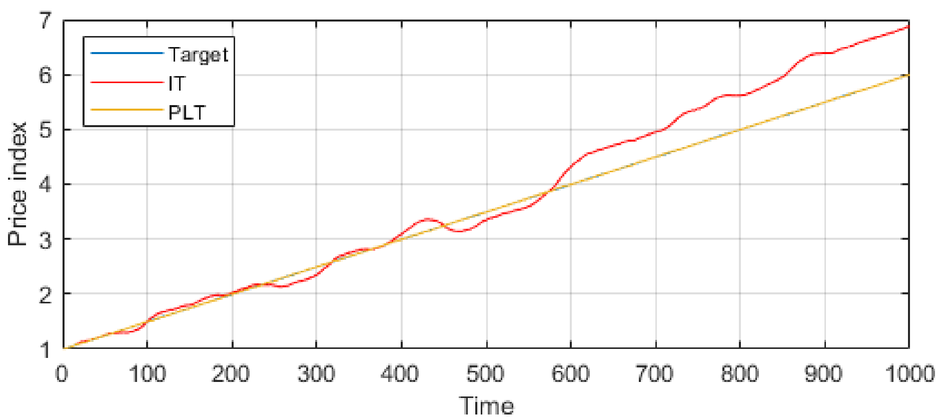


Figure 6: Price Level Development



price level—this rule in fact corresponds to the fundamentalist rule under the IT regime).[7] Such an approach allows the model to decide which rule will prevail.

Note that inflation targeting will be modelled in the same way as before using two rules only; the focus will be on price level targeting. As discussed above, let us consider one fundamentalist rule (19), one partial credibility rule (26) and one purely extrapolative rule (20). All the remaining aspects of the model are the same as before. The computation of utilities from the rules, the probability of using each of them, and the resulting market expectations are computed using a straightforward extension of the approach described in section 3.

When the model is simulated, it turns out that the PLT regime is now stable and superior to IT, as Table 3 shows—divergence is no longer a problem. Figure 7 shows that the share of economic agents using each rule fluctuates around one third. Other variables behave similarly as before (not shown in detail for brevity)—*animal spirits* are concentrated mostly around 0 for PLT and more at the extremes for IT; inflation persistence under PLT is now even lower than before (0.5), and the price level is again very predictable under PLT. The results of the three-rule analysis thus provide solid support for price level targeting.

Table 3: Loss Functions with Three Forecasting Rules

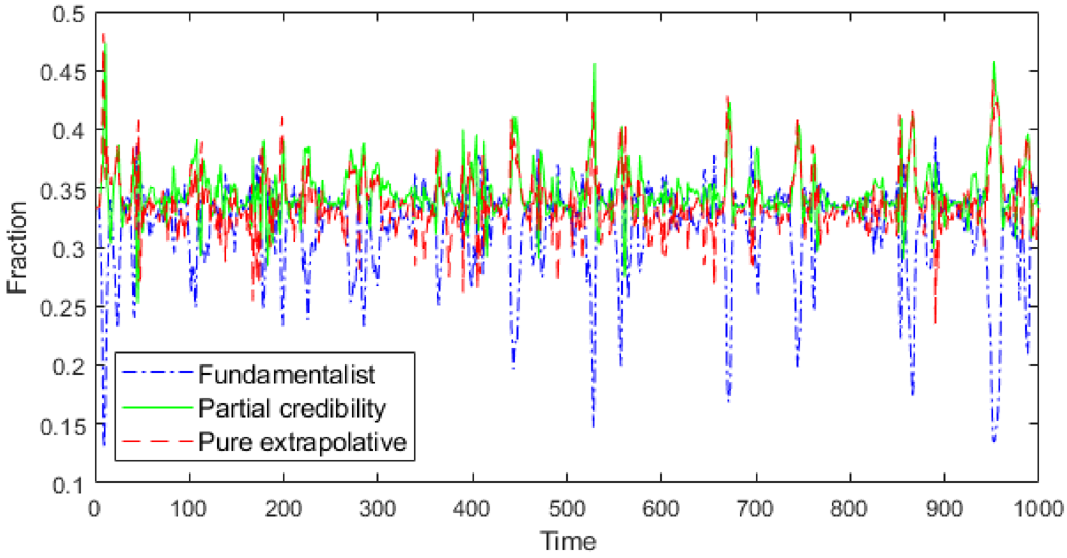
	IT	PLT
Mean loss	0.41	0.04
Higher loss	1000	0
Standard deviation of inflation	0.018	0.004
Standard deviation of the output gap	0.014	0.009

So far, the analysis has focused on case where economic agents form expectations about inflation under IT and expectations about the price level are formed under PLT. The next subsection examines the case where agents make expectations about inflation even in the PLT regime. Such duality captures the uncertainty of the agents in terms of understanding the policy regime. Creating expectations about inflation under IT seems intuitive, as inflation is a phenomenon commonly described in mainstream media, and the price level is a much more abstract variable for the general public than the inflation rate—it is not difficult to imagine what 2% year-on-year inflation means, but a price level index of, say, 140 does not mean anything without putting it into context. Thus, the key question is to what extent agents' expectations would adjust after the implementation of price level targeting. That is why more emphasis is put on expectations formation under PLT than under IT in this analysis.

At the same time, it may still make sense to examine the results of the IT regime when it is assumed that agents form expectations about the price level, i.e. the last possible combination of the monetary policy regime and expectations formation. Such model specification should not be interpreted as the

[7] If this rule was used separately in conjunction with the fundamentalist rule only, the PLT regime would give clearly superior outcomes. This has in fact been shown by Ho et al. (2019) in the case of a constant targeted price level and zero inflation target—such an approach is probably the key reason that this paper has reached different conclusions as opposed to that reached by Ho et al. (2019). But such results are rather artificial, as even the less stabilizing rule under PLT would correspond to the more stabilizing rule under IT. On the other hand, when all three mentioned rules are incorporated into one model, agents may still opt for the purely extrapolative rule, so they just have more degrees of freedom in their choice and the results are not caused by artificial restrictions on possible rules.

