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# Learning and Cross-Country Correlations in a Multi-Country DSGE Model

Volha Audzei \*

## Abstract

International spillovers in estimated multi-country DSGE models with trade are usually limited. The correlation of nominal and real variables across countries is small unless correlation of exogenous shocks is imposed. In this paper, I show that introducing adaptive learning (AL) with time-varying coefficients as in Slobodyan and Wouters (2012b and 2012a) increases the international correlation. I use an estimated large-scale model as in de Walque et al. (2017), which has reasonable forecasting performance under rational expectations (RE). The model features the euro area, the US, and an exogenous rest of the world, with endogenous exchange rate determination. I show that the increase in international correlation stems from the varying coefficients and the use of simple forecasting models. The increase in the correlation of international variables goes through two channels: larger shock spillovers through the exchange rate, and correlated adjustment of agents' forecasting model coefficients.

## Abstrakt

Přelévání ekonomických efektů mezi zeměmi je v odhadnutých modelech DSGE zahrnujících více zemí a mezinárodní obchod zpravidla omezené. Korelace nominálních i reálných veličin napříč zeměmi je nízká kromě případů, kdy je v modelu zavedena korelace exogenních šoků. V tomto článku dokládám, že zavedení adaptivního učení s časově proměnlivými koeficienty jako ve Slobodyan a Wouters (2012b a 2012a) mezinárodní korelaci posiluje. Využívám odhadnutý rozsáhlý model jako v de Walque et al. (2017), který vykazuje dobrou predikční schopnost za předpokladu racionálních očekávání. Tento model zahrnuje eurozónu, USA a jako exogenní veličinu zbytek světa, přičemž měnový kurz je určován endogenně. Dokládám, že nárůst mezinárodní korelace vyplývá z proměnlivých koeficientů a použití jednoduchých prognostických modelů. Nárůst korelace mezinárodních proměnných se odehrává prostřednictvím dvou kanálů: přelévání větších šoků prostřednictvím měnového kurzu a korelovaného přizpůsobení koeficientů prognostických modelů ekonomických subjektů.

**JEL Codes:** D83, D84, E17, E31.

**Keywords:** Adaptive learning, Bayesian estimation, Multi-Country DSGE.

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## 1. Introduction

Following Smets and Wouters (2003 and 2007), dynamic stochastic general equilibrium (DSGE) models have become a widely used tool for monetary policy analysis. Yet there are certain dimensions along which such models are criticized for failing to match the data. They do not capture the variability of nominal variables well, and when it comes to international models, standard DSGE models fail to match the comovement of nominal and real variables.<sup>1</sup>

In this paper, I contribute to the literature by studying international correlation through the lens of the formation of agents' expectations in a self-referential framework. I show how relaxing the assumption of rational expectations (RE) can improve international correlations.<sup>2</sup> I introduce adaptive learning (AL) in an estimated medium-scale two-country DSGE model developed by de Walque et al. (2017). The model features the euro area (EA), the United States (US), and an exogenous rest of the world sector, linked through trade in goods and international bonds. I introduce adaptive learning with restricted perceptions and time-varying coefficients as in Slobodyan and Wouters (2012b and 2012a). Agents deviate from RE in three dimensions: they do not know the true coefficients of the model but learn them using standard forecasting techniques, they only observe the variables usually observed by forecasters, that is, the set of observables in the model estimation, and they run simple models – autoregressive of order two, AR(2) – to form their predictions. Agents use identical rules to forecast domestic and foreign variables.

In the underlying model with trade and without financial links, the exchange rate is one of the main channels of shock spillovers between countries, while the exchange rate pass-through is limited by a distributional sector. Under RE, agents use an underlying structural model to form their expectations, so their forecasting rules – including the one for the exchange rate – take into account uncovered interest rate parity (UIRP) and the structural relationships between the policy rate and other variables. Such expectations result in a larger role of the policy rate and a UIRP-consistent exchange rate reaction in the model dynamics. Under the AL specification I use, agents forecast future variables by estimating AR(2) models, including one for the exchange rate. These forecasts then enter the model through the expectational terms, thus inducing different dynamics than in the RE case, and amplify the shock spillovers between countries.

I show that introducing AL with time-varying coefficients results in greater international correlation of both nominal and real variables, as well as larger shock spillovers. I document larger cross-country spillovers in a historical shock decomposition, in particular for nominal exchange rate growth. The model with learning demonstrates a larger country shock contribution to exchange rate volatility and a larger exchange rate shock contribution to the model variables, resulting in stronger cross-country spillovers than under RE. I further show that time-varying coefficients in restricted perceptions forecasting rules are important for improving the correlation of real variables, while nominal variables demonstrate higher correlation under learning even when the coefficients are fixed. The contributions of domestic monetary policy surprises are stronger under RE, some-

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<sup>1</sup> See Del Negro et al. (2007), Lindé et al. (2016), King and Watson (1996), and Alpanda and Aysun (2014). Justiniano and Preston (2010) document the failure of an open-economy DSGE model to account for the influence of foreign shocks.

<sup>2</sup> Other approaches to bringing the models closer to the data include introducing trade in intermediate goods and services: Bergholt and Sveen (2014), Eyquem and Kamber (2014), and Miyamoto and Nguyen (2017); and introducing various financial variables: long-term interest rates in Chin et al. (2015), the financial accelerator in Alpanda and Aysun (2014), the value of foreign financial assets in Born and Enders (2018), trade finance in Patel (2016), and common shocks in Aysun (2016). Gelfer (2021) improves forecasting quality, including international correlation, by means of rich-data estimation.

times having the opposite sign than under learning, mostly due to an opposite exchange rate reaction. A different and sometimes opposite reaction to monetary policy surprises underlines the importance of the formation of expectations in monetary policy conduct.

To demonstrate what drives the results, I estimate the model under two alternative assumptions about the formation of expectations. In the first model, while agents use AR(2) models to form expectations about future variables, they add marginal costs and the real interest rate differential to the inflation and real exchange rate equations, respectively. Under this formulation, the agents' forecasting models resemble the New Keynesian Phillips curve and UIRP. When the agents use the UIRP condition in their forecasting rules, the model responses to domestic monetary policy shocks have the same sign as under RE through the exchange rate reaction, but a smaller magnitude. In the second model, agents use simple AR(2) models but with fixed coefficients. When the coefficients are fixed, the model generates smaller cross-country correlations for real variables, but, unlike in the RE model, these correlations have the correct sign. I conclude that exchange rate expectations are important factors behind the increased correlation of nominal and real variables, while time-varying parameters are important contributors to improving the correlation of real variables.

My work is related to the literature on learning in macroeconomics. Under adaptive learning, agents do not know the parameters of the model and gradually update them using standard econometric techniques.<sup>3</sup> Without knowledge of the deep model parameters, agents form their perceived law of motion (PLM) based on their parameter estimates. The agents' forecasts enter the model through the expectational terms, resulting in the actual law of motion (ALM) and inducing feedback from agents' expectations to the model dynamics. The literature either assumes that agents know the structure of the model and the PLM has the same structural form as the RE minimal state variable (MSV) solution, or allows for mis-specified forecasting rules – over or underparametrized relative to the RE MSV solution. When agents' forecasting rules are underparametrized, restricted perceptions equilibria (RPE) arise. Because agents' expectations affect the data-generating process of the model, the restricted rule can outperform the RE-consistent rule in equilibrium. In my learning specification, I restrict the agents to using simple AR(2) models to form their forecasts. The restriction is motivated by a stream of empirical and theoretical literature showing that agents' behavior is consistent with using a small number of regressors.<sup>4</sup>

The papers most closely related to mine are Slobodyan and Wouters (2012b and 2012a), who introduce adaptive learning with time-varying coefficients in a medium-scale closed economy DSGE model. In their model, agents are restricted to using simple autoregressive processes. They show that the improvement in the model fit is mostly the result of restricted perceptions with simple forecasting models. In this paper, I build on their results and use their framework to study international spillovers. There is now a growing literature on small and medium-scale DSGE model simulation and estimation under adaptive learning. Iliopoulos et al. (2019) simulate a two-country DSGE model with a financial sector and assume home bias in the way agents process information. There are a number of papers showing better model fit under AL: Milani (2007, 2007, and 2014), Vázquez and

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<sup>3</sup> A survey of this approach to learning in macroeconomics can be found in Evans and Honkapohja (2009).

<sup>4</sup> Branch and Evans (2006), Adam (2007), Hommes (2014), and Pfajfar and Žakelj (2014) show that simple AR(1) rules might be used by agents to forecast inflation in survey and experimental settings. Papers that estimate DSGE models with adaptive expectations, such as Slobodyan and Wouters (2012b) and Ormeno and Molnar (2015), show that assuming agents use very simple forecasting rules leads to superior model fit. A range of papers study the conditions under which a mis-specified forecasting rule prevails in equilibrium: Evans et al. (2012), Adam (2005), Branch and Evans (2006, 2007, 2010), and Hommes and Zhu (2014) assume that agents use a simple autoregressive process in their forecasts, with the parameters of the process determined by autocorrelation in the data.

Aguilar (2021) in a model with a term structure of interest rates, and Rychalovska (2016) with financial frictions. Markiewicz and Pick (2014) show that the AL model fits the survey of professional forecasters better. The literature shows that AL introduces endogenous persistence into DSGE models, reducing the need for rigidities to capture the data (Orphanides and Williams, 2005; Slobodyan and Wouters, 2012b; Milani, 2007). My results broadly support these findings. My estimates on the persistence of mark-up and exchange rate shocks are significantly smaller under AL than under RE, while I do not find important differences in the estimated rigidities.

The paper is organized as follows. I start with a description of the underlying model of de Walque et al. (2017) and the learning framework. I proceed by presenting the estimation setup and the results. I assess the performance of the model under RE and AL using the model fit and forecasting performance. I then present cross-country correlations and a historical shock decomposition. Finally, I address alternative model specifications. The last section concludes.

## 2. A Model of the Euro Area and the United States under Adaptive Learning

The underlying model is developed by de Walque et al. (2017). It consists of two economies with exogenous oil supply and exogenous demand from the rest of the world. Each country resembles the model in Smets and Wouters (2003), but with foreign goods and oil products entering both the consumption basket and the intermediate goods production function. There is a set of frictions standard to New Keynesian models. The countries are linked through trade in intermediate goods and international bonds. Some foreign intermediate goods are used in the production of domestic intermediate goods, while others are consumption goods. The main sources of spillovers are international trade and the exchange rate, though the pass-through to prices is limited by a distributional sector.

**Households.** Households are risk-averse. Each of them maximizes an infinite sum of expected discounted utility. They extract utility from the consumption of a combined domestic and foreign good and oil products, and they have disutility from labor. Their utility features external habit in the combined consumption good. Households are monopolistic competitors in supplying a differentiated labor type. Unions represent households with the same type of labor in negotiating wages. Nominal wages demonstrate rigidity à la Calvo, with only a fraction of unions able to renegotiate. The wages that are not renegotiated are updated by consumer price inflation, trend inflation, and the deterministic growth rate of the economy. Households invest in domestic and foreign investment goods. Each investment has its own adjustment costs. There are also adjustment costs for imported products, so that it is costly to change the consumption of imported goods. Households can issue and buy international bonds. The demand for bonds pins down the exchange rate as the uncovered interest rate parity condition. I assume that only US households can issue international bonds and that those bonds pay the US interest rate adjusted for a wedge for foreign buyers. There assumed to be a complete set of securities and perfect risk-sharing so that household labor income does not depend on the labor type.

**Intermediate goods producers.** Monopolistically competitive intermediate goods producers use the Cobb-Douglas production function to combine domestic labor and capital. The output from the Cobb-Douglas production function is combined with oil and foreign production goods in fixed proportions. The producers set their prices in the destination currency and using the destination country rigidities. Each of the differentiated goods can be sold at home, exported as a consumption good, or exported as a production good. The producers take into account that consumption goods are

processed through the distributional sector, and price production and consumption goods differently. Nominal prices demonstrate rigidity à la Calvo, with only a fraction of producers able to reset their prices. The prices that are not reset are indexed using the inflation rate. The producers export their consumption and production goods to foreign homogeneous goods assemblers, and sell the consumption goods to domestic goods assemblers.

**Assemblers of homogeneous goods.** The differentiated intermediate goods are combined by competitive homogeneous goods assemblers, who also import differentiated consumption and production goods from abroad. A combined foreign production good is then sold to intermediate goods producers as a production input. Domestic and foreign homogeneous consumption goods are sold to the distribution sector.

**Distribution sector.** Foreign and oil consumption goods are processed by a competitive distributional sector, where they are combined with domestic final goods in fixed proportions via the Leontief production function. This limits the exchange rate pass-through without the complexity of modeling a non-tradable sector.

**International trade.** The rest of the world is represented as an exogenous demand process. A country's exports are determined by other countries' demand for imports, scaled by the presence of exogenous rest of the world demand. Total imports and exports are adjusted by their respective shares in the country's exports and imports. Oil supply is exogenous.

**Central bank.** There is an inflation-targeting central bank, which reacts to deviations of inflation from the target and to the output gap using a Taylor-like rule.

The key model equations are given in Appendix A. For a detailed description of the model, I refer the reader to de Walque et al. (2017) and Smets and Wouters (2003).

Trade and exchange rate adjustments are the channels through which shocks can transmit between countries. However, these channels are rather limited, as noted in de Walque et al. (2017). I next describe the learning setup.

## 2.1 Learning

The model equations linearized around the steady-state (described in Appendix A) can be represented as:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+1} + B_0 \varepsilon_t = C, \quad (1)$$

where  $w_t$  contains exogenous processes, AR(1) or ARMA(1,1), and lagged innovations  $\varepsilon_{t-1}$ ;  $\varepsilon_t$  is a vector of independent and identically distributed innovations with zero mean, and  $C$  is a vector of constants. Matrices  $A$  and  $B$  are functions of the model parameters. The vector of endogenous variables,  $y_t$ , consists of state variables  $y_t^s$  and forward-looking variables  $y_t^f$ :  $y_t = [y_t^s, y_t^f]$ . Forward-looking variables are variables appearing in the model with a lead; they are the variables agents have to forecast to make their decisions. In my model, for each country, agents have to forecast the following variables: consumption, investment, exports, labor, the price of capital, the return on capital, wages, consumer price inflation, home inflation, and imported and exported inflation for consumption and production goods; and a common variable: the real exchange rate. State variables are variables appearing in the model with a lag. Note that the model dynamics in (1) depend on agents' expectations about future variables through the expectational term  $A_2 E_t y_{t+1}$ .

Under the assumption of rational expectations, I can write the model solutions as

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu + T \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R\varepsilon_t, \quad (2)$$

where details on the Dynare solution for matrices  $T$  and  $R$  can be found in Adjemian et al. (2011) and Villemot (2011). The matrices are non-linear functions of the model parameters, and  $\mu$  is a vector of constants. For a model linearized around the steady state, the constants are zero, except for the observable variables. In this paper, I relax the assumption of rational expectations and incorporate adaptive learning, following Marcet and Sargent (1989) and Evans and Honkapohja (2009). I build my learning setup extensively on Slobodyan and Wouters (2012b). In particular, I assume that agents know neither the model parameters nor the model structure and use reduced-form models to forecast future variables, learn from their mistakes, and update the coefficients using the Kalman filter. I further assume that agents use a restricted set of variables in their forecasting rules. These forecasting rules are called perceived laws of motion (PLMs). My agents use simple autoregressive models with a constant in their PLMs:

$$y_t^f = X_t^T \beta_t, \quad (3)$$

where  $X_t = [\text{const}, y_{t-1}^f, y_{t-2}^f]$ . The coefficients  $\beta_t$  are updated each period using the Kalman filter described below. Moreover, agents believe that the coefficients follow a vector autoregressive process around their mean  $\bar{\beta}$ :

$$\text{vec}(\beta_t - \bar{\beta}) = F \text{vec}(\beta_{t-1} - \bar{\beta}) + v_t,$$

where  $F$  is a diagonal matrix with  $\rho \leq 1$  on the diagonal, and  $v$  are i.i.d. errors. If  $\rho = 1$  the coefficients become a random walk, and if  $\rho = 0$  the coefficients are constant over time. Thus  $\rho$  influences the magnitude of the variation in the parameters.

One can represent agents' forecasting model as seemingly unrelated regression equations (SURE) and rewrite:

$$\begin{bmatrix} y_{1t}^f \\ y_{2t}^f \\ \cdot \\ \cdot \\ y_{mt}^f \end{bmatrix} = \begin{bmatrix} X_{1,t-1} & 0 & \dots & 0 \\ 0 & X_{2,t-1} & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & X_{m,t-1} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \cdot \\ \cdot \\ \beta_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \cdot \\ \cdot \\ u_{m,t} \end{bmatrix}. \quad (4)$$

The resulting errors  $u_{i,t}$  are combinations of true innovations  $\varepsilon_{i,t}$ . Note that the errors can be correlated across variables and countries. Plugging the estimates of  $\beta_t$  and the resulting  $E_t y_{t+1}^f$  into (4) and combining them with (1), I get the actual law of motion (ALM), with time-varying matrices  $T$  and  $R$ :

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \varepsilon_t. \quad (5)$$

As agents learn from their mistakes and new observations arrive, the estimates of  $\beta$  are updated, new forecasts are formed, and matrices  $T$  and  $R$  are recalculated. Unlike in the model under the RE assumption, the constants in  $\mu$  are time-varying and can be different from zero.



As agents' PLMs under learning differ from those under RE, the model dynamics (5) and (2) differ. The RE model with trade and without financial links and global financial factors features limited shock spillovers; agents form their forecasts based on the model framework and incorporate limited spillovers into their forecasts. Under learning with simple AR(2) models, agents re-estimate their coefficients as new data arrive; their estimates for both countries then reflect the movements observed in the data. The correlation of the PLM coefficients affects the actual law of motion through the expectational terms. To see this, compare (5) with (2). The covariance between two variables  $y_t^i$  and  $y_t^j$  under learning can be written as:

$$Cov\left(\mu_t^i + T_t(i,:) \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t(i,:) \varepsilon_t, \mu_t^j + T_t(j,:) \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t(j,:) \varepsilon_t\right), \quad (6)$$

where  $T_t(i,:)$  and  $R_t(i,:)$  correspond to element  $i$  of matrices  $T_t$  and  $R_t$ . It is straightforward to see that the covariation between any two variables is different under learning through the comovement in the time-varying coefficients. Under RE, the comovement of the coefficients is zero, because the coefficients are constant.

I now formalize the Kalman updating equations for the coefficients and the variance-covariance matrix. Agents update their estimates of the model coefficients  $\beta$  using information from new observations:

$$\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1} X_{t-1} \left[ \Sigma + X_{t-1}^T P_{t|t-1} X_{t-1} \right]^{-1} \left( y_t^f - X_{t-1}^T \beta_{t|t-1} \right), \quad (7)$$

$$(\beta_{t+1|t} - \bar{\beta}) = F(\beta_{t|t} - \bar{\beta}),$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} X_{t-1} \left( \Sigma + X_{t-1}^T P_{t|t-1} X_{t-1} \right)^{-1} X_{t-1}^T P_{t|t-1}, \quad (8)$$

$$P_{t+1|t} = F P_{t|t} F^T + V.$$

The variance-covariance matrix of errors,  $u_t$ ,  $\Sigma = E[u_t u_t^T]$ ;  $V = E[v_t v_t^T]$ , is the variance-covariance matrix of the errors of the vector autoregressive processes, and  $P$  is the variance-covariance matrix of the belief coefficients. To initialize the Kalman Filter, I use the theoretical moments from the RE solution as in Slobodyan and Wouters (2012b). I use  $E(X^T X)$  and  $E(X^T y)$  from the RE solution to form a vector of mean parameter estimates  $\bar{\beta}$ . I use the mean estimate as an initial value for  $\beta$ :

$$\beta_{1|0} = \bar{\beta} = (E[X^T X])^{-1} E[X^T y]. \quad (9)$$

Given  $\beta_{1|0}$  and the theoretical moments under RE, I calculate the variance-covariance matrix of forecast errors and VAR errors:

$$\Sigma = E\left(\left[y_t^f - X_{t-1}^T \beta_{1|0}\right] \left[y_t^f - X_{t-1}^T \beta_{1|0}\right]^T\right), \quad (10)$$

$$V = \sigma_v \left(X^T \Sigma^{-1} X\right)^{-1}. \quad (11)$$

Then the initial variance-covariance matrix of the belief coefficients is:

$$P_{1|0} = \sigma_0 \left(X^T \Sigma^{-1} X\right)^{-1}. \quad (12)$$

I estimate  $\rho$  using Bayesian techniques, and I fix  $\sigma_0$ ,  $\sigma_v$  as in Slobodyan and Wouters (2012b)<sup>5</sup>.

