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Alexis Derviz*

Abstract

We develop a two-country dynamic optimization model with investment and labor mobility and calculate its full-distribution Markov solution without relying on non-stochastic steady-state shortcuts. Agents have access to so-called sovereign capital (an extension of the inside equity notion) as well as the usual outside equity in their own country, but only to outside equity in the other country. This friction creates two distinct categories of partially non-tradable investment goods. Their price ratio can be viewed as an analogue of the real exchange rate in the Balassa-Samuelson model, but with consumption goods replaced by assets. In equilibrium, this asset-based real exchange rate is more sensitive to the stock ownership split between residents and non-residents in each country's production capacity than to the ratio of national physical capital stocks. In a similar model without sovereign capital exclusivity, the order of the sensitivity is reversed. Along with the real exchange rate, we also analyze equilibrium net investment positions and financial account levels as functions of the physical capital ratio and the stock ownership splits. This allows us, in the dynamic equilibrium environment modeled, to point at the underlying regularities behind the seemingly irregular interplay between the external balance and the exchange rate.

Abstrakt

Vyvinuli jsme dynamický optimalizační model dvou zemí s mobilitou investic a práce a vypočítali jeho markovovské řešení pro úplné rozdělení bez použití zjednodušujících nestochastických rovnovážných hodnot. Ekonomické subjekty mají ve své zemi přístup k tzv. suverénnímu kapitálu (rozšíření konceptu vnitřního kapitálu) i k běžnému vnějšímu kapitálu, ale ve druhé zemi pouze ke vnějšímu kapitálu. Tato frikce vytváří dvě samostatné skupiny částečně neobchodovatelných investičních statků. Jejich cenový poměr lze vnímat jako analogii k reálnému měnovému kurzu v Balassově-Samuelsonově modelu, ale při nahrazení spotřebního zboží aktivy. V rovnovážném stavu je tento reálný měnový kurz založený na aktivech citlivější na rozdělení akciového vlastnictví produkční kapacity dané země mezi rezidenty a nerezidenty než na poměr stavu fyzického kapitálu v obou zemích. V podobném modelu bez exkluzivity suverénního kapitálu je pořadí citlivosti opačné. Společně s reálným měnovým kurzem analyzujeme také rovnovážné saldo investiční pozice vůči zahraničí a výši finančního účtu platební bilance jako funkce poměru fyzického kapitálu a rozdělení akciového vlastnictví. To nám umožňuje v modelovaném prostředí dynamické rovnováhy ukázat regularity v pozadí zdánlivě nepravidelné interakce mezi vnější rovnováhou a měnovým kurzem.

JEL Codes: C61, C63, F36, F41, F65.

Keywords: Capital, dynamic optimization, real exchange rate, sovereignty, tradability.

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1. Introduction

Innumerable economists for more than 200 years have studied, individually and inside specialized teams and institutions, the causes and consequences of international capital flows as well as their links to both real investment and the exchange rate. However, only a small proportion of them have explicitly defined what it was, in their opinion, that made external investment economically different. Even fewer have tried to explain what the terms "domestic" and "foreign" actually mean in the context of investment decisions, to begin with. To this day, although everyone seems to know and accept that investor nationality plays some role (Stulz, 2005), occasionally a significant one, formalization of the concept remains elusive.

The present paper intends to investigate the role of national borders separating investment opportunities, and investor nationality, for capital formation and financial wealth distribution across countries. In particular, we are interested in finding out whether the optimal response of nationality-constrained rational agents to an initial international asymmetry, either in physical capital endowment ("economy size") or in financial wealth distribution, results in some form of compensating behavior, or whether the initial differences tend to be reinforced in a dynamic equilibrium. Further, we relate this general research question to a variant of a conventional summary statistic that constitutes a traditional object of interest for international economists – the real exchange rate – in order to find out how well this statistic actually reflects the underlying cross-border imbalances.

However, the notion of the real exchange rate employed here is "asset-based", meaning that it operates with the ratio of asset price indices instead of goods price indices. It is well known that meaningful international comparisons of consumer spending are impossible without taking into account differences in the tradability of individual consumed commodities.[1] In the present paper, we try to apply the tradability concept to investment goods. In short, we put together an international investment model that defines investor nationality in terms of access to a specific investment opportunity within a given country. This investment opportunity will play the role of non-tradable commodity.

More specifically, we set up a model of optimal consumption and investment, in the form of common stock, in two production facilities, one in each of the countries considered, under technological risks in both. The model will fulfill the following tasks:

- generate a dynamic stochastic general equilibrium two-country environment in which each asset has a "nationality" in the sense of an inalienable characteristic exclusively tying it to residents of one country,
- provide an asset-pricing framework for this environment in a way that highlights the role of the nationality of an asset in its valuation,
- trace the nationality friction in the pricing framework responsible for exchange rate effects broadly analogous to the classical Balassa-Samuelson hypothesis in the goods markets.

A non-trivial choice between investment opportunities and equity markets presumes the existence of recurrent aggregate uncertainty. The latter entails dynamic stochastic optimization models for

[1] This has long been the essence of the so-called Balassa-Samuelson hypothesis; see Balassa (1964) and Samuelson (1964) for the original references, or Samuelson (1994) for a more up-to-date survey.

individual agents and equilibrium conditions that are generically lacking closed-form representations and are difficult to solve numerically (Fernández-Villaverde et al., 2016; Miftakhova et al., 2020). A popular shortcut provided by the so-called perturbation methods around an artificial "non-stochastic steady state" puts a severe restriction on the result interpretation possibilities, especially when an optimal portfolio choice is involved, since the stochastic dynamic systems in question do not have a steady state. If well-defined, they possess a (multivariate) ergodic distribution, which is the correct analogue and generalization of the steady-state notion that exists in deterministic models. Therefore, if one looks for macro model solutions able to inform the user about asset-pricing regularities, finding a global (or full-distribution) solution to the model cannot be avoided. In this paper, we apply a numerical optimization method that provides the necessary full-distribution solution.

When invoking the analogy with the classical Balassa-Samuelson effect (BSE),[2] we mean various degrees of correspondence between certain features of our model and the following four elements of the standard BSE:

1. The law of one price holds for tradable goods.
2. Wages are equalized between the T and NT sectors within each country thanks to national labor market arbitrage, whereas across countries, labor markets are autarchic.
3. The marginal products of labor in the NT sectors are roughly similar in the two countries, i.e., there is no apparent labor productivity advantage on the NT side in either country. (This is the usual narrative concerning emerging markets' catch-up, which is traditionally assumed to happen on the T side, where international competition supports it, whereas "barbers provide haircuts in the same way the world over.")
4. The marginal product of labor in the T sector of the poorer country gradually catches up with that of the richer country.

Under these conditions, practically any model with a neoclassical production function would predict faster NT price growth in the poorer country as long as the catch-up continues (this collection of formal features can be summarily called the Balassa-Samuelson model, or BSM). Accordingly, the real exchange rate of the catching-up country should also strengthen. On the other hand, this clear-cut result quickly breaks down as soon as one abandons one of the four specific assumptions listed above. In addition, even when it is supposed to hold, the BSE is not really important quantitatively (MacDonald and Ricci, 2005; Gubler and Sax, 2019). This should not be all that surprising given that, in reality, a sharp distinction between tradable and non-tradable goods is not easy to draw. Indeed, in the modern world, examples of pure tradable consumption goods are fairly hard to come by, and it is even rarer for those that exist to be traded in parallel in comparable quantities in two distinct national currencies. Therefore, the T-side arbitrage of the classical BSE story appears to be either insignificant or altogether absent. The whole argument thus loses ground.

In the exercise to follow, we exploit the exactly inverse situation that is typical of international markets for assets as opposed to goods. Securities are generically tradable cross-border, and, even when some authority tries to put a check on such trade with non-residents, the restrictions can

[2] Also known as the Harrod-Balassa-Samuelson effect due to a much earlier contribution by Harrod (1933) in the same vein.

always be circumvented at some cost. They are usually overcome if sufficient outside demand exists. In asset markets, the challenge is not to find tradables, but to identify non-tradables.

Our answer is to associate non-tradability with the notion of *sovereign capital* (sometimes shortened to SC in the sequel). Namely, within each national economy there are assets, tangible and intangible, whose control by the nation (regardless of who represents it) is part of the very definition of a sovereign country: infrastructure, public healthcare, and education systems, defense, law enforcement, border control, and so on. Most of them are public goods that enter value creation implicitly, as opposed to standard physical capital carried on the books of economically active entities. The value of this sovereign capital changes over time (on average growing, unless one is talking about a major crisis or a failed state) and can be taken over by foreigners only if the state itself ceases to exist, for example, if occupied, annexed by, or merged with, a foreign power. This is different from a mere stock acquisition by a foreign investor in a public utility company. The latter is a transaction in tradable assets, whereas the former cannot be meaningfully transacted at all. Accordingly, sovereign capital is not a specific portion of the physical capital at work within a country's borders, but a set of specific control rights over domestic economic activity not available to non-residents. The concept of sovereign capital is related to the distinction between controlling and minority shareholders, specifically the notion of outside equity (Myers, 2000) in corporate finance. Naturally, there are many differences in the use of the outside equity term in the present paper compared to its original content exploited by corporate finance theory.[3] Among other things, contrary to a dissolvable private corporation or a partnership, a nation, as long as it remains one, must always have its "inside investors" (citizenry and rulers) with inalienable control rights. Also, the observability of the inside investor contributions and the earnings generated by the "operating venture" (the national economy) is much higher than in a private company. Therefore, the incentives for "outside equity holders" (private domestic and foreign investors) to contribute their funds are much less fragile than in a classical agency cost (Jensen and Meckling, 1976; Fama and Jensen, 1983) or CSV (Townsend, 1979) narrative. The meaning of the presently invoked analogy with inside equity is mainly to stress the separation of stock ownership and control of operations.

The key elements of the proposed asset-based BS model will offer some analogy with properties 1–4 of the aforementioned classical goods-based BSM, but also exhibit qualitative differences. The latter will make the present approach more generally applicable. Namely

- Tradable goods in the present model are outside equity shares in the two national production capacities.
- In each of the two countries considered, the role of non-tradable commodity is assigned to control rights (by the country's nationals) over that country's physical capital. As already mentioned, these control rights resemble inside equity, but with no explicit dividend payments. The earnings of the country's residents are determined by the proportion of stock in the production capacity not sold to non-residents; this proportion is the choice

[3] In the present context, it would be misleading to compare our insider/outsider investor separation with the traditional distinction between foreign direct and portfolio investment. Indeed, in our model, even a standard FDI is an action of an outside investor, because, to keep things simple, we assume that strategic investment decisions always require some form of accord with residents (more precisely, their representative agent). Naturally, in practice such an accord can be anything in the range between perfect and non-existent. However, to keep the analysis tractable, the present model excludes the case of foreign agents taking their own strategic investment actions in full disregard of the residents.

variable of the residents. This means that, ultimately, they do decide upon their dividends quite similarly to the usual inside equity holders, even though indirectly.

