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Tibor Hlédik, Jan Vlček

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The Czech Case

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Tibor Hlédik and Jan Vlček [*]

Abstract

We identify the natural rate of interest in the Czech Republic as the real rate consistent with output at its equilibrium level and inflation at the target. To identify the rate, we use a (semi-)structural model featuring rational expectations and a forward-looking interest rate rule. Compared to the mainstream literature, the model provides a comprehensive set of cross-restrictions with respect to unobserved variables, including that of the natural rate. Furthermore, we argue that the natural rate of interest in a small open economy is a function of equilibrium real growth adjusted for equilibrium real exchange rate appreciation. Our findings suggest that the natural interest rate in the Czech Republic was around 1 percent in 2017. The current decline of the natural rate from its peak in 2015 mainly reflects the renewed appreciation of the equilibrium real exchange rate on the back of robust real GDP growth.

Abstrakt

Tato studie identifikuje přirozenou úrokovou míru pro Českou republiku, jako reálnou úrokovou sazbu konzistentní s reálným výstupem na rovnovážné (potenciální) úrovni a inflací na cíli. K identifikaci přirozené úrokové míry používáme (semi-)strukturální model s racionálními očekáváními a vpředhledícím úrokovým pravidlem. Na rozdíl od přístupu běžně používaného v literatuře poskytuje strukturální model úplnou množinu omezení na nepozorované veličiny včetně přirozené úrokové míry. V této studii také argumentujeme, že přirozená úroková míra pro malou otevřenou ekonomiku je funkcí rovnovážného (potenciálního) reálného růstu, který je očištěn o zhodnocování reálného rovnovážného kurzu. Výsledky naznačují, že přirozená úroková míra se v České republice v roce 2017 pohybovala na úrovni kolem 1 procenta. Identifikovaný pokles přirozené úrokové míry z jejího vrcholu v roce 2015 reflektuje především obnovené zhodnocování reálného rovnovážného kurzu na pozadí robustního růstu reálného HDP.

* Tibor Hlédik, Joint Vienna Institute, Vienna. Email: thledik@jvi.org
Jan Vlček, Czech National Bank, Prague. Email: jan.vlcek@cnb.cz, janvlcekatcz@gmail.com

Nontechnical Summary

Interest rates have declined substantially since the early 1990s and this decline cannot be explained solely by central banks having kept inflation at low and stable levels, since the downward trend has been observed in both nominal and real interest rates. As central banks have started to phase out their unconventional monetary policy tools, the fundamental role of interest rates as a monetary policy instrument has been restored. The low or even significantly negative level of real interest rates accompanied by a prolonged period of suppressed inflation rates calls for an answer to the key monetary policy question what the level of the natural rate of interest is. The natural level is defined as the real interest rate consistent with stable inflation implicitly at the target and with output at its equilibrium level. If the natural rate has declined compared to its pre-crisis level, central banks will also have to keep their nominal rates lower so as not to hinder real growth. On the contrary, if the natural rate has remained at its pre-crisis level, monetary policy might be too accommodative. Hence, the question about the level of the natural interest rate is moving to the center of policy interest.

In order to identify the natural rate of interest in the Czech economy, we modify the commonly used identification methodology of Laubach and Williams (2003) in two respects. First, the natural rate of interest is not only linked to equilibrium GDP growth, but also adjusted for appreciation of the equilibrium real exchange rate. This adjustment is needed because GDP growth only measures yields from production and ignores the convergence of the Czech economy to its advanced European peers. Second, we use a (semi-)structural model which is closed by a monetary policy rule.

The structural model allows us to work with model-consistent expectations and use the comprehensive set of restrictions, i.e., model equations, determining the natural rate of interest. The model resembles the Czech National Bank's (CNB) Quarterly Projection Model (QPM), see Benes et al. (2003). However, compared to the CNB's original QPM, the model contains several modifications. First, headline CPI is decomposed into core inflation excluding food and energy, energy price inflation, food price inflation, and change in administrative prices. Trends in relative prices are taken into account and explicitly modeled. Second, the effects of foreign prices and exchange rate dynamics on inflation are captured using the real exchange rate in real marginal costs, not through import price inflation as in the CNB's QPM.

The model is calibrated to match stylized facts of the Czech economy and transmission mechanism. The calibration of the model is verified using several simulation exercises. First, model-based forecasts (in-sample simulations) are used to check the ex-post forecasting power of the model. Second, the model structure is used to identify structural shocks and unobserved variables to interpret the Czech data. The filtration is done by a multivariate Kalman filter (KF), which minimizes the expected least squares of the shocks. A key aspect of the KF is that cross restrictions arising from the structural model equations are used to replicate the data. Consequently, the natural rate of interest identified is conditional on the framework used.

Our estimates suggest that the natural rate of interest hovers around 1 percent. Prior to the world financial crisis in the 2004–2007 period, the natural rate was quite stable and close to 1 percent p.a. This relative stability was followed by a period of increased volatility despite the fact that the Czech economy was not directly hit by the financial crisis. It did, however, face lower foreign demand and a volatile nominal exchange rate during the 2008–2010 period. Our estimation results suggest that the natural rate declined in the aftermath of the crisis and subsequently returned to 1 percent. The natural rate peaked in 2015, a period of rapid equilibrium GDP growth supported by high investment inflows under EU structural programs. At the same time, the real exchange rate appreciation was

still slow. The natural rate fell to 1 percent in 2016–2017 as potential GDP growth declined and the pace of real equilibrium exchange rate appreciation picked up.

Although the methodology relies on a semi-structural model, the natural rate of interest identified is obviously subject to uncertainty. The 90 percent and 60 percent confidence bands are about about ±200 and ±100 basis points wide, respectively. The assessment of the width of the confidence bands, however, should take into account the following three key aspects. First, all the shocks used to match the historical data are taken into account. Second, the period for which the standard deviations are inferred includes the world financial crisis, which was characterized by a high volatility of most macroeconomic and financial variables. Third, the bands only capture the uncertainty related to underlying shocks and ignore model uncertainty.

1. Introduction

Interest rates have declined substantially since the early 1990s. This decline cannot be explained solely by central banks having kept inflation at low and stable levels, since the downward trend has been observed in both nominal and real interest rates (see Figures 1 and 2). Similarly, the recent slowdown in world economic growth, which has brought about a need to ease monetary policy by pushing nominal interest rates into negative territory and/or by introducing unconventional policy measures in many advanced countries, is a phenomenon of the last decade only. Regardless of differences in macroeconomic performance and stage of development, the decline in interest rates is widespread across regions, including both advanced and emerging market countries.

The natural rate of interest (or the equilibrium real rate) is the real interest rate level consistent with stable inflation implicitly at the inflation target and with output at its equilibrium level. The concept of the natural rate goes back to Wicksell, who defined it as the rate consistent with a stable price level. Later, Woodford revived attention in the concept, linking it to a monetary policy rule within a New Keynesian framework. In his work Woodford (2003) defined the neutral rate as the rate which would prevail under flexible price equilibrium. He also showed that it is positively correlated with productivity and consumer preference shocks.

Declining nominal and real interest rates accompanied by very low inflation rates renewed central bankers' interest in determining the level of the natural rate of interest. As central banks have started to phase out their unconventional monetary policy tools, the role of interest rates as a standard monetary policy instrument has been re-established. If the natural rate has declined, central banks will have to keep their nominal interest rates below the pre-crisis level, otherwise their monetary policy actions will curb real growth. Conversely, if the natural rate remains at the pre-crisis level, monetary policy might be too expansionary. Hence, the question about the level of the natural rate of interest is moving to the center of the policy debate all over the world. In fact, the natural rate of interest is an integral part of any Taylor-type monetary policy rule. It determines the level of the policy interest rate consistent with both inflation being at the target and output being at the equilibrium level. Furthermore, the difference between the real interest rate and the natural rate – the real interest rate gap – is commonly used as a part of the monetary conditions index. It measures the effects of monetary policy on the real economy, specifically on intertemporal substitution. Observing the short-term real rate below its natural level means that the real interest rate stimulates the economy as measured by the output gap.

This paper is organized as follows. The second section provides a brief survey of the literature on the natural rate of interest. It is followed by a description of the methodology used to identify the natural rate. The fourth section discusses the findings and the robustness of the estimates. The last section concludes.

2. Literature Review

The literature on the natural rate of interest follows two streams. The first stream identifies the natural rate of interest without discussing the factors behind its dynamics. The second stream, by contrast, focuses mainly on the causes and/or consequences of low interest rates. However, it does not identify any particular level of interest rates; it just identifies the factors underlying the general tendency of real interest rates to decline. Our survey focuses mainly on the first stream, covering the identification of the natural rate of interest.
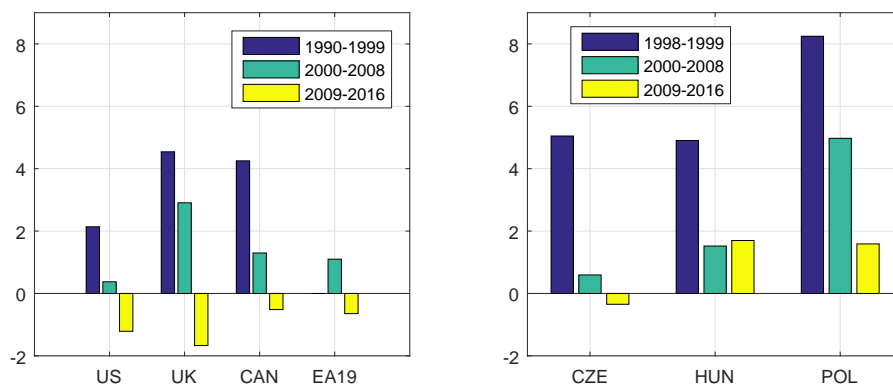
Figure 1: Average Nominal Interest Rates – Advanced and Emerging Market Countries
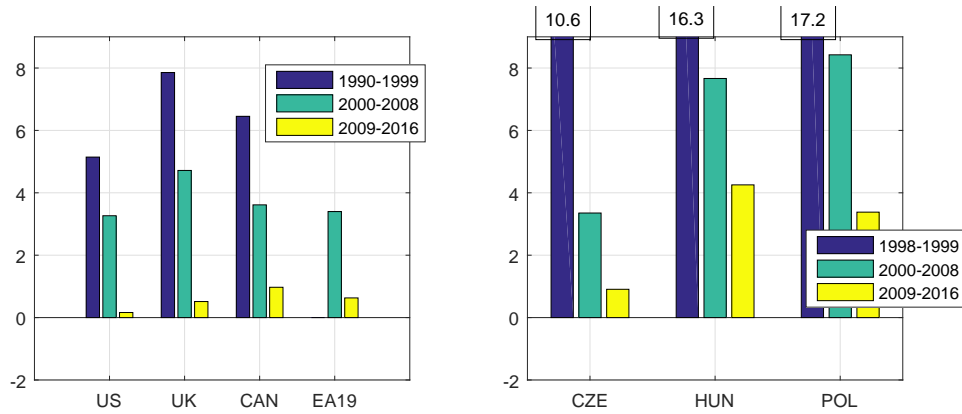
Note: Monetary policy rates and 3M interbank rates are used.
Source: Authors' calculations.