Figure 7: Probabilities of Using Each of the Three Rules



key result of the paper, but can serve as a robustness check about the properties of the IT regime after a certain deviation from the baseline version of expectations formation. More details are presented in Appendix B.3, but the crucial outcome of the robustness check is that inflation targeting still performs well and does not lead to divergence, which has been shown (and will be shown in the next subsection) to occur in some versions of the price level targeting model.

4.2 Expectations about Inflation

Now let us assume that even under price level targeting agents still create expectations about inflation at $t+1$, while price level expectations are now determined only ex post. The reasoning behind such an approach may be that agents still care about inflation rather than the price level (which is captured also by loss function 6, which contains deviations of inflation from the target, not of price level). The PLT regime merely serves as a tool to deliver potentially more stable inflation, which is still the ultimate goal. Agents hence still think about how high the inflation rate will be in the next period and not what the price level will be.

The extrapolative rule is straightforward:

$$\widetilde{E}_t^{e,PLT}\pi_{t+1} = \pi_{t-1}. \tag{27}$$

This is the same as the IT extrapolative rule. Extrapolative agents do not really distinguish between policy regimes here, they simply use adaptive expectations. Only fundamentalists actually take the monetary policy regime into account. This is a somewhat favorable feature of this approach as opposed to that with expectations formed about the price level.

On the other hand, there are now more potential specifications of the fundamentalist rule. The goal is to have the inflation rate at such a level that the targeted price level is met in the next period, but this is not directly possible as the agents only know the past values of variables. Therefore, an assumption about current inflation and the current price level needs to be made. Two reasonable specifications are either to assume that current inflation is equal to the long-term target ($\widetilde{E}_t^f\pi_t = \pi^*$)

and the deviation of the price level will be corrected for in the next period only, or to assume that the deviation is distributed over the current and next period with the same inflation rate. This leads to the following rules:[8]

$$\widetilde{E}_t^{f,PLT}\pi_{t+1}^q = \overline{p}_{t+1} - \widetilde{E}_t^f p_t = \overline{p}_{t+1} - p_{t-1} - \widetilde{E}_t^f \pi_t^q = \overline{p}_{t+1} - p_{t-1} - \pi^{*q} = \overline{p}_{t-1} - p_{t-1} + \pi^{*q}, \quad (28)$$

$$\widetilde{E}_t^{f,PLT}\pi_{t+1}^q = \frac{\overline{p}_{t+1} - p_{t-1}}{2}. \quad (29)$$

Thus, we have two different specifications of the fundamentalist rule. Nevertheless, it turns out that regardless of which one is used, PLT leads to divergence of the model in all 1000 simulations; the divergence occurs within 102-367 periods (without any significant difference between the two rules). The extrapolative rule always gains dominance over time, which is then very difficult to reverse. On the contrary, the IT model remains reasonably stable as before.

It is again very straightforward to extend the analysis by combining both versions into one model with three rules. But since the extrapolative rule prevails in both individual scenarios, we can expect such a model to again lead to divergence. Furthermore, actual simulations confirm this intuition. When people create expectations about inflation regardless of the monetary policy regime, price level targeting leads to divergence of the economy regardless of the particular model specification.

5. Discussion

The results presented in section 4 crucially depend on the assumptions about expectations formation, but they are fairly robust to particular values of model parameters. The only exceptions are a NKPC equation 2 shock (with increasing standard deviation of the shock and with its increasing autoregressive component, both monetary policy regimes deteriorate, but the susceptibility of PLT to instability grows substantially, while IT remains stable) and the intensity of choice parameter γ (as γ decreases, PLT becomes more stable, but it still performs worse than IT in cases where it was previously unstable). The sensitivity analysis is thus in accordance with the overall message of the results. The details of the sensitivity analysis are presented in Appendix B.2.

Overall, the results of the analysis are mixed. When economic agents form expectations about the price level, price level targeting is generally superior to inflation targeting. Such a conclusion is broadly in line with the results of Ho et al. (2019) that PLT leads to higher social welfare than IT in the heuristic model in a specific setting with a zero target. At the same time, PLT may diverge with a probability of 21.3% in the baseline version of the model presented here. This is not as high as it might seem since it captures a very long time span (the number is based on 1000 time periods, corresponding to 250 years), but it is not negligible either. This feature disappears when the model is expanded to contain one additional forecasting rule.

[8] Note that for some initial price level p_1 (leading to π_2 as the first inflation rate considered), price levels can be expressed by repeated substitution as $\overline{p}_{t-1} = p_1 + (t-2)\pi^{*q}$ and $p_{t-1} = p_1 + \sum_{j=2}^{t-1}\pi_j^q$. The rule in equation 28 can then be rewritten as

$$\widetilde{E}_t^{f,PLT}\pi_{t+1}^q = \overline{p}_{t-1} - p_{t-1} + \pi^{*q} = \pi^{*q} + \sum_{j=2}^{t-1}(\pi^{*q} - \pi_j^q).$$

This formulation nicely illustrates how price level targeting compensates for the past deviations of the inflation rate from the long-term target.

On the other hand, the outcomes reached in subsection 4.1 are based on a particular assumption about expectations. In an alternative case, where expectations are created about inflation, it turns out that price level targeting is very prone to losing credibility and it diverges in all simulations. Furthermore, it has been discussed that even in the first case of expectations about the price level, PLT starts to behave poorly with high standard deviation of the shock in the NK Phillips curve or with an autoregressive specification of that shock. Therefore, while the potential benefits of price level targeting have been confirmed in some cases, the overall results somewhat favor the inflation targeting regime, which is a significantly more robust choice. The overall message of the analysis is hence different to that of Ho et al. (2019), who provide stronger support for PLT.

