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Longer-term Yield Decomposition: An Analysis of the Czech Government Yield Curve

Adam Kučera, Michal Dvořák, Luboš Komárek and Zlatuše Komárková [*]

Abstract

The term structure of yields is an important source of information on market expectations about future macroeconomic developments and investors' risk perceptions and preferences. This paper presents the methodology used by the Czech National Bank to obtain such information. It describes the decomposition of the Czech government bond yield curve into its components. The evolution of those components is interpreted in relation to the macro-financial environment, as embodied by selected variables. The practical use of the decomposition in estimating and interpreting the responses of the Czech government bond yield curve to macroeconomic and financial shocks is presented using a vector autoregression model.

Abstrakt

Časová struktura výnosových měr je významným zdrojem informací ohledně očekávání trhu o budoucím makroekonomickém vývoji a vnímání rizik ze strany investorů a jejich preferencí. Článek představuje metodologický aparát, který Česká národní banka využívá k získání těchto informací. Popisuje rozklad výnosové křivky českých státních dluhopisů na dílčí komponenty. Vývoj těchto komponent je interpretován ve vztahu k makrofinančnímu prostředí reprezentovanému vybranými proměnnými. Následně článek představuje využití dekompozice k odhadu a interpretaci reakcí výnosové křivky českých státních dluhopisů na makroekonomické a finanční šoky v rámci modelu vektorové autoregrese.

JEL Codes: G11, G12, G23

Keywords: Affine model, decomposition, government bond, yield curve

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Nontechnical Summary

The yield curve is an important indicator of the economic cycle, as it reveals market participants' expectations about future economic and financial conditions. The factors that affect the shape of the yield curve manifest themselves differently in different circumstances. Correct interpretation of the evolution of the yield curve requires identification of the individual components of bond yields relating to individual factors. This paper presents the method used by the Czech National Bank (CNB) to decompose the Czech government bond (GB) yield curve.

We decompose the Czech GB yield curve into four components: a risk-neutral yield, a term premium, a credit risk premium, and a portfolio effect. The *risk-neutral yield* reflects market participants' expectations about future monetary policy and economic developments. The *term premium* reflects investors' uncertainty about the future path of the risk-neutral yield. In other words, it is compensation for bearing interest rate risk. The *credit risk premium* is compensation for the possibility that bond coupons and principal will not be duly repaid. The *portfolio effect* reflects specific demand for GBs as an investment asset. Many investors prefer GBs to other assets, mainly because of their low credit risk, their relatively high market liquidity, their low haircuts when used as financial collateral, and their preferential regulatory treatment.

The components are extracted from bond yields in the following way. The risk-neutral yield and the term premium are obtained by decomposing the zero-coupon koruna swap curve using an affine model. The credit risk premium is estimated from credit default swap quotations for Czech GBs. The portfolio effect forms the residual, i.e., the difference between the zero-coupon bond yield and the sum of the previous three components. We confirmed the strong theoretical interpretation of the components by comparing the four estimated components with selected macroeconomic and financial variables. As the theory had anticipated, for example, the risk-neutral yield matched analysts' expectations about future short-term policy rates, and the portfolio effect became highly negative as the removal of the CNB's exchange rate floor neared.

Using a simple vector autoregression model of the Czech economy, we also measure the sensitivity of the yield curve to changes in macroeconomic conditions (GDP and expected inflation) and global market uncertainty (the VIX index). According to our estimates, a one-time 1 percentage point increase in the quarterly GDP growth rate increases the short end of the yield curve by about 7 basis points six months from the impulse. This is caused by growth in the risk-neutral yield for short maturities, reflecting expectations of a monetary policy reaction. Longer yields, on the other hand, decrease by up to 35 basis points. This is caused by (i) a drop in the term premium for longer yields, since the positive economic results reduce long-term uncertainty, and (ii) a decrease in the perceived credit risk premium, together with a decrease in the portfolio effect, as lower risk and good prospects attract investors to Czech GBs. This effect lasts about two years. It gradually dies out and, instead, risk-neutral yields increase. This rise is related to the monetary policy response to the favorable economic conditions.

A 1 percentage point increase in the inflation expected one year ahead causes the yield curve to increase at its short end by about 60 basis points six months after the impulse. This is mostly due to an increase in the short-term risk-neutral yield.

An exogenous increase of the VIX from 12 to 30 points, which corresponds to a transition from a tranquil period to a period of elevated uncertainty, causes the slope of the yield curve to increase by about 10 basis points six months after the impulse. The uncertainty shifts the expected monetary policy rate path downward and risk-neutral yields decrease slightly. On the other hand, the shock increases the credit risk premium of Czech government bonds and reduces their attractiveness to investors, which pushes both the credit risk premium and the portfolio effect upward. As a consequence, the yield curve rotates.

The results demonstrate that the response of the yield curve is truly complex and that movements in its individual components may offset each other. The drop in the term and credit risk premia and the response of the portfolio component after a positive economic shock may have important implications for both the monetary and financial stability policies of the CNB. Similarly, the combination of macroeconomic and financial responses to a market uncertainty shock needs to be accounted for when considering possible policy measures.

1. Introduction

Yields on government bonds (GBs) have been falling across a wide range of countries for more than a decade. This trend is due to several common global factors: savings surpluses in emerging economies and a related build-up of foreign exchange reserves in central banks' balance sheets, global portfolio shifts toward safer assets, and a fall in nominal interest rates linked with the anchoring of inflation expectations at low levels. The decline in yields accelerated after the outbreak of the global financial crisis, when some central banks responded to the adverse economic outlook and deflation pressures by introducing unconventional measures targeted directly at lowering long-term yields.

Czech GB yields, too, have been on a downward trend on average since the global financial crisis started. They have been negative at maturities of up to six years since the beginning of 2016. This can hardly be explained solely by market expectations of continued low rates or by the lower Czech sovereign risk premium. One of the aims of this paper is to explain the causes of this trend by decomposing the Czech GB yield curve into components determining the bond yields represented in the yield curve. Another aim is to analyze the behavior of those components over time, as each component gains significance under different conditions. Last but not least, the aim is to share with the economic community the method used by the Czech National Bank (CNB) to obtain information on the term structure of yields.

The term structure of yields, or the slope of the yield curve, is also an important indicator of the economic cycle. The yield curve aggregates market participants' expectations about the future development of short-term rates, economic activity, inflation, and financial risks. Quantifying the effects of various factors on the shape of the yield curve allows us to better understand the extent of possible changes in yields if economic trends continue as they are or change direction and to better estimate the impacts of different economic scenarios on financial market participants. By decomposing the yield curve, the CNB gains an important source of information for monetary and prudential policy purposes.

The paper is structured as follows. Section 2 presents the method used to decompose yield curves. In section 3, the zero-coupon Czech GB yield curve is decomposed into four components and the factors that influence them are empirically analyzed. In section 4, the analysis focuses on the use of the components to estimate and interpret the response of the Czech GB yield curve to macroeconomic and financial shocks in a vector autoregression framework. The final section concludes.

2. Yield Curve Decomposition Methodology

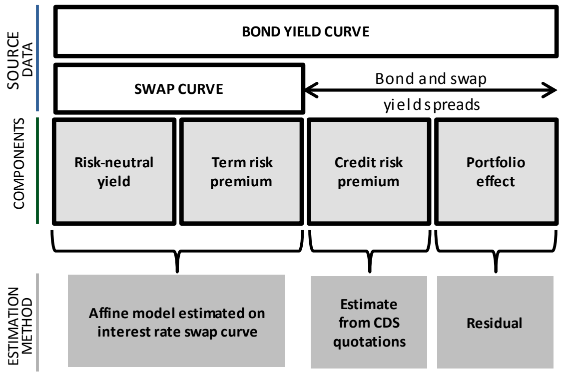
2.1 Decomposition Rationale and Approach

The yield curve is made up of yields on bonds with various residual maturities at a specific point in time. The shape of the curve is determined by its level (the position of the short end of the curve), its slope (the difference between yields on short- and long-maturity bonds) and its curvature (the maturity-yield relationship is not necessarily linear, but can be concave or convex). The relative level of short-term and long-term yields should depend on market expectations about the future path of short-term rates. According to the pure expectations hypothesis, a risk-neutral investor should attain the same yield from investing in a long-term bond as from a series of investments in a short-term bond over a period equal to the residual maturity of the long-term bond. The pure expectations hypothesis offers a simple and attractive interpretation of the yield curve. However, it does not hold in reality, as it does not take risk-averse investors into consideration. In other words, investors perceive long-term investment as uncertain and demand a risk premium.