Figure 2: Average Real Interest Rates – Advanced and Emerging Market Countries

Note: Real interest rates are nominal rates adjusted for YoY inflation.
Source: Authors' calculations.

The methodology for identifying the natural rate of interest presented in Laubach and Williams (2003) is the most widespread approach used in the literature. It relies on a simple structural model in which the natural rate is a function of equilibrium real GDP growth. On top of that, there are two main equations in the Laubach-Williams model. The first specifies the output gap, derived from the Euler equation. It links the output gap (the percentage deviation of output from its equilibrium level) with the real interest rate gap (the deviation of the real rate from the natural rate of interest). The second is the Phillips curve, linking inflation with the output gap and some relative price measures. There are no expectations in the model, as there is no policy rule closing it.

Most papers relying on the Laubach-Williams identification approach find that the natural rate of interest has been declining on average since the 1990s. Estimates based on Laubach and Williams (2003) indicate that the natural rate of interest for the US was close to zero in 2016. Holston et al. (2016) found a similar decline in the natural rate of interest for Canada, the Euro Area, and the United Kingdom. Given the co-movement across countries, Holston et al. (2016) suggested that global factors play an important role in shaping the natural rate of interest. The decline in the natural rate is robust regardless of the stage of economic development, as it is identified in both advanced and emerging market countries (Zhu, 2016). This finding further supports the hypothesis that global factors might play a role in shaping the dynamics of the natural rate.

Besides the Laubach Williams approach, there are other identification methodologies. Lubik and Matthes (2015) compared the natural rate estimated using the Laubach and Williams approach with an estimate based on time-varying VAR models. They conclude that the two approaches provide similar results for the period since the 1980s. However, prior to this date there is a significant difference in the results. Similar conclusions are presented by Pescatori and Turunen (2015) and Kiley (2015). The authors do not elaborate on the reason for the difference but suggest that the specification of the structural model identifying the natural rate may play an important role.[1] Without structural equations based on behavioral relationships, the model is not able to capture regime changes properly prior to the 1990s. The importance of identifying the natural rate of interest within a structural model is highlighted in Laubach and Williams (2015). The paper compares estimates based on a structural model and univariate filters. It shows significant differences in the estimates for several historical periods.

Estimates of the natural rate are necessarily conditional on its definition. This leads some authors to conclude that the natural rate of interest has not declined. For example, Juselius et al. (2016) suggested enlarging the definition of the natural rate to include financial factors. Doing so, the paper argues that monetary policy has a first-order effect on the financial cycle and that financial busts can have permanent effects on output. They call the rate defined in this way the "finance-neutral" natural interest rate. Once they take financial factors into account, the natural rate of interest is higher and, after the outbreak of the global economic crisis, falls by less than prevailing empirical approaches would suggest. Consequently, they find the actual real policy interest rate to be persistently below the natural rate, especially for the most recent period.

3. Methodology

We modify the commonly used identification methodology of Laubach and Williams (2003) in two respects. First, the natural rate of interest is linked to equilibrium GDP growth, which

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[1] By structural model we mean a model derived from micro foundations and encompassing forward-looking expectations.

is adjusted for real exchange rate appreciation. This adjustment is needed because GDP growth only measures yields from production and ignores the effect of koruna appreciation. Second, we use a (semi-)structural model which is closed by a monetary policy rule. This model allows us to work with forward-looking model-consistent expectations and impose a comprehensive set of restrictions, i.e., model equations, determining the natural interest rate, to identify the natural rate of interest. Third, we use calibration instead of Bayesian estimation. More specifically, we calibrate the standard errors of the Kalman filter ourselves to obtain economically intuitive impulse responses, shocks, and unobserved variables as well as plausible in-sample simulation results. We rely on our long-term knowledge of the Czech economy and experience with using calibrated models in forecasting and policy analysis over the last 15 years. The final estimates of the natural rate depend mainly on the time-dependent estimates of the growth of potential GDP and real equilibrium appreciation of the exchange rate. The steady-state calibration of the model therefore plays only a limited role in the short run. At the same time, we estimate some of the key model parameters in the equation determining the dynamics of the natural rate of interest.

3.1 Determinants of the Natural Rate

The level of the natural rate of interest is determined by equilibrium GDP growth adjusted for real exchange rate appreciation identified within our model framework. Following Laubach and Williams (2003), a restriction (equation) is commonly used in the literature in order to determine the natural rate. The restriction is motivated by the Euler equation. According to this equation, the natural rate of interest, $R_{eq,t}$, where t is the time parameter, is proportional to equilibrium (non-inflationary) GDP growth, $\Delta Y_{eq,t}$, and can deviate from the growth by a shock, ε_t.

$$R_{eq,t} = c\Delta Y_{eq,t} + \varepsilon_t, \tag{1}$$

where $c > 0$ is a scaling parameter.

However, equation (1) is not suitable for a small open economy with free capital flows such as the Czech Republic. While equilibrium GDP growth accounts for capital yields from production, real exchange rate appreciation, which increases the yields on investments realized in foreign currency, must also be taken into account. Hence, in our setup, the natural rate of interest is proportional to equilibrium GDP growth adjusted for equilibrium real exchange rate depreciation $\Delta Q_{eq,t}$:

$$R_{eq,t} = c(\Delta Y_{eq,t} + \Delta Q_{eq,t}) + \varepsilon_t, \tag{2}$$

where $\Delta Q_{eq,t} < 0$ means real appreciation and $\Delta Q_{eq,t} > 0$ real depreciation. Both $Y_{eq,t}$ and $Q_{eq,t}$ refer to equilibrium levels, i.e., levels consistent with neutral effects on inflation. According to Laubach and Williams (2003), the scaling parameter c is equal to 1.

Our specification above differs from the one in Laubach and Williams (2003) in that it does not specify the natural rate of interest as a sum of two random walks. On the contrary, it links the natural rate with its two most important stationary determinants ΔY_{eq} and ΔQ_{eq}. In this sense our specification is closer to that of Mesonnier and Renne (2007) and Fries et al. (2016). The latter paper even provides a reason for avoiding the random walk specification. Specifically, it finds that for the sample period they cover (1999–2016) the random walk assumption of Laubach and Williams (2003), and the intricate estimation strategy required, appears to be inadequate.

There is a positive link between equilibrium real growth and equilibrium appreciation observed in the data, called the Penn effect. Whenever a country is growing fast and converging in per capita GDP to advanced countries, the growth is accompanied by real appreciation ($\Delta Q_{eq,t} < 0$). In our framework, the equilibrium real appreciation of the exchange rate curbs the contribution of potential GDP growth to the natural rate of interest. Without this adjustment, the natural rate of interest would be unrealistically high due to fast GDP growth, especially in emerging and developing countries.

3.2 Structural Model

A (semi-)structural model of the Czech economy is used to identify the neutral real interest rate. The structure of the model resembles the CNB's QPM[2] – see Benes et al. (2003). However, compared to the CNB's original QPM, the model contains several modifications. First, headline CPI is broken down in the model into a core index excluding food and energy prices, an energy price index, a food price index, and administrative prices. Trends in relative prices are taken into account and explicitly modeled. Second, the effects of foreign prices and exchange rate dynamics on inflation are captured using the real exchange rate in real marginal costs, as opposed to import price inflation in the QPM. More details on the structure of the model can be found in Appendix B.

The gap form of the model is suitable for identifying the natural rate of interest as defined in our paper. The gap form requires us to decompose all real variables into a trend (equilibrium) and an implied gap, i.e., the percentage deviation from the trend. The idea behind the decomposition is the following. The trend is driven by country fundamentals and is beyond the scope of monetary policy. The gap, on the contrary, captures the cyclical part of the variable, linked to the business cycle. If a real variable is at its equilibrium level, the gap of that variable is zero and there is no need for further adjustment. The equilibrium level is neutral with respect to inflation and real economic activity. In other words, if the real rate is at its equilibrium level, there are no pressures on real economic activity stemming from the real rate level.

The model is calibrated to match stylized facts of the Czech economy and transmission mechanism. The calibration involves an iterative strategy of finding model parameters bringing the model close to the data. To calibrate the model to the Czech data, we break down the model parameters into several categories. The first category of parameters consists of those which determine the model's steady state. These parameters can be easily calibrated using historical averages or policy objectives such as the inflation target. The second category of parameters comprises those which affect the dynamic properties of the model. These parameters can be broken down further into parameters of structural equations and parameters of the model equations without any structural background. The literature and other publicly available models can provide guidance on the parameters in structural equations along with the actual data. The parameters in non-structural equations are calibrated to match the observed data along with reasonably smooth trends. Finally, the last category of parameters contains those determining the stochastic properties of the model. They are the standard deviations of the shocks and are calibrated to match the observed variance in the data. The calibrated values of all the parameters are listed in Appendix C.

The calibration of the model is verified using several simulation exercises. First, in-sample simulations are used to check the ex-post forecasting power of the model. Second, data filtration is performed to interpret the historical data and to decompose the observed variables into structural shocks. The results of these exercises are presented in Appendix C4.

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[2] QPM – Quarterly Projection Model.

The model is applied to interpret the Czech data and to identify unobserved variables, with special emphasis put on the natural rate of interest. To identify the shocks, a multivariate Kalman filter (KF) is used. It seeks to replicate the observed data using the model and its structural shocks. That is done by minimizing the expected least squares of the shocks. A key aspect of the KF is that cross-restrictions arising from the structural model equations are used to replicate the data. Consequently, the natural rate of interest identified is conditional on the framework used. However, given the structural character of the model, the estimates are more robust than any univariate estimate. Univariate approaches, including the HP and band-pass filters, merely smooth the data. The cross-restrictions applied in the estimation of the natural rate of interest along with the whole model structure are crucial to correctly determining the rate.