Here, one obvious yet crucial question arises—what in fact causes the model to deliver such contradictory results depending on whether the agents form expectations about the price level or inflation? For a potential explanation, first let us clearly state the actual difference between the two approaches. When it is assumed that expectations are created about the price level, people simply forecast the future price level based on the past with no regard to the present. Fundamentalists just assume that the price level will be at the target and they do not care whether a past inflation deviation will be compensated for in the current time period, in the next one, or gradually in both. There is hence more flexibility and the rule remains viable and stabilizes the economy.

On the other hand, when expectations are formed about inflation, the fundamentalist rule requires an implicit assumption about the current inflation rate in order to forecast the future rate. Even when price level targeting is working well and delivers the targeted price level, if the core of the compensation for past inflation deviations occurs in the current period, while the assumption was made that it would occur in the next period (or vice versa), the fundamentalist rule becomes imprecise. Moreover, for some levels of the current inflation rate, the fundamentalist rule can in fact become inconsistent with the policy regime and thus not stabilizing for the economy. Therefore, in order for the fundamentalist rule to perform well, a sufficiently precise forecast of both the present and future state of the economy is needed. This poses stronger requirements on the economic agents. The extrapolative rule, given its simple nature, may then become more attractive and predominate over time. This part of the answer hence explains why price level targeting in the case of expectations created about inflation leads to a higher prevalence of the extrapolative rule than under the assumption of expectations formed about the price level.[9]

The second part of the answer is related to the actual performance of the policy regimes under the extrapolative rules. To examine this issue, the model has been generated without the fundamentalist rules for inflation forecasting. In other words, all economic agents use adaptive expectations and the two regimes therefore differ only in central bank's rule; monetary policy has no credibility. The fundamentalist rule comprises a stabilizing element in the original model, so the performance of inflation targeting is worse without it. That said, the regime still leads to a relatively stable development of the economy. In fact, some of the typical inflation targeting studies by Svensson (1997, 1999b) were conducted under adaptive expectations. On the other hand, price level targeting is unstable under adaptive expectations. Such a conclusion holds regardless of whether the adaptive expectations are about the price level or inflation; and it does not depend on the value of the c_1^{PLT} parameter in the policy rule either. Furthermore, the outcome is the same whether it is assumed that output gap forecasts are created according to the heuristic switching mechanism just as before, or whether adaptive expectations are assumed even in the case of the output gap.

[9] In some cases, the extrapolative rule prevails even in the case of expectations formed about price level, which then leads to divergence in some simulations, as shown in subsection 4.1. Nonetheless, the extrapolative rule prevails much more often in the case of expectations formed about inflation.

To describe the actual dynamics of the model under adaptive expectations, let us assume that the economy is in a boom due to a shock; inflation is above the target and the output gap is positive; the central bank then reacts by increasing the nominal interest rate. As a result, inflation and the output gap fall and return to their steady state values; but this takes a few time periods due to inflation persistence, so the deviation of the price level from its targeted path accumulates. When inflation and the output gap are at the steady state, central bank targeting inflation also returns the nominal rate to its steady state value and the economy remains stable. However, it is necessary for the central bank targeting price level to depress inflation further in order to meet its price level target, so the nominal rate remains high. The output gap in turn declines as a consequence. In this case, the nominal rate does not smooth the cycle caused by shocks, but on the contrary, enhances it. When the price level finally meets the target and the nominal rate declines, inflation is below the steady state. Moreover, due to inflation persistence, it remains there for some time, generating the negative deviation of the price level from its target. The central bank then decreases the nominal rate below the steady state level and the mechanism is repeated. Each time correcting for the deviation in price level requires a larger deviation of inflation from its long-term target, which then generates an even larger deviation in the price level in the opposite direction, and so on.

In addition, even though inflation persistence is a key part of the mechanism, the final outcome does not depend on the b_1 parameter in the NKPC (equation 2). As $\widetilde{E}_t \pi_{t+1} = \pi_{t-1}$ under adaptive expectations, term $b_1 \widetilde{E}_t \pi_{t+1} + (1 - b_1)\pi_{t-1}$ in that equation is independent of the value of b_1. The inflation persistence is thus inherent in the adaptive expectations framework.

To summarize, price level targeting performs well as long as it retains certain credibility. However, first, it is more vulnerable to losing credibility than inflation targeting (and the vulnerability is higher in the case of expectations formed about inflation), and second, it behaves much worse when this occurs and adaptive expectations prevail. In order for PLT to function well, the response to the price level in the policy rule needs to be combined with regime-consistent expectations.

On the other hand, the analysis presented in this paper may be somewhat unjust to price level targeting, which constitutes a certain limitation of the applied method. Firstly, the higher vulnerability of PLT under the assumption of expectations created about inflation may be partly caused artificially by the sequential nature of the model. If people knew the current state of the economy, their expectations about future inflation and the price level would be equivalent; and in reality, people presumably have more information about the present than the model suggests. (On the other hand, we can expect the information to be imperfect, and official macroeconomic data usually come with a lag.) Secondly, in terms of behavior under adaptive expectations, the PLT rule contains only the current values of economic variables; forward-looking central bank could take inflation persistence into account and normalize the interest rate before the price level reaches its target.

Therefore (and considering that inflation targeting faces difficulties in the form of the zero lower bound), it could be a mistake to disregard price level targeting completely based on its low stability, as it offers a potential solution to at least partially deal with the zero lower bound problem. At the same time, the model does reveal the sensitivity of the PLT regime to particular model specifications and to certain model parameters. It also implies that PLT leads to substantially higher requirements on forward-looking behavior of central banks. Thus, while completely disregarding price level targeting may be a mistake, at the very least the results call for a high level of caution before the potential implementation of PLT in practice, and they also corroborate the paramount importance of credibility for the proper conduct of the regime.