The importance of yield decomposition became broadly recognized in the first decade of this century. Between 2004 and 2006, the Fed increased its target funds rate, but this was not reflected in longer yields. In contrast, 10-year U.S. Treasury yields decreased over the period.[1] This divergence could not be explained by the expectations hypothesis. After a wide discussion among researchers, a drop in the risk premium (which until then had been considered to be roughly constant) was identified as the source of the decline in 10-year yields (Backus and Wright, 2007). Consequently, since then, the ability of central banks to influence the longer part of the yield curve has been admitted to be weaker than originally thought. Proper risk premium modeling and forecasting can help overcome this gap.

A frequent approach in the literature (for example, Wright, 2011) is to consider the risk premium to be related solely to the uncertainty about the future evolution of yields (i.e., the term premium as described below). However, in case of Czech GB yields, the premium for credit risk has also historically been an important source of yield variation. At the same time, Czech GBs have an exclusive position among Czech financial assets, representing either the one and only liquid asset available in Czech koruna or a tool in speculative schemes. Consequently, we go beyond the traditional approach and, instead of decomposing the yield into two components (a risk-neutral yield and a risk premium), we propose a novel extension of the decomposition. More specifically, we decompose the Czech GB yield curve into four components (see Figure 1): a risk-neutral yield, a term premium, a credit risk premium, and a portfolio effect. The last three components together form the full risk premium of Czech GBs. The yields on Czech GBs are calculated additively as the sum of the four components; we prefer this to the multiplicative approach to keep the model parsimonious.

Figure 1: Components of the Swap and Bond Yield Curves



Note: CDS = credit default swap.
Source: Authors

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[1] This divergence is called Greenspan's conundrum, after a famous remark made by Alan Greenspan in a speech in February 2005 (Greenspan, 2005).

The risk-neutral yield reflects expectations about future monetary policy and economic developments (i.e., the expectations hypothesis). If investors expect the monetary policy rate to rise in the future, they also expect the rate of return on holding and regularly reinvesting short-term bonds to go up gradually. The term premium relates to the maturity of the bond and is compensation for interest rate risk. It takes into account investors' uncertainty about the future path of the short-term rate. Committing to long-term bonds will turn out to be relatively less (more) advantageous if future short-term rates are higher (lower) than originally expected.

Regarding our approach to the decomposition, as an underlying assumption we consider the risk-neutral yield and the term premium to be common to both Czech GB yields and Czech koruna interest rate swap rates (CZK IRSs). This assumption follows the intuition that expectations about, and the uncertainty related to, future interest rates are not dependent on financial instruments. A consequence of this assumption is that the spread between the Czech GB yield and the CZK IRS is formed exclusively by the other two components: the credit risk premium and the portfolio effect.

To simplify the method, we put both the credit risk premium and the portfolio effect of IRSs equal to zero. This simplification is in line with the nature of the IRS: IRSs contain only a very limited credit risk premium, as no principal is paid, coupon payments are netted, and the way IRSs are traded mitigates counterparty risk. An IRS is meanwhile not an investment asset, because it cannot be used to deposit liquidity. The portfolio effect of an IRS is therefore negligible.

The absence of any other components allows us to use the affine class of models (Duffie and Kan, 1996; the affine model is described in section 2.2) for the IRS data to separate the two components. The application of the affine model to the IRS data can be seen as its application to a set of yields on composite bonds which are risk free in terms of credit risk and are traded on a perfect market (i.e., the portfolio effect is not present). In contrast, the portfolio component of Czech GB yields can be affected in certain circumstances by specific market effects such as flight to quality, flight to liquidity, search for yield, and various types of speculation caused, for example, by unconventional monetary policies. However, these specific effects could disrupt the affine model's assumption of market efficiency and the impossibility of arbitrage. The risk-neutral yield and the risk premium estimated using the affine model from GB yield curves could thus be distorted.

The credit risk premium is compensation for the risk that bond coupons and principal will not be paid on time and/or in full. This premium tends to increase with increasing maturity. The issuer's position can worsen significantly over time, so, for example, the one-year probability of default in five years' time (i.e., the probability of default between the fifth and sixth years) is usually higher than the current one-year probability of default, i.e., the probability of default between now and 12 months from now (Moody's, 2016). The credit risk premium is estimated from credit default swap (CDS) quotations for Czech GBs.[2] The volatility of the CDS quotations was reduced by smoothing them using the three-month moving average. Furthermore, to obtain quotations for

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[2] The advantages of this approach are the objective existence of quotations (which should represent the direct cost of hedging credit risk), its forward-lookingness, and the availability of any periodicity. On the other hand, we also need to take into account certain sovereign CDS market anomalies that may limit the use of CDS quotations as a sovereign solvency indicator (see Komárek et al., 2013, Box 4 of CNB, 2010, and Box 4 of CNB, 2012). Short time series are another potential limitation for some maturities.

each maturity, the Nelson-Siegel function (Nelson and Siegel, 1987) was fitted to these averaged quotations.

To the best of our knowledge, such explicit calculation of the credit risk component from CDS quotations has not been used so far in term structure modeling. The literature mostly focuses on yields on U.S. government bonds, where the credit risk premium is considered to be negligible and rather constant over time and hence interpreted as being part of the total risk premium. By contrast, in the Czech GB yield curve, an increase in the credit risk premium significantly affected yields on longer maturities between 2009 and 2012 (see Figure 2). As a competitive approach, it would also be possible to estimate the credit risk premium from Czech GB yields by using intensity-based modeling of credit risk as in Lando (2009). This approach extends the risk-neutral pricing used in the affine model to include the credit risk premium as well. However, since this approach would further increase the technical demands of building and estimating the model, we leave this possibility for further research and stick to estimating the credit risk premium directly from CDS quotations.

The portfolio effect of the yield reflects demand for GBs as an investment asset or a tool usable for speculative trades (see also Kladívko, 2010). Many investors prefer GBs to other assets, mainly because of their low credit risk, their relatively high market liquidity, their low haircuts when used as financial collateral, and their preferential regulatory treatment. Additionally, GBs can serve as a tool for performing speculative or arbitrage trades. The portfolio effect is calculated as a residuum – the difference between the GB yield and the rate of an IRS rate of identical maturity minus the credit risk premium. The average portfolio effect in the model therefore depends on the estimate of the credit risk premium. The portfolio effect can take positive (negative) values if the yield demanded by the investor for holding the bond is higher (lower) than the expected short-term rate plus the term premium and the credit risk premium.

2.2 Interest Rate Swap Curve Decomposition

To decompose the IRS curve we use the affine model (Duffie and Kan, 1996, or Málek, 2005), which belongs to the category of factor models. The basic building block of this model is its assumption that there are several underlying factors (X_t) that determine the entire term structure of rates. The affine model presented here uses three factors, in line with the standard approach employed in the literature (Litterman and Scheinkman, 1991). Depending on their dynamic realizations, the factors can be frequently considered as the level, slope, and curvature of the term structure. We consider these factors to be unobservable and estimate them within the model. In the model, their dynamics are set in the form of a mean-reverting (Ornstein-Uhlenbeck) process (Krippner, 2015):

$$\mathrm{d}X_t = \kappa(\theta - X_t)\mathrm{d}t + \sigma \mathrm{d}W_t \tag{1}$$

where W_t is a three-dimensional independent Wiener process, θ is a 3×1 vector representing the level of mean reversion, κ is a 3×3 matrix determining the speed of mean reversion, and σ is a 3×3 matrix allowing the innovations to X_t to be correlated.

The process in equation (1) is valid for the physical (data-generating) $\mathbb{P}$ measure. To determine the swap curve, it is essential to convert the $\mathbb{P}$ measure into a risk-neutral $\mathbb{Q}$ measure. The risk-neutral $\mathbb{Q}$ measure offers an elegant approach to pricing the term risk, since it incorporates the price of risk into the adjusted path of the factors. To do so, the so-called market price of risk is established as an affine function of the factors:

$$\lambda_t = \sigma^{-1}(\gamma + \Gamma X_t) \tag{2}$$

where γ and Γ are a vector and a matrix of parameters. To incorporate the market price of risk into the factor process, we put $\tilde{\kappa} = \kappa + \Gamma$, $\tilde{\theta} = \tilde{\kappa}^{-1}(\kappa\theta - \gamma)$, and $\mathrm{d}\widetilde{W}_t = \mathrm{d}W_t + \lambda_t \mathrm{d}t$. Then the factor process under the risk-neutral $\mathbb{Q}$ measure can be written as:

$$\mathrm{d}X_t = \tilde{\kappa}\big(\tilde{\theta} - X_t\big)\mathrm{d}t + \sigma \mathrm{d}\widetilde{W}_t \tag{3}$$

From the path of the factors (under either the $\mathbb{P}$ or $\mathbb{Q}$ measure), the short (instantaneous) interest rate[3] r_t is calculated as their affine transformation:

$$r_t = \alpha_0 + \alpha_1 X_t \tag{4}$$

From the short rate, the longer rates $R_t(\tau)$ of any maturity τ are derived by applying the expectations hypothesis. This means that under the $\mathbb{P}$ measure, the expected path of the factors is first obtained from equation (1). From it, the expected short rate path can be calculated using equation (4). The longer rates $\hat{R}_t(\tau)$ are derived from the short rates over the expected path. However, the expectations hypothesis is not sufficient under the $\mathbb{P}$ measure. Investors are risk-averse, so they require a term premium to lock their investments over the maturity τ; otherwise, they would prefer to invest in a sequence of short-term investments with reinvesting. However, the term premium cannot be easily defined in the model.

