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Updating the Ultimate Forward Rate over Time: A Possible Approach

Diana Žigraiová and Petr Jakubík *

Abstract

This study proposes a potential methodological approach to be used by regulators when updating the Ultimate Forward Rate (UFR) for the evaluation of insurers' liabilities beyond the last liquid point observable in the market. Our approach is based on the optimisation of two contradictory aspects – stability and accuracy implied by economic fundamentals. We use U.S. Treasury term structure data over the period 1985-2015 to calibrate an algorithm that dynamically revises the UFR based on the distance between the value implied by the long-term growth of economic fundamentals in a given year and the regulatory value of the UFR valid in the prior year. We employ both the Nelson-Siegel and Svensson models to extrapolate yields over maturities of 21-30 years employing the selected value of the UFR and compare them with the observed yields using the mean square error statistic. Furthermore, we optimise the parameters of the proposed UFR formula by minimising the defined loss function capturing both mentioned factors.

Abstrakt

Tento článek navrhuje metodologický přístup, který může být využit regulátory pro aktualizaci tzv. konečné forwardové sazby (UFR) pro ohodnocení pasiv pojišťoven, která mají splatnost nad rámec výnosů pozorovatelných na trhu. Navržený koncept je založen na optimalizaci dvou protichůdných aspektů – stability a přesnosti implikované tržními fundamenty. Ke kalibraci algoritmu, který dynamicky aktualizuje UFR na základě vzdálenosti mezi hodnotou implikovanou dlouhodobým růstem ekonomických fundamentů v daném roce a regulatorní hodnotou UFR platnou v předchozím roce, jsme použili data o časové struktuře amerických vládních dluhopisů pro roky 1985–2015. Pro extrapolaci výnosových křivek pro splatnosti mezi 21 až 30 lety jsou použity modely Nelson-Siegel a Svensson využívající vybrané hodnoty UFR. Získané hodnoty jsou pak porovnány s pozorovanými výnosy prostřednictvím statistiky střední odchylky čtverců. Dále optimalizujeme parametr navrženého vzorce pro UFR pomocí minimalizace definované ztrátové funkce zohledňující oba zmíněné faktory.

JEL Codes: E43, G22, L51, M2.

Keywords: Extrapolation, Nelson-Siegel, Svensson, term structure of interest rates, Ultimate Forward Rate.

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Nontechnical Summary

As part of the Solvency II regulatory package an interest rate for the valuation of long-term contracts (the Ultimate Forward Rate) was set and its underlying methodology was presented. The importance of the Ultimate Forward Rate (UFR) lies in its use for calculating the present value of long-term commitments on the liability side of insurers' balance sheets. As a result, the setting of, and changes in, the UFR have valuation effects for insurance companies. In this paper we propose one possible dynamic approach to UFR revision.

Our approach relies on updating the interest rate for long-term contracts (the UFR) in a dynamic way that establishes the long-term evolution of economic fundamentals as measured by the average 20-year growth rate of nominal GDP as a benchmark for the UFR. We use U.S. Treasury term structure data over the 1985-2015 period and U.S. nominal GDP data over the 1965-2015 period to construct an algorithm that revises the UFR based on the distance between the long-term growth of economic fundamentals in a given year from the 1985-2015 period and the regulatory UFR from the prior year. If this distance is greater than some threshold value *p*, the UFR in the given year is set to the value given by economic fundamentals. On the other hand, a distance smaller than the threshold makes the regulatory UFR from the prior year valid in the given year as well. The threshold *p* takes values from 0.1% to 3.5%.

In the second part of our approach, we employ the Nelson-Siegel and Svensson models to extrapolate U.S. Treasury yields over maturities of 21-30 years and compare them with the historical yields at these maturities presented in the paper by Gurkaynak et al. (2007) using the mean square error statistic (MSE). The mean square error statistic measures the precision of the yield extrapolation by capturing the distance between the actual yields over maturities of 21-30 years and the estimated yields over the same maturities from both models.

We then combine the two aspects – the UFR stability for each value of *p* captured by the ratio of UFR changes to the total number of years in the time period, and the extrapolation precision as measured by the MSE – into a loss function for each value of *p* over the 1985-2015 period. The regulator's preference for each component of the loss function is expressed by assigned weights.

We search for such *p* (the distance between long-term growth of economic fundamentals and the regulatory UFR set in the prior year) that minimises the loss function.