<sup>5</sup> It was shown in Slobodyan and Wouters (2012b) that the three belief parameters are not all identified simultaneously.

### 3. Estimation

#### 3.1 Data and Calibration

For the US and the euro area, I closely follow the data choices of de Walque et al. (2017). In total, there are 22 observation series including the exchange rate and oil price series. Note that the exchange rate is defined as euro per US dollar, and appreciation corresponds to a stronger euro. The data is listed in Table C1 in Appendix C. The euro area data is from the area-wide model database. The US data is from the Bureau of Economic Analysis and the US Department of Labor. For US and euro area short-term interest rates I use information from the Fed and the ECB, respectively. I use a total of 98 quarterly observations from 1992Q3 to 2016Q4.<sup>6</sup> For labor supply, I use employment divided by the working age population and multiplied by hours worked. Following Smets and Wouters (2003), for the euro area I construct an employment variable,  $e$ , where:  $\hat{e}_t = \hat{e}_{t-1} + \bar{\beta}\gamma(\hat{e}_{t+1} - \hat{e}_t) + (1 - \xi_e)\frac{1 - \xi_e\bar{\beta}\gamma}{\xi_e}(\hat{l}_t - \hat{e}_t)$ , where  $\hat{e}$  is the number of employed people and  $\hat{l}$  is labor supply featuring Calvo adjustment of employment with probability  $\xi_e$ . I use the net exports to GDP ratio for the EA series to minimize intra-union trade in the data.

The observables are linked to the model equations as shown in Table C4 in Appendix C. Note that the observables have a constant trend. In our estimation there are 22 observables, with 22 structural shocks. A description of the shocks is given in Table C5 in Appendix C. The domestic productivity shock is allowed to impact government spending and rest of the world demand. As the focus of this study is to assess how much cross-country correlation the model can generate without imposing cross-country correlation of shocks, I do not allow for cross-country correlation of shocks when estimating the model under either RE or AL.

Some of the parameters of the model are hard to identify using the available data. I thus fix them at the calibrated or implied values. I report those parameters in Tables C2–C3. The consumption, investment, and imports to GDP ratios are fixed at their historical averages. Capital depreciation is set at 0.025 quarterly, corresponding to 0.01 annually. Following de Walque et al. (2017) and Smets and Wouters (2007), I fix the wage mark-up parameter,  $\lambda_w$ , at 0.25 and the Kimball curvature – the demand curvature from homogeneous goods assemblers –  $\varepsilon$  at 10. I also assume that demand for transit goods comoves with demand for exported goods and therefore set  $\lambda_x$  at 0. The oil demand elasticity and the shares in consumption and exports for the EA and the US are set in accordance with de Walque et al. (2017).

To initialize the Kalman-filter learning, I scale the variance-covariance matrices estimated under RE. I follow the choices of Slobodyan and Wouters (2012b) and fix the scaling parameters for the variance-covariance matrix of beliefs coefficients,  $\sigma_0$ , and the variance-covariance matrix of shocks to beliefs coefficients,  $\sigma_v$ , at 0.03 and 0.003, respectively. The choice was motivated to have a large initial variance of beliefs relative to shocks to beliefs. Slobodyan and Wouters (2012b) assessed the sensitivity of their results to these initial values and found that the influence of  $\sigma_0$  vanishes after several periods and the choice of  $\sigma_v$  does not have a significant impact on the marginal likelihood.<sup>7</sup>

The priors used in the Bayesian estimation procedure are set in accordance with the long-standing tradition in the literature and closely follow the choices of de Walque et al. (2017). The standard

<sup>6</sup> I start my sample in the 1990s to avoid changes in inflation trends in the US and the EA. This restriction is helpful in improving the RE estimates and avoids additional forward-looking variables for the AL model. I also avoid the period of negative interest rates in the EA.

<sup>7</sup> As the focus of this paper is on international correlation, I do not repeat the robustness checks for the estimation initialization here and refer the reader to Slobodyan and Wouters (2012b).

deviations of shocks are assumed to follow an inverse gamma distribution with mean 0.2 and 2 degrees of freedom. The ARMA parameters follow a beta distribution with mean 0.5 and standard deviation 0.2. For the shock correlations, I assume normality around zero with a standard deviation of 0.3.

### 3.2 Estimates

I estimate the model using the Bayesian technique. The posterior estimates are presented in Tables D1, D3, and D4 and in Appendix D.<sup>8</sup> Under AL, the most striking difference is in the estimated low persistence of the price mark-up shocks and the nominal exchange rate shock in Table D1. The mark-up shocks are estimated to be about twice as small for the home and import price mark-up shocks, the European wage mark-up shock, and the oil price shock, while the moving average components do not demonstrate significant differences. The mode persistence of the US wage mark-up shock is somewhat larger under learning (0.421 [0.209] vs 0.516 [0.179]), but the percentile intervals overlap. The risk premium for both countries and the investment shock for the US are significantly less persistent under AL.

When studying the trade-off between the estimated shock persistence and the model rigidities in Table D3, I note that the Calvo rigidities under AL are smaller, except for the foreign price shocks for both countries; this is the shock whose persistence decreases the most – it is about three times smaller under learning. The Calvo rigidities for US home prices are estimated to be larger under AL, though the difference is not very significant: 0.798 (0.038) as against 0.961 (0.028). The wage rigidities – both Calvo and indexation parameter – and EA Calvo employment have a smaller estimated mode, but the difference is not dramatic, and the uncertainty bands overlap. Interestingly, habit persistence and investment adjustment costs are estimated as larger under AL, with the intervals either overlapping (US adjustment costs and EA habit persistence) or close to each other (EA adjustment costs and US habit). The persistence in the UIRP condition is twice as large under learning, making the exchange rate more backward-looking and less forward-looking. One might also suspect that there is a trade-off between shock variance and persistence. In my estimates, this pattern can only be observed for risk premium shocks, the variance of which more doubles for both countries. The exchange rate shock and the EA home price have larger variance under AL, but the confidence interval overlaps with the RE values in Table D4. The consumer price, imported price, and wage mark-up shocks are all smaller under learning, but the difference is only significant for the imported price mark-up.

My results are broadly in line with the literature showing that adaptive learning generates internal persistence in DSGE models (Slobodyan and Wouters, 2012b, Milani, 2006, Milani, 2007, Orphanides and Williams, 2005), although my results are limited to lower persistence of exogenous processes. The model estimated under RE attributes this persistence of exogenous processes to higher rigidities or persistence of exogenous shocks. Similarly to Slobodyan and Wouters (2012b), I estimate the persistence of mark-up shocks as being about twice as small under learning, while the estimates of the model rigidities are mostly comparable under the two specifications.

A model's goodness-of-fit is often measured with marginal density statistics. I report the marginal density in Table 1. I present the marginal likelihood at the mode for my baseline AL specification with time-varying parameters, for the AL specification with fixed parameters, and for RE. Con-

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<sup>8</sup> I used 1 million posterior draws for each model. The model was estimated using Dynare 4.5.1. and the AL code by Slobodyan and Wouters (2012b).

sistent with the literature, learning results in an improvement in likelihood, albeit a rather small one.

**Table 1: Marginal Likelihood at the Mode**

AL	RE
-1496.38	-1518.0

Both the parameters and the shock estimates differ under AL and RE. Moreover, the impulse responses under learning are time-varying. However, to illustrate the model dynamics it is instructive to compare selected impulse responses under RE and AL, as I do in Appendix E. For the model with learning, I calculate the median impulse responses. For the productivity and risk premium shocks, the model with learning demonstrates smaller short-to-medium term responses to the shocks, as shown in Figure E1 for the US productivity shock. For the monetary policy shocks, the transmission to the rest of the variables, including consumer price inflation, is delayed under learning. The domestic policy rate reacts more strongly and persistently under learning, even though the shock itself is less persistent. A stronger policy rate response triggers a delayed but stronger real exchange rate reaction, which transmits the shock to the foreign economy, as shown in Figure E2 for the US monetary policy shock. The home price, consumer price, and imported price mark-up shocks trigger opposite reactions of the exchange rate under AL and RE, at least in the short term, causing a difference in the models' reactions, as shown in Figure E3. The real exchange rate shock is less persistent under learning (see Figure E4), while causing an exchange rate reaction of a similar size as under RE. Most of the variables are less responsive under learning than under RE. Output is an exception; under learning, a currency depreciation has a clear expansionary effect on EA output, while US output contracts in response to appreciation of the US dollar. Both effects are ambiguous under RE.

## 4. International Correlation and Spillovers

### 4.1 Correlation Parameters

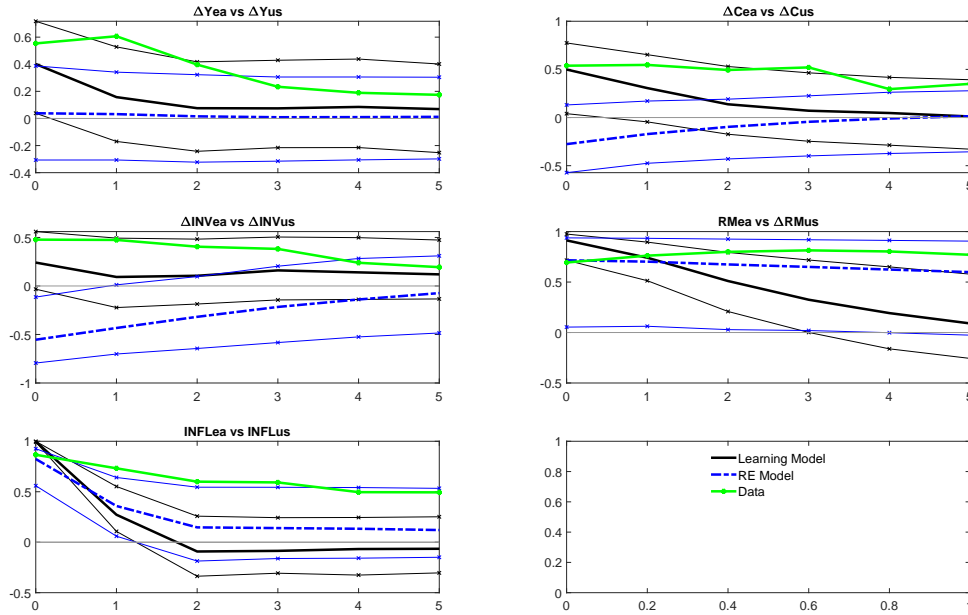
Having estimated the model under AL and RE, I continue with an international correlation analysis. I first draw 60 parameter draws from the posterior distribution. I then simulate the model 100 times for 98 periods under each draw. In the simulations under adaptive learning, the time-varying transition matrices  $T$  and  $R$  and the vector of constants  $\mu$  are calculated for each parameter draw. As a result, I have 6,000 time series for the variables of interest. To get the correlation coefficients, I run VAR(4) for the following variables: real GDP growth, real investment growth, real consumption growth, consumer price inflation, the 3M interest rate, and growth in the real exchange rate. I run VAR(4) on the empirical counterparts to get moments from the data.

Figure 1 shows the median correlation functions together with the 90 percentile intervals for the models with RE and AL and the correlation estimated from the data.<sup>9</sup> In the model with learning and time-varying coefficients, the mean of the cross-correlation is larger for both nominal and

<sup>9</sup> A similar empirical correlation for nominal and real variables was reported in, for example, Aysun (2016), who found a GDP growth correlation of 0.51 between the US and the EA and inflation correlations of 0.67 between the US and Germany and 0.76 between the US and France. Studies have documented larger correlations for nominal variables than for real variables (Henriksen et al., 2013; Wang and Wen, 2007) as well as an increasing international correlation over time due to deeper financial linkages or global factors (Pintus et al., 2019; Óscar Jordà et al., 2017).

real variables for the contemporaneous and short-term correlations. The RE model predicts a negative correlation for investment and consumption and an almost zero correlation for output growth. Learning “corrects” for this: the 90 percentile lies above zero for the contemporaneous correlation. In later periods, the 90 percentile interval widens under both concepts, inducing a larger overlap. As for the nominal variables (consumer price inflation and 3M interest rates), the 90 percentile intervals are narrower under AL and demonstrate persistence that overshoots the data.

**Figure 1: Cross-Country Correlations**

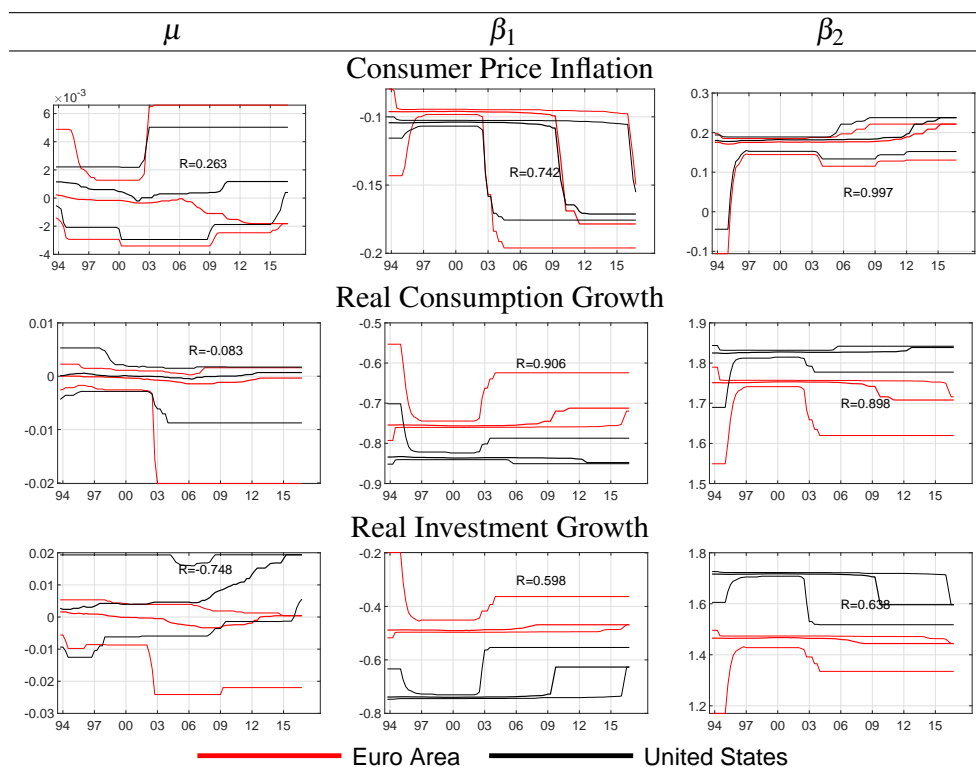


**Note:** The thick lines show the medians of the simulated correlations. The thin lines show the 5 and 95 percentiles of the distributions.

To highlight the effect of time-varying coefficients, I plot the correlations for alternative estimations with constant coefficients ( $\rho = 0$ ) in Figure ?? in Appendix G. The estimation with fixed coefficients yields little improvement in correlation relative to RE for real variables, while still resulting in the correct sign. As for the nominal variables (consumer price inflation and nominal interest rates), learning with fixed coefficients increases the international correlation relative to RE, reflecting larger spillovers through the exchange rate channel. To explore the role of time-varying parameters further, I plot the cross-country comovement in the estimated medians and 90 percentile intervals of coefficients  $\beta_1$  and  $\beta_2$ <sup>10</sup> from agents’ AR models for real consumption and investment growth and consumer price inflation in Figure 2, where I also show the correlation between the estimated time-varying coefficients. To account for posterior uncertainty, I use 60 random posterior parameter draws for 98 observations each.

The first thing to note is that the movements of the estimated AR(2) coefficients are synchronized and strongly correlated across countries, except for the constant terms. The 90 percentile intervals shift in a synchronized way, which is reflected most strongly in the median values. The correlation coefficients for consumer price inflation are 0.742 and 0.997 for the first and second lag, respectively. The coefficients in the agents’ models for real consumption growth and real investment growth have correlations as high as 0.906 and 0.598 for the first lag and 0.898 and 0.638 for the

<sup>10</sup> I do not plot the coefficients for the AR models for output and interest rates, as these are not forward-looking variables and are not predicted by agents.

**Figure 2: Comovement in Estimated AR(2) Coefficients**

**Note:** The lines are the 5, 50, and 95 percentiles of the time-varying coefficient estimates calculated for 60 posterior draws for 98 periods.  $R$  is the correlation coefficient between the 60 series of 98 observations. The first five periods were dropped to account for the excess volatility of the coefficients at the start of the updating.

second lag, respectively. The movements in the median coefficients are rather small in the first part of the sample before the Great Financial Crisis (GFC), but they become more pronounced when reflecting agents' perceptions of the crisis. In particular, agents reflect the negative trend in consumer price inflation by revising  $\beta_1$  downward, while  $\beta_2$  is revised upward, reflecting the perceived fall in inflation.<sup>11</sup>

In the linearized model, the constants, which are zero under RE, are estimates around a variable steady-state value. However, it is visible that agents estimate the drop in the steady-state value of consumer price inflation starting from 2000. After the GFC, the constant for consumer price inflation demonstrates a downward trend in the EA but not in the US. The estimated median constants for real consumption fluctuate around zero, but there is a downward revision of the percentile interval already during the Great Moderation. The constants for investment growth demonstrate a negative trend in the EA during the Great Moderation, with some revival in 2012. In the US, the steady-state estimate is growing over the entire sample. There is a small correlation for the consumer price constant of 0.263, which may be a reflection of the Great Moderation and the lack of inflation after the GFC. There is almost no comovement in the real consumption growth constant, which seems to be diverging with correlation coefficient -0.083. There is a very strong negative correlation (-0.748) for the real investment growth constant across countries, consistent with the diverging estimates.

<sup>11</sup> One can rewrite the AR(2) process for a forward-looking variable  $i$  as  $y_t^i = \mu_t^i + (\beta_{t,1}^i + \beta_{t,2}^i)y_{t-1}^i - \beta_{t,2}^i(y_{t-1}^i - y_{t-2}^i) = \mu_t^i + (\beta_{t,1}^i + \beta_{t,2}^i)y_{t-1}^i - \beta_{t,2}^i\Delta y_{t-1}^i$ .

## 4.2 Historical Shock Decomposition

As the focus of this study is on international spillovers, I start by presenting the contributions of bilateral shocks: how much euro area shocks contribute to the variation in US variables and vice versa. However, some variables and shocks demonstrate differences in variation under RE and AL. Therefore, I show the share of bilateral shocks in the total absolute variation of consumer price inflation and output growth in Figures 3–4.<sup>12</sup> Some of the shocks are estimated to have different variances under AL and RE, but their signs mostly agree over the sample. To highlight the role of shocks in driving international correlation, in each figure I also show the contribution of the same shocks in their economy of origin. The complete historical shock decompositions are plotted in Appendix F.