To overcome this, the expectations hypothesis is applied under the $\mathbb{Q}$ measure. In this case, the term premium is – through the λ_t process (equation (2)) – directly incorporated into the $\mathbb{Q}$ measure path of the factors X_t. The expected path of the factors is in this case obtained from equation (3). The expected short rate path under the $\mathbb{Q}$ measure is then again calculated by the transformation in equation (4). The longer rates $\hat{R}_t(\tau)$ are derived by integrating the short rate over the expected path:

$$\hat{R}_t(\tau) = \frac{1}{\tau}\int_0^\tau \big[E_t^{\mathbb{Q}}(r_{t+u}|X_t) - VE(u)\big]\mathrm{d}u \tag{5}$$

where $VE(u)$ is a volatility effect which corrects the expectations with respect to Jensen's inequality (Heath et al., 1992). Jensen's inequality results from the nature of the expectations hypothesis in terms of bond prices and the convexity of the exponential function:

$$e^{-R_t(\tau)\tau} = E_t^{\mathbb{Q}}\left[e^{-\int_0^\tau r_{t+u}\mathrm{d}u}|X_t\right] \le e^{-E_t^{\mathbb{Q}}[\int_0^\tau r_{t+u}\mathrm{d}u|X_t]} \tag{6}$$

The $VE(u)$ value requires calculation of a double integral (Heath et al., 1992) and depends only on the maturity and the parameters from equation (3).

Equation (5) can be solved by plugging in for $E_t^{\mathbb{Q}}(r_{t+u}|X_t)$, which is calculated using equation (4) from the expected dynamics of the factors given by equation (3). After combining with the

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[3] The short-term (instantaneous) risk-free rate is not observable in the market. However, it is linked to some extent with the overnight rate on the interbank market and the monetary policy rate.

equation for the $VE(u)$ calculation, the whole observed term structure $R_t(\tau)$ can be expressed in general as a term-structure function of X_t and the maturity:

$$R_t(\tau) = \hat{R}_t(\tau) + e_t(\tau) = F(X_t, \tau) + e_t(\tau) \tag{7}$$

where $F(\bullet)$ is some function parametrized by the parameters from equations (1), (2), and (4) and $e_t(\tau)$ is the measurement error of the rate with maturity τ.

Since the factors X_t are unobservable, it is necessary to estimate them. To do so, we specify the affine model in the state space representation, where the term structure function (7) represents the measurement equation, while the $\mathbb{P}$ measure factor process (1) represents the state equation. It can be shown (Meucci, 2010) that the solution to the state equation (1) forms a first-order vector autoregression process for some discrete time step Δt (equation (9)). The final state-space representation is thus:

$$R_t(\tau) = F(X_t, \tau) + e_t(\tau) \tag{8}$$

$$X_t = \theta + exp(-\kappa \Delta t)(X_{t-1} - \theta) + \upsilon_t \tag{9}$$

where υ_t is a 3×1 vector of random innovations to state variables. Consequently, the affine model in the state space representation can be estimated using maximum likelihood, with a Kalman filter (Durbin and Koopman, 2012) used to obtain the factors.

If interest rates are already close to their lower bound, the probability of them falling further is lower than the probability of them rising. This leads to a violation of the assumptions underlying the affine model – the process W_t in equation (1) is no longer a Wiener process. To take this asymmetry into account, the model uses the concept of shadow rates (Krippner, 2013). In this concept, which builds on the model of Black (1995), the yield on investing in a bond equals the sum of the yield on investing in a shadow bond whose yield is not bounded below by zero and the yield from the sale of a bond option. The option bears a right to purchase the bond at a price such that its yield is equivalent to the lower bound value. For details, see Krippner (2013).

In practice, in the shadow-rate affine model, equation (4) holds for the shadow short rate (rs_t). The expected value of the observed short rate (r_{t+u}) entering equation (5) is then obtained as the sum of the expected shadow short rate and an option effect:

$$E_t^{\mathbb{Q}}(r_{t+u}|X_t) = E_t^{\mathbb{Q}}[rs_{t+u}|X_t] + E_t^{\mathbb{Q}}[\max(r_L - rs_t, 0)|X_t] \tag{10}$$

where r_L is the value of the lower bound. Krippner (2013) derives a closed-form solution for $E_t^{\mathbb{Q}}(r_{t+u}|X_t)$ in terms of parameters from equations (1)–(4). After plugging this into equation (5), the whole term structure is derived, i.e., the function $F(\bullet)$ is obtained. The final formulas are very complex and therefore omitted here for parsimony. A complete description can be found in, for example, Krippner (2015). Due to the option effect, the function $F(\bullet)$ is not linear in the case of the shadow-rate affine model, so an extended iterative Kalman filter (Durbin and Koopman, 2012) is employed in the estimation. However, the shadow rates can still be expressed as an affine transformation of the factors. For this reason, the model still falls into the category of affine models.

To estimate the shadow-rate affine model in its state space representation defined by equations (8) and (9), it is necessary to set identifying restrictions. In our case, we first tried to utilize the arbitrage-free Dynamic Nelson-Siegel (ADNS) model specification established by Christensen et al. (2011) and extended to include shadow rates by Christensen and Rudebusch (2013). The shadow-rate ADNS identification has a favorable consequence: the three factors X_t are determined to be considered as the level, slope, and curvature of the term structure, in line with the parameters of the Nelson-Siegel function. However, due to the relatively short maximum maturity included in the sample,[4] the shadow-rate ADNS model does not converge well and is very sensitive to the initial conditions. To overcome this weakness, we then relaxed the restrictions imposed by the shadow-rate ADNS model and instead used the shadow-rate Stationary Gaussian affine (SR-SGA) model described in Krippner (2015, section 4.4.4). The restrictions imposed by the SR-SGA model on the parameters from equations (3) and (4) are as follows:

$$\tilde{\kappa} = \begin{bmatrix} \tilde{\kappa}_1 & 0 & 0 \\ 0 & \tilde{\kappa}_2 & 0 \\ 0 & 0 & \tilde{\kappa}_3 \end{bmatrix}, \tilde{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{11}$$

Furthermore, σ is lower-triangular. The measurement errors $e_t(\tau)$ from equation (8) form a random normally distributed vector with zero mean and a time-invariant diagonal covariance matrix. The state innovations v_t from equation (9) are normally distributed with zero mean and a variance given by the σ matrix (the derivation can be found in Krippner, 2015). The two error vectors are assumed to have a zero covariance matrix.

After the SR-SGA model is estimated, it can be used to decompose the observed rates into the two components. The decomposition is done independently for each period *t* in the sample. The vector of factors X_t, which was extracted during the SR-SGA model estimation, is the starting point. Using equation (9), the expected path of the factors $\{E^{\mathbb{P}}[X_{t+j\Delta t}|X_t]\}_{j=1,2,\ldots,\mathrm{T}/\Delta\mathrm{t}}$ is calculated, where T represents the longest maturity in the sample (in years). In practice, Δt is made very small (for example 0.01), so that the numerical calculation approaches continuous time reasonably well. For Δt = 0.01, there are 100 calculation steps per year, i.e., to get the expected path of the states T years ahead, j ranges from 1 to $T * 100$. From the path of the state variables, the expected path of the short rate $\{E^{\mathbb{P}}[r_{t+j\Delta t}|X_t]\}_{j=1,2,\ldots,\mathrm{T}/\Delta\mathrm{t}}$ is calculated on an identical horizon, using equation (4). Finally, for any maturity $\tau < T$, the average expected short rate $\Delta t/\tau * \sum_{k=1}^{\tau/\Delta t} E^{\mathbb{P}}[r_{t+k\Delta t}|X_t]$ represents the risk-neutral component of the yield $R_t(\tau)$. Similarly to equation (5), volatility effect (VE) adjustment applies. Expectations are obtained under the $\mathbb{P}$ measure: the risk-neutrality of the component means that it is calculated as an average of future short rates (i.e., without any risk premia). The second estimated component – the risk premium (which is equal to the term premium in the case of swap rates) – is then obtained as the difference between the fitted swap rate $\hat{R}_t(\tau)$ and the risk-neutral component. Note that if expectations were obtained under the $\mathbb{Q}$ measure, the risk premium would equal zero (see equation (5)).