- Labor markets are partially open cross-border, with some frictions that make it, ceteris paribus, less easy and less attractive to get a job abroad than at home. This is captured by different disutilities of work effort depending on whether the worker is a citizen of the country of employment. This means that the labor markets in the present model exercise an arbitrage function broadly analogous to that of the classical BSM, but without the need to assume unrealistically frictionless cross-border workforce movement.
- The domestic/foreign physical capital ratio is one of the state variables of the present model and, consequently, an argument in agents' optimal policy functions. Its variation plays the same role in the present model as MPLs in the tradable sectors in the standard BSM. In particular, the catch-up process in the BSM is represented by the convergence of the developing country MPL^T to that of the advanced country. In our setup, this would correspond to the catching-up country's physical capital stock approaching the reference country's physical capital stock from below. More generally, one can look at the physical capital stock ratio as a proxy for the relative economy size.

The final tangency point of the classical and asset-based BSE arguments concerns the assumed similarities of the non-tradable sectors. To obtain the usual BSE, one considers the marginal products of labor and assumes $MPL^{NT*} \approx MPL^{NT}$. In our model, in which non-tradability is associated with control rather than stock ownership, similarity of NT sectors must be expressed differently. A distant analogue to the classical BSM can be found in the domestically held proportion of the own country production capacity. Accordingly, our quantitative analysis will concentrate on the case where this proportion, for both countries' representative agents, will be reasonably close to 50 percent (in the current exercise, between 40 and 60 percent). In the numerical simulations conducted, optimal policies typically show mean reversion to a reference value, which, for same-size countries, is not too far away from, even though not exactly equal to, 50 percent. This is why this calibration choice does not entail a serious loss of generality, all the more so since we concentrate on structurally symmetric countries.

Under the conditions listed, a higher domestic-to-foreign physical capital stock ratio corresponds to a higher relative price of domestic stock in foreign stock price terms. We call this relative price the asset-based real exchange rate and its relation to the relative physical capital stock the asset-based Balassa-Samuelson effect. In this variant of the model, the role of goods trade is deliberately played down, which means that there is no rival goods-based real exchange rate to take care of concurrently.

Thanks to the numerical solution engine developed, the model can be exploited to produce a collection of results much richer than the headlined analogue of the BSE. One can look at this model as a tool for inspecting quantitative economic sovereignty effects in general.

One surprising quantitative result of the model is the inverse relation between own capital size and the proportion of national stock sold as outside equity to foreigners: the bigger one is, the more non-residents benefit from participation in one's economy. In other words, the model generates foreign bias more often than the better-known home bias, which is all the more pronounced the bigger the economy in question is compared to the rest of the world.

Another conclusion drawn from the numerical analysis is that, typically, the sovereignty effect seems to work in the direction of dampening the BSE: the asset-based RER is less sensitive to capital size than a direct analogy with the goods-based BSE would suggest. This finding happens to be related to the aforementioned foreign bias effect, but cannot be easily obtained without a full-distribution numerical solution. Therefore, the ability to calculate the optimal policies in a dynamic equilibrium is the central enabling element of the analysis.

1.1 Related Literature

It is not easy to point at an asset market model in the literature with both direct capital investment and equity trade. Within the international real business cycle literature, the departing point of the present model can be found in Backus, Kehoe, and Kydland (1992), although that model only allows for trade in goods, while leaving asset trade out completely. In addition, all one finds there concerning the solution is the non-stochastic steady-state discussion. In parallel, the quickly developing stochastic asset-pricing techniques in continuous time found their early applications to problems of optimal saving (in the neoclassical growth context) under diffusion uncertainty (Merton, 1975, 1977). However, the literature then branched into separate studies of portfolio optimization under exogenous risk factor specification (in our understanding – of the decisions made by the outside investor) on the one hand, and the real option theory (corresponding to the problem of our inside investor) on the other. Although the real option theory produced explicit statements about what the no-arbitrage price of equity *should be*, it did not offer a consistency check as to whether such price *really prevails* in a reasonably defined general equilibrium for the same class of agents. Apparently, technical difficulties stood in the way of doing so in an economically comprehensive way even in the continuous time case, which is analytically somewhat easier than the discrete time one.

Within the presently employed discrete-time dynamic model class, there are some commonalities between our setup and that of Angeletos and Calvet (2005, 2006), although the latter is a closed-economy model and traded securities have no direct relation to the production technology. Additionally, Angeletos and Calvet (2005, 2006) define risk factors as completely idiosyncratic, for which reason it is formally acceptable to look for deterministic long-term equilibria. In our case, (TFP) risks are systemic and the full solution must rely on stochastic control methods.

Papers have also appeared that state a dynamic two-country macro model enlarged by a portfolio choice element (see the survey by Coeurdacier and Rey, 2013). The agent problem statements there are similar to the present one. Those papers mostly rely on discussing general properties of the solution, if and when one exists, whereas the equilibrium equations are not fully solved outside of special cases. The main goal of this strand of literature is to provide an explanation of the home bias phenomenon. In this vein, Coeurdacier et al. (2010) introduce both portfolio stock investment and direct investment effectuated by the same household (to simplify matters, they posit an ad hoc linear dividend rule; otherwise, the perturbation method wouldn't work). Besides that, they allow direct investment to come from both countries, i.e., they use something that in our terms would be called a one-world economy with an imperfect exogenous home bias. The home bias is introduced through a CES rule that mixes the domestic and the foreign investor inputs. To inform empirical verification, Coeurdacier et al. (2010) conduct a log-linearization exercise around the non-stochastic steady state (NSSS). In another example, Heathcote and Perri (2013) circumvent the need to solve a joint problem of stock purchases and physical investment by delegating the latter to a pair of separate agents (intermediate and finite goods-producing firms) with partly ad hoc preferences. Then, in special cases (logarithmic preferences for consumption; separation of consumption and leisure in the utility), at least some approximate summary statistics, such as portfolio weights and income covariances, can be obtained. This paper also uses NSSS perturbation analysis in simulations.

There is also an earlier strand of literature dealing with asset pricing in open economies, in which models are in continuous time and production technologies are, most of the time, simple linear ones. A series of papers in this line of research starts with Stulz (1987). Formally, the latter setup is a specialization (in terms of state variable choice) of a more general Cox-Ingersoll-Ross (1985) model of asset pricing in continuous time from which the same authors derived the well-known term structure of interest rates formula. However, as opposed to our approach, the differences between nations in the Stulz (1987) paper and its sequels are only manifested by the existence of non-tradable consumption goods, whereas capital is perfectly mobile internationally. This would roughly correspond to the counterfactual one-world case examined in the present paper in parallel with the main one. As already mentioned, our primary focus is exactly the opposite, namely, we are interested in the consequences of control rights restrictions across borders.

In a much later and considerably less technical piece, Stulz (2005) analyzes the specific adverse forces encountered by foreign investors even in formally barrier-free equity markets, with the discussion involving the issue of the state ruler behavior. In that paper, foreign investors face an explicit disadvantage due to conflicting incentives between them and the “grabbing hand” of both the domestic inside investors and their state. In our case, explicit benefit appropriation by insiders (based on nationality) or any similarly crass misalignment of incentives is absent; all we assume is that inside investor status is restricted to residents. Therefore, the workings of our sovereignty effect on decisions are more subtle. It is all the more noteworthy that, even then, insider nationality seems to matter.

Our paper is also able to address one long-standing empirical puzzle of international finance having to do with the expected direction of cross-border investment depending on wealth or development stage. At one time, there emerged a lot of empirical work – probably inspired by Lucas’ (1990) observation concerning “uphill” international capital flows (i.e., from emerging to developed countries) – studying the effect of financial liberalization on foreign investment. Eventually, enough evidence for both uphill and more intuitive downhill cross-border capital movements was assembled (Abiad, Leigh, and Mody, 2007). Our model demonstrates that, when the sovereignty effect is accounted for, and depending on the initial cross-country distribution of financial wealth, seemingly uphill investment flows can be perfectly consistent with a long-term dynamic general equilibrium.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 states the individual optimization problems and the equilibrium concept used. It also features the basic building blocks of the numerical solution method employed. Section 4 reviews the results obtained. Section 5 concludes.

2. Model

2.1 Basic Definitions

There are two structurally identical countries, h (home) and f (foreign), with one productive firm in each. The earnings of each firm are disbursed to the owners of their common stock shares. Stockholders can be both residents and non-residents, and the total share numbers are normalized to unity. There is one representative (small) agent in each country, acting as a household of four members: one working domestically, one abroad, one managing the firm, and one purchasing and selling both stocks. The household members do not influence each other's actions within a period, taking the decisions and earnings of the other members during that period as given.

The production functions in the two countries are identical Cobb-Douglas ones (with the same capital shares) combining the domestic capital and labor supplied by both countries

$$Y = A \cdot F(K, L) = A \cdot K^{\alpha} L^{1-\alpha}.$$

We omitted the country index (superscript h or f) in the above definition to stress that the two countries have the same production function; only the values of the inputs can differ from period to period and from country to country.

The total factor productivity stochastic processes are denoted by A^h and A^f and are assumed to be unit-mean iid and mutually independent. The two physical capital stocks have the same depreciation rate δ:

$$K_{t+1}^h = (1-\delta)K_t^h + I_t^h,\ K_{t+1}^f = (1-\delta)K_t^f + I_t^f, \tag{1}$$

with I^h, I^f being the new investment volumes in period t.

One representative household member in country h supplies L^h working hours to the domestic firm, while another member of the same household supplies L^f working hours to the foreign firm. The representative foreign household is organized similarly. One member of that household supplies L^{*h} working hours to the firm in country h, and another member supplies L^{*f} working hours to the firm in country f. Accordingly, in a given period, production in country h uses L^h+L^{*h} working hours and production in country f uses L^f+L^{*f} hours (period subscripts omitted).

Wages are the same regardless of the worker's origin: w^h in country h and w^f in country f.