In Figures 3 and 4, the upper panels show the contribution of bilateral shocks to EA and US real output growth and consumer price inflation, respectively, under both AL and RE. To illustrate which shocks contribute to larger cross-country comovements, the bottom panels show the contribution of the same shocks, but to the variables in their country of origin. Figure 3 shows a larger contribution of foreign risk premium shocks under learning in both economies. The panels on the left demonstrate the larger role of US risk premium shocks in EA output growth under learning, with the risk premium responsible for about 5% of the increase in the growth rate at the turn of the millennium and 10–15% of the decline in 2009–2015. The panels on the right show a similar impact of EA risk premium shocks on US output growth, with contributions of 2% and 4%, respectively. At the same time, risk premium shocks move domestic and foreign output mostly in the same direction, contributing to larger comovement across countries. Foreign monetary policy surprises are also more visible under learning, yet they affect domestic and foreign output with the opposite sign, suggesting negative monetary policy spillovers. Under RE, while foreign monetary shocks have a smaller impact, they tend to move output growth in the same direction, indicating positive monetary policy spillovers. Bilateral investment technology and productivity shocks are less important in terms of output growth volatility than the risk premium and monetary policy, but they often move the economies in the same direction, contributing to larger comovement. EA productivity negatively contributes up to 2% to US growth over the entire period, reflecting cross-country trade links.

When analyzing the spillovers of foreign shocks to consumer price inflation in Figure 4, note that the spillovers in the model occur through the UIRP condition (A9) and through both the imported and exported price differentials. As for EA consumer price inflation, in Figure 4 US monetary policy and risk premium shocks are the largest relative contributors under learning. US monetary policy surprises put downward pressure on EA inflation before 2005 and in 2008–2009, and increase it somewhat afterward. The US risk premium drives EA inflation down after 2003, the contribution becoming larger after the GFC. The impact of US mark-up shocks on EA inflation is negligible. Under RE, the contribution of US shocks is much smaller in the EA; risk premium, monetary policy, and productivity shocks are still estimated as the most important contributors, but with monetary surprises often having the opposite impact than under learning. Unlike the rest of the variables, US inflation under learning demonstrates a smaller contribution of foreign bilateral shocks. The most important EA shocks for US inflation under AL are productivity shocks, which negatively affect inflation over the whole period. The contributions of monetary policy shocks change signs over time, perhaps reflecting the movement of the interest rate differential in the UIRP condition. EA risk premium shocks contribute negatively to US inflation starting from 2003. The RE contribution of EA shocks to US inflation is dominated by the EA risk premium, which is negative over the whole period and accounts for about 15% of the inflation volatility. As for the contribution of domestic shocks under learning, domestic mark-up shock shocks are the most important in both countries. In

<sup>12</sup> I calculate the total variation as the absolute sum of the contributions over all the shocks.

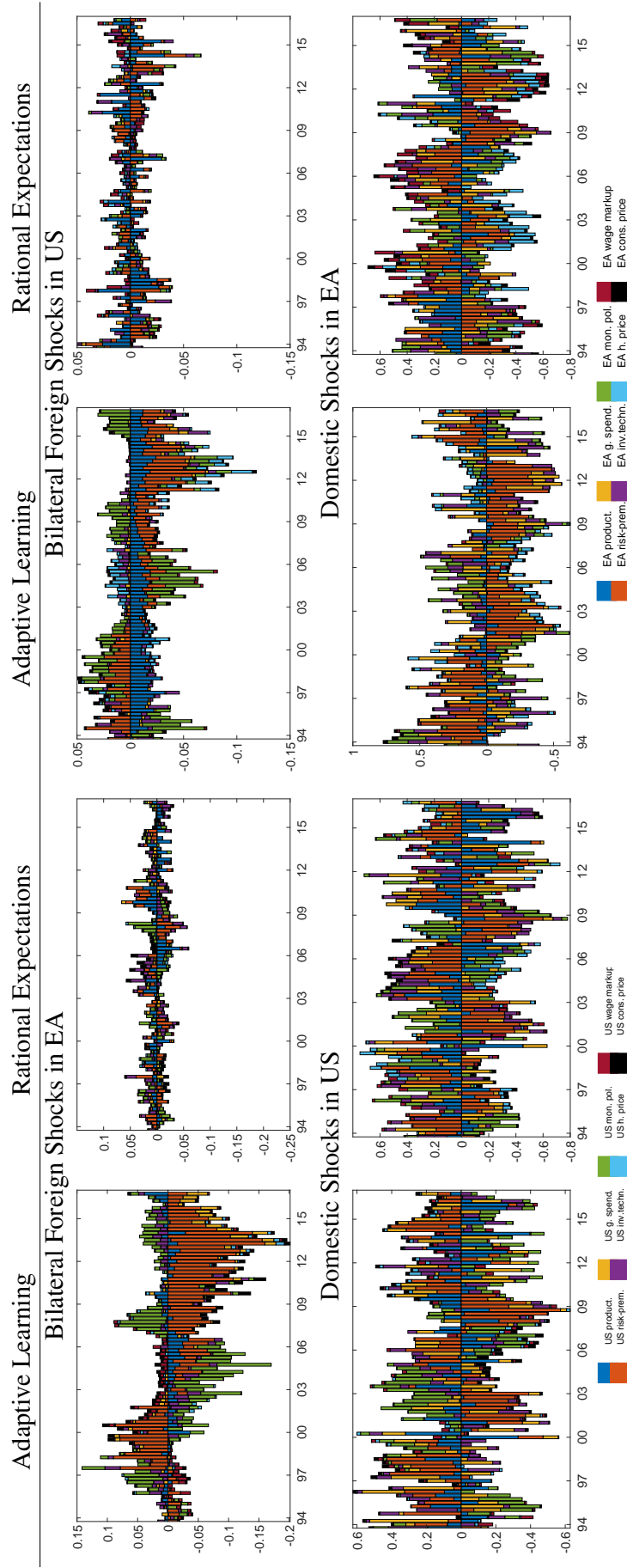
the US, a risk premium shock is seen to push inflation up from the start of the Great Moderation onward; in the EA, domestic productivity drives inflation down. Under RE, domestic monetary policy stimulates US inflation starting from 2003, while domestic risk premium and productivity shocks drive inflation down in both the EA and the US.

In Figure 5, I show the historical shock decomposition for the nominal exchange rate. In the upper panel, the figure shows the contributions of shocks grouped by their origin, while in the lower panel I show the three shocks with the largest total contributions over the period. Under learning, the impact of the countries' shocks is at least twice as large as under RE, contributing to larger cross-country shock spillovers. EA and US monetary policy shocks are among the most significant contributors under learning. Under RE the largest contributors are EA risk premium and US monetary policy shocks.

For all the variables except US consumer price inflation, under learning with time-varying coefficients foreign shocks are responsible for a larger share of the volatility of domestic variables. This is consistent with my previous finding that there is larger international correlation under learning. The relative importance of the different shocks differs under the two concepts. When it comes to the question of which shocks are responsible for larger comovement, it is not straightforward to select specific groups of shocks. For output growth volatility in both countries (Figure 3), the contribution of foreign risk premium and monetary policy shocks is significantly larger. The former may be responsible for an increase in the cross-country correlations. For consumer price inflation (Figure 4), there are periods when investment or mark-up shocks have a pronounced influence. These move output in the same direction in both countries. Productivity shocks have a material impact, though it is comparable with the model response under RE. EA and US shocks contribute more strongly to exchange rate movements under AL. This is one of the main channels of international cross-country spillovers in my open economy model.

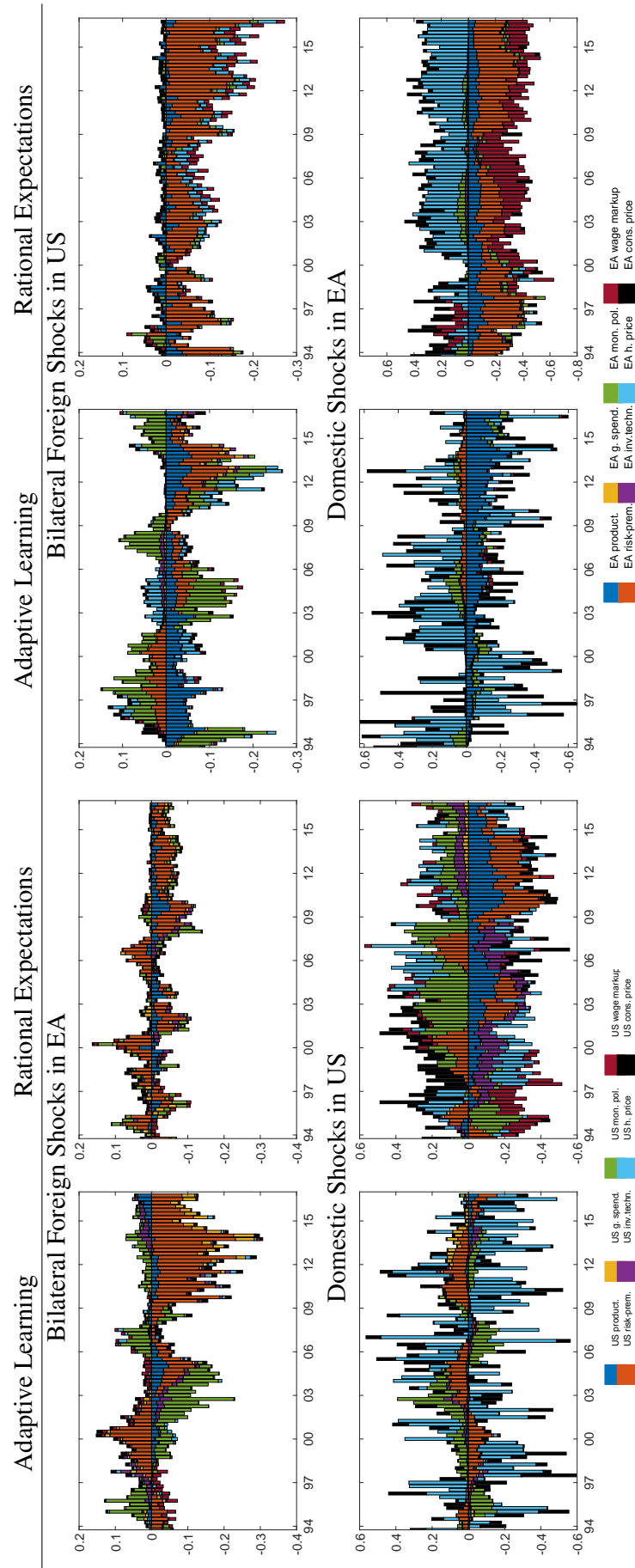


Figure 3: Historical Bilateral Shock Decomposition of Real Output Growth (relative to total absolute variation)



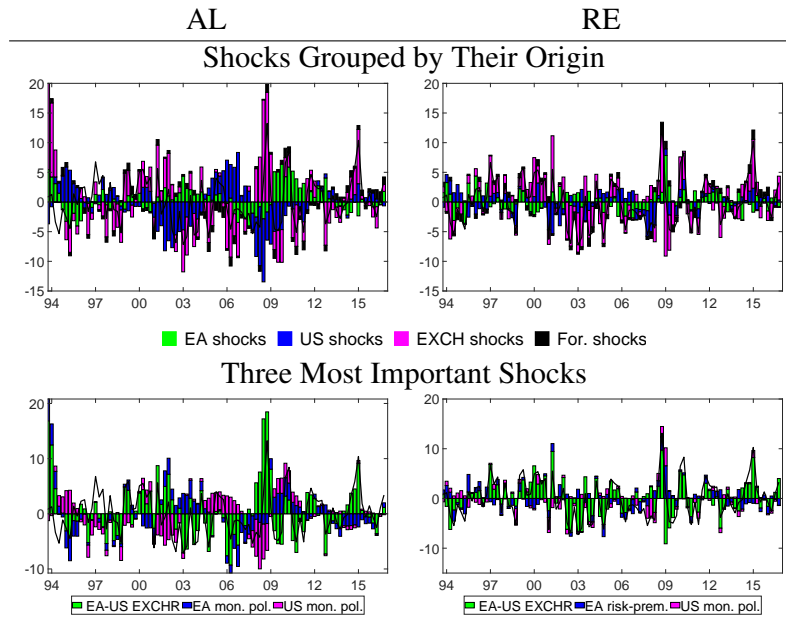
Note: Posterior mode.

**Figure 4: Historical Bilateral Shock Decomposition of Consumer Price Inflation (relative to total absolute variation)**



**Note:** Posterior mode.

Figure 5: Exchange Rate Historical Shock Decomposition



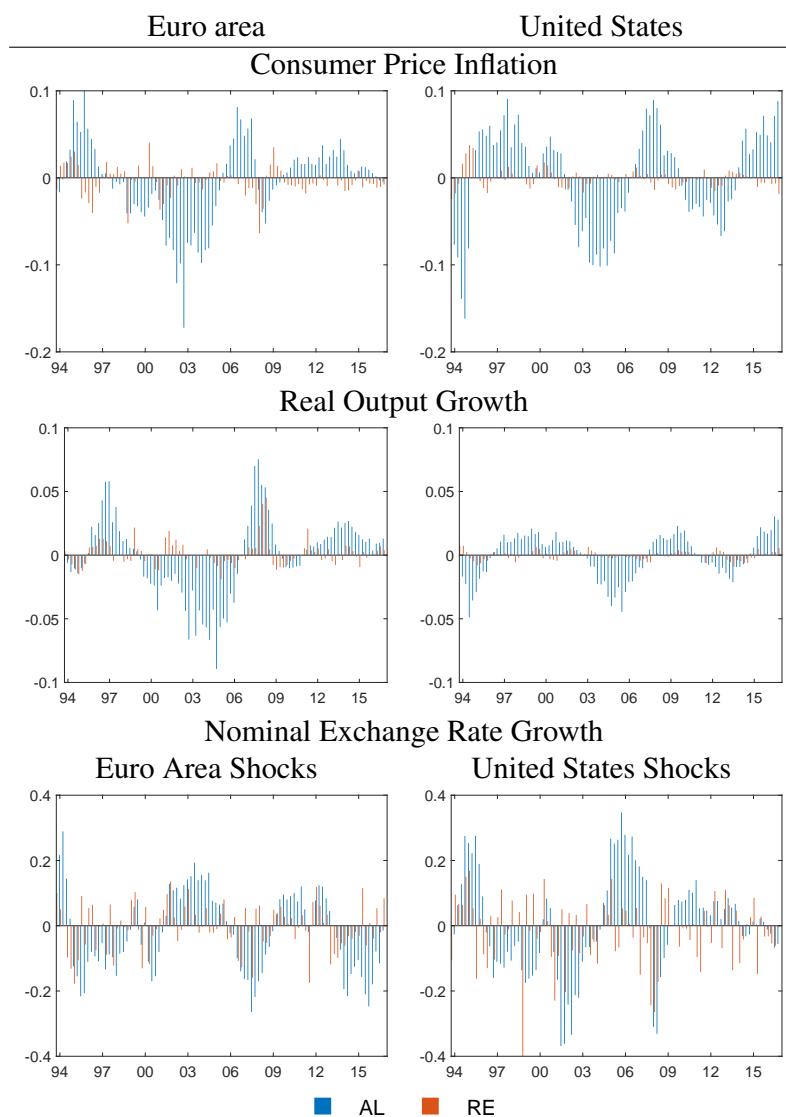
Note: Posterior mode.

Notably, the impact of monetary policy shocks is different under AL and RE, while the shocks are estimated to be very close to each other under both concepts (see Figure F4). Under RE, agents use a structural model to predict forward-looking variables and thus include policy rates in their law of motion in a theoretically consistent way. Under learning, agents use simple AR(2) models to form expectations about future variables, thus reducing the impact of domestic monetary policy surprises on the model dynamics. The impact of both domestic and foreign monetary policy shocks can have different signs under AL and RE. I illustrate this in Figure 6 for bilateral monetary policy shocks and in Figure F5 for both domestic and foreign monetary policy shocks. Figure 6 shows that the influence of foreign monetary policy shocks is more pronounced under learning than under RE for all the variables, including the nominal exchange rate. In the euro area, the dichotomy between the effects of US monetary policy surprises under RE and AL is most visible for consumer price inflation after the GFC, when the RE model predicts that US monetary policy shocks suppress EA inflation, while they are found to revive it under AL. For US consumer price inflation, the impacts of EA monetary policy shocks mostly disagree after 2012. The sign of the impact of US monetary policy shocks differs between AL and RE in accordance with their different contributions to the nominal exchange rate, as shown at the bottom of Figure 6. Note that a fall in the exchange rate corresponds to appreciation of the euro. EA monetary policy shocks mostly contribute to exchange rate volatility with the same sign under RE and AL, while in the AL model the magnitude of the contributions is larger.

Figure F5 shows that US consumer price inflation and output growth in both countries are more affected by domestic monetary policy shocks under RE than under AL. For euro area consumer price inflation, there are episodes when the contribution of a US monetary policy shock is stronger than that of a domestic one: during the GFC under both RE and AL, and prior to 2014 under AL. Under RE, US monetary policy shocks are found to stimulate US inflation starting from 2000, while under learning, the contribution to US inflation is negative in 2003–2008 and from 2010 onward. US monetary policy shocks are found to stimulate US output growth under RE in 2007–2009 and episodically afterward, while they mostly have a negative impact under learning. As for

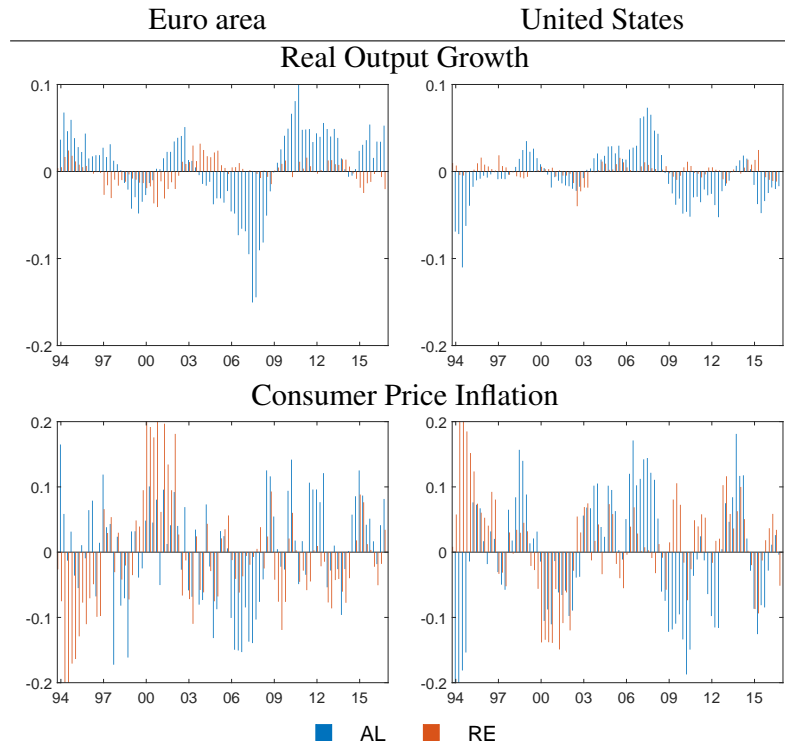
euro area monetary policy shocks, learning and rational expectations mostly agree on the sign of the impact on inflation and output growth. The model features neither the effective lower bound nor quantitative easing, and it is linearized around the non-stochastic steady state, so I do not intend to evaluate the effects of monetary policy surprises during crisis episodes. I rather highlight in Figure 6 that monetary policy surprises of a similar size and the same sign can work in different directions depending on how agents form their expectations.

**Figure 6: Impact of Foreign Bilateral Monetary Policy Shocks**



**Note:** Posterior mode, contributions of bilateral shocks – only US shocks for the euro area, only euro area shocks for the US.

As cross-country spillovers occur through trade and the exchange rate, it is instructive to consider the contributions of exchange rate shocks under learning and RE (see Figure 7). The contributions of exchange rate shocks are larger under learning and sometimes show the opposite sign than in RE model, while the smoothed exchange rate shocks look very similar in both models (see Figure F3 in Appendix F). Under learning, the sign of the contributions of real exchange rate shocks to output growth variability mimics the dynamics of the real exchange rate (see Figure F3 in Appendix F). Before 2008, the euro appreciates, then there are episodes of depreciation and appreciation, fol-

**Figure 7: Contribution of Exchange Rate Shocks**

**Note:** Posterior mode.

lowed by depreciation in 2011–2016. Appreciation of the euro contributes negatively to EA output growth and positively to US output growth. Unlike in the AL model, under RE the depreciation in 2014–2016 contributes mostly negatively to EA output growth, while also having episodes of negative contributions to US output growth.<sup>13</sup> The contribution of exchange rate shocks to consumer price inflation are more pronounced under AL, and in both models reflect the dynamics of the estimated exchange rate shocks, contributing to an increase (decrease) in EA inflation and a decrease (increase) in US inflation when the estimated shocks are positive (negative), i.e., when the euro depreciates (appreciates).