This analysis has focused on a comparison of pure price level and inflation targeting. It is worth noting, however, that these are not the only options. For example, average inflation targeting could be a potential way to utilize the advantages of price level targeting without endangering the credibility of monetary policy; under certain assumptions, it can in fact outperform both IT and PLT (see Nessen and Vestin (2005)). Its treatment is beyond the scope of this paper, but this framework surely merits further economic research.

6. Concluding Remarks

The paper has analyzed price level targeting, which has been previously found to outperform inflation targeting in various theoretical studies. Furthermore, the difference between the two regimes is even more pronounced when the zero lower bound on nominal interest rates is considered. Such results are, however, to a large extent obtained by a strong assumption of rational expectations, which seems to be unrealistic in practice. Therefore, a simple New Keynesian 3-equation model, which abandons the rational expectations hypothesis, has been applied here. Economic agents instead form expectations about future inflation (or the future price level) and the output gap using heuristics—they choose between a few simple forecasting rules based on their past performance. Two rules have been used in the baseline setting—the fundamentalist rule consistent with the given monetary policy regime, and the extrapolative rule corresponding to adaptive expectations. Furthermore, two main approaches to PLT have been discussed—one with the assumption that people form expectations about the future price level, and the other assuming expectations about inflation (which is not equivalent ex ante due to the sequential nature of the model).

It has been shown that price level targeting mostly outperforms inflation targeting under the assumption that expectations are formed about the price level, but there is a risk of instability. On the other hand, expectations about inflation cause PLT to diverge regardless of the particular specification of the fundamentalist forecasting rule. The regime is more prone to losing credibility when compared to IT, so the extrapolative rule begins to prevail. And if that happens, the PLT regime behaves very poorly, leading to a self-fulfilling wave of optimism and pessimism and setting the economy on an explosive path, while IT remains relatively stable. Moreover, even in models in which PLT is stable and superior under the baseline calibration, it is very sensitive to the characteristics of a shock in the NK Phillips curve. Inflation targeting is hence the safer choice under model uncertainty and, according to the results of the analysis, is therefore preferred overall to price level targeting (even though the preference is not unequivocal). The results are generally robust to the remaining model parameters and hinge especially on the actual assumptions about expectations formation.

The results are thus rather contradictory to much of the existing research, which favors price level targeting—but the research is largely determined by the assumption of rational expectations. On the other hand, the paper is in line with the approach common among policymakers, who admit the theoretical advantages of PLT, but who are also aware that the rational expectations hypothesis might be too strong an assumption and that the regime might not gain and retain sufficient credibility. This is what has been shown here.

At the same time, it is important to note that the model used here is just a simple closed economy model; it takes into account neither the length of the monetary policy horizon nor many of the other characteristics or sectors of the economy, and central bank policy rules are backward-looking. Tackling all of these issues may be a path for future research. Therefore, it would be premature to claim based on results of this paper that price level targeting is definitely not a good option for monetary policy, which would in practice lead to a high risk of the economy taking an explosive path. The policy regime surely offers substantial potential benefits, which might be especially useful in the presence of the lower bound on nominal interest rates. But at the very least, the results suggest that policymakers should be very cautious before actually adopting PLT in practice.

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Appendix A: Mathematical Derivations

This appendix shows several explicit derivations, which were skipped in the main body of the paper.

A.1 Model Solution

The applied model consists of three key equations, as presented in section 3. These are for the inflation targeting regime:

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1-a_1) y_{t-1} + a_2 (i_t - \widetilde{E}_t \pi_{t+1} - \bar{r}) + \varepsilon_t, \tag{A1}$$

$$\pi_t = b_1 \widetilde{E}_t \pi_{t+1} + (1-b_1)\pi_{t-1} + b_2 y_t + \eta_t, \tag{A2}$$

$$i_t = c_3 \left[\pi^* + \bar{r} + c_1(\pi_t - \pi^*) + c_2 y_t\right] + (1-c_3) i_{t-1} + u_t. \tag{A3}$$

For the given expectations of the output gap and inflation, the model can easily be solved by plugging equation A3 into A1, which leads to:

$$\begin{aligned} y_t = a_1 \widetilde{E}_t y_{t+1} + (1-a_1) y_{t-1} + a_2 c_3 \pi^* + a_2 c_3 \bar{r} + a_2 c_1 c_3 (\pi_t - \pi^*) + a_2 c_2 c_3 y_t + \\ a_2(1-c_3) i_{t-1} + a_2 u_t - a_2 \widetilde{E}_t \pi_{t+1} - a_2 \bar{r} + \varepsilon_t. \end{aligned} \tag{A4}$$

This equation together with equation A2 can be rearranged further:

$$\pi_t - b_2 y_t = b_1 \widetilde{E}_t \pi_{t+1} + (1-b_1)\pi_{t-1} + \eta_t, \tag{A5}$$

$$\begin{aligned} -a_2 c_1 c_3 \pi_t + (1 - a_2 c_2 c_3) y_t = -a_2 \widetilde{E}_t \pi_{t+1} + a_1 \widetilde{E}_t y_{t+1} + (1-a_1) y_{t-1} + a_2 (1-c_1) c_3 \pi^* + \\ a_2 (1-c_3)(i_{t-1} - \bar{r}) + a_2 u_t + \varepsilon_t. \end{aligned} \tag{A6}$$