Since labor inputs are used within one period, the household member operating the firm takes a standard intra-period hiring decision at each date by selecting the overall labor demand at a wage taken as given. The exact split between domestic and foreign workers meeting this demand depends on the leisure preferences in each country (see later).

The household member responsible for outside equity holdings chooses, at each date, the share of each of the two available national stocks to be held next period, under the tradeoff of spending out of today's disposable income against dividends to be received next period, and under uncertain future productivity realizations. The shares selected by the two households must add up to one, pinning down the market-clearing price at each date.

The budget constraint of the household in country h is

$$c_t^h = x_{t-1}^h\left(A_t^h(K_t^h)^\alpha(L_t^h + L_t^{*h})^{1-\alpha} - w_t^h(L_t^h + L_t^{*h})\right) + x_{t-1}^f\left(A_t^f\left(K_t^f\right)^\alpha\left(L_t^f + L_t^{*f}\right)^{1-\alpha} - w_t^f\left(L_t^f + L_t^{*f}\right)\right) + w_t^h L_t^h + w_t^f L_t^f - P_t^h(x_t^h - x_{t-1}^h) - P_t^f\left(x_t^f - x_{t-1}^f\right) - I_t^h \qquad (2)$$

and that in country f is

$$c_t^f = x_{t-1}^{*h}\left(A_t^h(K_t^h)^\alpha(L_t^h + L_t^{*h})^{1-\alpha} - w_t^h(L_t^h + L_t^{*h})\right) + x_{t-1}^{*f}\left(A_t^f\left(K_t^f\right)^\alpha\left(L_t^f + L_t^{*f}\right)^{1-\alpha} - w_t^f\left(L_t^f + L_t^{*f}\right)\right) + w_t^h L_t^{*h} + w_t^f L_t^{*f} - P_t^h(x_t^{*h} - x_{t-1}^{*h}) - P_t^f\left(x_t^{*f} - x_{t-1}^{*f}\right) - I_t^f. \qquad (3)$$

Here, x^i (i=h,f) with the corresponding time subscript is the number of shares in the production facility of country i held by the household in country h, and x^{*i} is the same number for the household in country f. The total numbers of shares are normalized to unity: $x^i+x^{*i}=1$. Share short-selling is not allowed (all xs must be non-negative). The price of one share in country i is denoted by P^i.

The budget constraint equations are the ones in which the sovereignty factor makes its first appearance in the model. Observe that additions to the capital stock of country i are taken entirely from the disposable income of household i. In return, this household is the one which gets to define, in every period, the investment policy, by optimizing control I^i in its objective function. This choice is made by the household member who performs the role of firm manager. In this way, the said manager chooses what accounting calls retained earnings. (Note that, from the manager's perspective, it is not a "donation" of funds by the domestic shareholder to the domestic firm generically co-owned by foreigners, since this household member serves the firm as a whole and does not internalize the income structure of other household members.)

Residents of the other country can only participate in the operation of the country i firm by adjusting their holdings of outside equity x^i (besides working in country i, of course, but this concerns another household member).

The actual values of the structural constants employed in the numerically solved version of the model are mostly conventional; the same or similar values appear throughout the RBS literature and beyond. For instance, the capital share of 1/3 is probably among the most frequently used in the literature. Further, the intertemporal elasticity of substitution σ (set at 3) is at the upper bound of the values routinely used when one wants high but realistic risk aversion. The time preference and capital depreciation rate choices are also standard. The exceptions, perhaps, are the parameter values of the labor disutility terms, as their functional form chosen for the present exercise seems to be relatively novel. Those parameters were selected with the objective to generate aggregate labor demands in each country close to unity in the benchmark case (identical economy sizes, average TFP realizations). The chosen parameter values are listed in Table A1.

2.1.1 The One-World Economy

To gauge the importance of sovereign capital, we shall also consider a counterfactual modification of this model in which sovereignty has been abolished. This economy without sovereignty friction will be referred to as the one-world (OW) model. In the OW case, residents of both countries are allowed to make contributions to the capital stocks of both national production facilities; $I^i=I^{ih}+I^{if}$. That is, one component comes from the same-country households, and the other from non-residents. These two posts also appear additively in the budget constraint as expenditures: instead of subtracting a single quantity I^h in (2) or I^f in (3) to be invested in the own country production, one now has to split the (inside equity-like) investment between both country facilities. Accordingly, the firm in country *i* receives new investment in the amount $I^{hi}+I^{fi}$.

Each household takes the contribution from the other country as given. Accordingly, the two households (more exactly, their firm manager members) play an intra-period simultaneous-move subgame in each of the national production facilities. Their moves (the choices of the (I^{hi},I^{fi}) pair made by the household from country *i*) are subject to natural physical capital and consumption non-negativity constraints analogous to those imposed in the SC case (for more details, see subsection 3.2).

The parametrizations needed to apply our numerical solution method will be selected as appropriate modifications of the ones applied to the main, i.e., SC model. We shall then solve both models, SC and OW, in parallel.

2.2 Labor Markets

Labor supply in the present model is elastic. The labor markets are constructed in such a way as to generate well-behaved labor supply curves and, at the same time, allow for (imperfect) cross-border workforce mobility.

Whereas labor demand in the model comes from production optimization by the firms in a standard competitive way, labor supply is shaped by work disutility driven by two factors: the standard leisure preference parameter and the natural working hour limits. These will differ depending on whether the agent works at home or abroad.

In consumption units, the labor disutility of a household member selling L^h working hours at home/L^f working hours abroad is assumed to equal, respectively,

$$\nu^{own}(\lambda^{own}-L^h)^{-\varphi} \tag{4}$$

and

$$\nu^{ext}(\lambda^{ext}-L^f)^{-\varphi}, \tag{5}$$

for some positive φ (elasticity of leisure preference). This is true symmetrically for both parts of the world. λ^{own} and λ^{ext} denote the imaginary upper bound on physically possible working hours. It is assumed that $\lambda^{own} > \lambda^{ext}$, meaning that the natural working time limit when working abroad is hit earlier than at home. In the same vein, we assume $\nu^{own} < \nu^{ext}$, which means that the average effort disutility abroad is higher than that at home. The functional form of the effort disutility has been chosen so that, instead of a hard limit, there is an exploding preference for leisure in every national labor market segment, which is felt once working hours get closer to the corresponding λ. In this way, one obtains smooth labor supply curves, which are preferable in the calculations.

Within each period, the labor market in a given country clears when the wage equalizes the labor demand coming from the firm in that country with the sum of the labor supplies, as generated by the effort disutilities defined above, coming from both countries. At the same time, due to cross-border labor mobility, there exists a natural "current account imbalance": the difference between wages earned by domestic workers abroad and those paid to foreign workers employed domestically: $CA=w^f L^f - w^h L^{*h}$. That is, one is able to talk about current account values in this setting even if there is no direct cross-border trade in goods.[4]

For the purposes of discounting by the growth rate, and with the deflator in each country being the aggregate national physical capital stock, parameters λ and ν will be further rescaled as follows: $\lambda^{own,ext} = K\lambda_n^{own,ext}$, $\nu^{own,ext} = K^{1+\varphi}\nu_n^{own,ext}$. The normalized parameters λ_n and ν_n (separately for the own and the foreign labor market; superscripts are omitted here for simplicity) will be fixed constants. Under these assumptions, the labor market fundamentals (working hours and wages) are functions only of the current period productivity realizations, the home/foreign ratio of physical capital stocks, and the parameters of the model. Specifically, they do not depend on the outside equity split between the nations or, generally, on the presence of sovereignty effects.

2.3 Preferences

Being in a growth environment, we use a simple variant of the so-called comparison utility (Carroll et al., 1997). Our choice is to use the necessary *reference stock* present in those specifications on the basis of aggregate capital, not consumption. The latter variant is also possible, but leads to an increase in the number of state variables: consumption growth rates need to be included. The numerical solution would be obtainable with the same method, but with considerably larger computational requirements, without providing indispensable additional insights in our chosen context.

Specifically, we define the period utility of the representative agent in a given country as

$$u(c^e) = \frac{\left(\frac{c^e}{z^\gamma}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{6}$$

with the reference stock importance parameter γ=1 and the reference stock z itself equal to the currently accumulated national physical capital stock: $z = \bar{K}$. That is, the agent values individual consumption relative to the nation's accumulated physical capital level:

$$u(c^e) = \frac{1}{1-\frac{1}{\sigma}}\left(\frac{c^e}{\bar{K}}\right)^{1-\frac{1}{\sigma}}. \tag{7}$$

[4] Actually, in the IMF terminology, *CA* defined here better corresponds to a portion of the *capital account* (remittances), whereas trade in outside equity is reflected in the *financial account*. We will later use the term financial account in this sense.

Alternative choices of the importance parameter are possible, but its exact value is not central to the current analysis.

Another distinguishing feature of our preferences, reflected in the notation c^e instead of the usual c, is the valuation of effective, instead of raw, consumption: the current consumption level is corrected for labor disutility expressed in national consumption units. The introduction of a preference for leisure in such a direct consumption-equivalent form proves convenient in the calculations, but, in principle, could be easily abandoned in favor of a separate term in the utility function itself. Conceptually, the exact choice is unimportant for the study at hand. The functional form of labor disutility was introduced earlier. The expression for efficient consumption (in either country; the country index is omitted for simplicity) is

$$c^e = \mathrm{c} - \nu^{own}(\lambda^{own} - L^h)^{-\varphi} - \nu^{ext}(\lambda^{ext} - L^f)^{-\varphi}. \tag{8}$$

3. Optimization Problems and Equilibrium

The decision problem of the representative household in a given country at a given starting date t is to select a sequence of labor supply, investment (direct in own production capacity and outside equity-like in both production capacities), and labor hiring decisions in all subsequent periods that maximizes the conventional separable inter-period utility

$$U_t = E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_t^e)\right] \tag{9}$$

subject to the budget constraint (2) (or (3) for the foreign country), the consumption and physical capital positivity constraints, the short-selling prohibition, and the leisure preferences modifying the effective consumption defined by (8).

More precisely, the stated optimization problem is a collection of four separate ones, each for an individual household member responsible for the given activity (work at home, work abroad, firm management, outside equity investment) who takes the decisions of the other members as given.