To address the role of the UIRP condition, I conduct an alternative estimation with agents using AR(2) models and the UIRP condition for the exchange rate and Phillips curves for the inflation variables, as discussed in Appendix G. In this case, the UIRP condition becomes self-fulfilling, and the effects of monetary policy shocks on the exchange rate have the same sign under learning and rational expectations as predicted by the UIRP condition. For example, larger policy easing surprises in the US (relative to the EA) exert appreciation pressure on the euro, while the situation is reversed when euro area monetary shocks fall short of US ones. Under AL, consumer price inflation and the output gap react to negative monetary policy consistently with the UIRP condition and the RE model. Note that the impact of euro area monetary policy shocks on US variables is smaller than under simple AR(2) rules, and the effects of domestic monetary policy shocks under AL are larger

<sup>13</sup> The literature shows that appreciation can also be expansionary (studies for the euro area include Lane and Stracca, 2018 and de Walque et al., 2017, Figure 3), as it reduces marginal costs and stimulates domestic demand.

for both countries' output growth and inflation. The increasing role of monetary policy shocks is consistent with agents adding interest rates to their PLMs.<sup>14</sup>

## 5. Conclusion

In this paper, I show that matching cross-country correlations in a large-scale two-country DSGE model can be improved by incorporating adaptive learning. In particular, I consider a learning setup with agents using AR(2) models to forecast future variables. They learn the coefficients of these models using the Kalman filter, and the coefficients, including constant terms, vary over time. I find that the EA-US correlations for both nominal and real variables are larger than in a model estimated under rational expectations. The improvement is mostly due to the time variability of the belief coefficients and to larger shock spillovers, in particular through the exchange rate channel.

Historical shock decompositions show that the contributions of bilateral and exchange rate shocks are larger under adaptive learning, with exchange rate and monetary policy shocks sometimes demonstrating the opposite sign than under rational expectations. I find that domestic monetary policy shocks have a larger influence under rational expectations than under adaptive learning, while foreign monetary policy surprises contribute more under learning. For example, US monetary policy shocks are found to have a stimulative effect on US inflation in the aftermath of 2007 under rational expectations, but not under learning, with the contributions to exchange rate movements sometimes having opposite signs under the two concepts. These differences are explained by the formation of agents' expectations. Under the assumption of adaptive learning, agents use simple autoregressive models of the second order, AR(2), for forecasting. Under rational expectations, agents use a structural model with the uncovered interest rate parity (UIRP) condition, thus inducing a UIRP-consistent reaction of the exchange rate to a policy rate differential. The exchange rate reaction then propagates to inflation and output through imported intermediate and final goods and foreign demand. Domestic monetary policy rates, when included in agents' forecasting equations, thus have a larger impact on the model dynamics under RE. To illustrate these differences, I run an alternative estimation with agents using AR(2) but including additional variables in the exchange rate and inflation forecasting equations to mimic UIRP and the inflation New Keynesian Phillips curve. Under this specification, domestic and foreign monetary policy shocks have the same sign under learning and rational expectations, while the size of the effects still differs.

It has been shown in the literature for small and medium-scale models that adaptive learning increases the fit of the model to the data. I consider a large-scale model carefully designed to deliver reasonable forecasting performance. In this model, adaptive learning still results in a somewhat better model fit as measured by marginal density statistics.<sup>15</sup> I further consider alternative specifications: with constant belief parameters and with agents using the New Keynesian Phillips curve to forecast inflation and uncovered interest rate parity to forecast the euro-US dollar exchange rate. I conclude that time-varying coefficients are the main factor behind the improved cross-country correlation for real variables.

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<sup>14</sup> Slobodyan and Wouters (2012b and 2012a) consider an estimation with alternative specifications of PLMs and initial beliefs. It is shown that PLMs adjusted for additional variables in the forecasting rules, such as adding marginal costs to the inflation equations, produces a similar or worse model fit, probably due to the overfitting problem. Slobodyan and Wouters (2012b) also find that their results are robust to the specification of initial beliefs in the Kalman filter, with the belief coefficients converging to the baseline version within several years and the marginal likelihood being similar to the baseline. As I do not expect the initialization of beliefs to influence international correlation, I do not consider alternative initialization of beliefs.

<sup>15</sup> Despite the increased model fit, models with adaptive learning are not usually used in central bank forecasting practice due to the long estimation period and the occurrence of explosive belief dynamics.

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## Appendix A: Model Equations

### A.1 Households

There is a continuum of households, indexed by  $h$ , with the following utility:

$$U_t(h) \equiv E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{(\mathbb{C}_{t+j}(h) - H_{t+j})^{1-\sigma_c}}{1-\sigma_c} \right) \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} l_{t+j}(h)^{1+\sigma_l} \right), \quad (\text{A1})$$

where  $\sigma_c$  is the degree of relative risk aversion,  $\sigma_l$  is the inverse of the Frisch elasticity of labor supply, and  $H_t$  is the external habit variable, such that  $H_t = \lambda_{hab} \mathbb{C}_{t-1}$ .

The consumption composite good,  $\mathbb{C}$ , consists of an oil good,  $O^D$ , and a non-oil good,  $C$ . The consumption index and the corresponding price index are:

$$\mathbb{C}_t(h) = \left[ (1 - \phi_{oil})^{\frac{1}{\lambda_{oil}}} C_t(h)^{\frac{\lambda_{oil}-1}{\lambda_{oil}}} + \phi_{oil}^{\frac{1}{\lambda_{oil}}} O_t^D(h)^{\frac{\lambda_{oil}-1}{\lambda_{oil}}} \right]^{\frac{\lambda_{oil}}{\lambda_{oil}-1}}, \quad (\text{A2})$$

$$P_{\mathbb{C},t} = \left[ (1 - \phi_{oil}) P_{c,t}^{1-\lambda_{oil}} + \phi_{oil} P_{oil,t}^D \right]^{\frac{1}{1-\lambda_{oil}}} \exp(\varepsilon_t^P), \quad (\text{A3})$$

where  $\varepsilon_t^P$  is the price shock, which is an AR(1) process with i.i.d. normal errors.  $P_{\mathbb{C}}$ ,  $P_c$ , and  $P_{oil}^D$  stand for the price of the total consumption aggregate, the price of non-oil consumption, and the price of oil products for direct consumption, respectively. Superscript  $D$  denotes that the good is ready for direct consumption.  $\lambda_{oil}$  is the price elasticity of demand in total consumption, and  $\phi_{oil}$  is the share of oil in consumption.

Similarly, the investment good,  $I$ , is a combination of domestic and foreign goods. It is further assumed that the price of the investment good equals the price of the non-oil consumption bundle,  $P_c$ , and that the same elasticity and home bias apply:

$$I_t(h) = \left[ \phi_H^{\frac{1}{\lambda}} I_{H,t}(h)^{\frac{\lambda-1}{\lambda}} + (1 - \phi_H)^{\frac{1}{\lambda}} (1 - \Omega_{I,t}) I_{F,t}(h)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}}. \quad (\text{A4})$$

### A.2 International Bond Market and Nominal Exchange Rate

Following de Walque et al. (2017) and Laxton et al. (2010) I assume that all foreign bonds are denominated in US dollars and pay the US interest rate. Then, the budget constraint for a household is:

$$\begin{aligned} & P_{\mathbb{C},t} (\mathbb{C}_t(h) + I_t(h)) + \frac{B_{H,t}(h)}{\exp(\varepsilon_t^b) R_t} + \frac{S_t B_{F,t}(h)}{\exp(\varepsilon_t^b) \tilde{R}_t^f} \\ & \leq W_t(h) l_t(h) + B_{H,t-1}(h) + S_t B_{F,t-1}(h) \\ & + R_t^k u_t(h) K_{t-1}(h) - \psi(u_t) K_{t-1}(h) + \int \text{Div}_i(i, h) di. \end{aligned} \quad (\text{A5})$$

On the revenue side, there is labor income,  $W_t(h) l_t(h)$ , the return on domestic bonds,  $B_{H,t-1}(h)$ , and the return on foreign bonds,  $B_{F,t-1}(h)$ . If the household is a borrower on the bond market, the return enters with a negative sign.  $S_t$  denotes the nominal exchange rate between the home currency and the US dollar (*USD*) in indirect quotation: home currency per unit of *USD*. The household receives

a return on the capital stock,  $R_t^k u_t(h) K_{t-1}(h)$ , minus utilization costs,  $\psi(u_t) K_{t-1}(h)$ , and dividends from domestic intermediate firms, indexed by  $i$ :  $\int Div_i(i, h) di$ .

On the expenditure side, there is total consumption, including consumption of the oil good,  $C_t(h)$ , investment in physical capital,  $I_t(h)$ , and the positions on the domestic and foreign bond markets.  $R_t$  and  $R_t^F$  stand for the domestic and foreign gross interest rate, and  $\varepsilon^b$  resembles the risk premium shock from Smets and Wouters (2007). It is assumed that households pay a premium over the foreign bond return to participate in the international bond market. Thus, the return on foreign bonds equals  $\tilde{R}_t^F = R_t^{US} \Theta_t$ . This wedge between the foreign interest rate and households' return on bonds is modeled as the real costs of holding foreign bonds, which are a function of total foreign bond holdings in the economy and changes in the nominal exchange rate:

$$\Theta_t = \exp \left( -\theta_a \frac{S_t^{\omega/USD} B_{F,t}(h)}{P_{C,t} \gamma^t} - \theta_s \left( \frac{E(S_{t+1}^{\omega/USD}) S_t^{\omega/USD}}{S_t^{\omega/USD} S_{t-1}^{\omega/USD}} - 1 \right) + \varepsilon^{s,\omega/USD} \right), \quad (\text{A6})$$

where  $\gamma$  is the deterministic growth rate of the economy and  $\theta_a$  and  $\theta_s$  are parameters capturing the persistence in the exchange rate data when the model is estimated.  $\varepsilon^s$  is an autoregressive process capturing the exogenous variation in the foreign bond market. Individual households take these costs as given in their optimal decisions.

Households' optimal bond holdings result in the following conditions:

$$[\partial B_{H,t}(h)] \quad \vartheta_t = \exp(\varepsilon_t^b) R_t \beta E_t \left[ \vartheta_{t+1} \frac{P_{C,t}}{P_{C,t+1}} \right], \quad (\text{A7})$$

$$[\partial B_{F,t}(h)] \quad \vartheta_t = \exp(\varepsilon_t^b) \tilde{R}_t^F \beta E_t \left[ \vartheta_{t+1} \frac{E S_{t+1}}{S_t} \frac{P_{C,t}}{P_{C,t+1}} \right], \quad (\text{A8})$$

where  $\vartheta$  is the associated Lagrange multiplier. Combining these two equations for the euro area, I get the equation for the nominal EA-US exchange rate – the uncovered interest rate parity condition.

$$E_t \left[ \frac{S_{t+1}}{S_t} \right] = \frac{R_t^{EA}}{\tilde{R}_t^F} = \frac{R_t^{EA}}{R_t^{US} \Theta_t}. \quad (\text{A9})$$

### A.3 Intermediate Goods Producers

Intermediate goods producers are monopolistic producers of differentiated intermediate goods. Producer  $i$  uses Leontief technology:

$$Y_t(i) = \min \left[ \frac{1}{1 - \rho_m - \rho_o} J_t(i); \frac{1}{\rho_m} Y_{F,t}^P(i); \frac{1}{\rho_o} O_t^P(i) \right] - \gamma^t \Phi, \quad (\text{A10})$$

$$J_t(i) = \tilde{K}_t^\alpha [\gamma^t L_t(i)]^{1-\alpha} \exp(\varepsilon_t^\alpha), \quad (\text{A11})$$

where  $O_t^P$  is oil used in production and  $Y_{F,t}^P$  denotes foreign production goods;  $\rho_m$  and  $\rho_o$  are the respective shares in production and  $\Phi$  is a fixed cost of production.  $L_t(i)$  is the aggregate labor input of the different types of labor used by the producer, and  $\tilde{K}_t^\alpha$  are the effective capital services.  $\varepsilon_t^\alpha$  is an AR(1) process with i.i.d. normal errors. There is also labor-augmenting deterministic growth,  $\gamma$ ,

which determines long-term growth in the economy. The Leontief production function implies the following relationships:

$$\frac{J_t(i)}{O_t^p(i)} = \frac{1 - \rho_m - \rho_o}{\rho_o}, \quad (\text{A12})$$

$$\frac{J_t(i)}{Y_{F,t}^p(i)} = \frac{1 - \rho_m - \rho_o}{\rho_m}, \quad (\text{A13})$$

$$\frac{W_t L_t(i)}{r_t^k \tilde{K}_t(i)} = \frac{1 - \alpha}{\alpha}. \quad (\text{A14})$$

Those relationships result in the following marginal costs of one unit of an intermediate good:

$$MC_t = (1 - \rho_m - \rho_o) \frac{W_t^{1-\alpha} (r_t^k)^\alpha}{\alpha^\alpha (1 - \alpha)^{1-\alpha} \varepsilon_t^\alpha} + \rho_m P_{F,t}^p + \rho_o P_{oil,t}. \quad (\text{A15})$$

The intermediate goods firm sets the prices of goods produced: domestic intermediate goods,  $Y_H$ , exported intermediate goods,  $Y_H^*$ , and exported production goods,  $Y_H^{*p}$ . When setting the prices, the firm maximizes its profit and faces Calvo price-setting rigidities. From the point of view of a euro area firm:

$$\begin{aligned} & \max_{\tilde{P}_{H,t}(i) \tilde{P}_{H,t}^*(i) \tilde{P}_{H,t}^{*p}(i)} E_t \sum_{j=0}^{\infty} (\beta \zeta_p)^j \frac{\vartheta_{t+j} P_{C,t}}{\vartheta_t P_{C,t+j}} [\tilde{P}_{H,t}(i) \chi_{t,j} Y_{H,t+j}(i) - MC_{t+j} Y_{H,t+j}(i)] \\ & + E_t \sum_{j=0}^{\infty} (\beta \zeta_{pF}^*)^j \frac{\vartheta_{t+j} P_{C,t}}{\vartheta_t P_{C,t+j}} [S_{t+j} \tilde{P}_{H,t}^*(i) \chi_{t,j}^* Y_{H,t+j}^*(i) - MC_{t+j} Y_{H,t+j}^*(i) \\ & + S_{t+j} \tilde{P}_{H,t}^{*p}(i) \chi_{t,j}^{*p} Y_{H,t+j}^{*p}(i) - MC_{t+j} Y_{H,t+j}^{*p}(i)], \end{aligned} \quad (\text{A16})$$

where  $\chi$  is the inflation indexation for each type of good:  $\chi_j$  for domestic goods,  $\chi_j^*$  for foreign consumption goods, and  $\chi_j^{*p}$  for foreign production goods.  $\iota_{pF}^*$  and  $\bar{\pi}_H^*$  are local inflation indexation parameters:

$$\chi_{t,j} = \begin{cases} 1 & \text{if } j=0, \\ \prod_{k=1}^j \pi_{H,t+k-1}^{\iota_p} \bar{\pi}_H^{1-\iota_p} & \text{if } j=1, \dots, \infty \end{cases}, \quad (\text{A17})$$

$$\chi_{t,j}^* = \begin{cases} 1 & \text{if } j=0, \\ \prod_{k=1}^j \pi_{H,t+k-1}^{*\iota_{pF}} \bar{\pi}_H^{*1-\iota_{pF}} & \text{if } j=1, \dots, \infty \end{cases}, \quad (\text{A18})$$

$$\chi_{t,j}^{*p} = \begin{cases} 1 & \text{if } j=0, \\ \prod_{k=1}^j (\pi_{H,t+k-1}^{*p})^{\iota_{pF}} (\bar{\pi}_H^{*p})^{1-\iota_{pF}} & \text{if } j=1, \dots, \infty \end{cases}. \quad (\text{A19})$$

Prices with  $\tilde{\cdot}$  denote intermediate goods prices, to distinguish them from final goods prices. Variables with  $*$  are exported variables. The price of domestic goods,  $\tilde{P}_{H,t}$ , is set in domestic currency and the prices of exported consumption and production goods,  $\tilde{P}_{H,t}^*$  and  $\tilde{P}_{H,t}^{*p}$ , are set in foreign currency. Export prices are rigid in the destination currency. Parameters  $\zeta_p$ , and  $\zeta_{pF}^*$  are the Calvo probabilities of not being able to re-optimize home and foreign prices, respectively. Prices that are not optimized are indexed to past inflation with a weight  $\iota_p$  and  $\iota_{pF}^*$  and to trend inflation with a weight  $1 - \iota_p$  and  $1 - \iota_{pF}^*$  for domestic and export prices, respectively. There are three types of trend inflation: domestic inflation,  $\bar{\pi}_H$ , exported intermediate goods inflation,  $\bar{\pi}_H^*$ , and exported production goods inflation,  $\bar{\pi}_H^{*p}$ .

#### A.4 Homogeneous Goods Assemblers

The assemblers are perfectly competitive and produce homogeneous domestic goods,  $Y_{H,t}$ , and imported goods,  $Y_{F,t}$  and  $Y_{F,t}^P$ , using  $Y_{H,t}(i)$ ,  $Y_{F,t}(j)$ , and  $Y_{F,t}^P(l)$ , respectively. They have Kimball production functions:

$$1 = \int_0^1 G\left(\frac{Y_{H,t}(i)}{Y_{H,t}}\right) di, \quad (\text{A20})$$

$$1 = \int_0^1 G\left(\frac{Y_{F,t}(j)}{Y_{F,t}}\right) dj, \quad (\text{A21})$$

$$1 = \int_0^1 G\left(\frac{Y_{F,t}^P(l)}{Y_{F,t}^P}\right) dl, \quad (\text{A22})$$

where  $G$  has the following properties:  $G(1) = 1$ ,  $G'(x) > 0$ , and  $G''(x) < 0$  for  $x > 0$ .

Assemblers select the optimal inputs and output levels, taking prices as given. The first-order conditions for domestic goods imply:

$$Y_{H,t}(i) = G'^{-1}\left(\frac{P_{H,t}(i)}{P_{H,t}}\mathbb{I}_t\right)Y_{H,t}, \quad (\text{A23})$$

$$\mathbb{I}_t = \int_0^1 G'\left(\frac{Y_{H,t}(i)}{Y_{H,t}}\right)\frac{Y_{H,t}(i)}{Y_{H,t}}di. \quad (\text{A24})$$

Similarly, for imported goods:

$$Y_{F,t}^j(i) = G'^{-1}\left(\frac{P_{F,t}^j(i)}{P_{F,t}^j}\mathbb{I}_{F,t}^j\right)Y_{F,t}^j, \quad (\text{A25})$$

$$Y_{F,t}^{j,P}(i) = G'^{-1}\left(\frac{P_{F,t}^{Pj}(i)}{P_{F,t}^{Pj}}\mathbb{I}_{F,t}^{Pj}\right)Y_{F,t}^{Pj}, \quad (\text{A26})$$

where the prices are determined by demand from final goods firms. The homogeneous goods assemblers sell domestic and foreign goods,  $Y_{H,t}$  and  $Y_{F,t}$ , to final goods producers and production goods,  $Y_F^P$ , to domestic intermediate goods producers.