which can be rewritten in matrix notation as

$$\begin{aligned} \begin{pmatrix} 1 & -b_2 \\ -a_2 c_1 c_3 & 1 - a_2 c_2 c_3 \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ -a_2 & a_1 \end{pmatrix} \begin{pmatrix} \widetilde{E}_t \pi_{t+1} \\ \widetilde{E}_t y_{t+1} \end{pmatrix} + \begin{pmatrix} 1-b_1 & 0 \\ 0 & 1-a_1 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \end{pmatrix} \\ + \begin{pmatrix} 0 \\ a_2(1-c_1)c_3 \end{pmatrix} \pi^* + \begin{pmatrix} 0 \\ a_2(1-c_3) \end{pmatrix} (i_{t-1} - \bar{r}) + \begin{pmatrix} \eta_t \\ a_2 u_t + \varepsilon_t \end{pmatrix}, \end{aligned} \tag{A7}$$

or simply

$$\mathbf{A}\mathbf{Z}_t = \mathbf{B}\widetilde{E}_t \mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{b}\pi^* + \mathbf{c}(i_{t-1} - \bar{r}) + \upsilon_t. \tag{A8}$$

The solution is then given by

$$\mathbf{Z}_t = \mathbf{A}^{-1} \left[\mathbf{B}\widetilde{E}_t \mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{b}\pi^* + \mathbf{c}(i_{t-1} - \bar{r}) + \upsilon_t\right]. \tag{A9}$$

The solution exists if $\mathbf{A}$ is not singular, i.e. if $1 - a_2 c_2 c_3 - a_2 b_2 c_1 c_3 \neq 0$, which can be rewritten as $1 \neq a_2 c_3 (c_2 + b_2 c_1)$. The solution yields y_t and π_t, which can be plugged into the interest rate rule A3 to generate i_t.

Under the assumption that agents forecast inflation even under PLT (as examined in subsection 4.2), the same approach can be applied to price level targeting, which differs only in the interest rate rule:

$$i_t = c_3\left[\pi^* + \bar{r} + c_1(p_t - \overline{p}_t) + c_2 y_t\right] + (1-c_3)i_{t-1} + u_t. \tag{A10}$$

As equation A2 determines the current inflation rate, we need to put inflation into the interest rate rule as well. This can, however, be done easily as $p_t = p_{t-1} + \pi_t^q$. But as the model equations contain annualized inflation, we need to use the form $p_t = p_{t-1} + \frac{1}{4}\pi_t$. The interest rate rule can therefore be written in the form

$$i_t = c_3\left[\pi^* + \bar{r} + c_1\left(\frac{\pi_t}{4} + p_{t-1} - \overline{p}_t\right) + c_2 y_t\right] + (1-c_3)i_{t-1} + u_t, \tag{A11}$$

where p_{t-1} and $\overline{p}_t$ are known values. This rule can be then be plugged into equation A1 and rearranged just as in the case of inflation targeting above. This yields

$$\begin{pmatrix} 1 & -b_2 \\ -\frac{a_2c_1c_3}{4} & 1-a_2c_2c_3 \end{pmatrix}\begin{pmatrix}\pi_t \\ y_t\end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ -a_2 & a_1\end{pmatrix}\begin{pmatrix}\widetilde{E}_t\pi_{t+1} \\ \widetilde{E}_t y_{t+1}\end{pmatrix} + \begin{pmatrix}1-b_1 & 0 \\ 0 & 1-a_1\end{pmatrix}\begin{pmatrix}\pi_{t-1} \\ y_{t-1}\end{pmatrix}$$
$$+\begin{pmatrix}0 \\ a_2c_3\end{pmatrix}\pi^* + \begin{pmatrix}0 \\ a_2(1-c_3)\end{pmatrix}(i_{t-1}-\bar{r}) + \begin{pmatrix}0 \\ a_2c_1c_3\end{pmatrix}(p_{t-1}-\overline{p}_t) + \begin{pmatrix}\eta_t \\ a_2u_t+\varepsilon_t\end{pmatrix}, \tag{A12}$$

or

$$\widetilde{\mathbf{A}}\mathbf{Z}_t = \mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\bar{r}) + \mathbf{d}(p_{t-1}-\overline{p}_t) + \upsilon_t, \tag{A13}$$

which differs from the case of inflation targeting by adjustments in the matrix $\widetilde{\mathbf{A}}$, vector $\widetilde{\mathbf{b}}$ (the tilde captures that it is slightly different than matrix $\mathbf{A}$ and vector $\mathbf{b}$ in equation A8), and by the term $\mathbf{d}(p_{t-1}-\overline{p}_t)$. The solution is given by

$$\mathbf{Z}_t = \widetilde{\mathbf{A}}^{-1}\left[\mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\bar{r}) + \mathbf{d}(p_{t-1}-\overline{p}_t) + \upsilon_t\right] \tag{A14}$$

on condition that $1 \neq a_2c_3(c_2 + \frac{b_2c_1}{4})$.

The derivation for the PLT model is slightly different when it is assumed that people form expectations about the price level (as in subsection 4.1). Again, we need to explicitly account for the difference between annualized and quarterly inflation and rewrite the model equations using $\widetilde{E}_t\pi_{t+1} = 4(\widetilde{E}_t p_{t+1} - p_t)$ and $\pi_t = 4(p_t - p_{t-1})$. This leads to:

$$y_t = a_1\widetilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2 i_t - 4a_2\widetilde{E}_t p_{t+1} + 4a_2 p_t - a_2\bar{r} + \varepsilon_t, \tag{A15}$$

$$4(p_t - p_{t-1}) = 4b_1\widetilde{E}_t p_{t+1} - 4b_1 p_t + 4(1-b_1)(p_{t-1} - p_{t-2}) + b_2 y_t + \eta_t, \tag{A16}$$

$$i_t = c_3\left[\pi^* + \bar{r} + c_1(p_t - \overline{p}_t) + c_2 y_t\right] + (1-c_3)i_{t-1} + u_t. \tag{A17}$$