We assume that the natural rate of interest converges to equilibrium real GDP growth adjusted for equilibrium real exchange rate appreciation with some persistence. Therefore, the model includes an equation as follows:

$$R_{eq,t} = \rho R_{eq,t-1} + (1-\rho)c(\Delta^4 Y_{eq,t}^{yoy} + \Delta^4 q_{eq,t}^{yoy}), \tag{3}$$

where *yoy* stands for YoY rates. Notice that compared to equation (2) we have introduced persistence but also removed the stochastic term. Hence, the natural rate cannot diverge from the real fundamentals. Furthermore, for the purposes of estimating the parameters in this equation, we re-write it as follows:

$$R_{eq,t} = \rho R_{eq,t-1} + (1-\rho)2c(w^{\bar{y}}\Delta^4 Y_{eq,t}^{yoy} + (1-w^{\bar{y}})\Delta^4 q_{eq,t}^{yoy}), \tag{4}$$

where $w^{\bar{y}}$ is the weight on equilibrium real GDP growth. If $w^{\bar{y}}$ equals 0.5 we get exactly the equation above.

There are three key cross-restrictions determining the natural rate of interest. The real rate is computed from the nominal interest rate and expected inflation using the Fisher equation. The real rate is then broken down into the natural rate and the gap. Any equation in the model containing either the natural rate or the real interest rate provides an identification cross-restriction. The equations are as follows.

The first is the *output gap equation*. It assumes that the output gap – the proxy for the domestic business cycle – is driven by the real monetary conditions and foreign demand. On top of that, the output gap exhibits persistence resembling habit formation. Overall, the demand equation is similar to an Euler equation.

$$Y_{gap,t} = \alpha_1^1 \cdot Y_{gap,t-1} - \alpha_2^1 \cdot (\alpha_3^1 \cdot R_{gap,t-1} - (1-\alpha_3^1) \cdot Q_{gap,t}) + \alpha_4^1 \cdot Y_{gap,t-1}^W + \varepsilon_t^{Y_{gap}} \tag{5}$$

where $Y_{gap,t}$ is the output gap, $Q_{gap,t}$ is the real exchange rate gap, $R_{gap,t}$ is the real interest rate, $Y_{gap,t-1}^W$ is the foreign output gap, and $\varepsilon_t^{Y_{gap}}$ is a demand shock.

Second, the *Phillips curves (PCs)* specified for core price inflation (excluding food, energy, and regulated prices), food price inflation, and energy price inflation, provide another important cross-restriction. The PCs link the output gap, as a proxy for domestic demand, along with the real exchange rate, to inflation. Both the output gap and the real exchange rate gap are proxies for real marginal costs. Such a structure of real marginal costs assumes that labor and imported goods are

used for domestic production. The Phillips curves are described in detail in Appendix B – Model Equations – and look as follows

$$\pi_t^i = \alpha_1^i \cdot \pi_{t+1}^i + (1 - \alpha_1^i) \cdot \pi_{t-1}^i + \alpha_2^i \cdot (\alpha_3^i \cdot Y_{gap,t-1} + (1 - \alpha_3^i) \cdot Q_{gap,t}^i) + \varepsilon_t^{\pi^i} \qquad (6)$$

where π_t^i is the inflation of a CPI sub-component ($i \in \{2,3,4,5\}$) and $\varepsilon_t^{\pi^i}$ is a supply shock. The PCs provide cross-restrictions to identify the unobserved output gap and the real exchange rate gap from their structural relationship with observed inflation.

Compared to Laubach and Williams (2003) we use an additional restriction for the natural rate of interest based on the monetary policy rule. As mentioned earlier, the natural rate of interest is an integral part of the monetary policy rule. Together with expected inflation it defines the so-called neutral (nominal) interest rate. However, through parametrization of the Kalman filter we make sure that this equation is not binding, as there is uncertainty about the parameters in the policy rule.

$$i_t = \alpha_1^{16} \cdot i_{t-1} + (1 - \alpha_1^{16}) \cdot (R_{eq,t} + \pi_{t+4}^{yoy} + \alpha_2^{16} \cdot (\pi_{t+4}^{yoy} - \pi^{tar}) + \alpha_3^{16} \cdot Y_{gap,t}) + \varepsilon_t^i \qquad (7)$$

where i_t is the short-term policy rate, $R_{eq,t} + \pi_{t+4}^{yoy}$ is the nominal policy-neutral rate, $\pi_t^{yoy} - \pi^{tar}$ is the deviation of expected inflation from the inflation target four quarters ahead, $Y_{gap,t}$ is the output gap, and ε_t^i is the structural shock.

The observed data include all key macroeconomic variables. Specifically, the set of observed variables contains inflation (headline inflation and its sub-components), real GDP, the 3M PRIBOR, and the nominal CZK/EUR exchange rate. These domestic variables are accompanied by their foreign counterparts. The effective EU measures used in the CNB's Forecasting and Policy Analysis System (FPAS) are taken as a proxy for the foreign block. See Appendix D for details on the observed variables.

4. Findings

Our estimates suggest that the natural rate of interest hovers around 1 percent (see Figure 3). Prior to the world financial crisis, in the 2004–2007 period, the natural rate was quite stable and close to 1 percent p.a. The crisis naturally increased the volatility of the natural rate. Although the Czech economy was not directly hit by the financial crisis, it did face considerably lower foreign demand and increased nominal exchange rate volatility throughout 2008–2010. Our estimate indicates that the natural rate declined initially and returned to 1 percent afterward. It peaked in 2015. The increase was most significant in the period of rapid equilibrium GDP growth fueled by high investment inflows under EU structural programs. At the same time, the real exchange rate appreciation was still slow. The subsequent fall in the natural rate to 1 percent during 2016–2017 was driven by a decline in equilibrium real GDP growth and renewed real exchange rate appreciation.

The natural rate of interest (NRI) is determined by the often mutually compensating interaction between potential growth and real exchange rate appreciation. This is the main reason why, after the outbreak of the global economic crisis in 2008, the natural rate is not falling in the Czech Republic proportionately with estimated potential GDP growth, in contrast to the NRI estimates for many other economies. Our results, using decomposition based on equation (4) and depicted in

Figure 3: Real Interest Rate and Natural Rate (in percent p.a.)
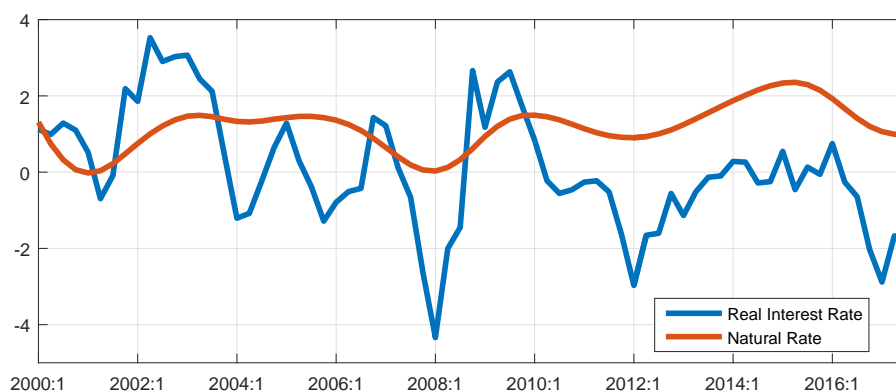
Source: Authors' calculations.

Figure 4, show that the fall in potential GDP growth after 2008 was accompanied by a significant slowdown in real equilibrium exchange rate appreciation. That, in turn, mitigated the decline in the natural rate induced by the fall in potential GDP growth (similarly to the situation during the 2000–2007 period).[3] As the economic recovery began in 2014 whereas the real appreciation started only during mid-2015, the natural rate of interest reached its peak in mid-2015.[4] It started to fall afterwards as the equilibrium real exchange rate appreciation gradually speeded up.

Our results obtained for the Czech Republic point to the importance of the real exchange rate appreciation in the NRI estimation for small open economies, in addition to the assessment of standard domestic factors. More specifically, the Czech results suggest that high potential GDP growth combined with steady real exchange rate appreciation can result in relatively low and stable NRI estimates despite high potential economic growth. In the Czech case, the highest NRI level for the 2000–mid-2017 period was achieved in 2015Q1–2015Q3, when potential growth had started to pick up but the change in the equilibrium real exchange rate was only minor.

The Kalman filter estimation methodology enables us to decompose the natural rate into observed variables. Besides real interest rates, real growth and real appreciation should be the most important factors. The decomposition of the natural interest rate into the contributions of observed variables is depicted in Figure 5. The decomposition shows the factors driving the dynamics of the natural rate. In technical terms, the decomposition depicts the contributions of the observed variables to the prediction errors. It shows that in our filtration results three variables dominate the level of the natural rate: real GDP growth, real exchange rate appreciation, and the observed real interest rate. Other factors play only a minor role in determining the natural rate of interest.

Given our estimate of the natural rate of interest, real interest rates have been accommodative since 2010 (see Figure 3). We observe the real interest rate below the natural rate at the end of our sample (in 2010–2016). The estimation results suggest that interest rates set by the central bank were supportive to real growth in 2010–2016. However, the magnitude of the accommodative

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[3] The correlation between potential output and real exchange rate appreciation depends, among other things, on the fundamental forces driving potential growth, for instance, productivity increases in the tradable and non-tradable sectors of the economy.

[4] The fast real growth observed in 2015 was partly driven by inflows of EU funds and hence was not accompanied by real appreciation.