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[4] As described in the next section, we used maturities of up to 15 years. By contrast, in the case of the U.S. and EA yield curves, maturities of 20–30 years are usually available. Apart from the sample, the shape of the Czech yield curve is also slightly different than in the case of other countries – the Czech yield curve was never inverted in the sample period.

3. Decomposition of the Czech Government Bond Yield Curve

The yield curve is decomposed using the yields on zero-coupon bonds of relevant maturities, since those yields are not affected by the size and distribution of the coupons over the life of the bond and hence are an exact indicator of the rate of return demanded for investing for the relevant time period.[5] For this reason, a zero-coupon curve was constructed using Czech government bonds in Czech koruna. As the risk-neutral yield and the term premium are estimated using swap rates, it was also necessary to construct a zero-coupon koruna swap curve. The two zero-coupon curves were constructed for maturities of 1 to 15 years[6] as of the end of each month over the period of 7/2003–12/2016. The Fama-Bliss bootstrap method (Fama and Bliss, 1987), which assumes constant forward rates among the closest maturities, was used for the construction. The advantage of this method over the alternatives (such as Nelson and Siegel, 1987, or Svensson, 1994)[7] consists in its ability to replicate any yield curve shape exactly, which eliminates problems with imperfect fit on some segments of the curve.

The maturity spreads for zero-coupon GB yields and swap rates in 2003–2016 show mixed developments (see Figure 2). Until the outbreak of the global financial crisis in September 2008, yields and rates followed similar patterns. This is consistent with the findings in Kladívko (2010). From then until the second half of 2009, yields were affected by the fear of the emerging debt crisis in Europe. Owing to the responses of the various relevant authorities to the crisis, yields began to trend downward in mid-2009 and a positive gap opened up between yields and rates at longer maturities. At the end of 2013, yields started falling faster than rates – until 2015 for long maturities and then exclusively for short maturities. It is clear from this simple historical excursion that yields and rates were affected by different factors with different intensity, including for individual maturities.

The estimated affine model from which the risk-neutral yield and the term premium are obtained is presented in the Appendix. The overall fit of the model is good (see Figures A1 and A2). The measurement error contains residual autocorrelation and heteroscedasticity (see the Appendix), which is a consequence of the changing ability of the model to fit various shapes of the yield curve. However, the measurement error is low; its absolute value does not exceed 2 basis points on average. Therefore, the consequences of the residual patterns are expected to be negligible. The first factor obtained resembles the average level of the yield curve (see Figure A3), whereas

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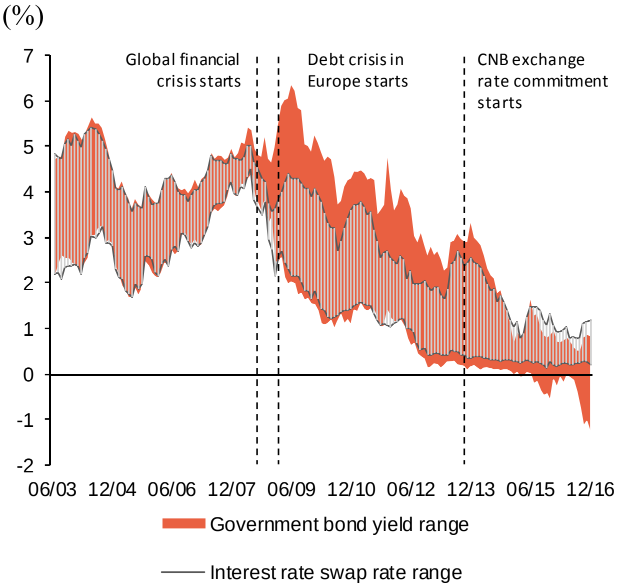
[5] The use of coupon bonds could potentially lead to underestimation of the yields demanded for a given maturity (Livingston and Jain, 1982).

[6] The range of maturities considered was chosen with regard to data availability and quality. Bonds with maturities of less than one year are not used in such studies because their prices can be distorted by specific effects due to lower liquidity (BIS, 2005). In addition, koruna interest rate swaps are not available for maturities of less than one year. The time series for bonds and swaps with maturities of over 15 years are shorter and their prices may be less reliable due to their lower trading volumes. The empirical strategy assumes that the quotes contributed for both bonds and swaps are reasonable. Surveys conducted by CNB and market intelligence suggest good liquidity of koruna swaps up to 10 years. There is also certain turnover above 10 years, which allows market participants to deal with market makers at their contributed quotes. The liquidity of swaps is comparable with that of Czech government bonds, which are also less liquid at longer maturities. The usability of swaps for yield curve construction is further confirmed by EIOPA's current use of swaps (up to 15 years) rather than bonds for discounting insurers' liabilities for regulatory purposes.

[7] Previous studies on yield curve construction from Czech government bonds that use the Nelson and Siegel and Svensson models include Hladíková and Radová (2012), Kladívko (2010), and Slavík (2001).

the second and third factors cannot be easily interpreted, which is a consequence of relaxing the ADNS identification.

Figure 2: Ranges for Zero-coupon Czech Government Bond Yields and Koruna Interest Rate Swap Rates



Note: Vertical lines mark the last monthly observation before the event described. The start of the global financial crisis is related to the collapse of Lehman Brothers in September 2008. The start of the debt crisis is related to the negative assessment of Greek public finances by the International Monetary Fund and the European Commission in February 2009.

Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations

To obtain the credit risk premium, month-end CDS quotations for maturities of 1–5, 10, 20, and 30 years were used in the estimation. We included CDS quotations with 20-year and 30-year maturities (which were not included in the yield sample) in the estimation because of the absence of quotations for 15-year CDS. After smoothing them by the three-month moving average, the Nelson-Siegel function was used to obtain the credit risk premium values for all the required maturities. Due to the limited liquidity on the Czech GB CDS market,[8] the CDS quotations for Czech public debt of shorter maturities are close to those of longer maturities. However, this was not reflected in the yields on Czech GBs of short maturities. The Nelson-Siegel function was therefore specified so that the credit risk premium converged to zero with decreasing maturity.

The zero-coupon Czech GB yield curve was decomposed into the four introduced components for one-year and ten-year maturities (see Figures 3 and 4). In the case of the one-year bond, it is clear that yield was made up predominantly of the risk-neutral yield until the global financial crisis broke out in 2008 (see Figure 3). From the end of 2008 onward, the one-year bond yield declined due to a falling risk-neutral yield. The decline in this component was linked with market expectations that short-term rates would stay very low. In addition, starting in the second half of

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[8] Data from trade repositories available to the CNB indicate that the Czech sovereign CDS turnover is rather limited. The most frequent maturity is five years (about 35% of total turnover by notional principal), followed by maturities of one to four years. Maturities longer than five years are rare (accounting jointly for about 15% of the turnover) and maturities longer than ten years are virtually non-existent. The turnover declined substantially once the market calmed after the debt crisis in Europe subsided. Nevertheless, low turnover does not necessarily imply illiquidity as long as market makers are willing and ready to transact at the quotes they contribute.

2008, key central banks gradually released large amounts of liquidity as part of their monetary and lender-of-last-resort policies. For reasons of flight to quality and search for yield, Czech GBs represented an attractive opportunity for foreign investors. Owing to the negligible risk of sovereign default over such a short time scale, the credit risk premium was relatively low in the period under review. The negative portfolio component was linked with investors' preference for holding shorter-maturity bonds at a time of market stress. In 2015, the portfolio component exceeded all the other components combined for the first time and the one-year bond yield thus turned negative. Since then, the yield on short-maturity Czech GBs has reflected strong interest among foreign investors speculating on appreciation of the Czech koruna against the euro upon the exit from the CNB's exchange rate commitment (see CNB, 2017, section 2.1).