#### A.5 Final Goods Firms

There is a continuum of competitive final goods firms, indexed by  $m$ , that produce retail goods (with superscript  $D$ ) using homogeneous domestic and foreign goods,  $Y_{H,t}$  and  $Y_{F,t}$ , and taking all prices as given. They are assumed to have Leontief technology combining homogeneous domestic and foreign goods with the home product as an input. The home product used in the distribution channel is labeled as  $Y^d$ :

$$Y_{H,t}^D(m) = \min\left[(1 + \delta_f)Y_{H,t}(m); \frac{1 + \delta_f}{\delta_f}Y_{H,t}^d\right], \quad (\text{A27})$$

$$Y_{F,t}^D(m) = \min\left[(1 + \delta_f)Y_{F,t}(m); \frac{1 + \delta_f}{\delta_f}Y_{H,t}^d\right], \quad (\text{A28})$$

where  $\delta_f$  governs the share of home goods used in the distribution process. Consequently, it also limits the exchange rate and foreign inflation pass-through to domestic inflation. As it is optimal to have no unused inputs at equilibrium, the following conditions arise:

$$(1 + \delta_f)Y_{H,t}(m) = \frac{1 + \delta_f}{\delta_f} Y_{H,t}^d = Y_{H,t}^D(m), \quad (\text{A29})$$

$$(1 + \delta_f)Y_{F,t}(m) = \frac{1 + \delta_f}{\delta_f} Y_{H,t}^d = Y_{F,t}^D(m). \quad (\text{A30})$$

$Y_{H,t}^d$  is pinned down by the demand for consumption goods. The demand for inputs is then a linear function of the distributed goods:

$$Y_{H,t}(m) = \frac{1}{1 + \delta_f} Y_{H,t}^D(m), \quad (\text{A31})$$

$$Y_{F,t}(m) = \frac{1}{1 + \delta_f} Y_{F,t}^D(m). \quad (\text{A32})$$

The final goods firms also produce an oil product for final consumption using the same technology but with an oil-specific distribution parameter:

$$O_t^D(m) = \min \left[ (1 + \delta_o)O_t^c(m); \frac{1 + \delta_o}{\delta_o} Y_{H,t}^d \right], \quad (\text{A33})$$

$$P_{oil,t}^D = \frac{1}{1 + \delta_o} P_{oil,t} + \frac{\delta_o}{1 + \delta_o} P_{H,t}, \quad (\text{A34})$$

where  $P_{oil,t}$  is the oil price in home currency. That is, for the euro area, the oil price is given by:

$$P_{oil,t}^\omega = P_{oil,t}^{US} S_t. \quad (\text{A35})$$

## A.6 International Trade

Exports are driven by non-oil demand from foreign countries. For the euro area, exports are:

$$X_{H,t} = M_{F,t}^{\beta_x} \exp(\varepsilon_t^{nt}), \quad (\text{A36})$$

where  $\beta_x$  is the share of the US in EA exports,  $\varepsilon_t^{nt}$  is the exogenous demand from the rest of the world, modeled as an AR(1) process with i.i.d. errors, and  $M_F$  are US imports from the EA. For the US, the expression is symmetric.

The model also allows for transit goods, which are imported to be exported. The price of the transit goods is assumed to be equal to the price of foreign consumption goods. Employing the notation from de Walque et al. (2017), I define total imports as an aggregate of foreign goods,  $Y_F^T$ , and transit goods,  $X_F$  (superscripts  $\omega$  and  $j$  are dropped for convenience):

$$M_{H,t} = \left[ \phi_m^H \frac{1}{\lambda_m} (Y_{F,t}^T)^{\frac{\lambda_m-1}{\lambda_m}} + (1 - \phi_m^H) \frac{1}{\lambda_m} (X_{F,t})^{\frac{\lambda_m-1}{\lambda_m}} \right]^{\frac{\lambda_m}{\lambda_m-1}}, \quad (\text{A37})$$

$$Y_{F,t}^T = \left[ \phi_F \frac{1}{\lambda_F} (Y_{F,t})^{\frac{\lambda_F-1}{\lambda_F}} + (1 - \phi_F) \frac{1}{\lambda_F} (Y_{F,t}^P)^{\frac{\lambda_F-1}{\lambda_F}} \right]^{\frac{\lambda_F}{\lambda_F-1}}, \quad (\text{A38})$$

where  $Y_F^T$  stands for total demand for foreign goods. This total demand consists of foreign goods for consumption and production, with the elasticity of substitution between them denoted as  $\lambda_F$ . The total foreign good is then aggregated with the transit good with elasticity of substitution  $\lambda_m$ . Parameters  $\phi_m^H$  and  $\phi_F$  govern the relative shares of the total foreign good and the total consumption good in the corresponding aggregates.

The total exports of a country are then defined in a similar fashion by combining its total exports of goods for production and consumption with the transit good:

$$X_{H,t} = \left[ \phi_x^H \frac{1}{\lambda_x} (Y_{H,t}^{T*})^{\frac{\lambda_x-1}{\lambda_x}} + (1 - \phi_x^H) \frac{1}{\lambda_x} (X_{F,t})^{\frac{\lambda_x-1}{\lambda_x}} \right]^{\frac{\lambda_x}{\lambda_x-1}}, \quad (\text{A39})$$

where  $\lambda_x$  is the corresponding elasticity of substitution and  $Y_{H,t}^{T*}$  are total goods exported.

Constant elasticity of substitution between transit and non-transit goods implies the following price aggregators:

$$P_{M,t} = \left[ \phi_m^H (P_{F,t}^T)^{1-\lambda_m} + (1 - \phi_m^H) (P_{F,t})^{1-\lambda_m} \right]^{\frac{1}{1-\lambda_m}}, \quad (\text{A40})$$

$$P_{X,t} = \left[ \phi_x^H (P_{H,t}^{T*})^{1-\lambda_x} + (1 - \phi_x^H) (P_{F,t})^{1-\lambda_x} \right]^{\frac{1}{1-\lambda_x}}, \quad (\text{A41})$$

$$P_{F,t}^T = \left[ \phi_F (P_{F,t})^{1-\lambda_F} + (1 - \phi_F) (P_{F,t}^P)^{1-\lambda_F} \right]^{\frac{1}{1-\lambda_F}}. \quad (\text{A42})$$

Again,  $P_H^{T*}$  is defined symmetrically to  $P_F^T$ .

Combining total non-oil imports with oil imports, I get the price of total imports:

$$\mathbb{M}_{H,t} = \left[ \phi_m^{oil} \frac{1}{\lambda_m^{oil}} (M_{H,t})^{\frac{\lambda_m^{oil}-1}{\lambda_m^{oil}}} + (1 - \phi_m^{oil}) \frac{1}{\lambda_m^{oil}} (OIL_t)^{\frac{\lambda_m^{oil}-1}{\lambda_m^{oil}}} \right]^{\frac{\lambda_m^{oil}}{\lambda_m^{oil}-1}}, \quad (\text{A43})$$

$$P_{\mathbb{M},t} = \left[ \phi_m^{oil} (P_{M,t})^{1-\lambda_m^{oil}} + (1 - \phi_m^{oil}) (P_{oil,t})^{1-\lambda_m^{oil}} \right]^{\frac{1}{1-\lambda_m^{oil}}}, \quad (\text{A44})$$

with  $\lambda_m^{oil}$  being the elasticity of substitution between oil and non-oil imports and  $\phi_m^{oil}$  the relative share of non-oil imports in total imports.  $P_{oil}$  is the price of crude oil in domestic currency.  $OIL_t$  is the total oil imported. Total oil imports consist of oil for consumption and oil for manufacturing purposes:

$$OIL_t = O_t^c + O_t^p. \quad (\text{A45})$$

## A.7 Aggregation and Monetary Policy

Final users use distributed goods to invest and consume. It is assumed that government spending and utilization costs are paid only in terms of domestic goods. In other words:

$$Y_{H,t}^D = C_{H,t} + I_{H,t} + G_t + \psi(u_t)K_{t-1}, \quad (\text{A46})$$

$$Y_{F,t}^D = C_{F,t} + I_{F,t}. \quad (\text{A47})$$

Government spending,  $G_t$ , is not modeled explicitly but is assumed to be an exogenous AR(1) process with i.i.d. shock  $\mu_t^g$  and persistence  $\rho_g$ .

Each economy features several New Keynesian Phillips curves capturing the development of prices of specific products: imported consumption goods, imported production goods, exported consumption goods, exported production goods, home inflation, and consumer price inflation. To derive the Phillips curves, I first substitute the demand of final goods producers for final goods (A31) and (A32) and the demand of foreign producers for production goods into the demand of homogeneous goods assemblers for intermediate goods (A23), (A25), and (A26):

$$Y_{H,t}(i) = G'^{-1} \left( \frac{P_{H,t}(i)}{P_{H,t}} \mathbb{I}_t \right) \frac{1}{1 + \delta_f} Y_{H,t}^D, \quad (\text{A48})$$

$$Y_{H,t}^*(i) = G'^{-1} \left( \frac{P_{H,t}^*(i)}{P_{H,t}^*} \mathbb{I}_t^* \right) \frac{1}{1 + \delta_f^*} Y_{F,t}^{*D}, \quad (\text{A49})$$

$$Y_{H,t}^{*p}(i) = G'^{-1} \left( \frac{P_{H,t}^{*p}(i)}{P_{F,t}^{*p}} \mathbb{I}_t^{*p} \right) \frac{1}{\rho_m^*} Y_{F,t}. \quad (\text{A50})$$

Then, from the zero profit condition of assemblers, the price index for the domestic product is:

$$P_{H,t} = \xi_p \pi_{H,t-1}^{l_p} \bar{\pi}_H^{1-l_p} P_{H,t-1} G'^{-1} \left( \frac{\pi_{H,t-1}^{l_p} \bar{\pi}_H^{1-l_p} P_{H,t-1}}{P_{H,t}} \mathbb{I}_t \right) + (1 - \xi_p) \tilde{P}_{H,t} G'^{-1} \left( \frac{\tilde{P}_{H,t}}{P_{H,t}} \mathbb{I}_t \right). \quad (\text{A51})$$

The export price indices are the following:

$$P_{H,t}^* = \xi_p^* (\pi_{H,t-1}^*)^{l_p^*} (\bar{\pi}_H^*)^{1-l_p^*} P_{H,t-1}^* G'^{-1} \left( \frac{(\pi_{H,t-1}^*)^{l_p^*} (\bar{\pi}_H^*)^{1-l_p^*} P_{H,t-1}^*}{P_{H,t}^*} \mathbb{I}_t^* \right) + (1 - \xi_p^*) \tilde{P}_{H,t}^* G'^{-1} \left( \frac{\tilde{P}_{H,t}^*}{P_{H,t}^*} \mathbb{I}_t^* \right), \quad (\text{A52})$$

$$P_{H,t}^{*p} = \xi_p^* (\pi_{H,t-1}^{*p})^{l_p^*} (\bar{\pi}_H^{*p})^{1-l_p^*} P_{H,t-1}^{*p} G'^{-1} \left( \frac{(\pi_{H,t-1}^{*p})^{l_p^*} (\bar{\pi}_H^{*p})^{1-l_p^*} P_{H,t-1}^{*p}}{P_{H,t}^{*p}} \mathbb{I}_t^{*p} \right) + (1 - \xi_p^*) \tilde{P}_{H,t}^{*p} G'^{-1} \left( \frac{\tilde{P}_{H,t}^{*p}}{P_{H,t}^{*p}} \mathbb{I}_t^{*p} \right). \quad (\text{A53})$$

If an intermediate firm was last able to optimize its price at time  $t$ , its product will be priced in the distribution channel as:

$$P_{H,t+i}^D(m) = \frac{1}{1 + \delta_f} \tilde{P}_{H,t}(m) \chi_{t,i} + \frac{\delta_f}{1 + \delta_f} P_{H,t+i}. \quad (\text{A54})$$

Analogously, its export consumption good will be priced via the foreign distribution channel as:

$$P_{H,t+i}^{D*}(m) = \frac{1}{1 + \delta_f^*} \tilde{P}_{H,t}^*(m) \chi_{t,i}^* + \frac{\delta_f^*}{1 + \delta_f^*} P_{H,t+i}^*, \quad (\text{A55})$$



A similar relationship holds for imported goods for direct consumption:

$$P_{F,t+i}^D = \frac{1}{1+\delta_f} \tilde{P}_{F,t}(m) \chi_{t,i} + \frac{\delta_f}{1+\delta_f} P_{H,t+i}, \quad (\text{A56})$$

The zero-profit condition for final goods producers implies that on aggregation the final prices of foreign goods are influenced by domestic prices:

$$P_{H,t}^D = \frac{1}{1+\delta_f} P_{H,t} + \frac{\delta_f}{1+\delta_f} P_{H,t} = P_{H,t}, \quad (\text{A57})$$

$$P_{F,t}^D = \frac{1}{1+\delta_f} P_{F,t} + \frac{\delta_f}{1+\delta_f} P_{H,t}. \quad (\text{A58})$$

The log-linearized New Keynesian Phillips curves are described in Appendix B.

Domestic output in each economy is used for household and government consumption, investment, utilization of capital, and net exports, and as distribution channel inputs to create distributed domestic and foreign goods and oil products. The resource constraint then takes the form:

$$Y_t = \left( s_{H,t} \frac{1}{1+\delta_f} + s_{H,t}^d \frac{\delta_f}{1+\delta_f} \right) (C_{H,t} + I_{H,t} + G_t + \psi(u_t) K_{t-1}) + s_{H,t}^* Y_{H,t}^* + s_{H,t}^{D*} Y_{H,t}^{D*} + s_{H,t}^d Y_{F,t} + s_{H,t}^d \frac{\delta_o}{1+\delta_o} O_t^D, \quad (\text{A59})$$

where  $s$  terms reflect price dispersion and are calculated as  $s_x = \int G^{j-1} \left( \frac{P_{x,t}(m)}{P_{x,t}} \right) \mathbb{I}_{x,t}$ .

In each economy, there is a central bank that sets the nominal interest rate using the following rule:

$$\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_r} \left[ \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_t^f} \right)^{\phi_y} \right]^{1-\rho_r} \left( \frac{Y_t/Y_{t-1}}{Y_t^f/Y_{t-1}^f} \right)^{\psi_{\Delta y}} \varepsilon_t^r. \quad (\text{A60})$$

In the rule above, the central bank adjusts the policy rate relative to its steady-state value,  $\bar{R}$ , responding to deviations of inflation from the target,  $\bar{\pi}$ , and to the output gap. As in Slobodyan and Wouters (2012b), I drop the flexible economy counterpart and substitute output from the flexible economy with the process for productivity:  $Y_t^f = A_t$ . The strength of the response is governed by parameters  $\phi_\pi$  and  $\phi_y$ . The policy maker also reacts to the growth rate of output relative to its flexible counterpart with parameter  $\psi_{\Delta y}$ . In the rule, there is a stochastic AR(1) process,  $\varepsilon^r$ , with i.i.d. shock  $\mu_t^r$  and persistence  $\rho_r$ .

In my model, I allow all countries to buy and sell foreign bonds. Following de Walque et al. (2017), I keep the assumption that all countries have zero foreign bond positions in the steady state. Then the net foreign asset position is the difference between acquired foreign bonds in US dollars and issued bonds sold abroad, also in US dollars:

$$NFA_t = S_t B_{F,t}. \quad (\text{A61})$$

$B_{F,t}$  stands for total net acquisition of foreign bonds – bonds bought minus bonds issued.

**Definition 1.** *Equilibrium.* A monopolistically competitive equilibrium for an open economy has the following properties:

- *Households maximize utility over consumption of domestic and foreign goods, oil consumption, investment, labor, bond holdings, and wages; intermediate goods producers optimize profits over foreign inputs, oil, labor demand, domestic capital, and prices in each market; and homogeneous goods producers and final goods producers maximize profit over demand for home and foreign differentiated intermediate goods and home and foreign homogeneous goods and oil, respectively.*
- *All markets clear.*
- *The nominal exchange rate is determined by (A9).*
- *Set of domestic, exported and imported prices for final goods for consumption and production, and wages that clear the markets;*

## Appendix B: Log-Linearized Equations

In this section I describe the log-linearized equations, where small letters with a hat stand for log deviations from the steady state:  $\hat{x}_t = \log(\frac{X_t}{\bar{X}})$ . Because the model features deterministic growth,  $\gamma$ , I first detrend the real variables and divide nominal variables by a composite consumer price,  $P_C$ . Below, small letters,  $p$ , stand for relative prices – divided by the composite consumer price. Many of the equations are standard in the DSGE literature and resemble those from de Walque et al. (2017).

### B.1 Households

Under the assumption of external habit formation, households' total real consumption depends on its past and expected values, expected changes in labor supply, and the expected real interest rate:

$$\begin{aligned} \hat{\mathbf{c}}_t &= \frac{\lambda_{hab}/\gamma}{1 + \lambda_{hab}/\gamma} \hat{\mathbf{c}}_{t-1} + \frac{1}{1 + \lambda_{hab}} E_t \hat{\mathbf{c}}_{t+1} + \left[ (\sigma_c - 1) \frac{\bar{w}}{\bar{c}} (\sigma_c (1 + \lambda_{hab}/\gamma))^{-1} \right] (\hat{l}_t - E_t \hat{l}_{t+1}) \\ &\quad - (1 - \lambda_{hab}/\gamma) (\sigma_c (1 + \lambda_{hab}/\gamma))^{-1} (\hat{r}_t - E_t \hat{\pi}_{t+1}^c + \varepsilon_t^b), \end{aligned} \quad (\text{B1})$$

where  $\hat{\mathbf{c}}$  is the log deviation of total consumption,  $\mathbb{C}$ , from its steady state and  $\hat{\pi}_t^c = \hat{p}_{\mathbf{c},t} - \hat{p}_{\mathbf{c},t-1}$  is the corresponding deviation of consumer price inflation from its steady state. The term  $\frac{\bar{w}}{\bar{c}}$  stands for the steady-state ratio of the detrended real wage to labor and total consumption,  $\hat{r}_t$  is the nominal central bank policy rate, and  $\varepsilon^b$  is a risk premium AR(1) process with an i.i.d. shock  $\mu_t^b$ . The first-order conditions with respect to investment and capital result in the following equations:

$$\hat{i}_t = \frac{1}{1 + \bar{\beta}\gamma} \left( \hat{i}_{t-1} + \bar{\beta}\gamma E_t \hat{i}_{t+1} + \frac{1}{\Psi'' \gamma^2} (\hat{q}_t - \hat{p}_{\mathbf{c},t}) + \frac{1}{\Psi'' \gamma^2} \varepsilon_t^i \right), \quad (\text{B2})$$

$$\hat{q}_t = \bar{\beta}(1 - \tau) E_t \hat{q}_{t+1} + (1 - \bar{\beta}(1 - \tau)) E_t \hat{r}_{t+1}^k - \bar{\beta}(\bar{r}^k + (1 - \tau)) (\hat{r}_t - E_t \hat{\pi}_{t+1}^c + \varepsilon_t^b). \quad (\text{B3})$$

where  $\hat{i}$  is the log deviation of investment and  $\hat{q}$  is the log deviation of the real price of capital. Parameter  $\Psi''$  is the steady-state second derivative of the investment adjustment cost function,  $\bar{\beta} \equiv \beta\gamma^{-1}$  and  $\bar{\beta}(\bar{r}^k + (1 - \tau)) = 1$ , where  $\tau$  is the depreciation rate.  $\hat{p}_{\mathbf{c},t}$  is the deviation of the price of total consumption. As I divide nominal variables by the total consumption price,  $\hat{p}_{\mathbf{c},t} = 0$ . Capital is accumulated according to the rule:

$$\hat{k}_t = \frac{1 - \tau}{\gamma} \hat{k}_{t-1} + \left( 1 - \frac{1 - \tau}{\gamma} \right) (\hat{i}_t + \varepsilon_t^i). \quad (\text{B4})$$

With  $\hat{r}^k = \psi'(1)$ , capital utilization is determined as:

$$\hat{u} = \frac{\psi'(1)}{\psi''(1)} \hat{r}_t^k. \quad (\text{B5})$$

Wages are subject to Calvo stickiness with probability  $\xi_w$  and partial indexation with probability  $\iota_w$ . Real wages then depend on past and future wages, consumer price inflation, and the wage mark-up:

$$\begin{aligned} \hat{w}_t &= \frac{1}{1 + \bar{\beta}\gamma} (\hat{w}_{t-1} + \bar{\beta}\gamma E_t \hat{w}_{t+1} + \iota_w \hat{\pi}_{t-1} + \bar{\beta}\gamma E_t \hat{\pi}_{t+1} - (1 + \bar{\beta}\gamma \iota_w) \hat{\pi}_t \\ &+ (1 - \iota_w) \hat{\pi}_t - \bar{\beta}\gamma(1 - \iota_w) \hat{\pi}_{t+1}) \\ &+ \frac{(1 - \bar{\beta}\gamma \xi_w)(1 - \xi_w)}{\xi_w \left(1 + \frac{1 + \lambda_w}{\lambda_w} \sigma_l\right)} \left( \frac{1}{1 - \lambda_{hab}/\gamma} (\hat{c}_t - \lambda_{hab}/\gamma \hat{c}_{t-1}) + \sigma_l \hat{l}_t - \hat{w}_t \right) \\ &+ \varepsilon_t^w. \end{aligned} \quad (\text{B6})$$

In the expression above  $\hat{\pi}$  stands for the deviation of the inflation trend from the steady state.