3.1 Optimality Conditions

The necessary conditions of policy optimality in each country give rise to two Euler equations, reflecting the optimal choices of domestic and foreign outside equity holdings, and one physical investment optimization equation. These can be written in the usual form

$$1 = \beta E_t\left[\frac{u'(c_{t+1}^e)}{u'(c_t^e)} z_{t+1}\right], \tag{10}$$

$$1 = \beta E_t\left[\frac{u'(c_{t+1}^e)}{u'(c_t^e)} z_{t+1}^*\right], \tag{11}$$

$$u'(c_t^e) = \beta E_t\left[u'(c_{t+1}^e)\left\{x_t^h A_{t+1}^H F_K\left(K_{t+1}, L_{t+1}^h + L_{t+1}^f\right) + 1 - \delta\right\}\right], \tag{12}$$

with the latter MPK condition being somewhat more general than would be the case in models with an explicit presence of a risk-free interest rate. The gross domestic and foreign stock returns appearing in (10) and (11) are given by

$$z_{t+1} = \frac{P_{t+1}^h + \alpha A_{t+1}^h F\left(K_{t+1}, L_{t+1}^h + L_{t+1}^f\right)}{P_t^h}, \tag{13}$$

$$z_{t+1}^* = \frac{P_{t+1}^f + \alpha A_{t+1}^f F\left(K_{t+1}^*, L_{t+1}^{*h} + L_{t+1}^{*f}\right)}{P_t^f}. \tag{14}$$

3.1.1 The OW Case

Note that in the OW case, the FOCs for households in both parts of the world are formally the same as in (8)-(10), although the decision vectors have dimension 6 instead of 5 in the SC case. For instance, the country *h* household decides over (L^{hh},L^{hf},I^{hh},I^{hf},x^{hh},x^{hf}) in every period (the first part of the superscript refers to the household domicile and the second to the country of operation). The country *f* choice variables are defined symmetrically. Optically, (10), (11), and the stock return definitions (13), (14) remain the same, whereas the condition on each of the two MPKs, i.e., (12), applies twice, once to the household *h* decisions and once to the household *f* decisions, each time with a different stock share x^{ij}, *i*, *j*=*h*,*f*.

3.2 Policy Functions

Whereas the individual optimization problem statement is essentially standard, the form in which the optimal decisions are put will be adapted to the numerical solution method developed. Among other things, it is convenient to express the control variables in such a way that the consumption and physical capital non-negativity constraints and the stock short-selling constraints are satisfied by construction. In particular, if one defines auxiliary disposable household income statistics for the home and the foreign country as

$$q_t = P_t^h x_{t-1}^h z_t + P_t^f x_{t-1}^f z_t^* + (1-\delta)K_t^h + w_t^h L_t^h + w_t^f L_t^f \tag{15}$$

$$q_t^* = P_t^h x_{t-1}^{*h} + P_t^I x_{t-1}^{*f} z_t^* + (1-\delta)K_t^f + w_t^h L_t^{*h} + w_t^f L_t^{*f}, \tag{16}$$

consumption becomes

$$c_t^h = q_t - K_{t+1}^h - P_{t+1}^h x_t^h - P_{t+1}^f x_t^f \tag{17}$$

in country *h* and

$$c_t^f = q_t^* - K_{t+1}^f - P_{t+1}^h x_t^{*h} - P_{t+1}^f x_t^{*f} \tag{18}$$

in country *f*. Therefore, we choose to look for the optimal outside equity choices at date *t* that imply the following wealth shares: $P_{t+1}^h x_t^h = q_t s^h(y_t)$, $P_{t+1}^f x_t^f = q_t s^f(y_t)$, $P_{t+1}^h x_t^{*h} = q_t^* s^{*h}(y_t)$, $P_{t+1}^f x_t^{*f} = q_t^* s^{*f}(y_t)$. Here, y_t is the vector of period *t* state variables, to be specified shortly, and s^h s^f, s^{*h}, s^{*f} are the per-unit-of-disposable-income policy functions to be calculated. In the same vein, we look for inside equity (physical capital adjustment) decisions in the form

$$K_{t+1}^h = q_t\left(1 - s^h(y_t) - s^f(y_t)\right) v^h(y_t), \tag{19}$$

$$K_{t+1}^f = q_t^* \left(1 - s^{*h}(y_t) - s^{*f}(y_t)\right) v^f(y_t), \tag{20}$$

with v^h, v^f being policy functions normalized by disposable income and ex-outside equity investment revenue. The purpose of these rescaling transformations is to be able to restate consumption expressions (17), (18) in the form

$$c_t^h = q_t \left(1 - s^h(y_t) - s^f(y_t)\right)\left(1 - v^h(y_t)\right), \tag{21}$$

$$c_t^f = q_t^* \left(1 - s^{*h}(y_t) - s^{*f}(y_t)\right)\left(1 - v^f(y_t)\right). \tag{22}$$

This allows one to impose the physical capital and consumption positivity constraints on the optimal policies by simply requiring $s^h+s^f<1$, $s^{*h}+s^{*f}<1$, $v^h<1$, $v^f<1$ for all state variable values. Our numerical algorithm will then be designed so as to comply with these restrictions throughout the calculations.

3.2.1 Policies in the OW Case

For comparison, in the OW model, in which both representative households contribute to both country capital stocks, we have a different set of normalized policy functions (although, formally, they are still functions of the same vector of state variables). We distinguish these functions by means of double superscripts: s^{hh}, s^{hf}, etc., to stress the different rules that OW and SC agents use in equilibrium. The first superscript denotes the domicile of the household making the decision and the second superscript the country to which the decision applies. The outside equity shares and the analogues of the disposable income variables (15), (16) are denoted according to the same principle. Then we can put the inside equity decisions in the form:

$$K_{t+1}^h = q_t^h \left(1 - s^{hh}(y_t) - s^{hf}(y_t)\right) v^{hh}(y_t) + q_t^f \left(1 - s^{fh}(y_t) - s^{ff}(y_t)\right) v^{fh}(y_t), \tag{23}$$

$$K_{t+1}^f = q_t^h \left(1 - s^{hh}(y_t) - s^{hf}(y_t)\right) v^{hf}(y_t) + q_t^f \left(1 - s^{fh}(y_t) - s^{ff}(y_t)\right) v^{ff}(y_t), \tag{24}$$

Consumption expressions (21), (22) are modified accordingly:

$$c_t^h = q_t^h \left(1 - s^{hh}(y_t) - s^{hf}(y_t)\right)\left(1 - v^{hh}(y_t) - v^{hf}(y_t)\right), \tag{25}$$

$$c_t^f = q_t^f \left(1 - s^{fh}(y_t) - s^{ff}(y_t)\right)\left(1 - v^{fh}(y_t) - v^{ff}(y_t)\right), \tag{26}$$

with the disposable income statistics now looking like

$$q_t^h = P_t^h x_{t-1}^{hh} z_t^h + P_t^f x_{t-1}^{hf} z_t^f + w_t^h L_t^{hh} + w_t^f L_t^{hf} + (1-\delta)\left(x_{t-1}^{hh} K_t^h + x_{t-1}^{hf} K_t^f\right),$$

$$q_t^f = P_t^h x_{t-1}^{fh} z_t^h + P_t^f x_{t-1}^{ff} z_t^f + w_t^h L_t^{fh} + w_t^f L_t^{ff} + (1-\delta)\left(x_{t-1}^{fh} K_t^h + x_{t-1}^{ff} K_t^f\right).$$

Due to the outside equity market clearing, apparently, $x^{hh}+x^{fh}=x^{hf}+x^{ff}=1$. The consumption positivity and physical capital positivity constraints are the same as in the SC case for the outside investment functions s^{ij} ($i,j=h,f$), but a little stronger for the physical capital functions v^{ij} ($i,j=h,f$), in that one requires $v^{ih}+v^{if}<1$ for $i=h,f$. As a result, given the optimal OW policies, households from the two countries contribute to the new physical capital formation in each production facility in proportion to their last-recorded outside equity holdings in it.

Under the chosen period utility definition, we are able, at any selected date *t*, to minimize the number of state variables in each household member's optimization problem to five, regardless of the country of residence: $y_t = \left(A_t^h, A_t^f, x_{t-1}^h, x_{t-1}^f, rk_t\right)$, with the last component being the ratio of the foreign to the domestic physical capital stock levels:

$$rk_t = \frac{K_t^f}{K_t^h}\,.$$

In model extensions that use a more general form of utility, the number of state variables may have to be increased. For example, if one used a reference stock in (6) equal to the aggregate past consumption level instead of capital, the necessary additional state variable would have to be tied to the inter-period consumption growth rate. However, the additional insights provided by such an extension in the present context are not so dramatic as to justify the associated growth in the necessary calculation volume. Therefore, we do not discuss such extensions further.

3.3 Equilibrium

The dynamic equilibrium of the model is a collection of decision vectors (L^h,L^f,I^h,x^h,x^f) for the household in country h and (L^{*h},L^{*f},I^f,x^{*h},x^{*f}) for the household in country *f* in all periods. Both representative households act optimally in the above sense, while taking the decisions of the other household as given, with simultaneous clearing of both national labor markets and both outside equity markets.

There are four markets requiring explicit clearing conditions in this model: two national labor markets and two outside equity markets.

Thanks to the work disutility specification selected, labor and wage quantities are determined within each period according to

$$w^h = (1-\alpha)A^h(K^h)^\alpha(L^h + L^{*h})^{-\alpha}, \quad (27)$$

$$w^h = \varphi v^{own}(\lambda^{own} - L^h)^{-1-\varphi}, \quad (28)$$

$$w^h = \varphi v^{ext}(\lambda^{ext} - L^{*h})^{-1-\varphi} \quad 29)$$

for the labor market in country h and the corresponding three equations for country *f* (the time index is left out for simplicity). It is not hard to establish that, after the required balanced growth normalization, these conditions render growth-adjusted labor quantities and wages as functions of the model parameters, the current TFP levels, and the current *rk* value. Conveniently, they are independent of the outside equity share splits between the two country residents.

As regards the outside equity markets, the simple clearing conditions for them ($x^i+x^{*i}=1$, $i=h,f$) were already stated in section 2.1. In terms of our rescaled policy notation, they imply the following expressions for stock prices:

$$P^h = qs^h(y) + q^*s^{*h}(y),\; P^f = qs^f(y) + q^*s^{*f}(y) \quad (30)$$

(time subscripts omitted for simplicity).

Obviously, so far we have characterized the equilibrium in parallel for both the SC and the OW case. To complete the equilibrium definition, we require a simultaneous-move Nash equilibrium in the physical capital determination subgame in every period (cf. section 2.1.1) for the OW case, whereas this condition is moot in the SC case.

Further, although the consumption and physical capital non-negativity constraints are imposed independently of the sovereignty friction, their specific implementation in the assumed functional forms of the Markov policy functions is case-specific, as stated in (19)-(22) for the SC, and (23)-(26) for the OW economy.