Plugging the nominal interest rate from A17 to A15 just as before yields:

$$y_t = a_1\widetilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2c_3\pi^* + a_2(c_3-1)\bar{r} + a_2c_1c_3p_t - a_2c_1c_3\overline{p}_t + a_2c_2c_3y_t$$
$$+ a_2(1-c_3)i_{t-1} + a_2u_t - 4a_2\widetilde{E}_t p_{t+1} + 4a_2p_t + \varepsilon_t. \tag{A18}$$

Then let us put all the terms with y_t and p_t in equations A16 and A18 on the left hand side and all the remaining terms on the right hand side. This results in:

$$\begin{pmatrix} 4(1+b_1) & -b_2 \\ -a_2(4+c_1c_3) & 1-a_2c_2c_3 \end{pmatrix}\begin{pmatrix} p_t \\ y_t \end{pmatrix} = \begin{pmatrix} 4b_1 & 0 \\ -4a_2 & a_1 \end{pmatrix}\begin{pmatrix} \widetilde{E}_t p_{t+1} \\ \widetilde{E}_t y_{t+1} \end{pmatrix} + \begin{pmatrix} 4(2-b_1) & 0 \\ 0 & 1-a_1 \end{pmatrix}\begin{pmatrix} p_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} -4(1-b_1) & 0 \\ 0 & -a_2c_1c_3 \end{pmatrix}\begin{pmatrix} p_{t-2} \\ \overline{p}_t \end{pmatrix} + \begin{pmatrix} 0 \\ a_2(1-c_3) \end{pmatrix}(i_{t-1}-\overline{r}) + \begin{pmatrix} 0 \\ a_2c_3 \end{pmatrix}\pi^* + \begin{pmatrix} \eta_t \\ a_2u_t+\varepsilon_t \end{pmatrix}, \quad \text{(A19)}$$

or

$$\overline{\mathbf{A}}\widetilde{\mathbf{Z}}_t = \overline{\mathbf{B}}\widetilde{E}_t\widetilde{\mathbf{Z}}_{t+1} + \overline{\mathbf{C}}\widetilde{\mathbf{Z}}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \widetilde{\mathbf{d}}\mathbf{P}_t + \upsilon_t \quad \text{(A20)}$$

with $\mathbf{P}_t = \begin{pmatrix} p_{t-2} \\ \overline{p}_t \end{pmatrix}$. This can be solved again as

$$\widetilde{\mathbf{Z}}_t = \overline{\mathbf{A}}^{-1}\left[\overline{\mathbf{B}}\widetilde{E}_t\widetilde{\mathbf{Z}}_{t+1} + \overline{\mathbf{C}}\widetilde{\mathbf{Z}}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \widetilde{\mathbf{d}}\mathbf{P}_t + \upsilon_t\right] \quad \text{(A21)}$$

on condition that $4(1+b_1)(1-a_2c_2c_3) \neq a_2b_2(4+c_1c_3)$.

A.2 Forgetfulness

This part shows the equivalence of equations 9 and 11, as claimed in section 3. Let us start with the latter equation and substitute for $U^y_{f,t-1}$, $U^y_{f,t-2}$ etc.

$$\begin{aligned} U^y_{f,t} &= \rho U^y_{f,t-1} - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2}y_{t-1}\right]^2 = \\ &\rho\left[\rho U^y_{f,t-2} - (1-\rho)\left[y_{t-2} - \widetilde{E}^f_{t-3}y_{t-2}\right]^2\right] - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2}y_{t-1}\right]^2 = \\ &\rho\left\{\rho\left[\rho U^y_{f,t-3} - (1-\rho)\left[y_{t-3} - \widetilde{E}^f_{t-4}y_{t-3}\right]^2\right] - (1-\rho)\left[y_{t-2} - \widetilde{E}^f_{t-3}y_{t-2}\right]^2\right\} \\ &- (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2}y_{t-1}\right]^2. \end{aligned} \quad \text{(A22)}$$

Repeating the substitution $j-1$ times then leads to:

$$\begin{aligned} &\rho^j U^y_{f,t-j} - \rho^{j-1}(1-\rho)\left[y_{t-j} - \widetilde{E}^f_{t-j-1}y_{t-j}\right]^2 - \rho^{j-2}(1-\rho)\left[y_{t-j+1} - \widetilde{E}^f_{t-j}y_{t-j+1}\right]^2 - \\ &\ldots - \rho^0(1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2}y_{t-1}\right]^2 = \rho^j U^y_{f,t-j} - \sum_{k=0}^{j-1}\rho^k(1-\rho)\left[y_{t-k-1} - \widetilde{E}^f_{t-k-2}y_{t-k-1}\right]^2. \end{aligned} \quad \text{(A23)}$$

For j going to infinity, with $0 < \rho < 1$, and after defining $\omega_k = \rho^k(1-\rho)$, this expression yields:

$$U^y_{f,t} = -\sum_{k=0}^{\infty}\rho^k(1-\rho)\left[y_{t-k-1} - \widetilde{E}^f_{t-k-2}y_{t-k-1}\right]^2 = -\sum_{k=0}^{\infty}\omega_k\left[y_{t-k-1} - \widetilde{E}^f_{t-k-2}y_{t-k-1}\right]^2, \quad \text{(A24)}$$

which is identical to equation 9. The equivalence of equations 10 and 12 can be shown in entirely the same way.