## B.2 Producers and Prices

Intermediate goods producers' optimal conditions for output, demand for capital, and the resulting marginal costs are:

$$\hat{y}_t = \Phi_y \left( \alpha \hat{k}_t + (1 - \alpha) \hat{l}_t + \varepsilon_t^a \right), \quad (\text{B7})$$

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{l}_t, \quad (\text{B8})$$

$$\begin{aligned} \widehat{mc}_t &= \Phi_y \left( \frac{1}{\Phi_y} - \rho_m - \rho_o \right) \left( \alpha \hat{r}_t^k + (1 - \alpha) \hat{w}_t - \varepsilon_t^a \right) \\ &\quad \Phi_y \left( \rho_m \hat{p}_{F,t}^p + \rho_o \hat{p}_{oil,t} \right). \end{aligned} \quad (\text{B9})$$

In the lines above,  $\hat{k} = \hat{u}_t \hat{k}_{t-1}$  stands for log-linearized effective capital and  $\Phi_y = \frac{\bar{y} + \Phi}{\bar{y}}$  is the inverse of the share of the variables' costs in intermediate goods production. Dividing both sides of (A51) by  $P_{C,t}$  I obtain that in the steady state, the relative price of individual manufacturers equals the home price:  $\bar{p}_H(i) = \bar{p}_H$ . Using that expression, I derive the relative distribution prices of intermediate products in (A54):

$$\bar{p}_H^D(i) = \frac{1}{1 + \delta_f} \bar{p}_H + \frac{\delta_f}{1 + \delta_f} \bar{p}_H = \bar{p}_H = \bar{p}_H^D. \quad (\text{B10})$$

That is, in the steady state, all home goods have the same relative price, and that price equals the distribution price. I further normalize  $\bar{p}_H = \bar{p}_H^D = 1$  and  $\bar{p}_F^D = 1$  in each country. From exporters' point of view, the relative distribution prices of their goods in foreign countries are also unity in the steady state:

$$\bar{p}_F^D = \frac{1}{1 + \delta_f} \bar{p}_F + \frac{\delta_f}{1 + \delta_f} \bar{p}_H = 1, \quad (\text{B11})$$

which results in  $\bar{p}_F = 1$ . With home and foreign relative prices equal to unity, the steady-state relative price of non-oil consumption composite goods is also unity,  $\bar{p}_c = 1$ . It must then follow that the oil price relative to the total consumption price is also unity:  $\bar{p}_{oil} = 1$ , together with relative

import and export prices. Therefore, the normalization implies that all relative prices are unity in the steady state.

Linearizing the equation for the price of the total consumption aggregate,  $P_{C,t}$ , with  $\hat{p}_{C,t} = 0$ , I obtain the dependencies between home, foreign, and oil prices as:

$$0 = (1 - \phi_{oil}) \left[ \frac{\phi_c + \delta_f}{1 + \delta_f} \hat{p}_{H,t} + \frac{1 - \phi_c}{1 + \delta_f} \hat{p}_{F,t} \right] + \phi_{oil} \hat{p}_{oil}^D + \varepsilon_t^P, \quad (\text{B12})$$

$$0 = (1 - \phi_{oil}) \hat{p}_{c,t} + \phi_{oil} \hat{p}_{oil}^D + \varepsilon_t^P. \quad (\text{B13})$$

When linearizing (A51) in relative prices, I use that fact that in the steady state  $G'^{-1} = 1$  and  $\bar{\mathbb{I}} = G'(1)$  to obtain the New Keynesian Phillips curve for home inflation:

$$\begin{aligned} \hat{\pi}_{H,t} &= \frac{1}{1 + \beta \iota_p} (\beta \hat{\pi}_{H,t+1} + \iota_p \hat{\pi}_{H,t-1}) \\ &+ \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p(1 + \beta \iota_p)} \frac{\eta_H - 1 - \delta_f}{\eta_H + \varepsilon - 1} (\widehat{mc}_t - \hat{p}_{H,t}) + \varepsilon_t^{pH} - \nu \varepsilon_t^P. \end{aligned} \quad (\text{B14})$$

Parameter  $\eta_H$  is the steady-state price elasticity of demand in the home country, with  $\eta_{j,t} = -\frac{G'(z_{j,t})}{z_{j,t} G''(z_{j,t})}$  and  $z_{j,t} = G'^{-1} \left( \frac{P_{j,t}^D(i)}{P_{j,t}^P} \mathbb{I}_t \right)$ , where  $j = H, F$ .  $\varepsilon$  is the curvature of Kimball's aggregator  $G$ . Following de Walque et al. (2017) among others, I define the curvature as the steady-state elasticity of the price elasticity of demand with respect to the relative price. For home prices, it is defined as:

$$\varepsilon = \frac{\bar{p}_j / \bar{P}_j}{\eta_j(z_{j,SS})} \frac{\partial \eta_j(z_{j,SS})}{\partial \bar{p}_j} \Big|_{z_{j,SS}=1} = 1 + \eta_j \left( 1 + \frac{G'''(1)}{G''(1)} \right), \quad (\text{B15})$$

$$\frac{\left( 1 + (1 + \delta_f) \frac{G''(1)}{G'(1)} \right)}{\left( 2 + \frac{G'''(1)}{G''(1)} \right)} = \frac{\eta_j - 1 + \delta_f}{\eta_j - 1 + \varepsilon}. \quad (\text{B16})$$

The domestic inflation New Keynesian Phillips curve features two stochastic AR(1) processes, each with i.i.d. errors: the domestic price mark-up,  $\varepsilon_t^{pH}$ , and the feedback from the consumption price mark-up,  $\varepsilon_t^P$ , with a negative sign. The consumer price inflation index data contain some elements that introduce additional volatility into the index, but they are unmodeled here, so I use the feedback from  $\varepsilon_t^P$  to subtract them from home inflation.

The Phillips curves for the imported inflation indices, where superscript \* denotes foreign variables, are derived in a similar fashion:

$$\begin{aligned} \hat{\pi}_{F,t} &= \frac{1}{1 + \beta^* \iota_{pF}^*} \left( \beta^* \hat{\pi}_{F,t+1} + \iota_{pF}^* \hat{\pi}_{F,t-1} \right) \\ &+ \frac{(1 - \xi_{pF}^*)(1 - \beta \xi_{pF}^*)}{\xi_{pF}^*(1 + \beta^* \iota_{pF}^*)} \frac{1}{\eta_F + \varepsilon - 1} \left( (\widehat{mc}_t^* + \widehat{r}_t) + \delta_f \hat{p}_H - (\eta - 1) \hat{p}_{F,t} \right) \\ &+ \varepsilon_t^{pF}, \end{aligned} \quad (\text{B17})$$

$$\begin{aligned} \hat{\pi}_{F,t}^P &= \frac{1}{1 + \beta^* \iota_{pF}^*} \left( \beta^* \hat{\pi}_{F,t+1}^P + \iota_{pF}^* \hat{\pi}_{F,t-1}^P \right) \\ &+ \frac{(1 - \xi_{pF}^*)(1 - \beta \xi_{pF}^*)}{\xi_{pF}^*(1 + \beta^* \iota_{pF}^*)} \frac{\eta_F - 1}{\eta_F + \varepsilon - 1} (\widehat{mc}_t^* + \widehat{r}_t - \hat{p}_{F,t}^P) \\ &+ \varepsilon_t^{pF}, \end{aligned} \quad (\text{B18})$$

where  $\hat{\pi}_F$  and  $\hat{\pi}_F^p$  are imported inflation and imported production goods inflation, respectively. Foreign intermediate goods producers set their export prices directly in the currency of the importing (destination) country and take into account the destination distribution costs,  $\delta_f$ , and price elasticity,  $\eta$ . Calvo pricing parameters  $\xi_{pF}^*$  and  $\iota_{pF}^*$  stand for the probability and degree of price indexation of imported goods in the destination market. Note that these parameters can be different from those for goods produced locally in the destination country,  $\xi_p$  and  $\iota_p$ , even though they are applied on the same destination market. The real marginal costs in foreign currency,  $\widehat{mc}^*$ , are converted into local currency using the real exchange rate,  $\widehat{r}_s$ . Because production goods are not processed through the distribution channel, the distribution costs,  $\delta_f$ , do not enter the corresponding expression for inflation. Both imported inflation indices are affected by mark-up shocks, which are AR(1) processes with i.i.d. disturbances. These mark-up processes originate in the local market, but only affect foreign goods prices.

### B.3 Exchange Rates and International Trade

To derive the equation for the euro-US dollar real exchange rate I make use of the UIRP condition (A9). I follow de Walque et al. (2017) in assuming that the net foreign asset position is zero in the steady state. In the equation below, the net foreign asset position,  $nfa$  is linearized, but the rest of the terms are log linearized around the steady state:

$$\begin{aligned} \widehat{r}_s = & (1 - \theta_s)E_t \widehat{r}_{s,t+1} + \\ & \theta_s \widehat{r}_{s,t-1} + r_t^{US} - r_t^{EA} - E_t \widehat{\pi}_{t+1}^{US} + E_t \widehat{\pi}_{t+1}^{EA} - \rho_{nfa} nfa_t^{EA} + \varepsilon_t^s, \end{aligned} \quad (\text{B19})$$

where  $\widehat{r}_s$  is the real exchange rate in indirect quotation (euros per 1 US dollar),  $\theta_s$  is the parameter on exchange rate persistence obtained from the definition of the risk premium over foreign bonds ( $\Theta_t$ ) and  $\rho_{nfa} = \Theta'(0)\bar{y}\gamma'$  ensures stationarity and is set to be a very small number. Foreign consumption and investment goods from the point of view of the home country are derived using the expression for the non-oil consumption aggregate and the adjustment costs for foreign goods consumption.

$$\begin{aligned} \hat{c}_{F,t} = & \hat{c}_t - \lambda(\hat{p}_{F,t}^D - \hat{p}_{c,t}) - \Omega_c \lambda(\hat{c}_{F,t} - \hat{c}_t - \hat{c}_{F,t-1} + \hat{c}_{t-1}) \\ & + \beta \Omega_c \lambda(\hat{c}_{F,t+1} - \hat{c}_{t+1} - \hat{c}_{F,t} + \hat{c}_t), \end{aligned} \quad (\text{B20})$$

$$\begin{aligned} \hat{i}_{F,t} = & \hat{i}_t - \lambda(\hat{p}_{F,t}^D - \hat{p}_{c,t}) - \Omega_i \lambda(\hat{i}_{F,t} - \hat{i}_t - \hat{i}_{F,t-1} + \hat{i}_{t-1}) \\ & + \beta \Omega_c \lambda(\hat{i}_{F,t+1} - \hat{i}_{t+1} - \hat{i}_{F,t} + \hat{i}_t), \end{aligned} \quad (\text{B21})$$

$$\hat{c}_t = \hat{c}_t - \lambda_{oil} \hat{p}_{c,t}, \quad (\text{B22})$$

where  $\hat{c}_{F,t}$  and  $\hat{i}_{F,t}$  are, respectively, consumption and investment of foreign goods and  $\hat{c}_t$  is total non-oil consumption. The log-linear demand for transit goods is derived from (A39):

$$\hat{x}_{F,t} = \hat{x}_{H,t} - \lambda_x(\hat{p}_{F,t} - \hat{p}_{x,t}). \quad (\text{B23})$$

Log-linearization of non-oil imports (A37), together with (A38) and the use of  $Y_{F,t}^D = C_{F,t} + I_{F,t}$ , results in:

$$\begin{aligned} \hat{m}_{H,t} = & \phi_m^H \left( \frac{\bar{y}_F}{\bar{y}_F^T} \hat{y}_{F,t} + \frac{\bar{y}_F^p}{\bar{y}_F^T} \hat{y}_{F,t}^p \right) + (1 - \phi_m^H) \hat{x}_{F,t} \\ = & \phi_m^H \left( \frac{\bar{y}_F}{\bar{y}_F^T} \left[ \frac{\bar{c}_F}{\bar{y}_F^D} \hat{c}_{F,t} + \frac{\bar{i}_F}{\bar{y}_F^D} \hat{c}_{F,t} \right] + \frac{\bar{y}_F^p}{\bar{y}_F^T} \frac{1}{\rho_m} \frac{\bar{y}}{\bar{y}_F^p} \hat{y}_t \right) + (1 - \phi_m^H) \hat{x}_{F,t}. \end{aligned} \quad (\text{B24})$$

Employing the trick by de Walque et al. (2017) I express the coefficients in the above expression as:

$$\begin{aligned} \frac{\bar{c}_F}{\bar{y}_F^D} &= \frac{\bar{y} \bar{\mathbf{m}} \bar{m}_H \bar{y}_F^T \bar{y}_F \bar{c}_F \bar{c} \hat{\mathbf{c}} \bar{y}_F}{\bar{\mathbf{m}} \bar{m}_H \bar{y}_F^T \bar{y}_F \bar{y}_F^D \bar{c} \hat{\mathbf{c}} \bar{y} \bar{y}_F^T} \\ &= \alpha_m^{-1} (1 - \phi_m^{oil})^{-1} (\phi_m^H)^{-1} (1 + \delta_f)^{-1} (1 - \phi_c^H) (1 - \phi_c^{oil}) \alpha_c, \end{aligned} \quad (\text{B25})$$

$$\frac{\bar{i}_F}{\bar{y}_F^D} = \alpha_m^{-1} (1 - \phi_m^{oil})^{-1} (\phi_m^H)^{-1} (1 + \delta_f)^{-1} (1 - \phi_c^H) \alpha_i, \quad (\text{B26})$$

where  $\alpha_c$  and  $\alpha_i$  are the steady-state ratios of total consumption to GDP and total investment to GDP, respectively. Then, the expression for imports becomes:

$$\begin{aligned} \hat{m}_{H,t} &= \phi_m^H \left( \alpha_m^{-1} (1 - \phi_m^{oil})^{-1} (\phi_m^H)^{-1} (1 + \delta_f)^{-1} (1 - \phi_c^H) [(1 - \phi_{oil}) \alpha_c \hat{c}_{F,t} + \alpha_i \hat{i}_{F,t}] + (1 - \phi_F) \hat{y}_t \right) \\ &\quad + (1 - \phi_m^H) \hat{x}_{F,t}, \end{aligned} \quad (\text{B27})$$

$$\phi_F = \left( \frac{1 - \phi_H}{1 + \delta_f} ((1 - \phi_{oil}) \alpha_c + \alpha_i) \right) / \left( \frac{1 - \phi_H}{1 + \delta_f} ((1 - \phi_{oil}) \alpha_c + \alpha_i) + \rho_m \right). \quad (\text{B28})$$

Exports are driven by foreign demand normalized by the export share to account for the presence of the rest of the world:

$$\hat{x}_{H,t} = \beta_x^j \hat{m}_{F,t} + \varepsilon_t^{nt}. \quad (\text{B29})$$

Total imports, including imports of oil products, are log-linearized as:

$$\hat{\mathbf{m}}_t = (1 - \phi_m^{oil}) \hat{m}_{h,t} + \phi_m^{oil} \hat{oil}_t. \quad (\text{B30})$$

#### B.4 Resource Constraint and Monetary Policy

Log-linearized around the steady state, the resource constraint (A59) looks like:

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}} \hat{\mathbf{c}}_t + \frac{\bar{i}}{\bar{y}} \hat{i}_t + \frac{\bar{g}}{\bar{y}} \hat{g}_t + \frac{\bar{r}^k \bar{k}}{\gamma \bar{y}} \hat{u}_t + \frac{\bar{\mathbf{m}}}{\bar{y}} (\hat{x}_{H,t} - \mathbf{m}_t) + (\rho_o + \rho_m) \hat{y}_t. \quad (\text{B31})$$

Note that as  $\bar{S}\bar{B}_F = 0$  in the steady state, the trade balance is also zero in the steady state and  $\frac{\bar{x}}{\bar{y}} = \frac{\bar{\mathbf{m}}}{\bar{y}}$ . When log-linearizing (A61), I linearize the terms in *nfa* and log-linearize the rest:

$$\beta^* nfa_t = \gamma^{-1} nfa_{t-1} + \frac{\bar{\mathbf{m}}}{\bar{y}} (\hat{p}_{x,t} + \hat{x}_F - \hat{p}_{\mathbf{m},t} - \mathbf{m}). \quad (\text{B32})$$

The trade balance is then defined as:

$$\hat{tb}_t = \hat{p}_{x,t} + \hat{x}_F - \hat{p}_{\mathbf{m},t} - \mathbf{m}. \quad (\text{B33})$$

Monetary policy is set as the following rule:

$$\hat{r}_t = \rho^r \hat{r}_{t-1} + (1 - \rho^r) (\rho^\pi \hat{\pi}_t + \rho^y (\hat{y}_t - \hat{y}_t^f)) + \rho^\Delta (\hat{y}_t - \hat{y}_{t-1} - \hat{y}_t^f + \hat{y}_{t-1}^f) + \varepsilon_t^r. \quad (\text{B34})$$

## Appendix C: Estimation and Data

*Table C1: List of Data*

Data Series	Euro area	US
Real GDP	+	+
Real individual consumption	+	
Private consumption, current prices		+
Gross fixed capital formation	+	
Private investment, current prices		+
Real wage per head	+	
Hourly compensation		+
Hours worked or employment	+	+
Short-term interest rate, annualized	+	+
Consumption deflator	+	+
Import deflator	+	+
GDP deflator	+	+
Net exports	+	+
Working age population	+	+
Nominal exchange rate	+	
Oil price		+

*Table C2: Calibrated Parameters: Matched Data and Steady-State Ratios*

Name	Symbol	EA	US
Consumption to GDP	$\alpha_c$	0.564	0.645
Investment to GDP	$\alpha_i$	0.22	0.173
Imports to GDP	$\alpha_m$	0.2	0.14
Oil demand elasticity	$\lambda^{oil}$	0.3	0.3
Oil share in consumption	$\phi^{oil}$	0.04	0.06
Oil share in exports	$\phi_x^{oil}$	0.04	0.08

*Table C3: Calibrated Parameters*

Name	Symbol	EA	US
<b>Model parameters:</b>			
Capital depreciation	$\tau$	0.025	0.025
Wage elasticity	$\lambda_w$	0.25	0.25
Kimball curvature	$\varepsilon$	10	10
Substitution transit goods	$\lambda_x$	0	0
<b>Initial scaling for</b>			
variance-covariance matrix	$\sigma_0$	0.03	
VAR errors	$\sigma_v$	0.003	