Figure 3: Decomposition of the 1-year Zero-coupon Bond Yield

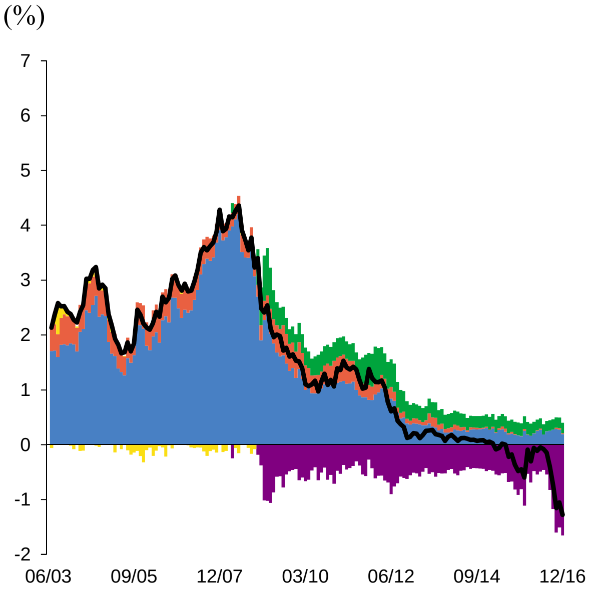
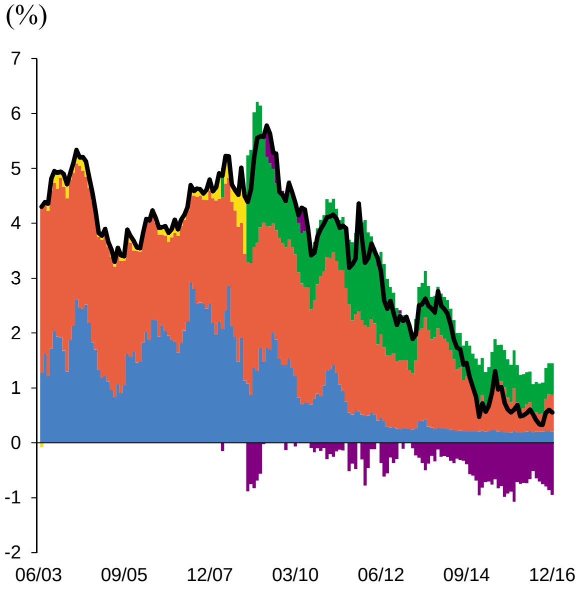


Figure 4: Decomposition of the 10-year Zero-coupon Bond Yield



Note: Reliable data on CDS quotations are not available until 2008. As a result, the difference between the bond yield and the swap rate could not be decomposed and is reported as *Unexplained.*

Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations

The significance of the different components in the level of the ten-year Czech GB yield changed substantially over the 13 years under review (see Figure 4). Until the global financial crisis broke out, ten-year bond yields were almost equal to swap rates of the same maturity. The risk-neutral yield and the term premium each made up around half of the yield. When the U.S. investment bank Lehman Brothers collapsed in mid-September 2008, the global financial market situation worsened sharply. Uncertainty and risk aversion increased, giving rise to higher market price volatility. Owing to the high level of global market integration, the market stress passed to the Czech GB market, as evidenced by growth in the credit risk premium. In mid-October 2008, market liquidity on the Czech GB market dropped sharply as a result of excess supply of Czech GBs from foreign institutional investors. The CNB responded by introducing extraordinary liquidity-providing repo operations in which Czech GBs were accepted as eligible collateral for the first time. This fostered a slight reduction in the credit risk premium. For the same reasons as for the one-year bond, the risk-neutral yield and the term premium began to fall in mid-2008. The term premium increased in late May/early June 2013 in response to a change in market expectations about the timing of the tapering of bond purchases by the U.S. Federal Reserve in the QE3 program. This change in expectations triggered an unusually sharp price adjustment in a whole range of asset categories across global markets, accompanied by market turbulence. In November 2013, however, the ECB reduced its base rate and in June 2014 it announced the use of other unconventional instruments, including a plan to purchase euro area GBs. In November 2013, the CNB started to use the koruna exchange rate as an additional monetary policy instrument. This combination of measures led not only to a fall in the term premium of the ten-year bond, but also to a negative portfolio component. From 2011 onward, the credit premium and the portfolio component were also affected by the debate about, and subsequent phasing in of, new financial market regulatory measures (Basel III, CRD IV/CRR). A signal of preferential treatment of GBs in the capital and liquidity requirements was sent out to the market.

In order to confirm the theoretical interpretation of the estimated components, their profiles were compared with those of selected macroeconomic and financial variables with which they should theoretically be closely linked. Besides that, we performed a correlation analysis between the components and the macroeconomic and financial variables. The correlation analysis was run on monthly changes of components and variables (reported in Table 1) and also on their levels (CNB, 2017).

The risk-neutral yield should match market expectations about future short-term rates. A comparison with analysts' expectations about the CNB's two-week repo rate one year ahead confirmed this theoretical assumption. The correlation between the dynamics of risk-neutral yields and those of expected monetary policy rates also turned out to be higher than that between risk-neutral yields and actual policy rates (see Table 1). With respect to the correlation of levels, the correlation between risk-neutral yields and expected policy rates was the highest among all the variables investigated (0.95).

The term premium should theoretically be closely correlated with the level of difficulty in forecasting future short-term rates at a given maturity horizon. Forecasting difficulty is hard to measure, so it was proxied by the variability of the last four monthly values of the forecasted variables. The correlation between the term premium and the variability of inflation, inflation expectations, and expectations about the CNB's two-week repo rate was relatively low (CNB, 2017). This may have been caused by the backward-looking nature of these imperfect measures of forecasting difficulty. Conversely, a relatively strong correlation between the term premium and the present and expected interest late level lent some support to the theoretical assumption. Generally speaking, when interest rates are low, their volatility is also low. This enables investors to make better forecasts and demand a lower term premium. Although the presented long-term correlation patterns of the term premium and the risk-neutral yield are similar, it should be noted that their short-term movements differ significantly. For instance, in the case of the ten-year GB, the volatility of the term premium was lower than the volatility of the risk-neutral yield. Additionally, whereas the largest decrease in the risk-neutral yield was observed over the years 2011–2012, the term premium decreased only slightly in this period. Instead, the largest drop in the term premium occurred between the end of 2013 and mid-2015, when uncertainty about future monetary policy rate movements was suppressed by the introduction of an unconventional monetary policy tool.

Table 1: Correlation between the Components of the 10-year Zero-coupon Bond and Economic and Financial Variables

Type of variable	Name of variable	Risk-neutral yield	Term premium	Credit risk premium	Portfolio effect
Macroeconomic	Inflation (CPI)	0.50 (***)	0.42 (***)	0.67 (***)	0.12
	GDP growth	0.27 (***)	0.15 (*)	-0.49 (***)	-0.54 (***)
	CZK/EUR exchange rate	0.29 (***)	0.45 (***)	-0.27 (***)	-0.53 (***)
Short interest rates and market expectations	CZEONIA index	0.86 (***)	0.73 (***)	0.69 (***)	0.38 (***)
	CNB 2W repo rate (current)	0.88 (***)	0.76 (***)	0.77 (***)	0.36 (***)
	3M PRIBOR	0.87 (***)	0.74 (***)	0.78 (***)	0.38 (***)
	3M OIS in CZK	0.88 (***)	0.76 (***)	0.74 (***)	0.37 (***)
	CNB 2W repo rate (1-year expectations)	0.95 (***)	0.79 (***)	0.57 (***)	0.53 (***)
	Inflation (1-year expectations)	0.75 (***)	0.65 (***)	0.39 (***)	0.40 (***)
Fluctuations in short interest rates and market uncertainty	Variability[#] of inflation	0.25 (***)	0.29 (***)	0.51 (***)	-0.01
	Variability[#] of 1-year inflation expectations	0.30 (***)	0.14 (*)	0.23 (**)	0.13
	Variability[#] of 1-year expectations about CNB 2-week repo rate	0.35 (***)	0.42 (***)	0.70 (***)	0.07
	VIX volatility index	0.14 (*)	0.17 (**)	0.72 (***)	0.13
Credit risk of Czech state and Czech interbank market	Czech GBs issued/GDP	-0.44 (***)	0.25 (**)	-0.02	0.37 (***)
	5-year CDS spread for Czech GB	-0.06	0.13	0.93 (***)	0.15
	Spread between 3-month PRIBOR and 3-month OIS	-0.14	0.01	0.59 (***)	0.24 (**)
	Spread between Czech and German 5-year GB yields	-0.13	-0.07	0.82 (***)	0.37 (***)
Investment flows	Czech GB trading volume	-0.05	0.11	0.28 (***)	-0.01
	Proportion of foreign holders of Czech GBs	-0.33 (***)	-0.73 (***)	-0.49 (***)	-0.62 (***)
	Profit on hedged investment in Czech GBs[##]	-0.45 (***)	-0.59 (***)	-0.38 (***)	-0.52 (***)
	Net portfolio and other investment in balance of payments	0.06	0.21 (***)	0.17 (*)	0.06

Colour scale for Pearson's correlation coefficient

-1 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 1

Note: The asterisks refer to statistical significance at the 1% (***), 5% (**), and 10% (*) levels. The explanatory power of the correlations may be limited by the short length and the existence of a trend for some of the time series.