3.3.1 Computation Techniques

Inspection of the equilibrium conditions shows that all agents' problems in our setup are time-homogeneous. This allows one to consider the equilibrium calculation as a problem of finding policy functions of the common state vector *y* belonging to a subspace of a (five-dimensional) Euclidean space. The collection of policy functions sought must satisfy the FOC (10)-(12), since they maximize utilities (9) and the maximum must be interior (recall the positivity conditions of section 3.2). One can also easily establish uniqueness by means of commonplace strict utility concavity arguments.

The numerical procedure developed to calculate policy function vectors *s* and *v* actually conducts a quasi-optimization search on a discrete grid of *y* values. Obviously, the precision depends on the grid density and can be enhanced at the cost of increasing calculation volume. Since we work with TFP variables distributed over a finite support, whereas stock fractions lie between zero and one, the domain to be discretized is finite along four out of five dimensions. The fifth state variable, *rk*, formally has the whole positive half-line as support: one cannot a priori exclude extreme disparities in country sizes. Therefore, the *rk* grid must include sufficiently big values as well as values close to zero. A direct approach would be to calculate an approximate solution on a grid with a fixed pair of trial boundaries for *rk* and then validate the solution by checking whether exceeding the boundaries happens with a sufficiently small probability. These ex post checks can slow down the algorithm substantially. However, in the present exercise we employ an innovation based on perceptron approximations of the policy function candidates. The biggest advantage of this neural network technique is that, formally, the approximating function is defined on the whole half-line. At the same time, the actual focus (and approximation precision), analogously to the human brain, on which the artificial perceptrons are modeled, is located in the finite interval of interest: one simply is not all that interested in precision goals for very small or very large values of *rk*. If the first trials show that the window of attention needs to be shifted somewhat, the grid itself can be easily redefined without any substantial loss of the accuracy gained in the previous calculation steps. Obviously, the applicability of this mechanism is not limited to the unbounded dimension of the policy function domain; we employ it for stock split variables as well, when needed.

3.3.2 External Balance Fundamentals

As mentioned earlier, the present model does not include explicit trade in consumption goods, for which reason the current account is always balanced by construction (although remittances, i.e., wage transfers by workers employed outside their country, exist and can be recorded as a capital account component). The external balance statistics that it makes sense to examine in the model are the net investment position, defined from the country h viewpoint as

$$P^f x^f - P^h(1-x^h),$$

and the financial account, defined for country h at time t as

$$P_t^h(x_{t-1}^h - x_t^h) - P_t^f(x_t^f - x_{t-1}^f)$$

in the SC case, with the OW-case definitions adjusted to the corresponding notation.

Finally, we define the statistic corresponding to what people usually associate (often adapted to the context of the particular model employed in their analysis) with the real comparison terms of two economies investigated jointly: the “real exchange rate” (RER). The asset-based real exchange rate (or the “terms of equity trade”) in the present model is defined as

$$R = \frac{P^f}{P^h}.$$

This is an analogue of the usual terms of trade, but with the nominal exchange rate component omitted, since the present model assumes a common global numéraire, but no cross-border trade in goods (only cross-border labor movements are possible, subject to frictions).

When the policy functions are known, (30) allows one to express the RER at any given date in terms of the current state variables. We comment on the RER properties obtained in this manner in the next section. Still, the RER in this model lies downstream from stock prices and the latter lie downstream from the state variables. Recall that the latter, besides the stationary i.i.d. TFP shocks, include the home/foreign ratio of physical capitals and the fractions of h- and f-owned stocks in national production facilities. Accordingly, our examination of the model outcomes will be focused on the physical capital and stock holding statistics.

4. Results

Visualizing the results of a full-distribution solution when policy functions have several variables is not associated with an established practice in the literature comparable to impulse response functions (IRF) for perturbation analysis. In the following, without any attempt to impose a canon on future analysis, we illustrate the outcomes of calculations by means of sectional graphs of a few key fundamentals.

4.1 New Investment

First, we find out how people add to the capital stock in their country depending on the size of already installed capital relative to the other country (which in this model is a proxy for the relative size of the economy). We also inspect the role of domestic and foreign financial wealth in this respect, in an economy in which domestic residents are the sole controlling owners of the national production facilities (i.e., when the sovereignty factor is present, called SC). Figure A1 reports the dependence of new capital stock (deflated by the current national physical capital level as our scaling factor in the balanced growth environment considered) on three state variables:

relative size, the fraction of domestically held shares in home production, and the fraction of domestically held shares in foreign production. As a reference, we also report the corresponding values for an economy in which control over either production facility is unrestricted by residence (the OW case).

One immediate qualitative difference between the preserved and ceded sovereignty cases is the intuitive growth of capital accumulation with economy size in the OW case (people simply allocate more funds to the capacity which is bigger). This is in evident contrast to the SC economy. There, residents optimally allocate relatively less funds to their own production capacity if it is big enough already compared to the foreign one. As we will explain shortly, they prefer to attract funds from non-residents by selling them relatively more outside equity shares. Given that this behavior is mirrored by the residents of the other country, there is an aggregate average tendency for the capital levels in the two countries to stabilize and converge instead of drifting further apart. On the contrary, there is no such tendency in the OW economy: capital differences there tend to be preserved in the dynamic equilibrium. Could this provide a clue to the diehard wealth disparities that exist between differently developed parts of large economies, a phenomenon for which regional economic theory has so far been unable to offer a convincing explanation?

The dependence of capital formation on the current fraction of domestically held outside shares (in either country) exhibits quite different properties with regard to the sovereignty factor. In the SC case, at least when the relative economy size is held constant, this fraction, as an indicator of financial wealth, is the main driving factor of the resource allocation decision. The wealthier domestic residents are, the more resources they allocate to their future production. This happens irrespective of whether the domestic or the foreign stock is the source of augmented financial wealth. Qualitatively, the OW case exhibits the same pattern, but the sensitivity to current financial wealth is much lower. This is not surprising given that, in the OW setting, each agent participates in investment decisions in both economies. Accordingly, some of the funds not supplied by agents in one country due to the wealth effect are compensated by the contribution from the other country. The size of this contribution, as was mentioned earlier, is an increasing function of relative economy size (the physical capital ratio).

4.2 Stock Prices and RER

Next, we look at how asset values depend on the same three state variables. For both stocks, domestic and foreign, (growth-discounted) asset values are decreasing functions of own capital. On the domestic side, the reason is the declining MPK, and on the foreign side it is the declining importance of foreign production in investors' decisions in the face of a growing weight of domestic production. More subtle is the fact that the two stock prices are differently sensitive to the home/foreign capital ratio, the sensitivity of the domestic stock being higher.

Finally, we can look at the ratio of foreign and domestic market capitalization values, i.e., the (asset-based) real exchange rate. Predictably, this exchange rate appreciates (domestic assets become relatively more valuable) with growing current domestic/foreign capital ratio. Appreciation can also be a consequence of higher financial wealth levels of domestic households, regardless of whether the source of this elevated wealth is higher domestic or foreign stock holdings. Qualitatively, these patterns are predictable. The actual role of sovereignty friction should be assessed by looking at the same dependencies in the OW setup and comparing the intensity of the effect between the SC and OW cases. Figure A3 provides such a comparison.

To begin with, observe that the "exchange rate" in the OW economy is not really dependent on the relative financial wealth (on either side of the border, which is now purely symbolic; in the OW model, the two former countries are only distinguished for data collection purposes). There is merely a very weak dependence on stock holding asymmetry: the country whose stock holdings are initially asymmetric between residents and non-residents undergoes a rebalancing that makes its assets slightly more expensive in that period. On the contrary, the SC economy exhibits the expected conventional wealth effect with regard to the real exchange rate (panels (b) and (c) of Figure A3).

More insights are offered by an inspection of the dependence of the RER on the capital ratio (panel (a) of Figure A3). Evidently, compared to the OW case, the presence of sovereignty dramatically reduces the sensitivity of the RER to the capital stock ratio. This is so because, in an SC economy, households prefer to hold more outside equity from abroad, while selling more own outside equity to foreigners, than in the OW economy (see Figure A4; the same phenomenon can also be made visible in net investment position terms – see Figure A5). If their capital stock is relatively big, they benefit more from their sovereign privilege by rebalancing their financial wealth from domestic toward foreign outside equity, but, at the same time, support a higher foreign stock price in the market than what simple portfolio optimization in an OW economy would generate. Due to this mechanism, the effect of size/capital disparity on the exchange rate is dampened. On the surface, this looks like an inverse of the BSE in the goods markets: instead of a stronger RER due to the presence of NT goods when catch-up takes place, one sees, when non-tradable sovereign rights are present, a much weaker trickling down of the real catch-up into the real exchange rate than in their absence. In other words, the BSE seems to be reversed. On closer inspection, though, one sees that the RER does not appreciate so strongly in reaction to convergence of physical capital levels simply because it was not so weak prior to the convergence the mentioned foreign bias in outside equity holdings strengthened the RER. In addition, much more complex effects emerge if one analyses more asymmetric initial conditions patterns involving more than one state variable.

4.3 External Balance Indicators

To give an idea of what can be inferred about the RER when considering the initial conditions of multiple state variables together, we inspect it in conjunction with the financial account, looking at the dependence of both these fundamentals on relative capital size under varying levels of non-resident ownership of domestic stocks. Figure A6 provides this view for the financial account, with the sign convention set so that a positive value means more domestic assets sold to foreigners than foreign assets purchased from foreigners. Figure A7 gives the same dependence for the RER, under a fixed 50 percent resident/nonresident split of foreign stock ownership and with the domestically retained outside equity share at 45, 50, and 55 percent.

First of all, the potentially unstable nature of the seemingly "perfect balance" state of $x^h=x^f=0.5$, $K^h/K^f=1$ transpires. As mentioned earlier, rational agents in this SC model have foreign bias in terms of outside equity holdings. Therefore, if they start with a 50/50 share split between them and foreign households, they shed own stock and acquire the foreign one, making the financial account under our sign convention positive. One can see this in more detail in Figure A8, for the domestic share choice in panel (a) and the foreign share in panel (b). Besides the extent of the foreign bias (exemplified by the comparison with the OW case, for which the selected *h* and *f* shares are identical under symmetry of other state variables), one can observe the already mentioned mean-reversion of optimal share choices by comparing their schedules with the 45-degree line. In a sufficiently large neighborhood of the initial 50/50 share split point, agents in the SC economy optimally choose a foreign bias for the next period. However, they also correct large initial portfolio imbalances by optimally reducing their magnitude.