Table C4: Measurement Equations

Name	Trend	Model Variables
<b>Common equations:</b>		
$\Delta$ Real GDP	$= \bar{\gamma}$	$+(1 - \rho_m - \rho_o)(\hat{y}_t - \hat{y}_{t-1})$
$\Delta$ GDP deflator	$= \bar{\pi}$	$+\hat{\pi}_t$
$\Delta$ Consumption deflator	$= \bar{\pi}$	$+\hat{\pi}_{c,t}$
$\Delta$ Import deflator	$= \bar{\pi}^*$	$+\hat{p}_{m,t} - \hat{p}_{m,t-1}$
$\Delta$ Real consumption	$= \bar{\gamma}$	$+\hat{c}_t - \hat{c}_{t-1}$
$\Delta$ Real investment	$= \bar{\gamma}$	$+\hat{i}_t - \hat{i}_{t-1}$
Net exports	$= \bar{n}\bar{x}$	$+\alpha_m(\hat{x}_{H,t} - \hat{m}_{H,t})$
Nominal interest rate	$= 4\bar{r}$	$+ 4\hat{r}_t$
$\Delta$ Real wage	$= \bar{\gamma}$	$+\hat{w}_t - \hat{w}_{t-1}$
<b>Specific equations</b>		
$\Delta$ US labor	$= \bar{l}$	$+\hat{l}_t - \hat{l}_{t-1}$
$\Delta$ EA labor	$= \bar{l}$	$+\hat{e}_t - \hat{e}_{t-1}$
$\Delta$ Nominal exchange rate	$= \bar{s}$	$+\hat{r}_{s,t} - \hat{r}_{s,t-1} - \hat{\pi}_{c,t}^{US} + \hat{\pi}_{c,t}^{EA}$
$\Delta$ Oil price	$= \bar{\pi}_{oil}$	$+\pi_{oil,t} - \pi_{oil,t-1} + \hat{\pi}_{c,t}^{US}$

Table C5: Description of Shocks

Name	Process	Shock
<b>Common shocks:</b>		
TFP	$\varepsilon^a, \text{AR}(1)$	$\mu^a$
Risk premium	$\varepsilon^b, \text{AR}(1)$	$\mu^b$
Government spending	$\varepsilon^g, \text{AR}(1) + \eta_{gy}\mu^a$	$\mu^g$
Investment	$\varepsilon^i, \text{AR}(1)$	$\mu^i$
Interest rate	$\varepsilon^r, \text{AR}(1)$	$\mu^r$
Home price mark-up	$\varepsilon^{pH}, \text{ARMA}(1,1)$	$\mu^{pH}$
Wage mark-up	$\varepsilon^w, \text{ARMA}(1,1)$	$\mu^w$
Consumer price	$\varepsilon^p, \text{AR}(1)$	$\mu^p$
Imported price mark-up	$\varepsilon^{pF}, \text{ARMA}(1,1)$	$\mu^{pF}$
Rest of the world demand	$\varepsilon^{nt}, \text{AR}(1) + \eta_a\mu^a$	$\mu^{nt}$
<b>Other shocks:</b>		
Oil price	$p_{oil,t}, \text{ARMA}(1,1)$	$\mu^{oil}$
Exchange rate	$\varepsilon^s, \text{AR}(1)$	$\mu^s$



## Appendix D: Posteriors

*Table D1: Estimated ARMA Parameters under AL and RE*

		Euro Area				United States			
		RE		AL		RE		AL	
		mode	std.error	mode	std.error	mode	std.error	mode	std.error
TFP	AR	0.982	0.007	0.996	0.003	0.981	0.021	0.932	0.023
risk premium	AR	0.963	0.009	0.543	0.016	0.883	0.029	0.587	0.062
gov. spending	AR	0.857	0.046	0.874	0.040	0.955	0.021	0.941	0.024
investment	AR	0.187	0.107	0.142	0.065	0.767	0.100	0.327	0.084
monetary policy	AR	0.204	0.073	0.205	0.073	0.536	0.063	0.462	0.072
home price mark-up	AR	0.994	0.055	0.407	0.157	0.803	0.173	0.436	0.163
home price mark-up	MA	0.069	0.027	0.063	0.027	0.067	0.029	0.063	0.033
wage mark-up	AR	0.936	0.216	0.369	0.081	0.421	0.209	0.516	0.179
wage mark-up	MA	0.084	0.032	0.057	0.027	0.055	0.025	0.052	0.021
imp. price mark-up	AR	0.993	0.060	0.382	0.058	0.982	0.061	0.276	0.106
imp. price mark-up	MA	0.448	0.133	0.344	0.113	0.304	0.117	0.455	0.106
ROW demand	AR	0.808	0.057	0.843	0.048	0.769	0.049	0.729	0.081
TFP in gov. spending		0.132	0.064	0.197	0.074	0.510	0.076	0.582	0.073
cons. infl. in home infl.		0.356	0.094	0.248	0.071	0.490	0.116	0.462	0.116
TFP in export		0.511	0.189	0.672	0.187	0.820	0.150	0.824	0.144
<b>Exch. rate and oil price</b>		-	-	-	-	-	-	-	-
nom. exch. rate	AR	0.905	0.037	0.444	0.058	-	-	-	-
oil price	AR	0.928	0.044	0.849	0.045	-	-	-	-
oil price	MA	0.373	0.087	0.425	0.077	-	-	-	-

*Note:* Posterior mode.

*Table D2: Estimated Trends under AL and RE*

	Euro Area				United States			
	RE		AL		RE		AL	
	mode	std.error	mode	std.error	mode	std.error	mode	std.error
labor	0.102	0.026	0.222	0.021	-0.079	0.018	-0.042	0.032
normalized disc. factor	0.333	0.067	0.460	0.075	0.322	0.076	0.324	0.060
real GDP growth	0.391	0.010	0.398	0.009	0.313	0.037	0.374	0.028
imp. price	1.749	0.393	1.986	0.463	-1.988	0.381	-1.843	0.396
net trade	0.305	0.043	0.314	0.036	0.352	0.055	0.403	0.037
<b>Exchange rate and Oil price</b>	-	-	-	-	-	-	-	-
exchange rate Euro/USD	0.151	0.110	0.031	0.084	-	-	-	-
oil price	1.654	0.317	1.575	0.241	-	-	-	-

*Note:* Posterior mode.

**Table D3: Estimated Parameters under AL and RE**

	Euro Area				United States			
	RE		AL		RE		AL	
	mode	std.error	mode	std.error	mode	std.error	mode	std.error
capital share	0.320	0.029	0.271	0.025	0.252	0.026	0.258	0.025
fixed costs	1.475	0.103	1.482	0.105	1.400	0.088	1.349	0.086
habit	0.777	0.049	0.881	0.014	0.655	0.054	0.862	0.025
inv. adj. costs	4.276	1.161	7.226	0.689	5.204	1.069	6.666	0.838
inv. Frisch elast.	2.308	0.698	2.699	0.554	0.554	0.644	2.206	0.678
rel. risk aversion	1.013	0.111	1.139	0.036	1.262	0.150	1.295	0.061
cap. util. adj. costs	0.367	0.104	0.315	0.109	0.209	0.087	0.294	0.093
Calvo price	0.773	0.071	0.875	0.020	0.798	0.038	0.961	0.028
price index.	0.152	0.068	0.243	0.082	0.242	0.092	0.249	0.091
Calvo wage	0.761	0.040	0.709	0.021	0.692	0.056	0.618	0.060
wage index.	0.153	0.066	0.136	0.049	0.480	0.141	0.325	0.119
Calvo im. price	0.346	0.116	0.746	0.047	0.213	0.084	0.420	0.065
im. price index.	0.517	0.132	0.398	0.120	0.451	0.136	0.407	0.111
Calvo empl	0.756	0.033	0.729	0.034	-	-	-	-
elast. of substitution	2.947	0.820	2.929	0.693	5.978	1.338	5.186	1.121
adj. costs	3.692	1.008	3.621	0.920	5.413	0.929	5.690	0.790
distrib. in cons.	0.110	0.249	0.117	0.147	0.222	0.290	0.268	0.222
distrib. in oil	2.612	0.479	2.439	0.403	3.970	0.664	3.778	0.612
bilateral imp. share	0.251	0.082	0.534	0.082	0.328	0.079	0.403	0.064
bilateral exp. share	0.363	0.087	0.391	0.070	0.283	0.080	0.329	0.076
imp. goods in prod.	0.050	0.013	0.042	0.012	0.043	0.008	0.044	0.007
oil in production	0.002	0.001	0.003	0.001	0.005	0.001	0.004	0.001
<b>Exch. rate and Oil price</b>	-	-	-	-	-	-	-	-
UIRP Euro/USD smooth. param	0.153	0.051	0.308	0.062	-	-	-	-
<b>Taylor rule parameters</b>	-	-	-	-	-	-	-	-
inflation	1.665	0.149	1.563	0.122	1.597	0.132	1.468	0.168
lagged int. rate	0.878	0.015	0.890	0.013	0.895	0.013	0.927	0.013
output gap	0.051	0.021	0.038	0.008	0.102	0.027	0.118	0.026
diff. in output gap	0.019	0.016	0.022	0.012	0.068	0.017	0.064	0.017

*Note:* Posterior mode.**Table D4: Estimated Standard Deviations of Shocks under AL and RE**

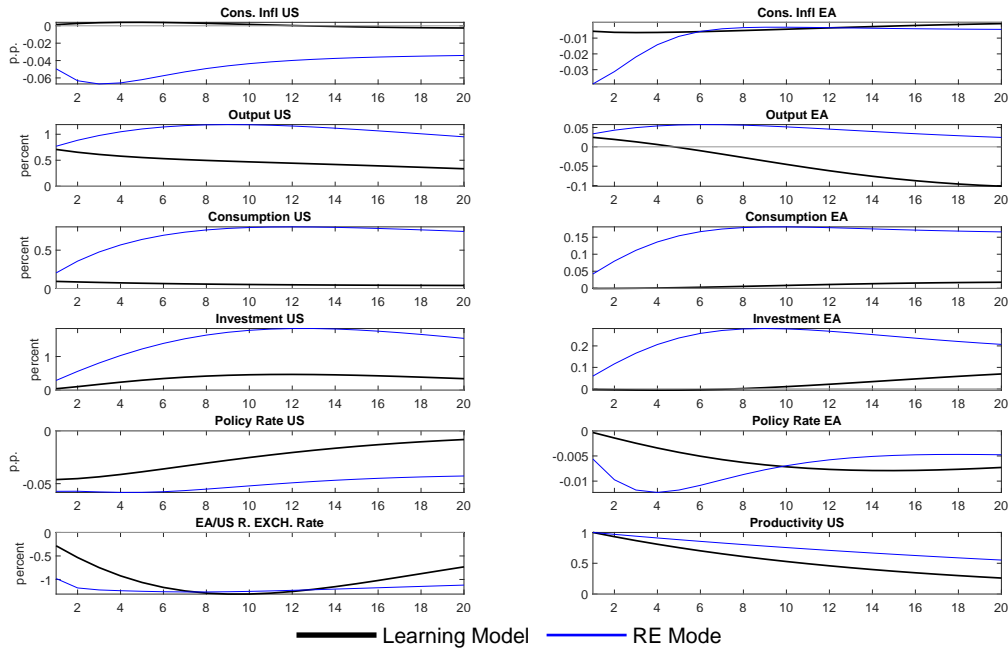
	Euro Area				United States			
	RE		AL		RE		AL	
	mode	std.error	mode	std.error	mode	std.error	mode	std.error
productivity	0.514	0.074	0.357	0.045	0.455	0.042	0.457	0.037
risk premium	0.108	0.028	0.877	0.038	0.266	0.076	0.727	0.132
gov. spending	0.312	0.026	0.306	0.021	0.321	0.027	0.307	0.027
inv. technology	0.434	0.056	0.253	0.029	0.291	0.057	0.252	0.043
monetary policy	0.075	0.007	0.065	0.005	0.089	0.009	0.074	0.007
home price	0.095	0.012	0.117	0.011	0.113	0.012	0.118	0.012
wage markup	0.100	0.013	0.094	0.009	0.516	0.047	0.396	0.046
consumer price	0.154	0.012	0.145	0.009	0.145	0.012	0.151	0.011
imp. price	0.551	0.100	0.389	0.050	1.581	0.225	1.012	0.133
ROW demand	1.982	0.165	1.971	0.153	1.522	0.132	1.482	0.133
<b>Exch. rate and Oil price</b>	-	-	-	-	-	-	-	-
oil price shock	0.146	0.012	0.145	0.012	-	-	-	-
EA-US EXCR	0.320	0.117	0.583	0.145	-	-	-	-

*Note:* Posterior mode.

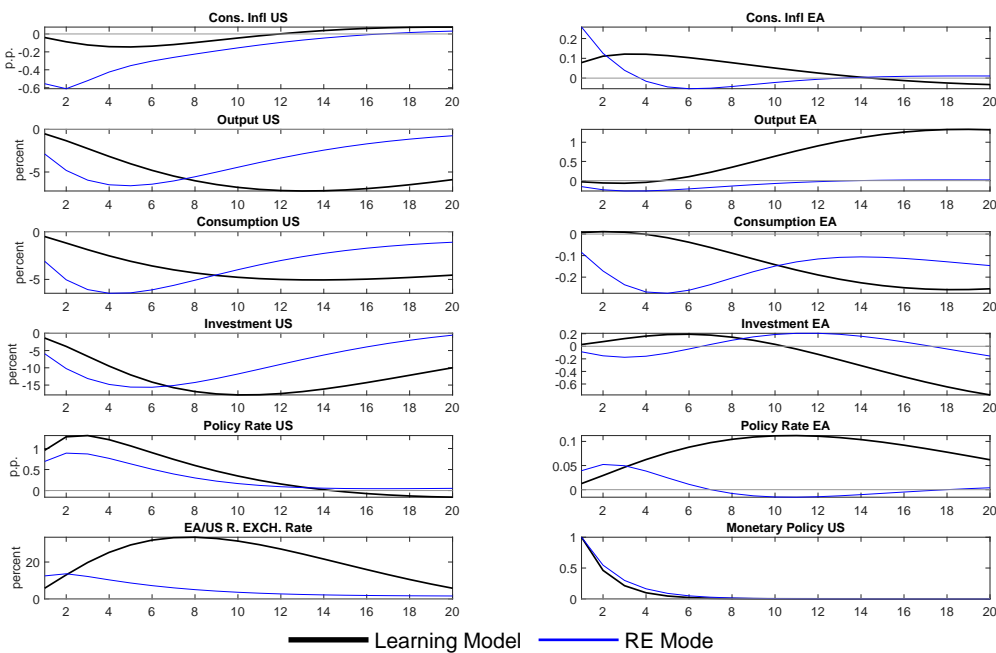
## Appendix E: Impulse Responses

In this section, I plot the impulse responses to shocks with standard deviations equal to one. For the time-varying response of the AL model, I plot the median responses. The rest of the impulse responses are available on request.

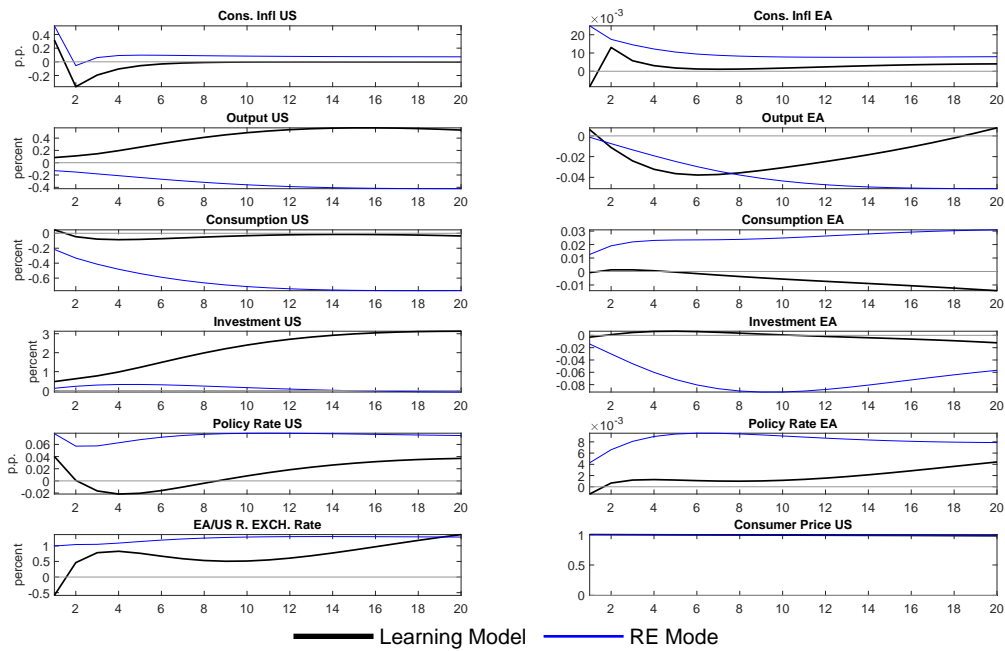
*Figure E1: Responses to 1 Standard Deviation US Productivity Shock*



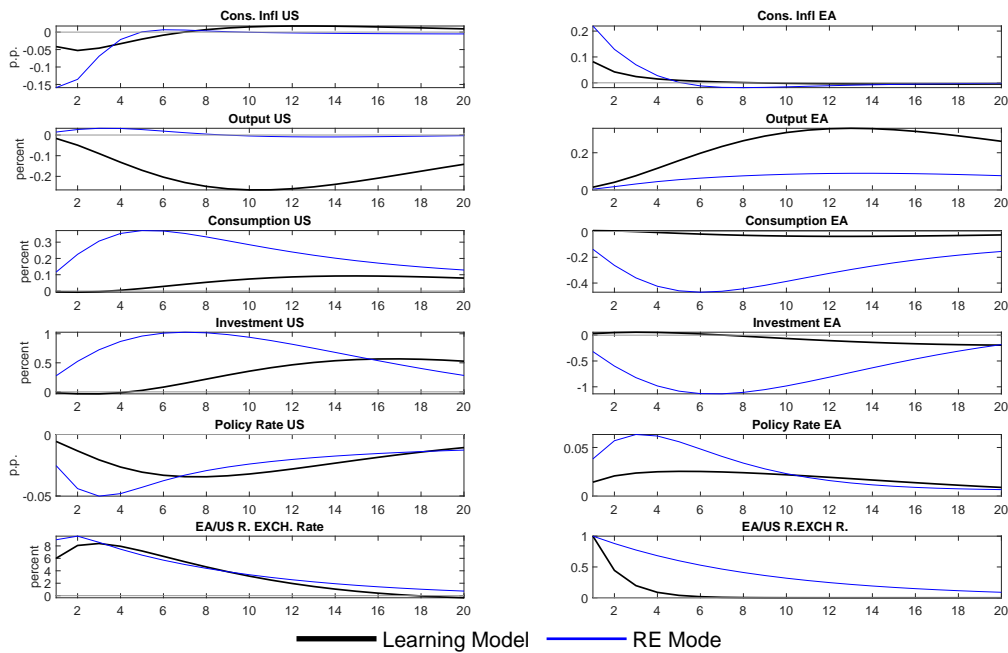
*Figure E2: Responses to 1 Standard Deviation US Monetary Policy Shock*