We find that once the distance between average 20-year nominal GDP growth in a given year and the regulatory UFR valid in the prior year exceeds 1.2% and 1.3% under preference neutrality for the Nelson-Siegel and Svensson models respectively, the UFR should be adjusted. This result changes in response to the regulator's preferences.

1. Introduction

The aim of this paper is to propose a methodological framework for updating the Ultimate Forward Rate (UFR) based on the regulator's preference between stability and accuracy reflecting a theoretical value. By establishing quantitative definitions of those two criteria, regulators would obtain a clear rule for setting the UFR. As interest rates on investment instruments with very long maturities cannot typically be observed in the market, the Ultimate Forward Rate (UFR) is essential for valuing insurers' long-term commitments.

The current low interest rate environment poses three types of risks for insurance companies (e.g. EIOPA Financial Stability Report, 2013). First, cashflow risks arise from narrowing yield spread, as new premiums and returns on maturing investment are reinvested at lower yields relative to the yields that insurers have committed to pay. The available margin on this business is thus gradually eroded by a low-yield environment if no action is taken to alter the underlying position. Second, valuation risks are linked to the calculation of the present values of insurance companies' assets and liabilities. Under low interest rates, a decline in benchmark interest rates will be reflected in the discount rate applied to liabilities. The fact that the duration of liabilities is typically greater than that of assets for life insurers in particular leads to the erosion of available net assets because the present value of liabilities would increase more than that of assets. Consequently, the insolvency risks of insurers are exacerbated.

At present, the UFR used for discounting insurers' long-term liabilities is not universal across countries. For instance, the European Insurance and Occupational Pensions Authority (EIOPA) recommends in its Technical Specification for the Preparatory Phase of Solvency II (2014) that the UFR be set to 4.2% until the end of 2016. In this specification, the UFR is defined as a function of long-term expectations of the inflation rate and of the long-term average of short-term real interest rates. Furthermore, variations in the recommended UFR are arranged for countries with different inflation expectations (EIOPA, March 2016). The UFR can take values of 3.2% for currencies with low inflation expectations (Swiss franc, Japanese yen), or 4.2% for EEA currencies and those non-EEA currencies that are not explicitly mentioned in any other category, or 5.2% for the Brazilian, Indian, Mexican, Turkish and South African currencies, for which there are higher inflation expectations. In contrast, some national supervisors have decided to implement their own UFR methodologies in their domestic financial markets. In this spirit, the Swiss Financial Market Supervisory Authority (FINMA) in July 2015 implemented a UFR of 3.9%, while at the same time the Dutch National Bank adjusted the UFR for the Dutch pension sector. In its 2015 field testing package for the insurance capital standard, the International Association of Insurance Supervisors (IAIS) chose to apply a UFR equal to 3.5% (EIOPA, April 2016). EIOPA's UFR framework is, however, currently undergoing revisions. A new methodology for the calculation of the UFR on an ongoing basis is expected to be implemented in 2017 (EIOPA, April 2016).

With regard to how frequently the UFR should be revised, in this paper we propose a quantitative approach that reflects on two contradictory aspects – the stability of the UFR over time versus its distance from a derived theoretical benchmark value based on economic fundamentals.

The paper is organised as follows. After a brief literature review in section 2, section 3 presents the term structure data used in our analysis, section 4 describes the methodology applied to the

UFR setting, section 5 presents our results, section 6 touches upon macroprudential features of a time-varying UFR, the UFR's standing within the concept of equilibrium rates and its linkages with accommodative monetary policy, as well as the implications of UFR changes for insurance companies, and section 7 concludes.

2. A Brief Literature Review

The low-yield environment resulting from the monetary policies pursued by European central banks poses the most prominent risk to the insurance sector at present. Despite the fact that such policies contributed to financial stability in the short term (IMF Global Financial Stability Report, 2013), lower yields on corporate and sovereign bonds in many European countries have unfavourable implications for insurer companies' profitability, solvency and sustainability.

Overall, insurance companies are seen as a relatively stable segment of the financial system. However, over time their interaction with other agents in the financial system, such as banks and pension funds, has intensified. The negative spill-overs and the risk of bi-directional contagion have led to increased acknowledgement of the importance of the insurance sector for overall financial stability (e.g. Bakk-Simon et al., 2012). This interconnectedness and the size of the insurance segment make insurance firms important from the financial stability point of view and lay the ground for further research in this area.