In general, the optimal portfolio choices in this model are internal (x^h<1, etc.) for standard portfolio diversification/international risk-sharing reasons. Still, regularities of the exact stock splits in equilibrium might go against some pre-existing conventional wisdoms. More specifically, let us look at the initial state with a 50/50 share split. Then, domestic and foreign stocks cost the same at this initial point, resulting in an RER equal to one. Deviations of the relative capital size from unity lead to an RER stronger or weaker than one, but the financial account stays positive for deviations in a fairly large neighborhood of this initial condition.

Further, it is confirmed that both the financial account and the RER are sensitive to the initial stock share splits. When domestic households start with too much of the home outside equity and not enough of the foreign equity, their rebalancing effort in the next period, in conjunction with the mirror-image effort of foreign households, makes home stocks more expensive and foreign stocks less expensive, implying, ceteris paribus, a stronger RER. Also the financial account is positive in such circumstances, satisfying the conventional wisdom ("improving external balance, ergo stronger currency"). However, the closer the initial condition for the share split gets to the desired one, the weaker this spurious regularity becomes. At the same time, when share rebalancing is – almost – not an issue, the capital stock ratio gains significance for the RER value. As a result, situations in which the financial account is positive, whereas the equilibrium RER is above unity (which may seduce one to call it undervalued), are perfectly normal in this model. The conventional wisdom about the external balance-exchange rate link becomes misleading.

Naturally, the obtained numerically calculable full-distribution solution of the model provides access to a lot of other information. Given the richness of the interpretation opportunities supplied by the available policy functions, we could have proceeded further and further in discussing equilibrium external balance fundamentals for other state variable constellations relevant for policy analysis. Instead, in this overview of assorted numerical outcomes, we have deliberately discussed only some of the less expected ones. More straightforward and intuitive results are either not reported here in detail, or are omitted altogether. For example, same-period productivity differences have fully predictable consequences for investment and asset prices (the more productive the economy, the more funds it receives and the higher its stock price becomes). Further, as expected, the "current account" (cf. section 2.2) shows an inverse dependence on both the home/foreign capital ratio and the home/foreign TFP differential: both bigger economies and more productive ones naturally hire more foreign workers, whose wages are sent abroad. Similarly, productivity differences have the expected effect on the real exchange rate: it appreciates in the momentarily more productive economy.

In this paper, we deliberately left out welfare analysis, since the simple SC-OW comparison conducted here proceeds along a different analytical dimension. There is the obvious welfare improvement one should expect from the hypothetical SC-to-OW transition, simply because one expands the agents' opportunity sets. However, it would not be all that useful at this point to associate policy exercises with hypothetical relaxations of the SC friction, given that the latter is a phenomenon outside the economic analysis area: the exercise conducted here was not intended to investigate the "self-dissolution" of a sovereign nation.

5. Conclusion

We have constructed a stochastic dynamic two-country model with sovereignty friction in the capital markets: control over investment and labor-hiring decisions in the production facility operating in one country cannot be exercised by residents of the other country. We have further developed a numerical procedure to obtain a full-distribution solution of this model. We present the solution in the form of calculable policy functions for representative agents in both countries and the resulting equilibrium asset prices as functions of current values of five fundamentals: domestic and foreign productivities, the domestic and foreign stock ownership split between residents and non-residents, and the ratio of country-level physical capital stocks.

We have found that the asymmetric status of investors depending on their nationality is reflected in the behavior of conventional fundamentals that describe external imbalances (the net investment position, the financial account, and the asset-based real exchange rate), but its workings are not as unidirectional as the analogy with the classical Balassa-Samuelson effect in the goods markets might suggest. On the one hand, catch-up in conventional financial wealth measures does indeed go hand in hand with real appreciation on the side of the catching-up country, in the same way as it happens in the usual BS story. There is, essentially, no such effect in a hypothetical world with identical production capacities but control not restricted to residents. On the other hand, catch-up in the physical capital level, although it qualitatively produces the same real appreciation effect, does so on a much lesser scale than in the mentioned "one-world" setup. In the latter, differences in physical capital levels in the two parts of the world result in a much stronger reaction of the asset-based real exchange rate, since those differences, which are in a one-to one correspondence with the relative asset market size, are the only differentiating factor that guides their investment. In its absence, individually optimal decisions would produce a standard risk-sharing solution. On the contrary, with the sovereignty factor present, there is a compensating effect on the side of the country with the lower capital stock. This effect takes the form of elevated sales of domestic (outside equity) shares to foreigners and a resulting fall in the net investment position, as opposed to the OW case. The effect of sovereignty on national asset prices is such that domestic market capitalization does not fall as much, and the foreign market capitalization does not rise as much, as the installed capital discrepancy would suggest based on what happens in the one-world case. As a result, the (asset-based) real exchange rate is closer to unity under the sovereignty friction, giving rise to something one might be tempted to call the "anti-Balassa-Samuelson effect."

The exercise described in the paper delivers a multitude of very diverse constellations of equilibrium policies implied by possible combinations of the initial capital stocks, asset prices, and productivities. Among other things, it provides several counter-examples to the popular belief about the ability of external balance fundamentals to explain exchange rate behavior (Égert et al., 2006; Cline and Williamson, 2008; Lee et al., 2008). We show that a simplistic causality of this kind has no place in the full-distribution equilibrium obtained.

More generally, as our analysis clearly demonstrates, the influence of sovereignty on capital formation, asset market capitalization, and the real exchange rate is both multifaceted and multidirectional. Therefore, its role cannot be easily summarized by a single catchphrase of the BSE type, instead producing a variety of (occasionally unexpected) effects. In the presented model, these effects are driven by the interplay of installed capital levels, patterns of cross-border ownership, and current productivity differentials between the two countries. One thing to be certain about is that sovereignty matters in both qualitative and quantitative terms.

Finally, in this exercise, we deliberately model a structurally symmetric case, not because it is realistic, but simply because any future taking the theory to the data would require a comparison to this symmetric benchmark as a pre-requisite. Otherwise, one would be unable to separate the effects of asymmetry from other, mainly quantitative, factors. Applying the model to relevant policy questions, such as the proper treatment of North-South or East-West imbalances, means introducing appropriate asymmetries as a natural next step. This is a topic of future research.

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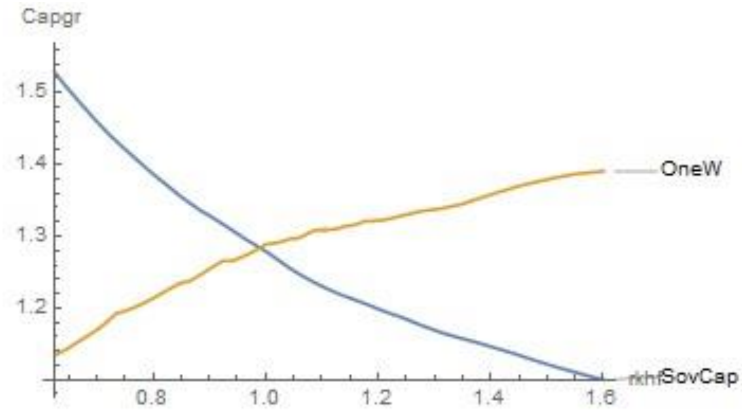
Appendix

Table A1: Parameter Values

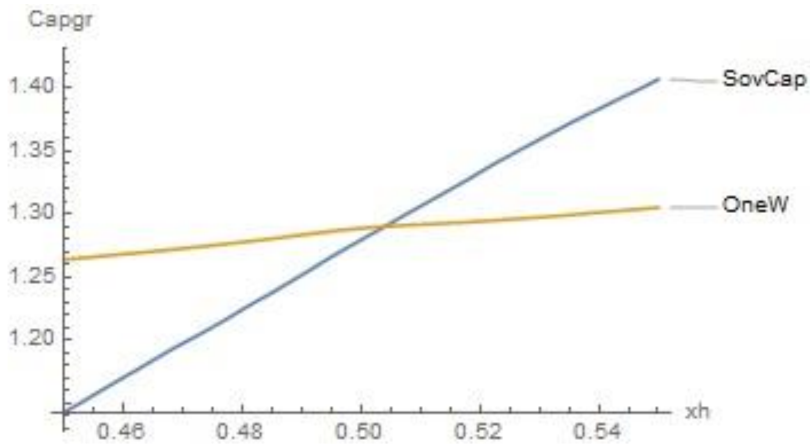
Notation	Description	Value
α	Capital share in production	0.33
β	Time preference rate	0.98
δ	Physical capital depreciation rate	0.03
σ	Elasticity of intertemporal consumption substitution	3
λ^{own}	Upper bound on working hours supply in own country	2
λ^{ext}	Upper bound on working hours supply abroad	1
ν^{own}	Average work effort disutility at home	0.04
ν^{ext}	Average work effort disutility abroad	0.08
φ	Elasticity of leisure preference	0.05

Figure A1: Capital Formation as a Function of

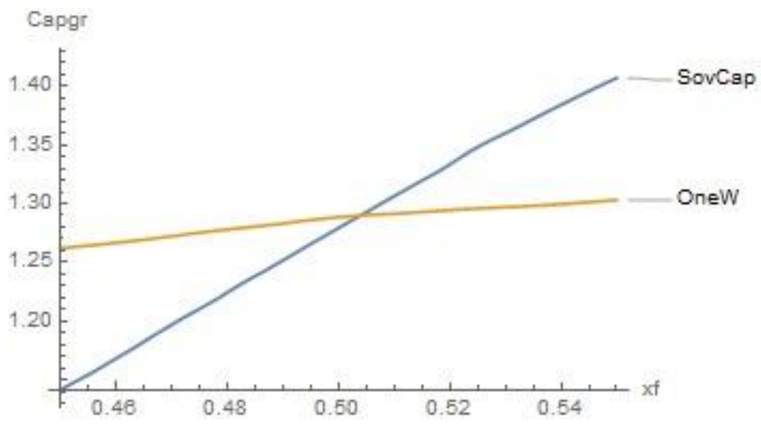
(a) Relative Home Country Past Installed Capital ("Size")