**Figure E3: Responses to 1 Standard Deviation US Consumer Price Mark-Up Shock**

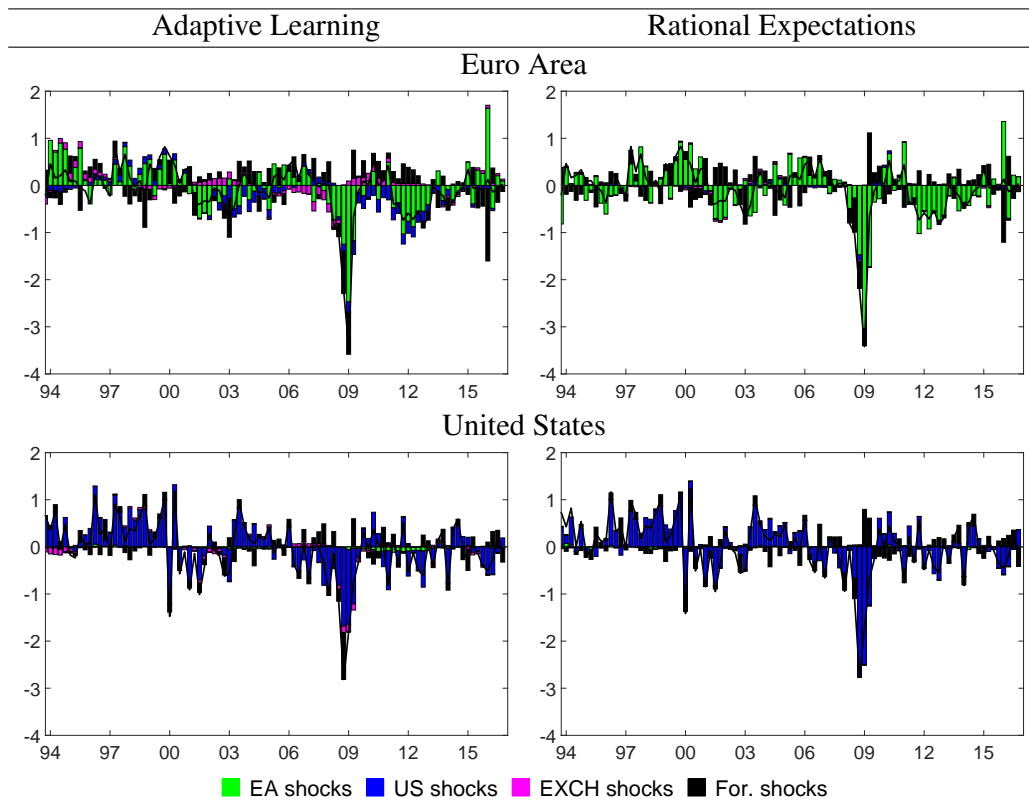


**Figure E4: Responses to 1 Standard Deviation Real Exchange Rate Shock**

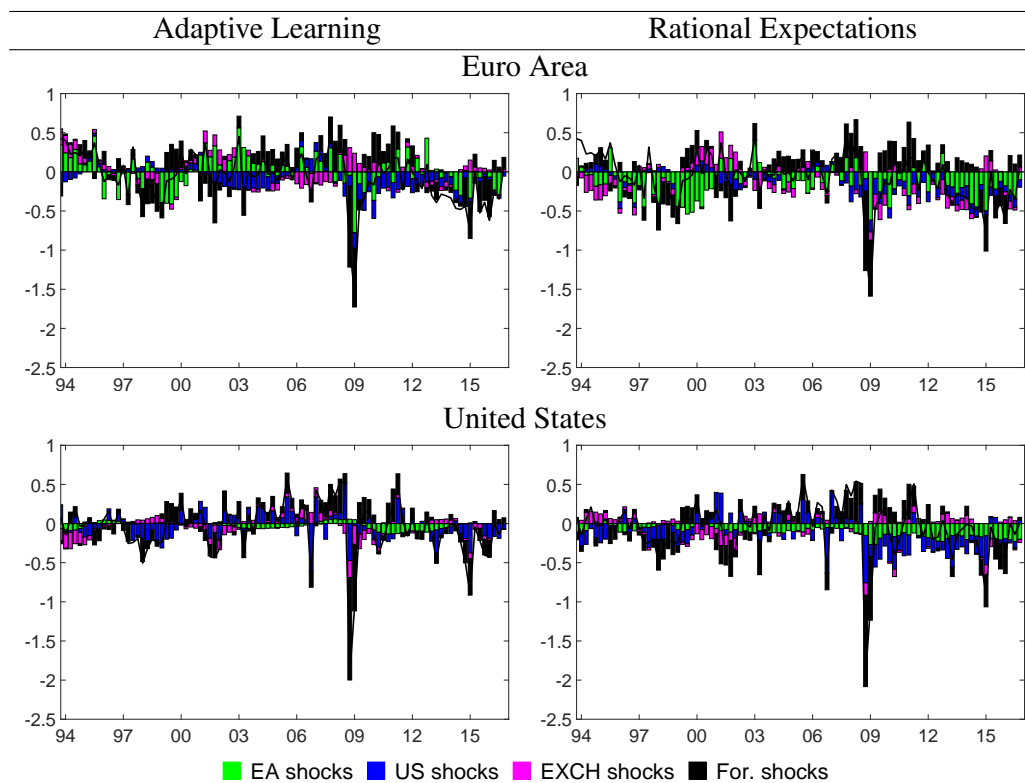


## Appendix F: Historical Shock Decomposition

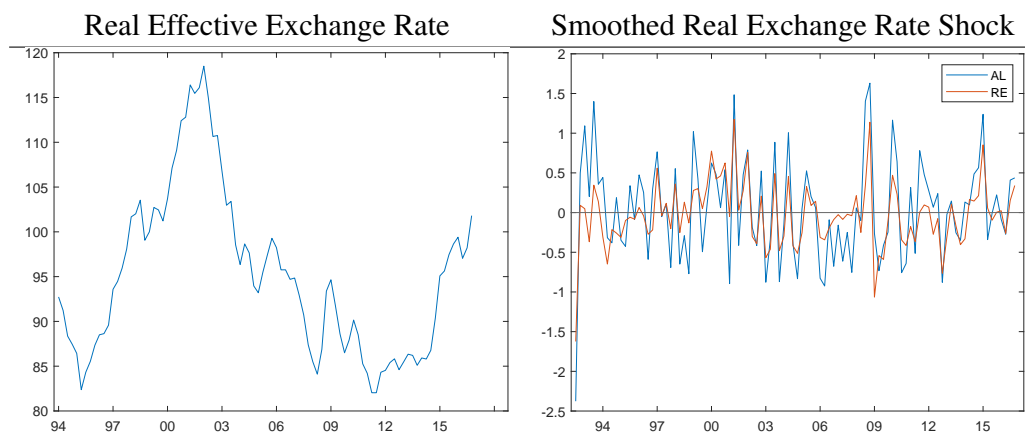
Figure F1: Historical Shock Decomposition of Real Output Growth



**Figure F2: Historical Shock Decomposition of Consumer Price Inflation**

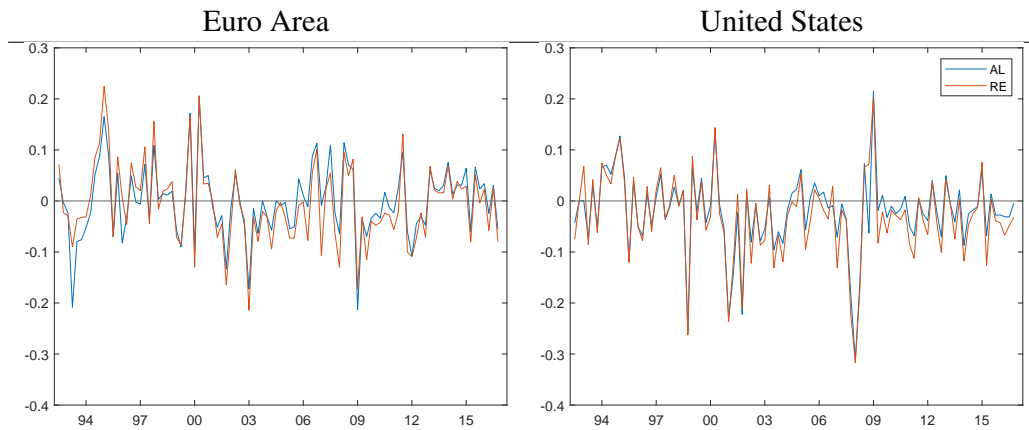


**Figure F3: Real Effective Exchange Rate and Smoothed Shocks to Real Exchange Rate**

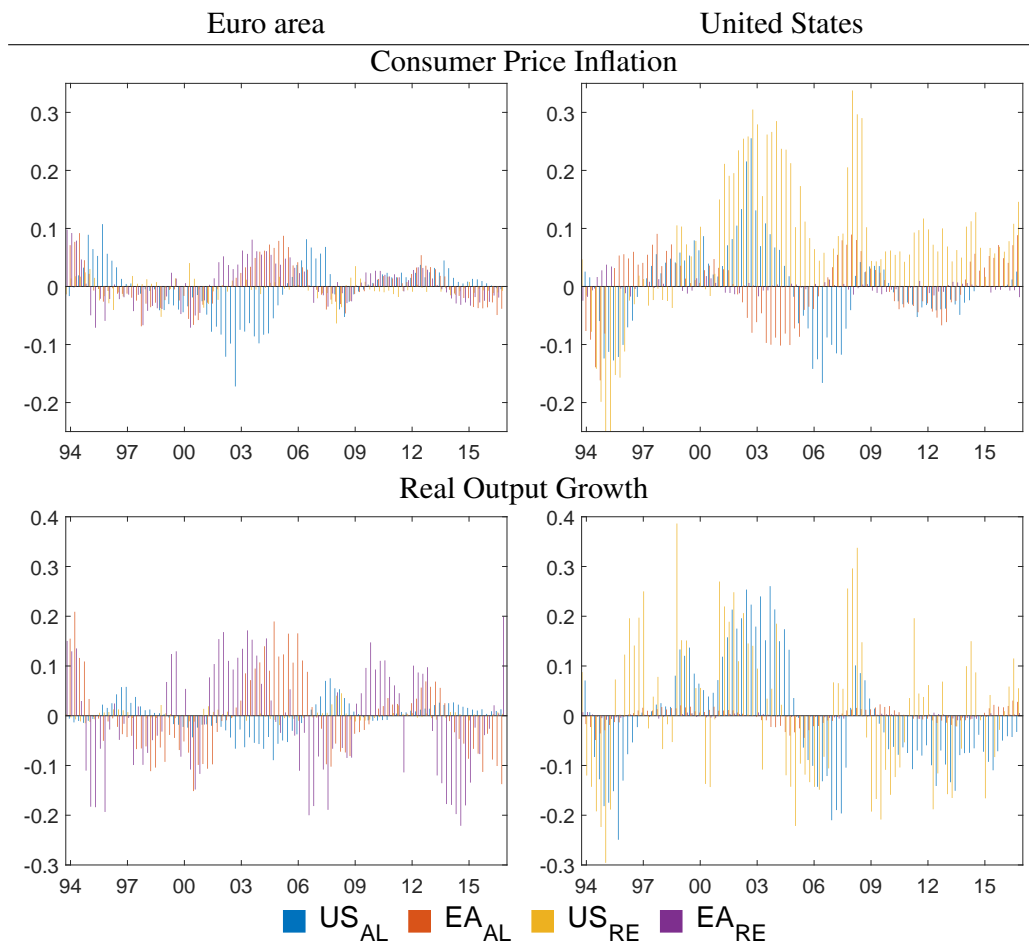


**Note:** The data is the CPI deflated EER-19/US dollar. Source: European Central Bank (ECB).

**Figure F4: Smoothed Monetary Policy Shocks**



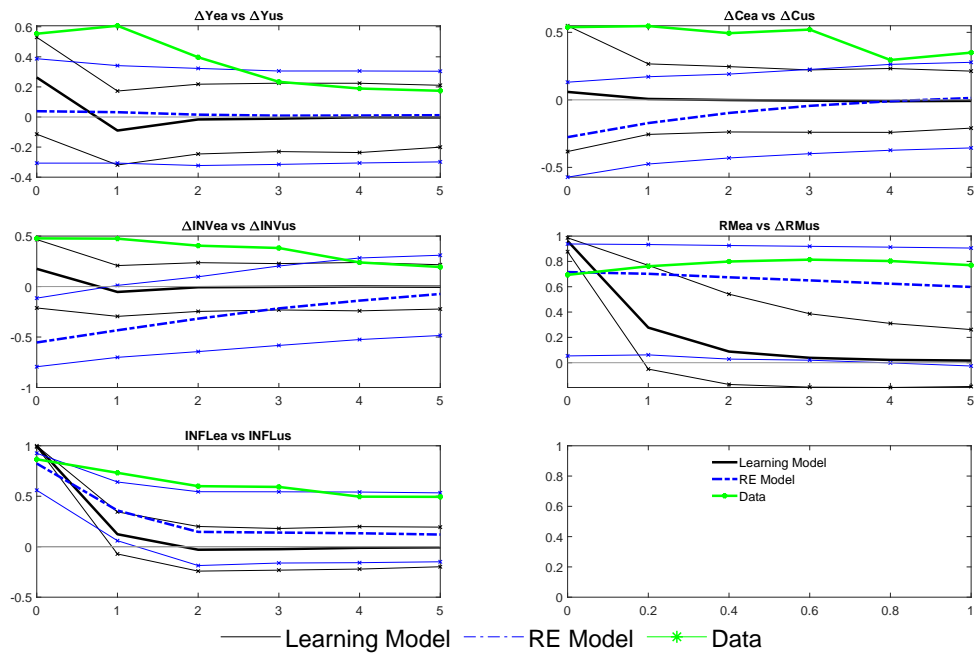
**Figure F5: Impact of Domestic and Foreign Monetary Policy Shocks**



**Note:** Posterior mode.  $US_{AL}$ ,  $US_{RE}$ ,  $EA_{AL}$ , and  $EA_{RE}$  correspond to US and EA monetary policy shocks under AL and RE, respectively.

## Appendix G: Alternative PLM Specifications

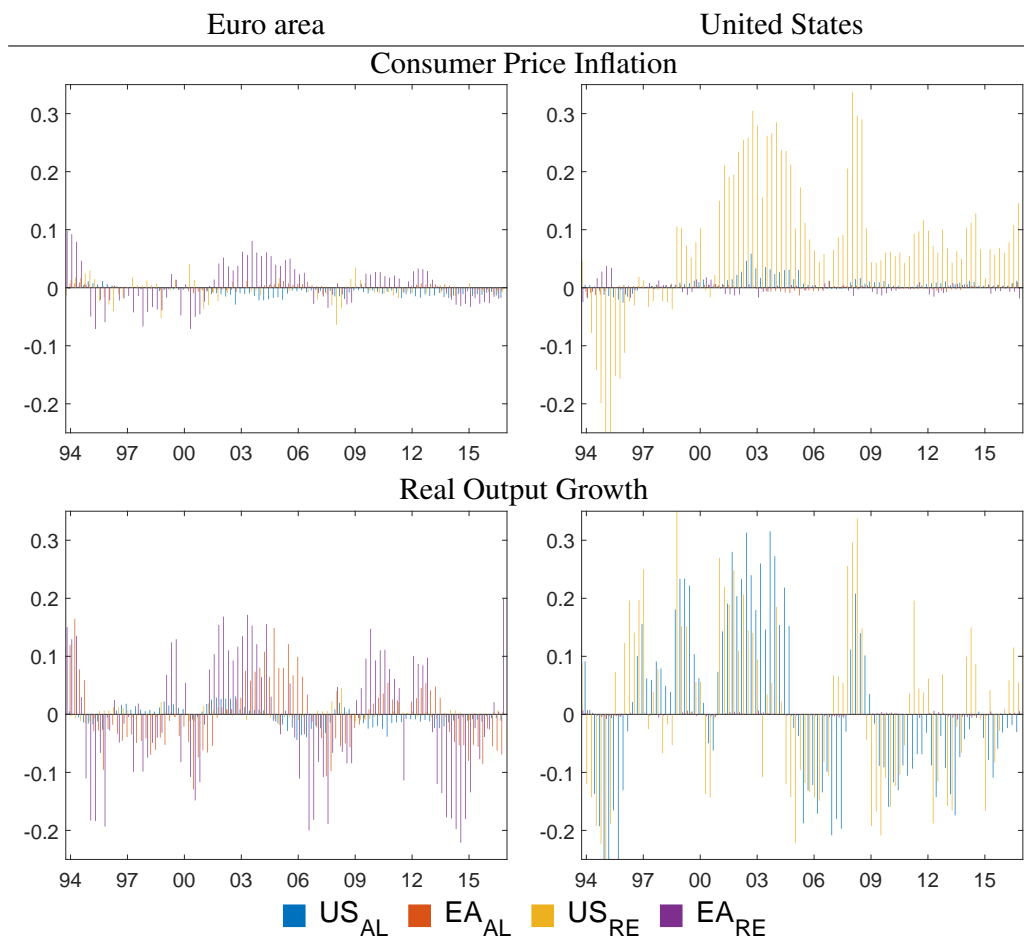
Figure G1: Sampling from Posterior Distribution with Fixed Parameters



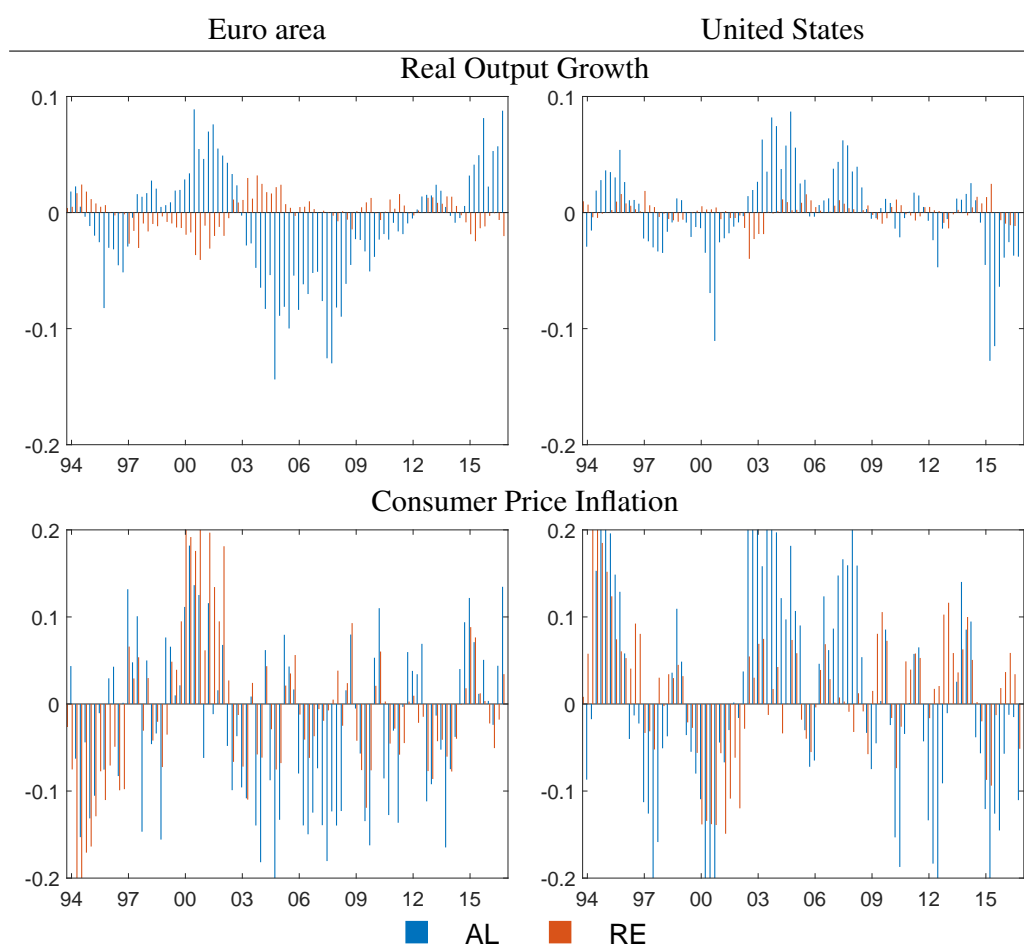
**Note:** The thick lines show the means of the simulated correlations. The thin lines show the 10 and 90 percentiles of the distributions.



**Figure G2: Impact of Domestic and Foreign Monetary Policy Shocks under UIRP and Phillips Curve-Consistent PLM**



**Note:** Posterior mode. US<sub>AL</sub>, US<sub>RE</sub>, EA<sub>AL</sub>, and EA<sub>RE</sub> correspond to US and EA monetary policy shocks under AL and RE, respectively.

**Figure G3: Exchange Rate Shock Contribution under UIRP and Phillips Curve-Consistent PLM**

*Note:* Posterior mode.

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