[#] Variability is measured by the standard deviation of the last four monthly observations.

[##] The average profit on (1) an investment consisting in converting euros into korunas, depositing them at the CNB deposit rate, and then converting them back into euros at the 3-month forward rate, and (2) an investment consisting in converting euros into korunas, buying a 2-year Czech GB and then converting it back into euros at the 2-year forward rate. The return that could have been achieved by depositing the funds for three months at the ECB deposit rate was deducted from the first investment and the 2-year German GB yield was deducted from the second.

Source: Bloomberg, CNB, MTS Czech Republic, Prague Stock Exchange, Thomson Datastream, authors' calculations

The credit risk premium should be correlated with investor perceptions about Czech GB credit risk. Given the method for estimating the credit risk premium, the correlation between it and CDS spreads was very high (even with CDS spreads of other maturities; see Table 1). Another market indicator of credit risk – the spread between Czech and German five-year GB yields – was also highly correlated with the credit risk premium. By contrast, a fundamental often used to express the level of sovereign credit risk – the ratio of GBs issued to GDP – had only a limited correlation with the credit risk premium. This may be due to the relatively low Czech public debt level. A closer relationship between the public-debt-to-GDP ratio and the sovereign credit risk premium does not usually surface until public debt exceeds a certain level. For many reasons, moreover, that level differs from country to country and for some countries is below the often cited 60% level (Banque de France, 2012).

The credit risk premium turned out to be closely correlated with inflation and the exchange rate and market uncertainty indicators (see Table 1). An increase in inflation and a weakening Czech koruna could signal potential macroeconomic instability and require foreign investors to monitor the fiscal position of the government more closely. Global uncertainty as measured by the VIX index was significantly correlated with the credit risk premium. When uncertainty rises, investors become more cautious and require higher compensation for bearing credit risk; sometimes this surge in credit premium is not fully justified by the evolution of Czech fundamentals.

The portfolio effect should theoretically be linked with investors' preference for Czech GBs over other assets – denominated in korunas or other currencies. We used four variables to express this preference. First, we looked at the correlations between the portfolio effect and the inflow of short-term foreign assets into the Czech economy and the Czech GB trading volume. In both cases, the correlation was low. Then we used the stock indicator of the proportion of Czech GBs held by non-residents. It was correlated strongly negatively with the portfolio effect when we analyzed the correlation of their levels. Strong correlation of levels was also found between the portfolio effect and the profit on investing in Czech assets with simultaneous exchange rate risk hedging (i.e., purchasing korunas on the spot market, depositing those korunas or buying a short-term bond, and then converting back at the forward exchange rate). Rising yields on this type of investment were associated with a lower portfolio effect. Both negative correlations can be interpreted as meaning that an inflow of foreign portfolio investment motivated by hedged profits boosts demand for Czech GBs as an attractive instrument, causing their yields to turn negative.

4. Czech GB Yield Curve Response Analysis

In this section, we estimate a simple vector autoregression model with an exogenous variable (VARX model) of the Czech economy to obtain some basic responses of the Czech GB yield curve to macroeconomic shocks. Using the presented decomposition methodology, we split the yield curve responses into the responses of individual components. This allows us to interpret the yield curve movements in a greater detail and obtain an insight into the monetary policy transition. The aim of the analysis is to illustrate the usefulness of the decomposition in relation to the interpretation of macroeconomic dynamics. The development of more sophisticated macroeconomic VAR models is left for future research. The model can be written in the following form:

$$V_t = A_0 + A_1 V_{t-1} + A_2 W_t + \epsilon_t \tag{12}$$

where V_t is a vector of endogenous variables, W_t is a vector of exogenous variables, A_0, A_1, and A_2 are parameter matrices, and ϵ_t is an i.i.d. vector of random disturbances. In the presented model, we use seven endogenous variables and a single exogenous variable, i.e., the matrices A_0, A_1, and A_2 have dimensions 7×1, 7×7, and 7×1, respectively.

The seven endogenous variables include two observable macroeconomic variables, three IRS latent factors, and the level and slope of the asset swap spread (ASWS). The ASWS represents the difference between the Czech GB yield and IRS rates, which means that it equals the sum of the credit risk premium and the portfolio effect. The reason we use the ASWS instead of its two components is to keep the model parsimonious. Additionally, separation of these two components was not possible until the end of 2008 due to the unavailability of reasonable CDS quotations. Modeling them jointly therefore allows us to use a longer sample period.

The two macroeconomic variables used in the model are the quarterly GDP growth rate[9] and the inflation expected one year ahead. Unlike a canonical monetary VAR (Bernanke and Blind, 1992, for instance), we do not use the monetary policy rate, since the monetary policy rate hit its lower bound during the sample period. As a result, the monetary authority used the exchange rate intervention as an alternative tool, which can be considered a structural change. Including monetary policy or money market rates would consequently lead to biased results.

By including the three IRS latent factors X_t extracted from the estimated affine model (SR-SGA model specification; see section 2) in the VARX model, we allow both an unbiased monetary policy proxy (the shadow short rate) and interest rates to enter the model. The shadow-rate specification of the affine model avoids the bias caused by the lower bound. The shadow short rate can be easily obtained in the SR-SGA model as the sum of the latent factors X_t (plus parameter α_0 from equation (4)). This shadow short rate can be seen as a proxy for overall monetary policy conduct, reflecting both interest rate tools and any unconventional tools (Krippner, 2015). The estimated affine model also allows us to translate the responses of the latent factors X_t to the responses of rates via equation (7) and to decompose these responses into two components: the risk-neutral yield and the term premium.

To be able to infer the responses of the Czech GB yield curve to macroeconomic shocks, we need to introduce another two components: the credit-risk premium and the portfolio effect. They are jointly expressed by the ASWS. Using the first two elements of the Nelson-Siegel function fitted over the ASWS, we represent the ASWS term structure by two variables: its level and its slope. These enter the VARX model as the sixth and seventh variables. Apart from the endogenous variables, we also consider the effect of general market uncertainty on the Czech GB yield curve. We assume these effects come from global financial markets, so we use the VIX index as a proxy. Since the Czech economic situation has a negligible effect on global markets, we do not assume any feedback loops, which support the exogeneity of this variable.

To identify the model, we utilize recursive identification with the Cholesky decomposition. In this case, the ordering of the endogenous variables is crucial. We order them in the way they are presented above. Ordering the macroeconomic variables first is reasonable because of their publication lag and the sluggish overall response of the real economy to financial shocks. The IRS factors are ordered afterward, since we assume that they react immediately to shocks to the macroeconomic variables but not to shocks to the ASWS. Finally, the ASWS factors are ordered at the end. We use a first-order VARX model. The single lag was chosen mostly in order to maintain parsimony. Taking into account the number of variables, a higher order would require the use of advanced techniques (factor-augmented VAR or Bayesian VAR, for instance), which we leave for future research.

For the estimation, we use the same sample period as in the rest of the paper, i.e., the 162 months from July 2003 to December 2016. We obtain the parameters using maximum likelihood estimation. To discuss the results of the model, we mainly focus on the impulse response functions. We proceed in three steps. First, from the model parameters, we calculate the responses of the yield factors (i.e., the three IRS factors and the level and slope of the ASWS) to the shocks