Appendix B: Calibration and Robustness Analysis

B.1 Calibration

Table B1 contains calibrated values of the parameters in the baseline version. The calibration is such that one time period in the model corresponds to one quarter, but the variables are presented in annualized form to be consistent with the usual convention. Most of the parameters are the same as in De Grauwe (2012) or Ho et al. (2019) and correspond to standard DSGE models. The memory parameter ρ with a value of 0.5 means that the last forecast error has a weight of one half, while all the previous forecast errors have a joint weight of the remaining half. The intensity of choice parameter γ corresponds to De Grauwe (2012) as well, although it has a different numerical value for technical reasons.[10]

Table B1: Calibration

Parameter	Value
a_1	0.5
a_2	-0.2
b_1	0.5
b_2	0.05
ρ	0.5
γ	20000
σ_ε	0.005
σ_η	0.005
σ_u	0.005
$\bar{r}$	0.02 (2%)
π^*	0.02 (2%)
λ	0.25
c_1^{IT}	1.5
c_1^{PLT}	0.3
c_2	0.7
c_3	0.3

Note: σ_ε, σ_η, and σ_u are standard deviations of random disturbances in model equations, i.e. in the dynamic IS curve, NKPC and interest rate rule, respectively. Shock u_t relates to both IT and PLT rules 3 and 4.

The neutral real interest rate $\bar{r}$ is based on Taylor (1993) and the weight of the output gap in the loss function λ on Walsh (2003). Parameters c_2 and c_3 are based on author's simulation and are the same for both monetary policy regimes (the model has been simulated for a sequence of reasonable values of these parameters; the loss function generally decreases as the value of these parameters increases, but the rate of decrease starts to be negligible under both regimes at the values presented in the table). The c_1 parameter has been calibrated in the same manner (the IT model diverges for values approximately below 1—the model is in fact stable even for slightly lower values, about

[10] De Grauwe (2012) sets the model in a way that inflation of, say, 2 corresponds to a 2% inflation rate. But as price level is an explicit part of the model in this case, a 2% inflation rate is captured by the value of 0.02 to make the log-linearized relationship between the price level and inflation work. To compensate for this in the calculation of utilities (and given their quadratic nature), the γ parameter, originally equal to 2, has to be multiplied by 10000. Similar logic applies for standard deviations of model shocks.

0.95; but the overall logic is in the spirit of the Taylor principle), but it differs by regime. The results presented in the main body of the paper were obtained using these values.

B.2 Robustness to Model Parameters

To assess robustness, we simulated the model repeatedly with always one departure from the baseline calibration—all coefficients were increased and decreased one by one by a reasonable amount compared to their baseline value, and the effect on the results were examined. This holds not only for parameters in model equations, but also for the memory parameter ρ. Furthermore, we introduced autocorrelation into the error terms in the model equations (with autoregressive coefficients of various sizes). Most of these changes did not influence the implications of the results under the baseline calibration. The probability of PLT divergence with rule 20 does not fluctuate much from the value for the baseline calibration (21.3%) and PLT is superior in cases where it is stable. The PLT model always diverges in the model used in subsection 4.2.

The following exceptions actually influence the results: the size of the standard deviation σ_η of shock η in the New Keynesian Phillips curve 2; adding the AR(1) process to the same shock (let us label the AR coefficient as ξ_η); and the intensity of choice parameter γ. The first two of these parameters influence only the results of two models in subsection 4.1 (one with the purely extrapolative rule 20 only and one with this rule in conjunction with the partial credibility rule 26). Further, the γ parameter also influences the divergence of the model in subsection 4.2. Allow us, therefore, to focus the sensitivity analysis on these cases only, starting with σ_η and ξ_η.

Table B2: Sensitivity to Characteristics of Shock η

	$\sigma_\eta = 0.001$	$\sigma_\eta = 0.01$	$\xi_\eta = 0.1$	$\xi_\eta = 0.2$	$\xi_\eta = 0.5$	$\xi_\eta = 0.9$
PLT divergence (20)	0	1000	410	745	1000	1000
PLT divergence (3 rules)	0	717	0	9	768	1000
Median loss (PLT 20)	0.019	n/a	0.047	0.05	n/a	n/a
Median loss (PLT 3 rules)	0.019	0.14	0.041	0.047	0.139	n/a
Median loss (IT)	0.023	1.61	0.5	0.65	1.76	35.7
Loss(IT) < Loss(PLT 20)	0	n/a	33	34	n/a	n/a
Loss(IT) < Loss(PLT 3 rules)	0	32	0	0	35	n/a

Note: The table analyses two versions of the PLT model, one using the extrapolative rule 20 and one using this rule in conjunction with 26, i.e. the model with three rules. A reminder: the standard deviation $\sigma_\eta = 0.005$ and the autoregressive coefficient $\xi_\eta = 0$ in the baseline calibration. Each case has been simulated 1000 times.

The results from Table B2 suggests that both monetary policy regimes deteriorate as the standard deviation of a NKPC equation shock and the autoregressive component in the same shock increase. Most importantly, the susceptibility of price level targeting to instability grows substantially; and while it still generally outperforms inflation targeting in cases where it is stable, PLT is clearly much less robust to the characteristics of the η shock. With only a small autoregressive coefficient in the shock, it already starts to perform poorly (especially in the specification using rule 20 only), and with a moderate size of 0.5 it diverges for most of the time regardless of the particular model specification.

Now let us turn to the intensity of choice parameter γ. As a brief reminder—the lower the value of the parameter, the greater the role of randomness in the choice of forecasting rules (as discussed in section 3, $\gamma = 0$ leads to a purely stochastic choice between the rules, regardless of their performance). The periods in which all agents use just one rule hence cease to exist as the value of the parameter decreases, and the probabilities of using one or the other rule are closer to 0.5 (with a slight preference for the extrapolative rule most of the time unless $\gamma = 0$). In this case, the sensitivity analysis again includes the model with the extrapolative rule 20, and now also the model with expectations about inflation in subsection 4.2 with the fundamentalist rule 29. The model with rule 28 would give similar results to the one with rule 29, so it is not shown here for brevity (and the model specification in subsection 4.1 with three forecasting rules was stable even for the initial $\gamma = 20000$).