(b) Share of Domestically Owned Home Stock



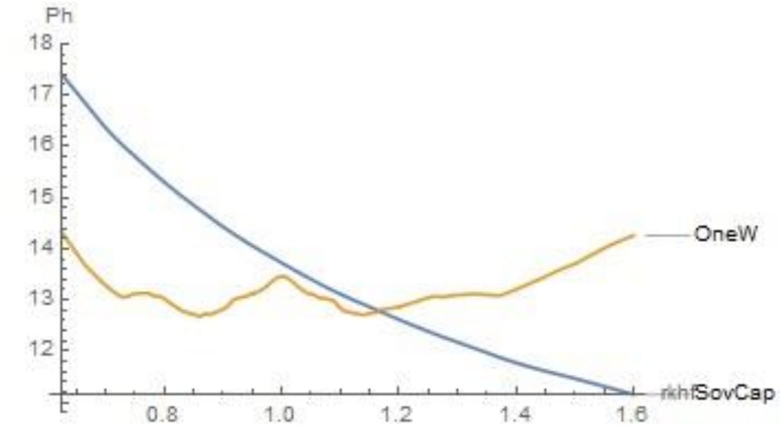
(c) Share of Domestically Owned Foreign Stock



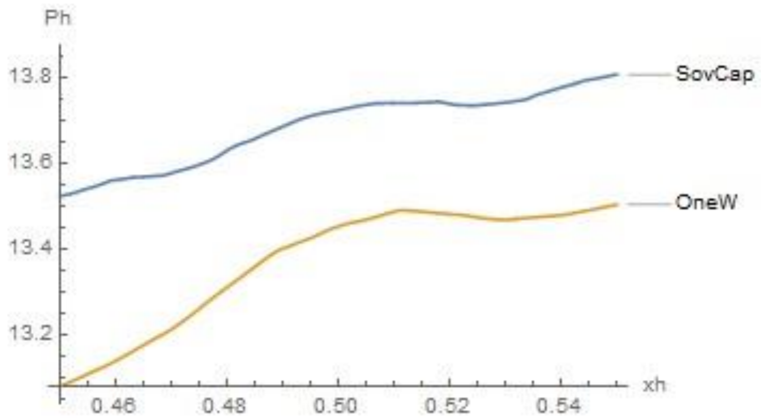
Note: SovCap – the case of a valid sovereignty factor, OneW – the case of no sovereignty; the reported values are deflated by the previous capital stock level

Figure A2: Domestic Stock Market Capitalization as a Function of

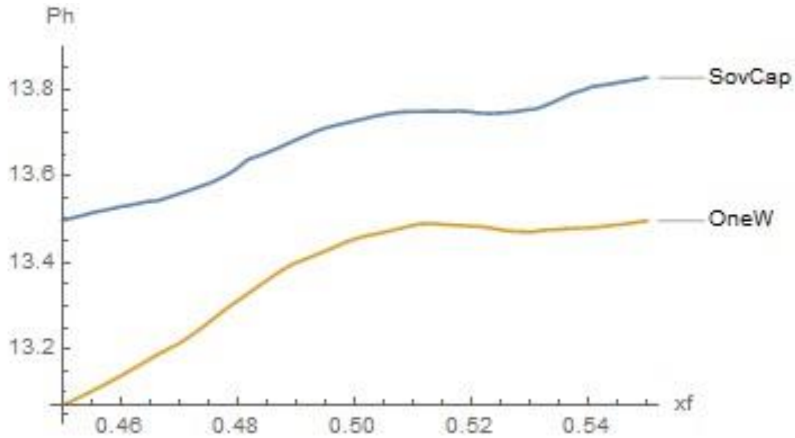
(a) Relative Home Country Past Installed Capital ("Size")



(b) Share of Domestically Owned Home Stock



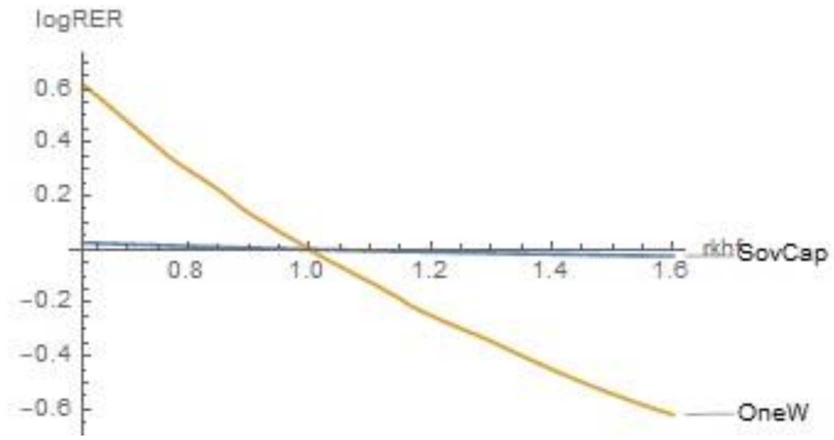
(c) Share of Domestically Owned Foreign Stock



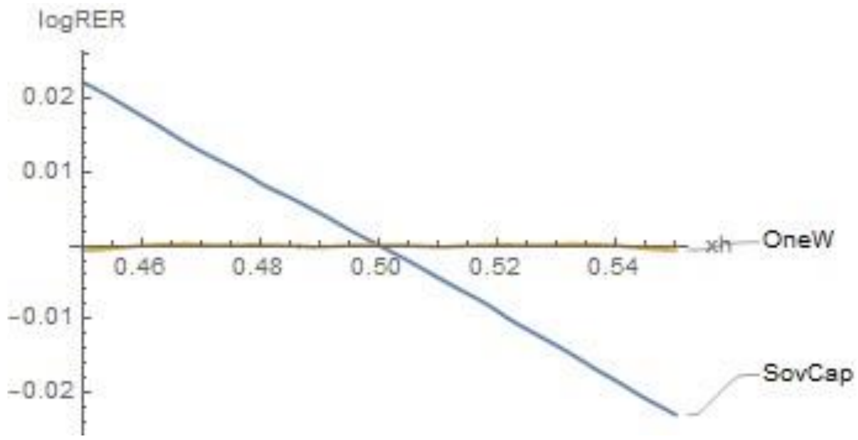
Note: SovCap – the case of a valid sovereignty factor, OneW – the case of no sovereignty

Figure A3: Asset-Based Real Exchange Rate as a Function of

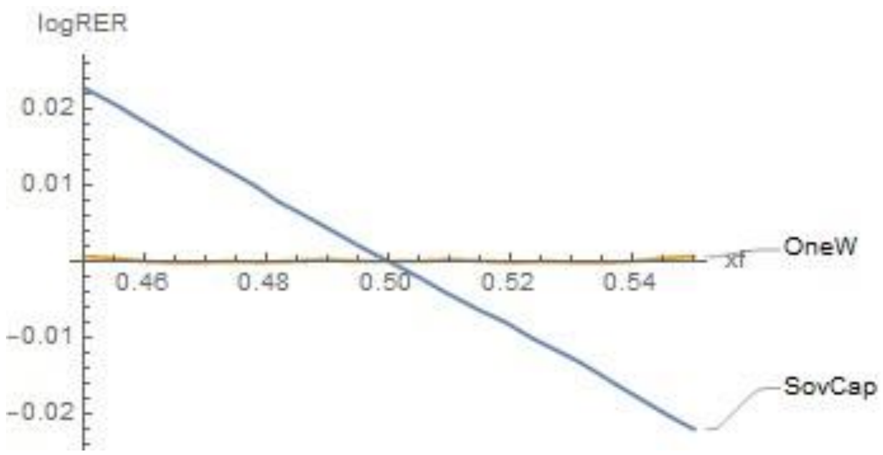
(a) Relative Home Country Past Installed Capital ("Size")



(b) Share of Domestically Owned Home Stock



(c) Share of Domestically Owned Foreign Stock



Note: SovCap – the case of a valid sovereignty factor; OneW – the case of no sovereignty

Figure A4: Shares of Home and Foreign Stock Holdings as a Function of Relative Physical Capital