Figure 4: Decomposition of the Natural Rate of Interest (in p.p.)
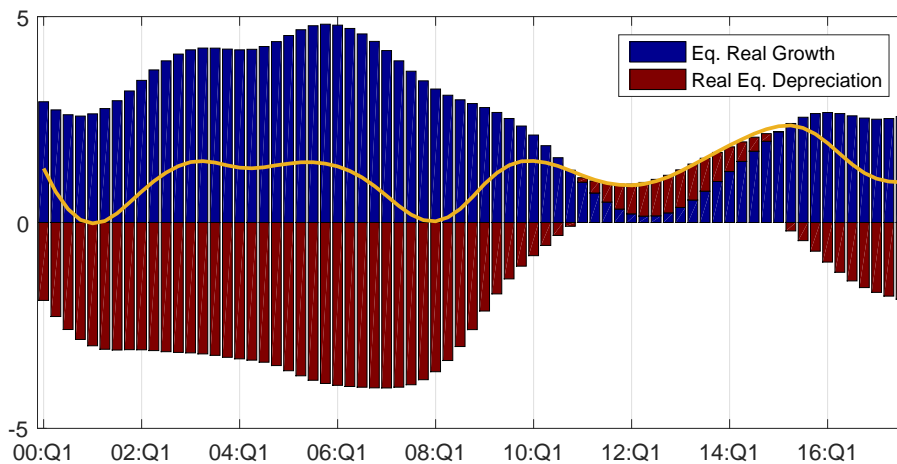
Note: Persistence is substituted out.
Source: Authors' calculations.

Figure 5: Natural Interest Rate and its Decomposition into Observed Variables (contributions in percentage points)
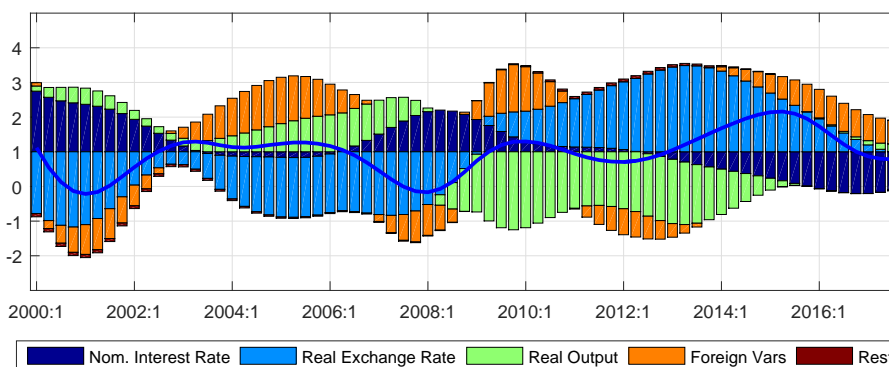
Note: The bars in each time period sum to the estimated level of the natural rate. The magnitude of each bar depicts the contribution of the observed variables to the shift of the estimated natural rate from its steady-state value. Factors are ordered by their importance, which is approximated by the sum of the squares of the contributions of each factor.
Source: Authors' calculations.

Figure 6: Real Monetary Conditions Index Gap (in percent)
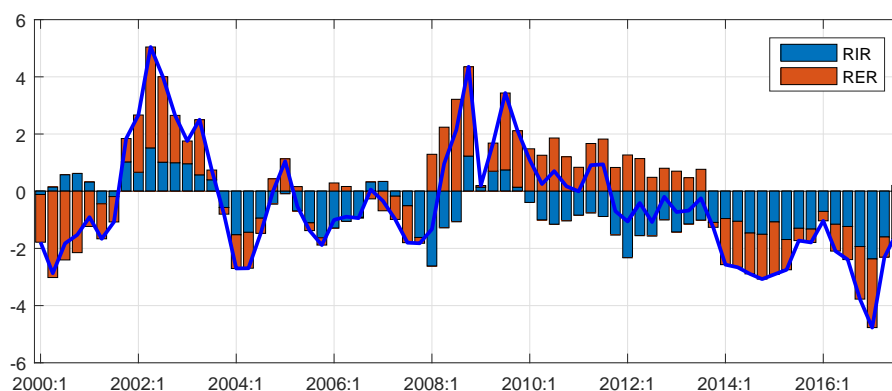
Note: The solid blue line is the real monetary conditions gap. At each point in time, the bars stand for the contributions of each factor to the monetary conditions index. RIR stands for the real interest rate gap and RER for the real exchange rate gap. Positive values of the monetary conditions index mean a tight monetary policy stance and negative values a loose stance.
Source: Authors' calculations.

monetary policy stance arising from the level of the real interest rate compared with the natural rate has varied over time. It peaked in early 2012 before shrinking significantly at the end of 2013. The difference between the real interest rate and its natural level in 2014–2015 was only about 150 basis points, due to very low inflation pushing the real interest rate up. Monetary policy easing through real interest rates started to increase when inflation began to accelerate in 2016.

The overall monetary conditions, defined as a weighted average of the real interest rate and the exchange rate, were also accommodative at the end of our sample. The model, as a set of simultaneous equations, provides us with estimates of all parts of the monetary conditions, including the real exchange rate gap. The contributions of the two factors to the real monetary conditions are shown in Figure 6. The estimates based on the model suggest that the monetary conditions have been relaxed since 2010. While in 2010–2013 the conditions were accommodative only in the real interest rate component of the MCI, which outweighed the slightly restrictive stance of the real exchange rate, both components of the real monetary conditions are relaxed in 2014–2016. Hence, the CNB's exchange rate commitment from November 2013 till March 2017 helped ease the monetary conditions significantly.

5. Robustness Analysis

The estimate of the natural rate of interest is obviously subject to uncertainty. The 90 percent and 60 percent confidence bands are about ±200 and ±100 basis points wide, respectively (see Figure 7). When assessing the widths of the confidence bands it is worth taking into account the following aspects. First, all the shocks used to match the historical data are taken into account. Second, the period for which the standard deviations are inferred includes the world financial crisis, which was characterized by high volatility of most macroeconomic and financial variables. Third, the bands only capture the uncertainty related to underlying shocks and ignore model uncertainty. Hence, any estimate is conditional on the framework/model used.

Figure 7: Natural Interest Rate with Confidence Bands (in percent p.a.)
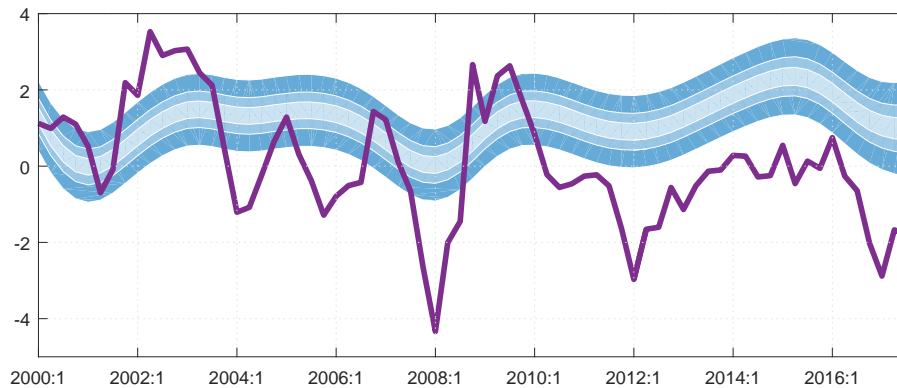
Note: The solid blue line is the natural rate of interest. The blue areas around the solid line depict the 90, 60, and 30 percent confidence bands. The solid magenta line in the second plot is the actual real rate.
Source: Authors' calculations.

The parametrization of the model, and in particular the process for the natural rate of interest, affects the natural rate identified. While the confidence bands presented above take into account the uncertainty about the underlying shock, we also demonstrate the effects of different parametrization. In this exercise we run Kalman filtration with different settings of parameters ρ, c, and $w^{\bar{y}}$ in equation (4). See Figure (8) for the implications for the level of the natural rate. We use a change of ± 2 standard deviations for the estimated parameters ρ and $w^{\bar{y}}$ and ± 0.25 for parameter c.

Appendix A compares our estimates of the natural rate with those using the Laubach and Williams model and univariate filters.

Figure 8: Natural Interest Rate with Confidence Bands (in percent p.a.)
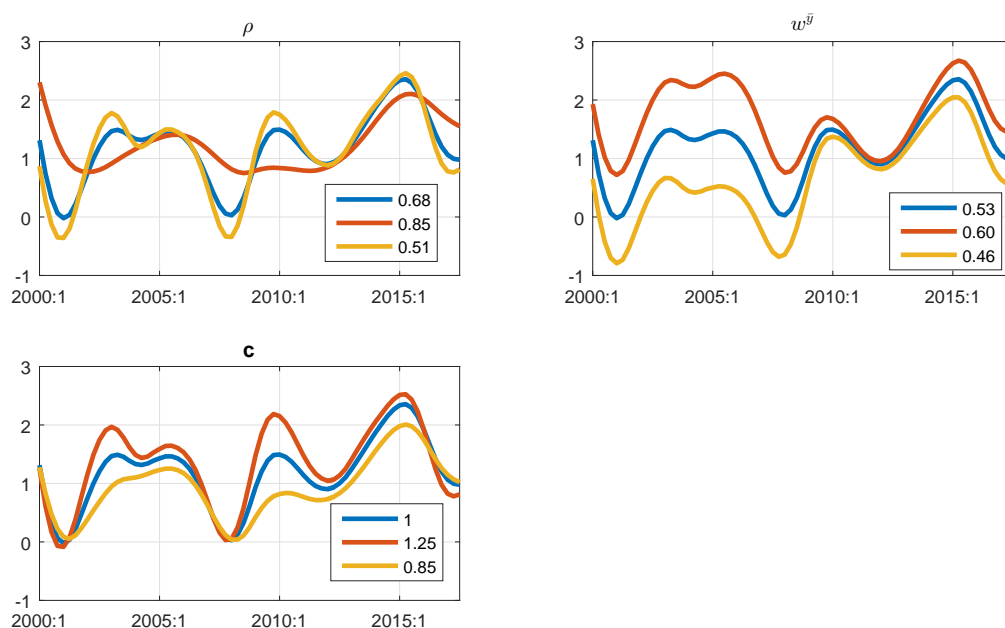
Source: Authors' calculations.

6. Conclusions

Real interest rates are currently at low levels around the world. As a result, central banks are faced with the fundamental challenge of determining the natural rate of interest so that they can assess their monetary policy stance. Many economists believe that natural rates are very low. If they remain low, policy rates are obviously likely to hit the zero lower bound more often and stay there for longer periods than in the past. In addition, the uncertainty about the natural rate calls for policy approaches that are more robust to the uncertainty surrounding natural rate estimates.

We identify the natural rate of interest for the Czech economy. The natural rate of interest in a small open economy with free capital flows is driven, in our framework, by potential GDP growth adjusted for equilibrium real exchange rate appreciation. To identify the natural rate of interest, we filtered the available data using a structural model. The model equations served as cross-restrictions on the estimates of unobserved variables, the natural rate of interest being one of them.