In terms of the performance of insurance companies, there are several papers focusing on modelling their profitability. In line with research on drivers of bank profitability (e.g. Staikouras and Wood, 2004; Macit, 2012; Ameur and Mhiri, 2013; Goddard, Molyneux and Wilson, 2004), Christophersen and Jakubik (2014) revealed a strong link between insurance companies' premiums, on the one hand, and economic growth and unemployment on the other. Similarly, Nissim (2010) argues that overall economic activity affects insurance carriers' growth, because the demand for their products is affected by the available income. Moreover, he underlines that investment income is highly sensitive to interest rates, both in the short and in the long run. D'Arcy and Gorvett (2000) argue that inflation heavily affects the liability side of property-liability insurers' balance sheets. As for insurer insolvencies, Browne et al. (1999) find a positive correlation between the number of insurers in the life-insurance industry, unemployment and stock market returns on the one hand and life insurers' insolvency on the other. Similarly, the failure rate of property-liability insurers was also found to be positively correlated with the number of insurers in the industry (Browne and Hoyt, 1995).

Since interest rates have been shown to affect the income and profitability of insurance companies in previous research, we propose to further investigate in this paper the setting of the long-term interest rate used for discounting insurance firms' long-term commitments, which has substantial valuation implications.

3. Data

In our analysis we use the U.S. Treasury term structure data presented by Gurkaynak et al. (2007). The advantages of using U.S. data as opposed to European data stem from the availability of long historical time series of yield curves with maturities of up to 30 years. The data set is compiled on a daily basis, with the first entry in 1961, and is regularly updated. This data set includes all U.S. Treasury bonds and notes with the exception of the following:

i. Securities with option-like features, i.e. callable bonds or flower bonds.
ii. Securities with less than three months to maturity due to the specific behaviour of yields on securities with such short residual maturities.
iii. Treasury bills that seem to be affected by segmented demand from money market funds and other short-term investors (Duffee, 1996).
iv. Twenty-year bonds in 1996 owing to their cheapness relative to ten-year notes of comparable duration.
v. Securities with maturities of two, three, four, five, seven, ten, twenty and thirty years issued in 1980 or later owing to the fact that they trade at a premium to other Treasury securities given their greater liquidity in the repo market.
vi. Securities excluded on an ad hoc basis to deal with other issues in the data.

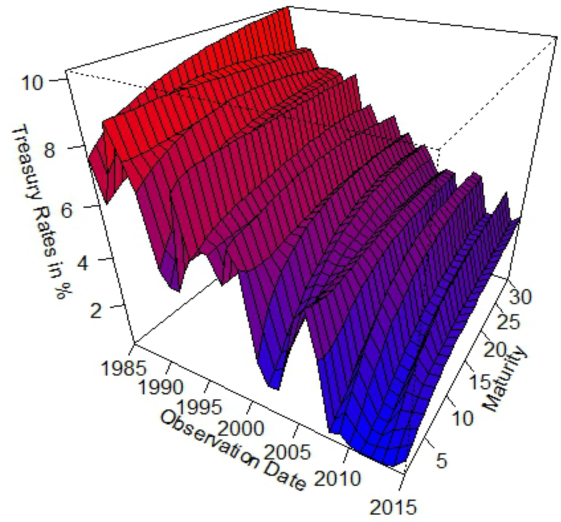
All in all, the Treasury yield curve provided in this data set is estimated in a way that ensures that the liquidity of the securities included is adequate and relatively uniform.

However, Gurkaynak et al. (2007) do not present raw market quotes on U.S. Treasury securities but rather give fitted values from the Svensson model with no restrictions imposed on the model parameters (i.e. with no prior assumptions made about the long-term forward rate level). This is due to the confidential nature of the original market data, which is not freely at researchers' disposal. Nevertheless, Gurkaynak et al. (2007) report a good fit of the original data with only small residuals, which can thus be considered a suitable representation of market data. In their effort to produce a benchmark yield curve suitable for further empirical analysis, Gurkaynak et al. (2007) also extrapolate yields and forward rates on U.S. Treasury securities, though they only employ horizons for which outstanding securities are available for estimation. For the above reasons we consider the data from Gurkaynak et al. (2007) to be an adequate proxy for market yield curves and treat it as such in the remainder of this paper.