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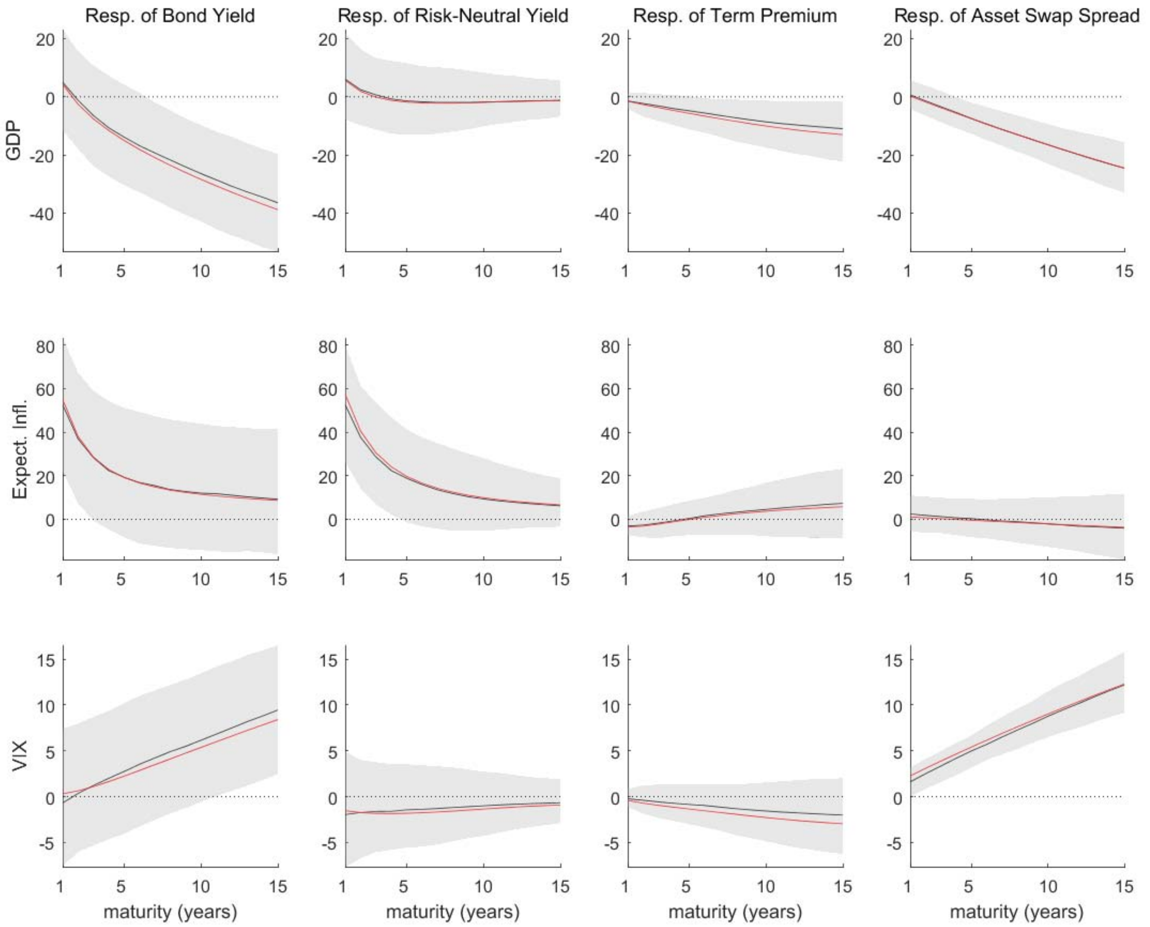
[9] In order to obtain monthly data, we interpolate the quarterly observations. Although we admit that this approach has multiple weaknesses, we prefer it to both the use of alternative series (to keep the variables canonical) and switching to quarterly data (which would reduce the number of available observations).

to the macro-financial variables (GDP growth, expected inflation, and the exogenous VIX index) over a three-year horizon. Afterward, we translate the responses of the yield factors to responses of the yield components. That means that (i) the responses of the risk-neutral yield and the term premium are calculated from the IRS factors' responses, and (ii) the ASWS responses are calculated from the responses of the ASWS level and slope. The ASWS responses aggregate the response of the other two components – the credit risk premium and the portfolio effect. Finally, we sum the response of the risk-neutral yield, the term premium, and the ASWS to obtain the response of the Czech GB yield curve.

Figure 5 shows the responses of the whole yield curve (in basis points) six months after the initial shocks. The left column of the figure shows the response of the whole yield curve, which equals the sum of the responses of the components in the other columns. Each row presents a response to a shock to the variable denoted on the left of the figure. Figure 6 shows the dynamics of the response for the ten-year Czech GB yield over a three-year horizon. In the figures, the red line shows the response estimated on the whole sample, the grey area displays the 80% confidence intervals obtained by bootstrapping, and the grey line displays the median response.

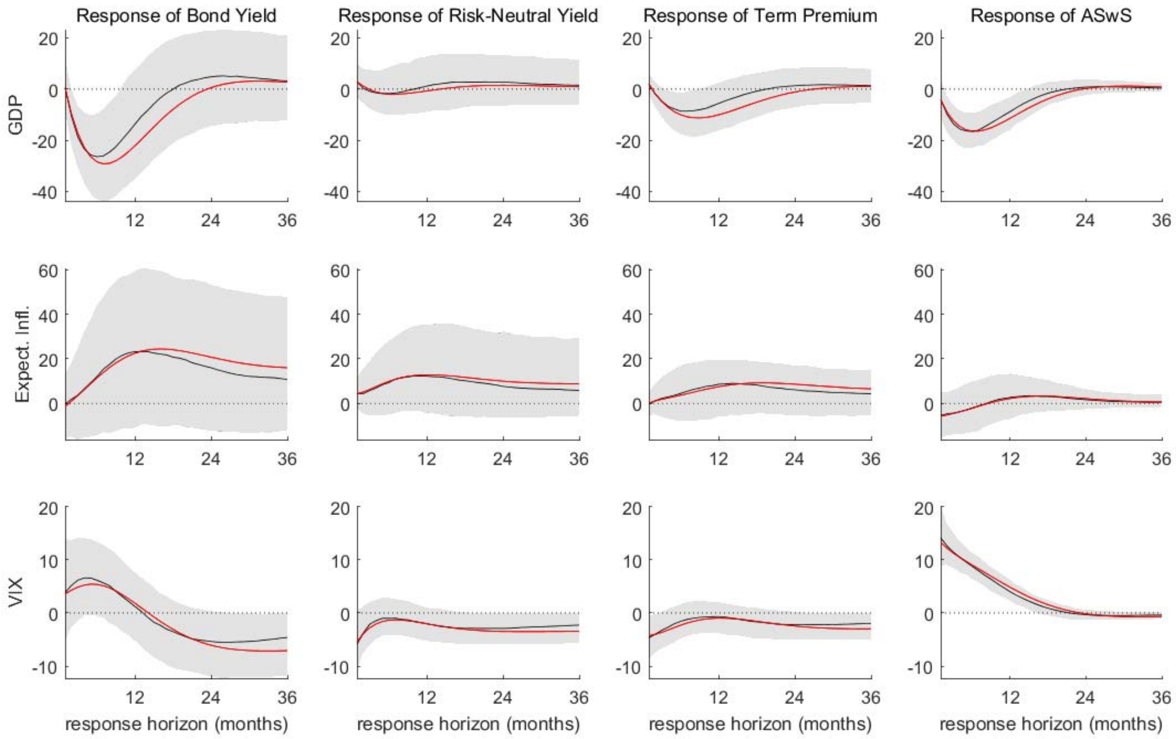
Figure 5: Response of the Whole Yield Curve after Six Months



Note: The figure displays the response of the whole Czech GB yield curve (1–15 years) to the shocks after 6 months. The shocks are defined as a 1 pp increase in GDP and expected inflation and as an increase in the VIX index from 12 to 30 points. The red line displays the estimated response; the grey area and the grey line display the 80% confidence intervals and the median response obtained by bootstrapping.

We put the shocks equal to a one-time 1 percentage point increase in the quarterly GDP growth rate and in expected inflation. The VIX entered the model in logarithmic form; we set the size of the response equivalent to an increase in the VIX index from 12 points (the historical level related to low risk) to 30 points (the average value in periods of increased uncertainty).

Figure 6: Response of 10Y Yield, Response Horizon up to Three Years



Note: The figure displays the dynamics of the response of the 10-year Czech GB yield to the shocks over a 36-month response horizon. See the note under Figure 5 for the description of the plot elements.

As Figure 5 shows, the response of the Czech GB yield curve takes various forms, including both a parallel shift and a rotation. In response to a 1 percentage point positive shock to the GDP quarterly growth rate, the short end of the yield curve increases because of an expected monetary policy reaction. The longer yields, on the other hand, decrease. This is caused by (i) a drop in the term premium for longer yields, since the positive economic results reduce long-term uncertainty; and (ii) a decrease in the ASWS, which can be seen as a combination of a decrease in the credit risk premium and the portfolio effect (growth in the attractiveness of Czech GBs). This effect lasts about two years (Figure 6) and gradually dies out.

The shock to expected inflation causes a significant increase in the yield curve at its short end (Figure 5). The short-term risk-neutral yield increases in response to the shock, peaking at 60 basis points around six months after the impulse. The responses of the term premium and the ASWS are not significant.