Our estimate of the natural rate of interest in the Czech economy is close to 1 percent. Despite a recovery in real economic activity, the natural rate has decreased from its peak in 2015 due to renewed appreciation of the equilibrium real exchange rate.

References

ANDRLE, M. (2010): "A Note on Identification Patterns in DSGE Models." ECB Working Paper 1235, European Central Bank

ANDRLE, M. AND J. BENES (2013): "System Priors: Formulating Priors about DSGE Models' Properties." IMF Working Papers 257, International Monetary Fund

BENES, J., T. HLEDIK, D. VAVRA, AND J. VLCEK (2003): *The Czech National Bank's Forecasting and Policy Analysis System,* chapter 4, pages 63–98. Czech National Bank. Chapter: The Quarterly Projection Model and its Properties

BLANCHARD, O. AND C. KAHN (1980): "The Solution of Linear Difference Models Under Rational Expectations." *Econometrica,* 48:1305–1311.

FRIES, S., J. S. MESONNIER, S. MOUABBI, AND J. P. RENNE (2016): "National natural rates of interest and the single monetary policy in the Euro Area." Banque de France Working Paper 611, Banque de France

GOLUB, G. H. AND C. F. V. LOAN (1996): *Matrix Computations.* Johns Hopkins University Press.

HOLSTON, K., T. LAUBACH, AND J. C. WILLIAMS (2016): "Measuring the Natural Rate of Interest : International Trends and Determinants." Finance and Economics Discussion Series 2016-073, Board of Governors of the Federal Reserve System (U.S.)

JUSELIUS, M., C. BORIO, P. DISYATAT, AND M. DREHMANN (2016): "Monetary policy, the financial cycle and ultra low interest rates." BIS Working Paper 569, BIS

KARAM, P., A. BERG, AND D. LAXTON (2006): "Practical Model-Based Monetary Policy Analysis; A How-To Guide." IMF Working Papers 81, International Monetary Fund

KILEY, M. T. (2015): "What Can the Data Tell Us About the Equilibrium Real Interest Rate?." Finance and Economics Discussion Series 2015-77, Board of Governors of the Federal Reserve System (U.S.)

LAUBACH, T. AND J. C. WILLIAMS (2003): "Measuring the Natural Rate of Interest." *The Review of Economics and Statistics,* 85(4):1063–1070.

LAUBACH, T. AND J. C. WILLIAMS (2015): "Measuring the Natural Rate of Interest Redux." Working Paper Series 2015-16, Federal Reserve Bank of San Francisco

LUBIK, T. AND C. MATTHES (2015): "Calculating the Natural Rate of Interest: A Comparison of Two Alternative Approaches." Economic Brief EB15-10, Federal Reserve Bank of Richmond

MESONNIER, J. S. AND J. P. RENNE (2007): "A time varying "natural" rate of interest for the euro area." *European Economic Review,* 51(7):1768–1784.

PESCATORI, A. AND J. TURUNEN (2015): "Lower for Longer: Neutral Rates in the United States." IMF Working Paper 15/135, IMF

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

ZHU, F. (2016): "Understanding the changing equilibrium real interest rates in Asia-Pacific." BIS Working Paper 567, BIS

Appendix A: The Natural Rate Identified Using Different Methods

We apply the model of Laubach and Williams, commonly used to quantify the natural rate of interest in the literature, to the Czech case. The Laubach Williams (LW) model is unchanged except for two modifications to its structure. First, the output gap now encompasses foreign demand and the real exchange rate gap. The real exchange rate gap is exogenous to the system and is not estimated simultaneously with the output gap. The same holds for the foreign demand gap. Second, the real exchange rate gap is also added to the Phillips curve as a part of real marginal costs, capturing the import price channel in a small open economy setup.

Applying the Laubach and Williams model, we illustrate the differences between the natural rates identified with and without correction for real exchange rate appreciation. The comparison is provided in Figure A1. There are two distinct periods in which our estimate differs from the LW one. First, prior to the world financial crisis the LW model suggests a higher natural rate. Second, the LW model provides a lower natural rate of interest for the period of 2012–2015. Both differences are given by observed real economic growth, which was fast before the world financial crisis and then slowed significantly.

Figure A1: Comparison of Natural Interest Rates Identified Using Different Models (in percent p.a.)
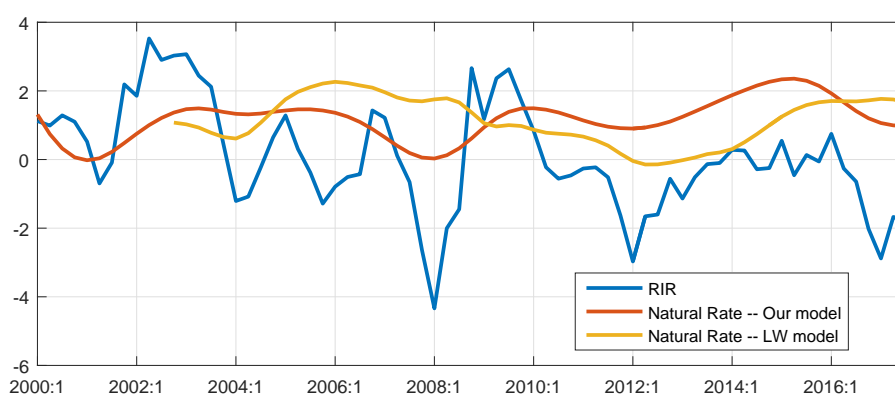
Source: Authors' calculations.

The natural rate identified in our paper also deviates from estimates based on common univariate filters. Estimates using the HP filter are captured in Figure A2 for the period of 2000–2017Q3. The univariate filter estimate is even negative, especially at the end of our historical sample. This comparison with the KF estimates points to the importance of our structural approach in the identification of the natural rate as an unobserved variable. While the structural approach ensures that the rate identified is consistent with the definition of the natural rate, univariate filters provide estimates close to a simple average of the data. Consequently, the results based on the use of univariate filters are not necessarily consistent with the definition of the natural rate based on economic theory.

Figure A2: Natural Interest Rate – Comparison of KF and HP Filter Estimates (in percent p.a.)
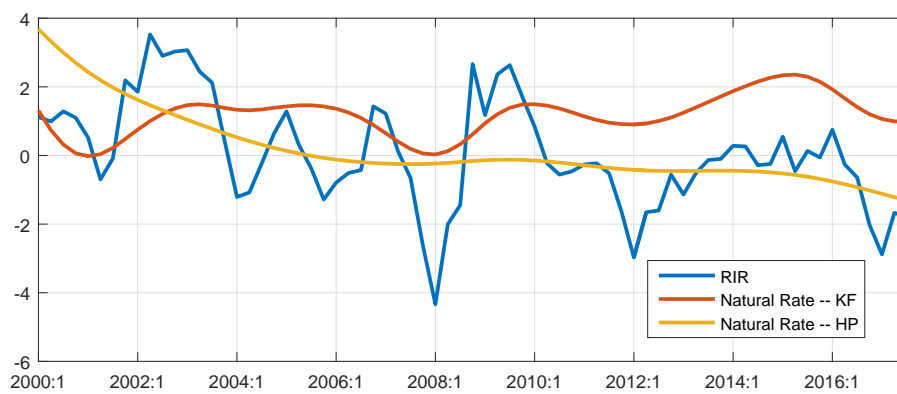
Source: Authors' calculations.

Appendix B: Semi-Structural Model of the Czech Economy

The model used in this paper is a New-Keynesian model of a small open economy with a floating exchange rate. The model is linear and calibrated to match the business cycle of the Czech economy. It captures the key macroeconomic relationships and transmission channels. In particular, inflation expectations are modeled as forward-looking (model-consistent) and monetary policy is represented by a forward-looking policy rule.

The model is in so-called gap form. This means that all real variables are broken down into a trend (equilibrium) and a gap. Trends are driven by country fundamentals and stay beyond the scope of monetary policy. By contrast, gaps, as percentage deviations from the trends, describe the business cycle dynamics. The unobserved trends (equilibria) and gaps are identified using Kalman filtration. All the model variables are in natural logarithms multiplied by one hundred, except for gaps and interest rates, which are both in percent.

Let's start with the description of the real economy block, which is highly aggregated in the model and represented by an IS curve:

$$Y_{gap,t} = \alpha_1^1 \cdot Y_{gap,t-1} - \alpha_2^1 \cdot (\alpha_3^1 \cdot R_{gap,t-1} - (1 - \alpha_3^1) \cdot Q_{gap,t}) + \alpha_4^1 \cdot Y_{gap,t-1}^W + \varepsilon_t^{Y_{gap}} \qquad \text{(B1)}$$

Equation (B1) is a law of motion for the domestic output gap.[5] The output gap depends on the output gap lagged by one quarter, capturing the inertia in the economy caused by real frictions in household consumption and investment behavior. The inclusion of the lagged short-term real interest rate gap, $R_{gap,t-1}$, in the IS curve approximates the intertemporal substitution effect of the real interest rate on consumption, motivated by the Euler equation. The real exchange rate gap, $Q_{gap,t}$, as the relative price captures substitution between domestic and foreign goods. Next, the lagged foreign effective output gap, calculated on the basis of Czech trade weights, is a proxy for foreign demand for Czech exported goods, and $\varepsilon_t^{Y_{gap}}$ is a demand shock.