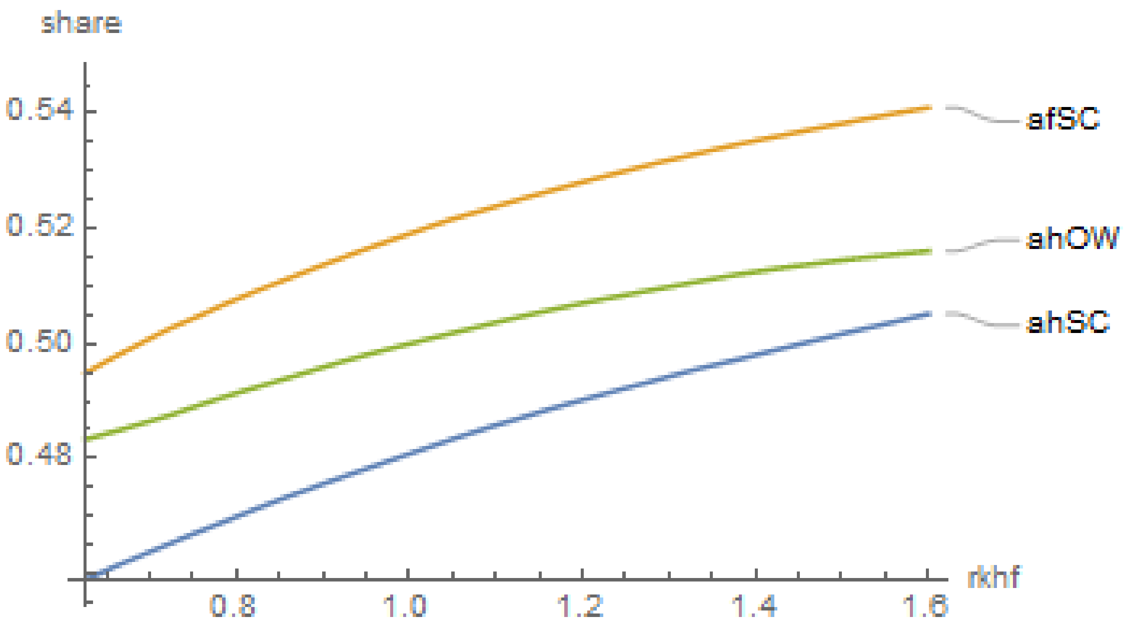


Figure A5: Net Investment Position as a Function of Relative Physical Capital

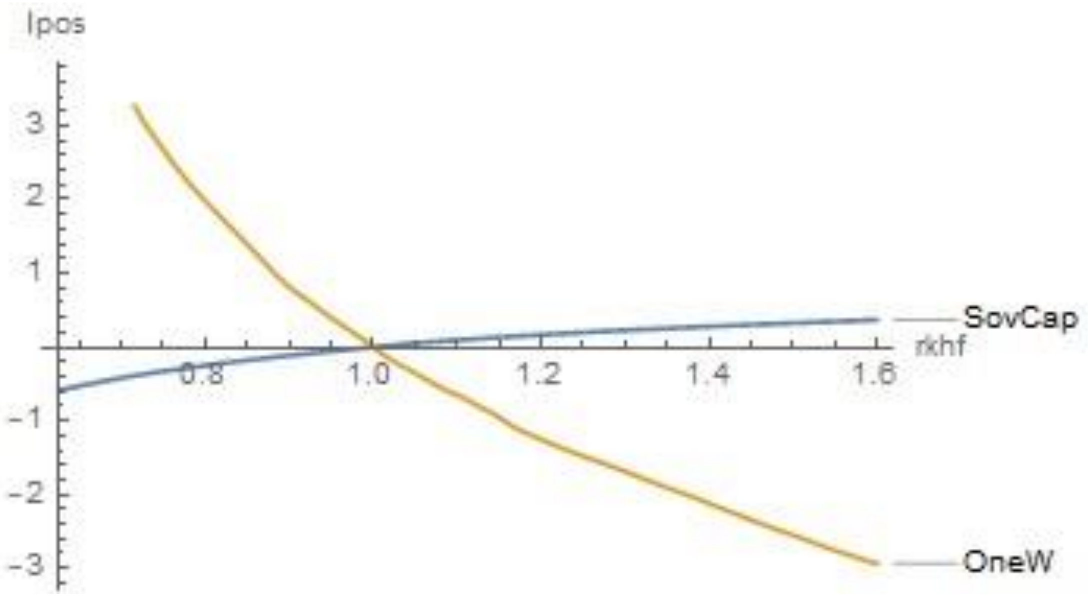


***Figure A6:** Financial Account as a Function of Relative Domestic Physical Capital Size under Varying Levels of Domestic Holdings of Home Outside Equity*

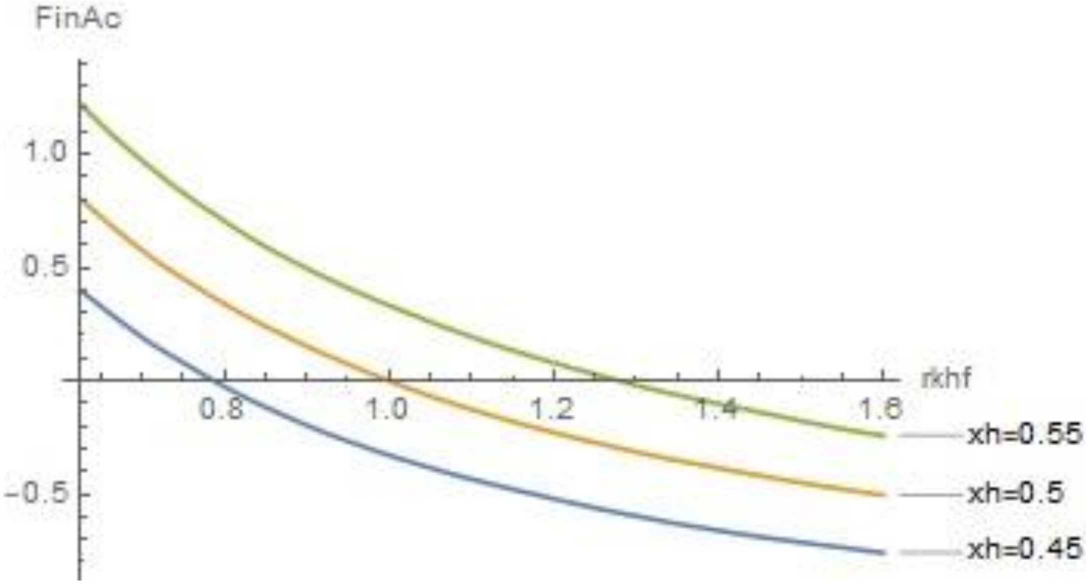


***Figure A7:** Log of Real Exchange Rate as a Function of Relative Domestic Physical Capital Size under Varying Levels of Domestic Holdings of Home Outside Equity*

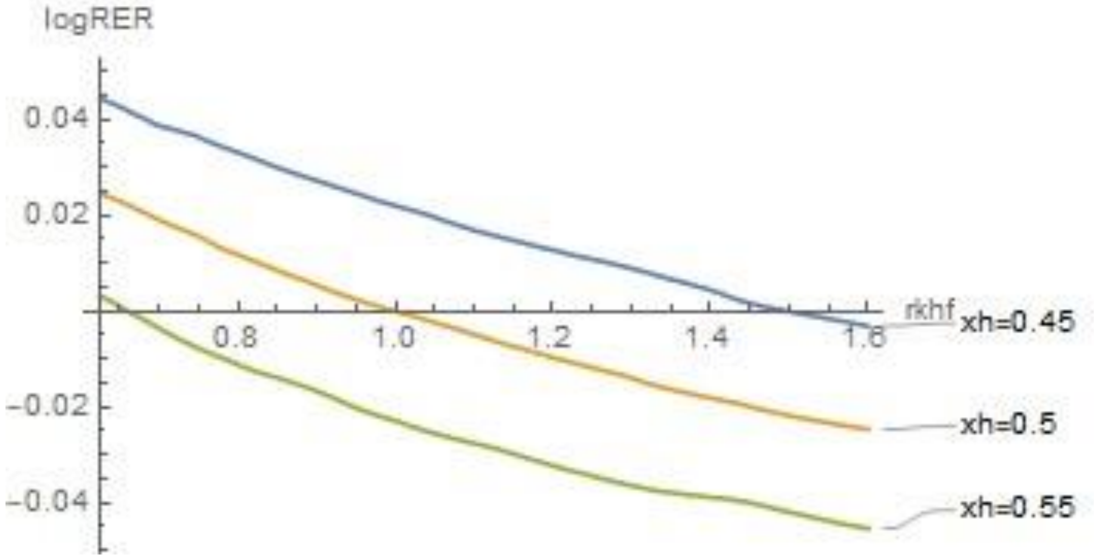
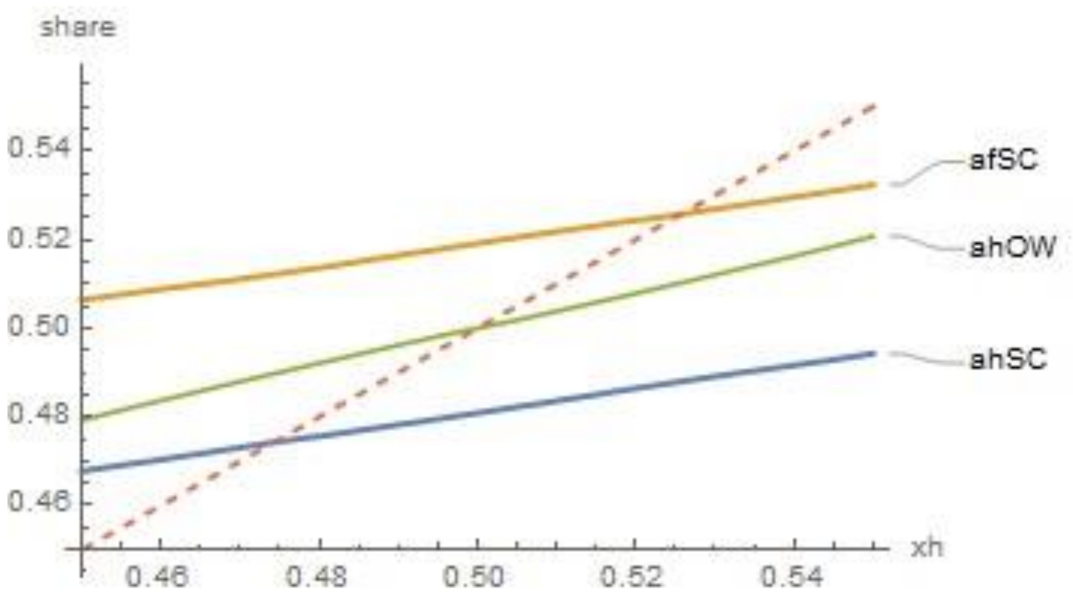
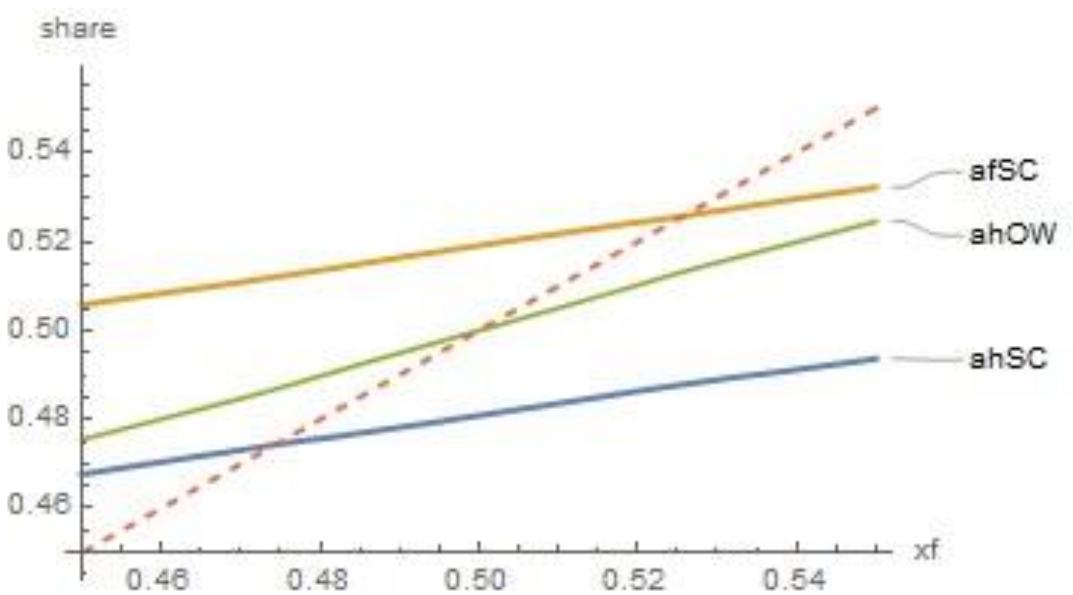


Figure A8: Next-Period Domestic and Foreign Outside Equity Share Selection as a Function of

(a) Initial Domestic Share



(b) Initial Foreign Share



Note: the dashed 45° line shows the unchanged share possibility

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