Finally, the exogenous shock to the VIX has a negative impact on the expected monetary policy rate path, i.e., risk-neutral yields decrease slightly (Figure 5). For longer yields, the impact increases over time and is quite persistent (Figure 6). On the other hand, the shock increases the credit risk premium of Czech GBs and reduces their attractiveness to investors, which pushes the

ASWS up. As a result, the yield curve rotates. The outflow of investors (presumably foreign), which causes the ASWS to increase after an increase in market uncertainty, signals that their perception of Czech GB bonds as a safe asset is limited. The role of foreign investors in determining the ASWS is dominant due to the high turnover these investors have in the Czech GB bond market relative to domestic investors (CNB, 2014).

The results demonstrate that the response of the yield curve is truly complex and that movements in its components may offset each other. The drop in the term and credit premia and the response of the portfolio component after a positive economic shock may have important implications for both the monetary and financial stability policies of the CNB. Similarly, the combination of macroeconomic and financial responses to the market uncertainty shock (VIX) needs to be accounted for when considering possible policy measures.

5. Conclusion

The yield curve is an important indicator of the economic cycle, as it aggregates the expectations of market participants. The factors that affect the shape of the yield curve do so to different extents in different circumstances. To interpret the evolution of the yield curve correctly, it is therefore useful to decompose it. This paper presented the method used to decompose the Czech government bond yield curve.

We decomposed the Czech GB yield curve into four components: a risk-neutral yield, a term premium, a credit risk premium, and a portfolio effect. The first two were obtained by decomposing the zero-coupon koruna swap curve using the affine model. The credit risk premium was estimated from credit default swap quotations for Czech GBs. The portfolio effect formed the residual.

A comparison of the four estimated components with selected macroeconomic and financial variables confirmed the strong theoretical interpretation of these components. As the theory had anticipated, for example, the risk-neutral yield matched analysts' expectations about future short-term policy rates, and the portfolio effect became highly negative as the removal of the CNB's exchange rate floor neared.

The above decomposition allowed for a more detailed interpretation of the responses of the Czech GB yield curve to macroeconomic and financial shocks. Using vector autoregression analysis, we showed that the yield curve responds both by changing its level and by rotating. Such responses result from a combination of various responses of the yield components to the shocks.

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Appendix

Figure A1: Affine Model Fit and Shadow Rate (1-year Swap Rate)

(%)

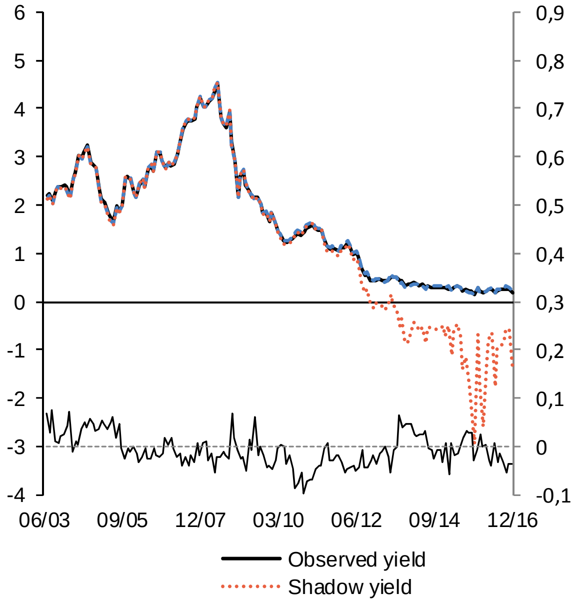
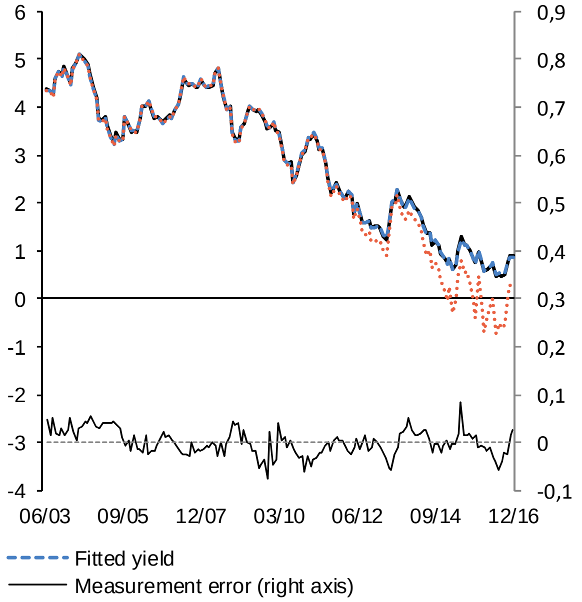


Figure A2: Affine Model Fit and Shadow Rate (10-year Swap Rate)

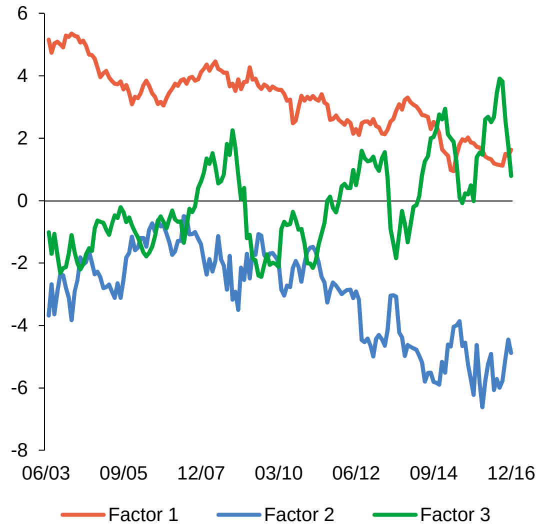
(%)



Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations

Figure A3: Affine Model Factors

(values of factors, multiplied by 100)



Source: Authors' calculations

Estimated Parameters

$$\tilde{\kappa} = \begin{bmatrix} -0.01 & 0 & 0 \\ 0 & 0.15 & 0 \\ 0 & 0 & 0.99 \end{bmatrix} \quad \tilde{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \alpha_0 = 0.01 \quad \alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\kappa = \begin{bmatrix} 0.052 & -12.350 & -0.071 \\ -0.001 & 4.800 & -0.008 \\ -0.059 & 29.620 & 70.630 \end{bmatrix} /10^2 \quad \theta = \begin{bmatrix} -0.001 \\ -0.002 \\ -1.310 \end{bmatrix} /10^2$$

$$\begin{aligned} std(e_t) = diag(&3.02, 3.09, 3.23, 3.30, 3.33, \\ &3.35, 3.36, 3.37, 3.38, 3.38, \\ &3.37, 3.37, 3.36, 3.34, 3.34)/10^4 \end{aligned}$$

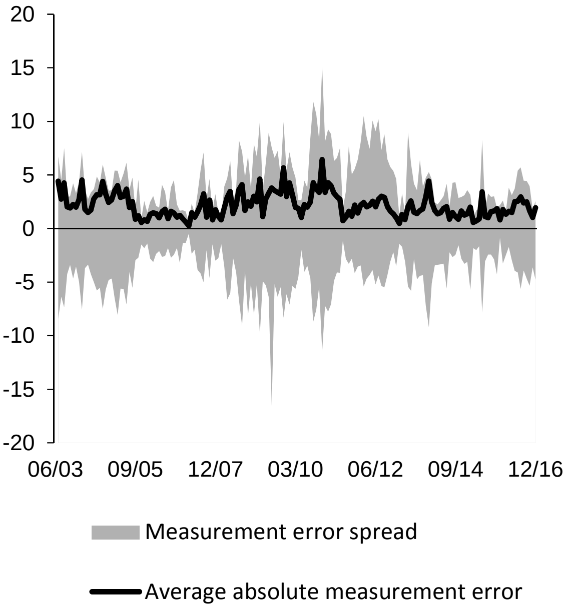
$$\sigma = \begin{bmatrix} 79 & 0 & 0 \\ -102 & 100 & 0 \\ 0.002 & 0.004 & 143 \end{bmatrix} /10^4$$

Lower bound value = 0.0019

Note: Small values of parameters and the lower bound reflect the fact that the model uses yields in decimal representation.

Figure A4: Measurement Error Series

(b.p.)



Source: Authors' calculations

Table A1: Measurement Error Diagnostics

Maturity	1	5	10	15
Normality (Shapiro-Wilk test)	YES	NO	YES	NO
Autocovariance (Ljung-Box)	YES	YES	YES	YES
Homoscedasticity (Ljung-Box on squares)	NO	NO	NO	YES
Cross- correlations (%):				
1	100	-10	68	-71
5	-10	100	-38	18
10	68	-38	100	-86
15	-71	18	-86	100

Source: Authors' calculations

Note: The tests were evaluated for the 5% significance level.

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