The price block, unlike the real economy part of the model, is quite detailed. It consists of the following four Phillips curves, captured by equations (B2)–(B5):

$$\pi_t^C = \alpha_1^2 \cdot \pi_{t+1}^C + (1 - \alpha_1^2) \cdot \pi_{t-1}^C + \alpha_2^2 \cdot (\alpha_3^2 \cdot Y_{gap,t-1} + (1 - \alpha_3^2) \cdot Q_{gap,t}^C) + \varepsilon_t^{\pi^C} \qquad \text{(B2)}$$

$$\pi_t^F = \alpha_1^3 \cdot \pi_{t+1}^F + (1 - \alpha_1^3) \cdot \pi_{t-1}^F + \alpha_2^3 \cdot (\alpha_3^3 \cdot Y_{gap,t-1} + (1 - \alpha_3^3) \cdot Q_{gap,t}^F) + \varepsilon_t^{\pi^F} \qquad \text{(B3)}$$

$$\pi_t^{Fu} = \alpha_1^4 \cdot \pi_{t+1}^{Fu} + (1 - \alpha_1^4) \cdot \pi_{t-1}^{Fu} + \alpha_2^4 \cdot (\alpha_3^4 \cdot Y_{gap,t-1} + (1 - \alpha_3^4) \cdot Q_{gap,t}^{Fu}) + \varepsilon_t^{\pi^{Fu}} \qquad \text{(B4)}$$

$$\pi_t^A = \alpha_1^5 \cdot \pi_{t+1}^A + (1 - \alpha_1^5) \cdot \pi_{t-1}^A + \alpha_2^5 \cdot (\alpha_3^5 \cdot Y_{gap,t-1} + (1 - \alpha_3^5) \cdot Q_{gap,t}^A) + \varepsilon_t^{\pi^A} \qquad \text{(B5)}$$

Equations (B2)–(B5) are Phillips curves specified for core, food price, fuel price, and administered price inflation. Their functional specification is identical in the sense that they are all expectations-augmented New-Keynesian Phillips curves quantifying the link between changes in the selected

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[5] The output gap $Y_{gap,t}$ is in percent and is defined as the difference between observed log-output Y_t and potential output Y_t^{eq}.

price index and their main determinants. More specifically, inflation for all of these Phillips curves is a linear combination of inflation expectations one quarter ahead, observed lagged inflation, the lagged output gap, and the relative price gap, where the relative price gaps are always specific to the Phillips curve as described below. Inflation expectations are fully model-consistent.[6] They are derived within the model and are a function of all exogenous and pre-determined variables. The output gap is a proxy for domestic demand-led price pressures. The relative price gap quantifies the effect of import prices on the inflation rate of the price sub-component of the CPI under consideration.

Let's specify the four relative price gaps $Q_{gap,t}^C$, $Q_{gap,t}^C$, $Q_{gap,t}^C$, and $Q_{gap,t}^A$ in equations (B6)–(B9), entering the corresponding Phillips curves equation above:

$$Q_{gap,t}^C = e_t + cpi_t^* - cpi_t^C - Q_{eq,t}^C \tag{B6}$$

$$Q_{gap,t}^F = e_t + pf_t^* - e_t^{usd} - cpi_t^F - Q_{eq,t}^F \tag{B7}$$

$$Q_{gap,t}^{Fu} = e_t + oil_t^* - e_t^{usd} - cpi_t^{Fu} - Q_{eq,t}^{Fu} \tag{B8}$$

$$Q_{gap,t}^A = e_t + cpi_t^* - cpi_t^A - Q_{eq,t}^A \tag{B9}$$

Equation (B6) defines the relative price gap $Q_{gap,t}^C$ for the price index measuring core inflation. The proxy for the corresponding foreign price level is $e_t + cpi_t^*$, the foreign level of the CPI expressed in domestic currency. $Q_{eq,t}^C$ is the equilibrium relative core price index. Equation (B7) defines the relative price for food prices, $Q_{gap,t}^F$. The term $e_t + pf_t^* - e_t^{usd}$ is the world food price expressed in foreign currency and $Q_{eq,t}^F$ is the equilibrium relative food price. The following two equations (B8)–(B9) exhibit the same logic, where $e_t + oil_t^* - e_t^{usd}$ is the world price of oil expressed in CZK and $e_t + cpi_t^*$ is the foreign counterpart for the administered price level.

The log-linearized approximation of the CPI identity, stating that the CPI is a weighted sum of its price sub-components, is:

$$cpi_t = \alpha_1^{10} \cdot cpi_t^C + \alpha_2^{10} \cdot cpi_t^F + \alpha_3^{10} \cdot cpi_t^{Fu} + (1 - \alpha_1^{10} - \alpha_2^{10} - \alpha_3^{10}) \cdot cpi_t^A + \varepsilon_t^{cpi} \tag{B10}$$

We constructed a foreign counterpart to the domestic CPI using the same weights as in the Czech CPI, but including the foreign price proxies as in equations (B2)–(B5):

$$cpi_t^{**} = \alpha_1^{10} \cdot cpi_t^* + \alpha_2^{10} \cdot (pf_t^* - e_t^{usd}) + \alpha_3^{10} \cdot (oil_t^* - e_t^{usd}) + (1 - \alpha_1^{10} - \alpha_2^{10} - \alpha_3^{10}) \cdot cpi_t^* \tag{B11}$$

Let's define the real exchange rate as

$$Q_t = e_t + cpi_t^{**} - cpi_t \tag{B12}$$

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[6] By fully model-consistent expectations we mean rational expectations. Our (linear) is transformed to a state space form and solved by applying the generalized Schur decomposition and the Blanchard-Kahn algorithm. For further details see Golub and Loan (1996) and Blanchard and Kahn (1980).

where the real exchange rate gap is

$$Q_{gap,t} = Q_t - Q_{eq,t} \tag{B13}$$

The equilibrium real exchange rate $Q_{eq,t}$ is defined as follows:

$$Q_{eq,t} = \alpha_1^{10} \cdot Q_{eq,t}^C + \alpha_2^{10} \cdot Q_{eq,t}^F + \alpha_3^{10} \cdot Q_{eq,t}^{Fu} + (1 - \alpha_1^{10} - \alpha_2^{10} - \alpha_3^{10}) \cdot Q_{eq,t}^A + \varepsilon_t^{Qeq} \tag{B14}$$

Hence, the equilibrium real exchange rate $Q_{eq,t}$ is expressed as a weighted average of the equilibrium relative prices of $Q_{eq,t}^C - Q_{eq,t}^A$, where the weights $\alpha_1^{10} - \alpha_3^{10}$ are the weights of core, food, and fuel prices in the Czech CPI.

The exchange rate of the koruna against the euro is determined by the staggered version of the uncovered interest rate parity condition:

$$e_t = \alpha_1^{15} \cdot e_{t+1} + (1 - \alpha_1^{15}) \cdot (e_{t-1} + 2/4 \cdot (\pi^{tar} - cpi_{ss}^{**} + Q_{eq}^{ss})) + (i_t - i_t^* + prem_t)/4 + \varepsilon_t^e \tag{B15}$$

where the current nominal exchange rate depends on the future expected exchange rate, the short-term interest rate differential $i_t - i_t^*$, and the risk premium $prem_t$. The term $2/4 \cdot (cpi^{tar} - cpi_{ss}^{**} + Q_{eq}^{ss})$ ensures the right steady-state dynamics.

The short-term policy rate follows a forward-looking, inflation forecast-based interest rate rule:

$$i_t = \alpha_1^{16} \cdot i_{t-1} + (1 - \alpha_1^{16}) \cdot (R_{eq,t} + \pi_{t+4}^{yoy} + \alpha_2^{16} \cdot (\pi_{t+4}^{yoy} - \pi^{tar}) + \alpha_3^{16} \cdot Y_{gap,t}) + \varepsilon_t^i \tag{B16}$$

The policy rate, i_t depends on the lagged policy rate (interest rate smoothing), the policy-neutral rate $R_{eq,t} + \pi_{t+4}^{yoy}$, the deviation of expected inflation four quarters ahead from the inflation target $\pi_t^{yoy} - \pi^{tar}$, and the output gap $Y_{gap,t}$. Any deviation of monetary policy from the rule is captured by the shock, ε_t^i.

The real interest rate gap, $R_{gap,t}$ is defined as follows:

$$R_{gap,t} = i_t - (\alpha_3^{17} \cdot \pi_{t+1}^{yoy} + (1 - \alpha_3^{17}) \cdot \pi_t^{yoy}) - R_{eq,t} \tag{B17}$$

The real interest rate gap $R_{gap,t}$ is a function of the short-term policy rate i_t, inflation expectations, which are assumed to be a weighted average of the current and expected future (model-consistent) y-o-y inflation rates, and the natural rate of interest $R_{eq,t}$. The natural rate of interest is defined by equation (4).

The unobserved equilibrium values X for non-stationary variables, where $X \in \{Q_{eq,t}^C, Q_{eq,t}^F, Q_{eq,t}^{Fu}, Q_{eq,t}^A, Y_{eq,t}\}$, are modeled as an AR(1) process of the following functional form:

$$\triangle X_t = \alpha_1^{18} \cdot \triangle X_{t-1} + (1 - \alpha_1^{18}) \cdot \triangle X^{ss} + \varepsilon_t^{\triangle X} \tag{B18}$$

where $\triangle$ is a difference operator and $\triangle X^{ss}$ is the steady-state growth rate of variable X.

The foreign block, i.e., the equations for all foreign variables, consists of stochastic AR(1) processes defined using levels for stationary variables and growth rates for non-stationary ones.

Appendix C: Model Parameters

We calibrate all the model parameters except those in equation (4) defining the dynamics of the natural rate of interest. We consider calibration to be a more efficient and robust approach to finding parameter values compared to estimation. In general, most of the parameters in structural models cannot be estimated, as they deliver observationally equivalent model dynamics, i.e., they are non-identifiable or subject to weak identification.[7] The Bayesian approach does not provide a remedy either, as the estimated means of the parameters stay close to the priors and the variance of the posteriors is not significantly reduced.

C.1 Estimated Parameters

We estimate the parameters in the key model equation determining the dynamics of the natural rate of interest using maximum likelihood. In order to reduce the number of parameters we reshape equation (3) as follows:

$$R_{eq,t} = \rho R_{eq,t-1} + (1-\rho)2c\left[w^{\bar{y}}\Delta Y_{eq,t}^{yoy} + (1-w^{\bar{y}})\Delta Q_{eq,t}^{yoy})\right], \tag{C1}$$

where c is the scaling parameter as in Laubach and Williams (2003) and $w^{\bar{y}}$ is the weight on YoY equilibrium real GDP growth in the determination of the natural rate. The scaling parameter c is multiplied by 2, as the weights on equilibrium GDP growth and real exchange rate appreciation are normalized to one.

Using identification tests suggested in the literature, we find that the only combination of parameters which can be estimated simultaneously consists of ρ and $w^{\hat{y}}$.[8] Therefore, we calibrate c equal to 1 in line with Laubach and Williams (2003) and we estimate the remaining parameters.

The parameters are estimated using maximum likelihood along with the penalty function, which reduces the likelihood of the natural rate being negative.[9] Table C1 reports the means and standard deviations of the estimated parameters.