For the purposes of our analysis we extract from the data set by Gurkaynak et al. (2007) one yield curve per year from the 1985-2015 period. We opt for the last available yield curve in each calendar year, typically from 31 December. Thus, our sample consists of 31 yield curves altogether. The starting date of our observed time period is conditional on the availability of Treasury zero coupon rates with maturities of up to 30 years. In the data set by Gurkaynak et al. (2007), 30 years is the maximum available maturity for U.S. securities, and the first year for which a yield curve with this maturity becomes available is 1985, which also marks the start of our sample.

The full U.S. Treasury term structure for the 1985-2015 period and maturities 1 to 30 years is shown below.

Figure 1: Term Structure Plot 1985-2015



Note: x axis shows maturities of U.S. Treasury securities, y axis indicates period of observation and z axis depicts Treasury zero rates in per cent.

Next, we use the yield curve data for the 1985-2015 period extracted from the data set by Gurkaynak et al. (2007) to calibrate a framework for setting a simple rule for when to revise the Ultimate Forward Rate (UFR).

4. Methodology

In this section we present a framework for setting the UFR and for providing a UFR revision mechanism using a benchmark value for the long-term rate that reflects economic conditions in the long run using extrapolation of the term structure based on two different models.

4.1 Setting the Ultimate Forward Rate

EIOPA's Technical Specification for the Preparatory Phase of Solvency II (2014) defines the UFR as the sum of the long-term average of short-term real interest rates and long-term expectations of the inflation rate, usually captured by the central bank's inflation target.

In our framework, we deviate from using the central bank's inflation target as a proxy for inflation expectations, as economic reality over the past years has been characterised by a low-yield environment with significant undershooting of the inflation target. Instead, we propose to set the UFR benchmark equal to average growth of nominal U.S. GDP over the previous 20 years.[1] We

[1] There are many alternative ways to set the benchmark for the UFR. For example, Laubach and Williams (2003) propose an approach to measuring the natural rate in the economy, which could potentially be used as the UFR

opt for the period of 20 years to simultaneously capture the economic cycle and mitigate any substantial impact of technological changes. Despite the fact that the business cycle literature has documented a certain lengthening of the economic cycle from the mid-1980s onwards (e.g. Creal, Koopman and Zivot, 2010), it still views 20 years as the long run (e.g. Stock and Watson, 2002).

We obtain the data from the Federal Reserve Bank of St. Louis and use Equation 1 to calculate the average 20-year growth rate of nominal GDP for each year up to 2015:

$$g_t = \left(\frac{GDP_t}{GDP_{t-20}}\right)^{1/20} - 1, \qquad (1)$$

where g is the average long-term growth rate and t indicates the year.

Next, we propose a UFR-updating rule based on the distance of the UFR from economic fundamentals, i.e. economic growth and inflation. The suggested rule offers a transparent way of updating the UFR, but embodies only one potential approach, which we readily acknowledge.

In this light, we set the initial regulatory UFR equal to the average growth rate of nominal U.S. GDP over the previous 20 years in 1980 using Equation 1. Subsequently, we calculate UFR_t for every year until 2015 using the following rule:

$$\begin{gathered} UFR_t = f(g_t, UFR_{t-1}) + UFR_{t-1} \\ f(g_t, UFR_{t-1}) = \begin{Bmatrix} g_t - UFR_{t-1} & if & \left|g_t - UFR_{t-1}\right| > p \\ 0 & if & \left|g_t - UFR_{t-1}\right| \leq p \end{Bmatrix}, \end{gathered} \qquad (2)$$

where g_t is obtained from Equation 1, t indicates a year from 1981 to 2015 and p is the distance between the long-term growth rate of nominal U.S. GDP at time t and the UFR at time *t-1*. Equation 2 thus resets the UFR at time t if the distance between long-term nominal GDP growth at time t and the regulatory UFR from the previous period *t-1* exceeds the value given by p. As we prefer to express g_t in percentages in our analysis, the values we assign to p are also in percentages. Hence, p takes values of 0.1%, 0.2%, 0.3%, 0.4% etc. up to 3.5% and we calculate the UFR in each year over the 1981-2015 period for every assigned value of p from Equation 2.

4.2 Extrapolation of Yield Curves

The next step in our framework for setting the UFR and its optimal adjustment frequency is extrapolation of zero rates on U.S. Treasury securities for maturities beyond 20 years, which are not provided by Gurkaynak et al. (2007) until 1985. Given that the EIOPA Technical Standards (2016) set the last liquid point (LLP), i.e. the maturity up to which yields on securities are quoted on the market, to 20 years, we also adopt this definition and extrapolate yields on securities with

benchmark. However, as this rate is unobservable, it might introduce additional uncertainty into the framework. Furthermore, the aim of this paper is to set up a framework providing a UFR revision rule rather than proposing the regulatory value.

maturities from 21 to 30 years, i.e. the maximum maturity available in the data set provided by Gurkaynak et al. (2007) from 1985.