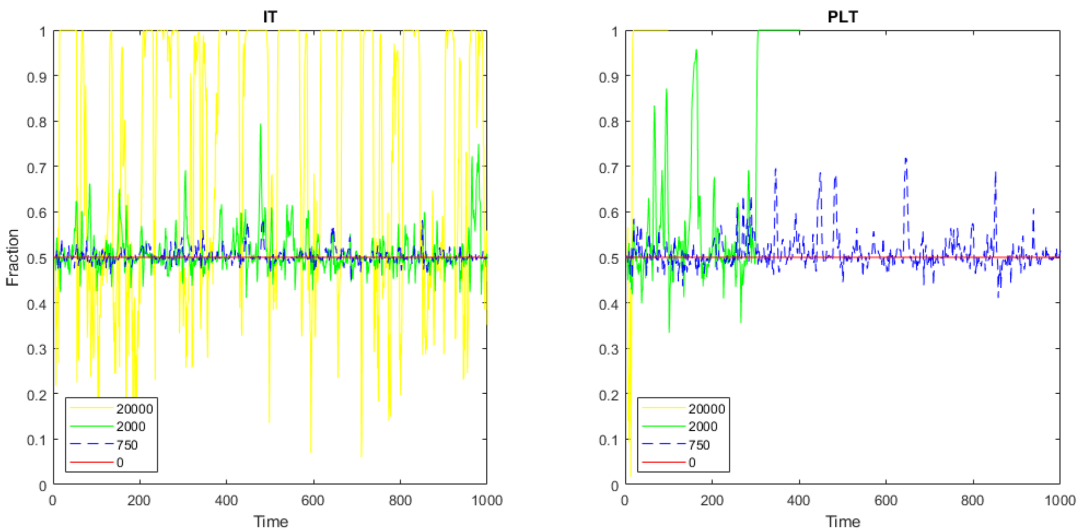
Table B3 compares simulations for different values of γ. The stability of PLT increases with decreasing γ, but it is worth noting that a lower parameter also improves the performance of IT, as it eliminates *animal spirits* and enhances the viability of the fundamentalist rule. The model under the expectations created about inflation, which previously diverged in all cases, starts to be stable with a high level of randomness in the choice of the forecasting rule. However, even when it is stable, IT still outperforms it in all simulations. Conversely, just as before, the other model using rule 20 dominates IT as long as it is stable. To gain more intuition about what the different values of γ mean, see figure B1, which depicts the fraction of extrapolative agents in forecasting inflation under several values of the parameter.

Table B3: Sensitivity to γ

γ	0	500	1000	5000	10000	20000
PLT divergence (20)	0	0	0	0	2	194
PLT divergence (29)	0	0	43	1000	1000	1000
Median Loss (PLT 20)	0.034	0.034	0.035	0.036	0.039	0.043
Median Loss (PLT 29)	0.078	0.078	0.078	n/a	n/a	n/a
Median Loss (IT)	0.069	0.07	0.07	0.11	0.26	0.4
Loss(IT)<Loss (PLT 20)	0	0	0	0	0	28
Loss(IT)<Loss (PLT 29)	959	942	885	n/a	n/a	n/a

Note: The last column with $\gamma = 20000$ corresponds to the baseline calibration. The particular numbers presented here may differ slightly from those presented in section 4, as the simulations for the sensitivity analysis have been conducted separately; however, the difference is negligible.

Figure B1: Fraction of Extrapolatives for Different Values of γ



B.3 IT Model with Expectations about the Price Level

In main part of the paper, we examined the PLT model under the assumption of expectations made either about the price level or about inflation, while examining the IT model only under the assumption of expectations formed about inflation. We have discussed the reasons for this asymmetry in main body of the paper. However, one might also ask whether it is not just this asymmetry which causes the IT regime to generally perform well, while PLT is more sensitive to model specification. Therefore, to check for the robustness of the IT model to an even greater extent, we simulated the IT model even under the assumption of expectations formed about the price level.

The fundamentalist rule can be defined as

$$\widetilde{E}_t^{f,IT} p_{t+1} = p_t + \pi^{*q} = p_{t-1} + 2\pi^{*q} = p_{t-1} + \frac{1}{2}\pi^*, \tag{B1}$$

and the extrapolative rule as

$$\widetilde{E}_t^{e,IT} p_{t+1} = p_{t-1} + 2\pi_{t-1}^q = p_{t-1} + \frac{1}{2}\pi_{t-1}. \tag{B2}$$

The exact derivations of the model are not shown here for brevity as they closely correspond to the derivations presented in Appendix A.1. We can simply take the same IT model as before, use the identities $\widetilde{E}_t \pi_{t+1} = 4(\widetilde{E}_t p_{t+1} - p_t)$, $\pi_t = 4(p_t - p_{t-1})$ and $\pi_{t-1} = 4(p_{t-1} - p_{t-2})$, as in case of the PLT model above (where $\widetilde{E}_t p_{t+1}$ is based on rules B1 and B2), and then solve for p_t and y_t.

The crucial result is that such a specification of the inflation targeting model still behaves in a stable manner and even better than in the baseline version—it now actually performs similarly as the PLT model (even when we consider only the stable simulations of the PLT model). The mean value of the loss function is 0.04, which is the same as the value presented in Table 2 for the PLT model. The inflation persistence of 0.66 is still slightly higher than for PLT, but much lower than in the IT baseline model (a reminder: in the baseline model inflation persistence was 0.60 for PLT and 0.95 for IT).

For reasons discussed in the paper, this specification seems less intuitive for IT than the specification with expectations formed about inflation, which is also why the results are not presented here in greater detail. Nonetheless, this robustness check confirms the conclusion of the paper that the inflation targeting regime is more robust to various model specifications than the price level targeting regime.

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