Table C1: Estimated Parameters

Parameter	Mean	Std Dev
ρ	0.68	0.086
$w^{\bar{y}}$	0.53	0.033

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[7] Andrle (2010) provides several examples of identifiable parameter sub-spaces in structural models. The paper shows that the subset of parameters which can be estimated is quite small.

[8] The parameters can be identified only if an additional restriction is applied – a penalty function on negative values of the natural rate.

[9] This estimation procedure resembles the system-based priors introduced and described in Andrle and Benes (2013).

C.2 Calibrated Parameters

Calibration is an adaptive strategy for finding model parameters. In calibrating the model, we used the parameter ranges suggested in Karam et al. (2006) and certain values reported in Benes et al. (2003) as the starting point. The parameters were amended in an iterative process aimed at obtaining (i) plausible dynamic impulse responses, (ii) unbiased model-based forecasts (in-sample simulations), and (iii) economically intuitive shock decompositions.

When calibrating the model, it is useful to distinguish two groups of parameters: (i) steady-state parameters and (ii) parameters affecting the dynamic properties of the model.

C.2.1 Steady-State Parameters

The steady-state parameters of the model reflect the historical means and our judgment about the medium-term growth rates of the non-stationary real variables of the model and the medium-term levels of the stationary variables. By medium-term we mean 3–5 years ahead.

The calibration of the steady-state parameters is shown in Table C2. The domestic inflation target is set to 2 percent, consistently with the CNB's target. The assumption of 3 percent potential GDP growth for the Czech economy reflects our judgment about medium-term equilibrium growth. The trends in relative prices ΔQ_{ss}^C, ΔQ_{ss}^F, ΔQ_{ss}^{Fu}, and ΔQ_{ss}^A are calibrated iteratively. The average slopes of the historical trends were used as first approximations. Those values were then slightly adjusted in the calibration process to obtain plausible gaps and economically intuitive shock decompositions. The resulting trend appreciation of the Czech real exchange rate is 2.1 percent.

The calibration of the foreign variables is mostly based on historical averages with some slight expert adjustment. For the Eurozone, inflation is set to the inflation target of 2 percent and the short-term steady-state real interest rate to 1.5 percent, a calibration of the economic area's potential growth. As for commodity prices, the growth rates of world food prices Δpf_{ss}^* and oil prices Δoil_{ss}^* are based on historical averages.

Table C2: Steady-State Parameters

Variable	Model Parameters	Values
Domestic inflation target	π^{tar}	2.0
Growth rate of domestic potential output	$\Delta \bar{Y}_{ss}$	3.0
Trend in relative prices (core indices)	ΔQ_{ss}^C	-1.5
Trend in relative prices (food prices)	ΔQ_{ss}^F	-2.0
Trend in relative prices (energy prices)	ΔQ_{ss}^{Fu}	1.5
Trend in relative prices (administrative prices)	ΔQ_{ss}^A	-4.5
EMU inflation rate	π^{tar*}	2.0
Foreign short-term real interest rates	R_{ss}^*	1.5
World food price (USD), change	Δpf_{ss}^*	2.0
World oil price (USD), change	Δoil_{ss}^*	5.0

C.2.2 Calibrating the Dynamic Properties of the Model

The calibration of the parameters determining the dynamic properties of the model follows an iterative process aimed at obtaining economically plausible impulse response functions and unbiased model-based forecasts – so called in-sample simulations. The parameters are reported in Tables C3–C11 and grouped according to the main equations (B1)–(B18).

The calibration of the IS curve assumes some inertia in the output gap (the lagged output gap parameter is set to 0.6), although considerably less than the first calibration of 0.9 in the CNB's first QPM model. The lower value reflects the structural changes and modernization of the Czech economy, resulting in more flexible product and labor markets. The importance of the real monetary conditions index (RMCI), consisting of the real interest rate gap and the real exchange rate gap, in determining the cyclical position of the economy is calibrated to 0.4, with a slightly higher weight on the real interest rate. These values imply a fairly efficient transmission mechanism from real exchange and interest rates to the real economy, with the exchange rate playing an important role due to the openness of the Czech economy. The elasticity of the output gap to foreign demand, set to 0.5, points to the high integration of the Czech economy into the European production chain.

The calibrations of the Phillips curve parameters (see Tables C4–C7) share some similarities. Most importantly, the coefficients on the expected inflation rates (and the lagged inflation coefficients) are set to 0.5 in all the Phillips curves. These values are suggested in micro-based Phillips curves á la Calvo, when full backward indexation is assumed. The weights on the output gap and the relative price gaps reflect the perceived domestic and import price content of goods and services in the corresponding sub-indices. The highest import content within the CPI is set for fuel prices (0.9), whereas for the other Phillips curves these values are set within a relatively narrow range of 0.25–0.30. Obviously, the share of domestic prices, approximated by the output gap, is the remainder after taking into account the import price share. The elasticities wrt real marginal costs (RMC), approximated as a weighted average of the relative price and output gaps, is set to 0.1, with the exception of the Phillips curve for administered prices, which is set to 0.3. These values were set to obtain plausible dynamic impulse responses of the model to various shocks and are further verified during testing of the model-based forecasts.

The calibration of the weights of the individual sub-indices (core, food, fuel and administered prices) in the total CPI are based on simple OLS estimation (see Table C8). The reason we do not use the constant weights reported by the Czech Statistical Office (CZSO) is the log-linear approximation of the CPI identity. The estimated weights, as reported in Table C8, result in a smaller approximation error ε_t^{cpi} compared with the equation using the official constant weights of the CZSO.

The UIP condition has only one parameter to calibrate (see Table C9) – the share of forward- and backward-looking exchange rate expectations. The parameter controls how "jumpy" the nominal exchange rate is. It is set to 0.5. This value results in a plausible exchange rate reaction to shocks and also helps achieve plausible shock decomposition results.

The policy rule parameters are fairly standard; they are included in Table C10. The coefficient on the lagged policy rate is set to 0.7, reflecting the quite high inertia in setting the short-term interest rate. At the same time, the parameters are approximately in line with the calibration of the policy rule in the CNB's core g3 model. The relatively small weight on the output gap, 0.2, reflects the fact that direct output smoothing is not considered in the CNB's current model, although implicit output gap stabilization is present through stabilizing inflation expectations via real marginal costs. The coefficient on inflation stabilization is 1.2.

The autoregressive coefficients are included in Table C11 and lie in a narrow range of 0.75–0.8. This means relatively high inertia for the unobserved real variables of the model. The implications of these parameters for the model's empirical properties, however, can only be assessed together with the calibration of the Kalman filter's standard deviations of the shocks (reported in the next subsection of this Appendix).

Table C3: IS Curve

Equation Coefficients	Model Parameters	Values
Lagged output gap	α_1^1	0.6
RMCI	α_2^1	0.4
Real interest rate gap in RMCI	α_3^1	0.6
Real exchange rate gap in RMCI	$1 - \alpha_3^1$	0.4
Foreign demand	α_4^1	0.5

Table C4: Phillips Curve – Core Index

Equation Coefficients	Model Parameters	Values
Expected core inflation	α_1^2	0.5
Lagged core inflation	$1 - \alpha_1^2$	0.5
RMC	α_2^2	0.1
Output gap in RMC	α_3^2	0.75
Relative price gap in RMC	$1 - \alpha_3^2$	0.25

Table C5: Phillips Curve – Food Prices

Equation Coefficients	Model Parameters	Values
Expected food price inflation	α_1^3	0.5
Lagged food price inflation	$1 - \alpha_1^3$	0.5
RMC	α_2^3	0.1
Output gap in RMC	α_3^3	0.7
Relative price gap in RMC	$1 - \alpha_3^3$	0.3

Table C6: Phillips Curve – Fuel Prices

Equation Coefficients	Model Parameters	Values
Expected fuel price inflation	α_1^4	0.5
Lagged fuel price inflation	$1 - \alpha_1^4$	0.5
RMC	α_2^4	0.2
Output gap in RMC	α_3^4	0.1
Relative price gap in RMC	$1 - \alpha_3^4$	0.9

Table C7: Phillips Curve – Administrative Prices

Equation Coefficients	Model Parameters	Values
Expected adm. price inflation	α_1^5	0.5
Lagged adm. price inflation	$1 - \alpha_1^5$	0.5
Elasticity wrt RMC	α_2^5	0.3
Output gap in RMC	α_3^5	0.7
Relative price gap in RMC	$1 - \alpha_3^5$	0.3

Table C8: CPI Identity

Weight of	Model Parameters	Values
Core inflation	α_1^{10}	0.533
Food price inflation	α_2^{10}	0.242
Fuel price inflation	α_3^{10}	0.030
Administrative price inflation	$1 - \alpha_1^{10} - \alpha_2^{10} - \alpha_3^{10}$	0.195

Table C9: Uncovered Interest Rate Parity

Equation Coefficients	Model Parameters	Values
Expected nom. exchange rate	α_1^{15}	0.4
Lagged nom. exchange rate	$1 - \alpha_1^{15}$	0.6

Table C10: Policy Rule

Equation Coefficients	Model Parameters	Values
Lagged nominal interest rate	α_1^{16}	0.7
Deviation of exp. inflation from target	α_2^{16}	0.2
Output gap	α_3^{16}	0.1

Table C11: AR(1) Equations

Coefficient	Model Parameters	Values
Persistence in domestic potential GDP growth	$\Delta \bar{Y}_{t-1}$	0.80
Persistence in eq. relative prices (core indices)	$\Delta \bar{Q}_{t-1}^{C}$	0.75
Persistence in eq. relative prices (food prices)	$\Delta \bar{Q}_{t-1}^{F}$	0.75
Persistence in eq. relative prices (energy prices)	$\Delta \bar{Q}_{t-1}^{Fu}$	0.75
Persistence in eq. relative prices (administrative prices)	$\Delta \bar{Q}_{t-1}^{A}$	0.75
Persistence in foreign short-term real interest rate	R_{t-1}^{*}	0.80
Persistence in foreign inflation rate	Δcpi_{t-1}^{*}	0.80
Persistence in foreign output gap	$\Delta Y_{gap,t-1}^{W}$	0.80

C.3 Calibration of Stochastic Properties

The identification of unobserved variables using Kalman filtration requires calibration of the stochastic properties of the model. The stochastic properties of the model are driven by the standard deviations of the shocks. The standard deviations are calibrated to get reasonable (i) dynamics of unobserved variables (trends smoother than gaps, etc.) and (ii) contributions of shocks in line with economic intuition and the story of economic developments as articulated in the CNB's Inflation Reports. Given the software used (IRIS), only the relative size, not the absolute value, of the standard deviations determines the outcomes.

Inflationary pressures identified based on the model and its key behavioral equations are one of the key indicators of calibration quality. Logically, the Phillips curve(s) plays a crucial role in identifying the inflationary pressures. At the same time, the equations determining the unobserved values for potential output and trends in relative prices are important as well, as they determine the

contribution of the domestic economic cycle on top of import prices to the inflationary pressures. Therefore, the standard deviations of the shocks in the Phillips curves are set low to ensure that there is a link between inflation and the pressures approximated by the output gap. Specifically, the magnitudes of the standard deviations of the supply shocks in the Phillips curves (especially for core inflation) are, ceteris paribus, relatively small to determine the output gap, which explains inflation. In particular, the standard deviation of the supply shocks in the core inflation Phillips curve is the smallest of all the standard deviations of the shocks in the Phillips curves. This is due to higher volatility of the change in food, fuel and administered prices compared with that of core inflation. The high standard deviation of the policy rule shock (calibrated to 10) seeks to avoid a situation where the policy rule and its calibration affect the natural rate. The calibrated standard deviations are reported in Tables C12 and C13 below.

Table C12: Behavioral Equations

Coefficient	Shock	Standard Deviation
IS curve	$\varepsilon_t^{Y_{gap}}$	1.0
Phillips curve – Core Inflation	$\varepsilon_t^{\pi^C}$	1.0
Phillips curve - Food Price Inflation	$\varepsilon_t^{\pi^F}$	3.0
Phillips curve – Fuel Price Inflation	$\varepsilon_t^{\pi^{Fu}}$	15.0
Phillips curve – Administered Price Inflation	$\varepsilon_t^{\pi^A}$	7.0
Uncovered Interest Rate Parity	ε_t^e	1
Policy Rule	ε_t^i	10
CPI Identity	ε_t^{cpi}	0

Table C13: Equilibrium Trends and Exogenous Processes

Coefficient	Shock	Standard Deviation
Domestic potential GDP growth	$\varepsilon_t^{\Delta \bar{Y}}$	0.6
Domestic eq. real interest rate	$\varepsilon_t^{\bar{R}}$	0.5
Equilibrium relative prices (core indices), change	$\varepsilon_t^{\Delta \bar{Q}^C}$	1.75
Equilibrium relative prices (food prices), change	$\varepsilon_t^{\Delta \bar{Q}^F}$	2.0
Equilibrium relative prices (energy prices), change	$\varepsilon_t^{\Delta \bar{Q}^{Fu}}$	3.5
Equilibrium relative prices (administrative prices)	$\varepsilon_t^{\Delta \bar{Q}^A}$	3.0
Foreign short-term real interest rate	$\varepsilon_t^{R^*}$	1.0
Foreign inflation rate	$\varepsilon_t^{\Delta cpi^*}$	2.0
Foreign output gap	$\varepsilon_t^{\Delta Y_{gap}^W}$	1.0

C.4 Calibration Checks

We run ex-post model-based projections – so-called in-sample simulations – to check the model calibration. We report only YoY headline CPI inflation here; other model variables can be provided upon request. When doing the simulations, we let the model produce the forecast iteratively. First, we run the Kalman filter to identify the initial conditions at each point in time in the past. We then use the model to generate a forecast, using only the external (exogenous) variables as inputs. We do not impose any other judgment. The foreign variables are treated as unanticipated. The model-based forecasts are subsequently compared with the actual data (see Figure C1). The model's dynamic simulations, relying on knowledge of the foreign exogenous variables, are rela-

Figure C1: Actual Data versus Model Projections – YoY Headline CPI Inflation (in percent p.a.)
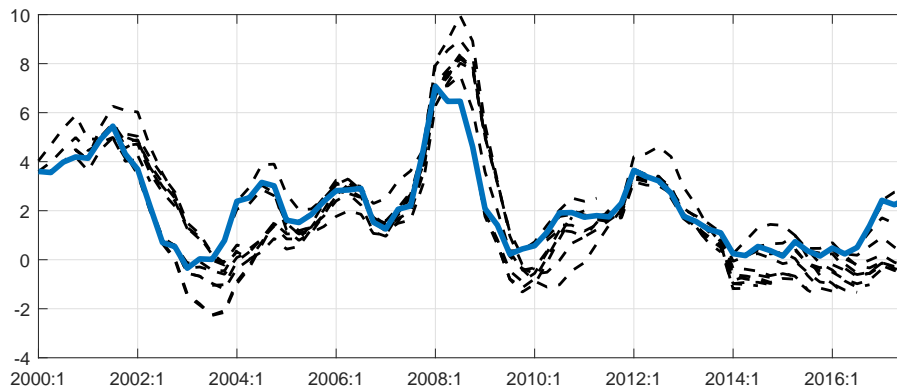
Note: The solid blue line depicts the actual data. The dotted black lines depict the model-based forecasts.
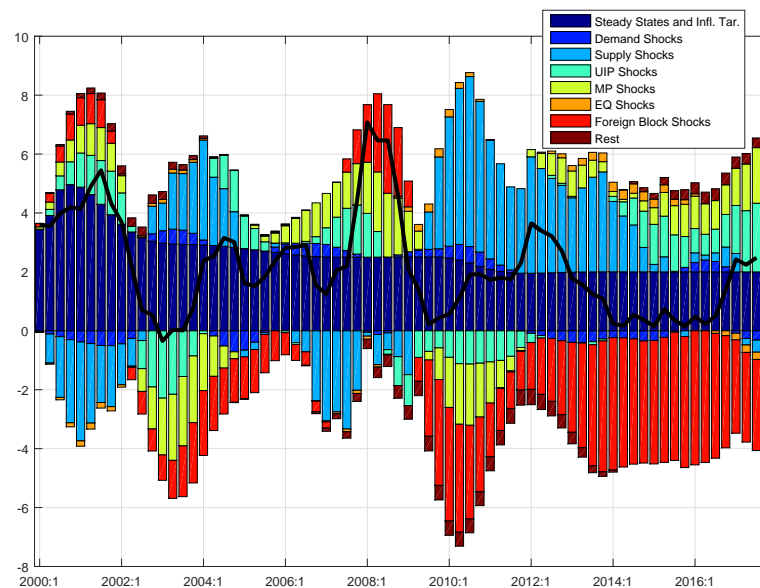*Source:* Authors' calculations.

tively close to the observed data and are unbiased on average, implying a reasonable calibration of the model.

Decomposition of the observed variables into structural shocks provides an additional check of the model calibration. The composition of the shocks has to be reasonable, i.e., in line with economic intuition about the main driving factors of inflation and in line with the CNB's Inflation Reports. The decomposition is presented in Figure C2.[10] The results suggest a significant effect of external sector shocks on inflation since 2010. The effects of these shocks on inflation were offset by monetary policy easing – positive effects of UIP shocks and monetary policy shocks – both mainly related to the CNB's exchange rate commitment. The positive supply shocks observed from the end of 2009 until the end of 2014 are related mainly to administrative prices, which stay unaffected by the business cycle. Despite the disinflationary environment and declining commodity prices, regulated price inflation was high in order to catch up with the level of non-regulated prices.

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[10] The decomposition is done as follows. First, the filter identifies all the structural shocks in the model. All of these shocks are subsequently used to generate impulse response functions. Due to the linear nature of our model, summing these impulse response functions allows us to construct the decomposition.

Figure C2: Decomposition of YoY Headline CPI Inflation (in percent p.a.) into Structural Shocks

Note: The bars in each period of time sum to the black line, which stands for YoY inflation.
Source: Authors' calculations.

Appendix D: Data Sources and Definitions

All the measurement equations are merely identities with appropriate state variables without any measurement below. We used the following data as measurement variables in the Kalman filtration:

Name	Description
gdp	Czech gross domestic product, constant prices, non-seasonally adjusted, source: Czech Statistical Office; transformation: seasonally adjusted (X13), log-transformed (taking the natural log of the variable) and multiplied by 100
cpi	Czech non-seasonally adjusted consumer price index, source: Czech Statistical Office; transformation: seasonally adjusted (X13), log-transformed and multiplied by 100
cpi^C	Czech CPI sub-index used for the calculation of the core inflation index, source: CNB calculation, transformation: seasonally adjusted (X13), log-transformed and multiplied by 100
cpi^F	Food price sub-index of the Czech CPI, source: CNB calculation; transformation: seasonally adjusted (X13), log-transformed and multiplied by 100
cpi^{Fu}	Fuel prices sub-index of the Czech CPI, source: CNB calculation; transformation: seasonally adjusted (X13), log-transformed and multiplied by 100
cpi^A	Czech CPI sub-index measuring administered-prices, source: CNB calculation; transformation: seasonally adjusted (X13), log-transformed and multiplied by 100
e	Nominal exchange rate of the Czech koruna against the euro, source: Czech National Bank, transformation: log-transformed and multiplied by 100
e^{USD}	Nominal exchange rate of the euro against the US dollar, source: Eurostat, transformation: log-transformed and multiplied by 100
i	3M Pribor, CNB's online ARAD database
i^*	3M Euribor, Eurostat
π^{tar}	The CNB's y-o-y inflation target, source: Czech National Bank
pf^*	World food price and beverage index, source: IMF commodity database, seasonally adjusted (X13), transformation: log-transformed and multiplied by 100
oil^*	Global price of Brent crude oil in USD, transformation: log-transformed and multiplied by 100;
cpi^*	Trade-weighted average of consumer price indices based on the 17 largest trading partners of the Czech Republic (DE, AT, IT, FR, NL, BE, ES, IE, FI, GR, PT, SK, SI, CY, EE, LV, LT), source: Eurostat and CNB calculations, transformation: seasonally adjusted (X13), log-transformed and multiplied by 100
$wgdp$	Trade-weighted average of the gross domestic products of the 17 largest trading partners of the Czech Republic (DE, AT, IT, FR, NL, BE, ES, IE, FI, GR, PT, SK, SI, CY, EE, LV, LT), constant prices, source: Eurostat and CNB calculations, transformation: seasonally adjusted (X13), log-transformed and multiplied by 100, HP filtered ($\lambda = 1600$)

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