For the extrapolation we use the model by Nelson and Siegel (1987) and its extension by Svensson (1994), which are frequently employed by central banks and other market participants (e.g. BIS, 2005) to fit term structures of interest rates. Furthermore, the studies by Diebold and Li (2006) and De Pooter, Ravazzolo and van Dijk (2007) provide evidence that these models are a useful tool in forecasting term structures of interest rates.

Despite these advantages, Bjork and Christensen (1999) showed that the Nelson-Siegel model is not theoretically arbitrage-free, i.e. the theoretical prices of securities resulting from the model and the actual prices observed on the market differ to such an extent that transaction costs do not prevent arbitrage. Since this condition between theoretical and observed prices is not hard-coded into the model, it was assumed that the model violates the no-arbitrage condition. However, Coroneo et al. (2011) show on U.S. yield curve data from 1970 until 2000 that the Nelson-Siegel model is statistically arbitrage-free. In this sense, another popular model, the Smith-Wilson (2001) model used by EIOPA to extrapolate the yield curve for very long maturities, is arbitrage-free, as it fits the yield curve exactly up to LLP. However, we do not use this model in the study, as our work aims to provide a framework for deriving a simple rule to update the UFR over time rather than assessing the optimal extrapolation method. Compared to the recent EIOPA work, we introduce a different approach using regulatory preferences as a parameter of the framework. Such preferences may differ among countries with different historical experiences, exposures and cultural backgrounds.

Nelson and Siegel (1987) model the term structure of interest rates using forward rates as follows:

$$f(\tau) = \beta_1 + \beta_2 \exp(-\tau/\lambda) + \beta_3 \frac{\tau}{\lambda} \exp(-\tau/\lambda), \tag{3}$$

while the spot yield curve can be obtained from Equation 3 by integration:

$$y(\tau) = \beta_1 + \beta_2 \left[\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right] + \beta_3 \left[\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda} - \exp(-\tau/\lambda)\right], \tag{4}$$

where $y(\tau)$ is the zero rate for maturity τ, and parameters β_1, β_2, β_3 and λ need to be estimated. β_1 is independent of time to maturity and as such indicates the long-term yield. Moreover, it is strictly positive and represents the value towards which the instantaneous forward rate $f(\tau)$ converges at infinity. Barrie and Hibbert (2008) equate β_1 to the UFR. β_2 decays exponentially to zero with increasing τ, so it only influences the short end of the yield curve, while β_3 first increases then decreases with increasing τ, which adds a hump to the yield curve.

Svensson (1994) extends the Nelson-Siegel (1987) model by adding a second hump to the yield curve, as expressed by Equation 5 in the form of forward rates:

$$f(\tau) = \beta_1 + \beta_2 \exp(-\tau/\lambda_1) + \beta_3 \frac{\tau}{\lambda_1} \exp(-\tau/\lambda_1) + \beta_4 \frac{\tau}{\lambda_2} \exp(-\tau/\lambda_2) \tag{5}$$

and by integrating it into the form of spot rates:

$$y(\tau) = \beta_1 + \beta_2 \left[\frac{1-\exp(-\tau/\lambda_1)}{\tau/\lambda_1}\right] + \beta_3 \left[\frac{1-\exp(-\tau/\lambda_1)}{\tau/\lambda_1} - \exp(-\tau/\lambda_1)\right] + \beta_4 \left[\frac{1-\exp(-\tau/\lambda_2)}{\tau/\lambda_2} - \exp(-\tau/\lambda_2)\right], \quad (6)$$

where $y(\tau)$ is again the zero rate for maturity τ, and six parameters, β_1, β_2, β_3, β_4, λ_1 and λ_2, need to be estimated. This model is able to better capture the shape of the yield curve, as it allows for a second hump that usually occurs at long maturities (i.e. 20 years or more). The occurrence of the second hump can be attributed to convexity, which pulls down the yields on long-term securities and as a consequence makes the yield curve's shape concave at long maturities.

In order to extrapolate U.S. Treasury yield curves for maturities 21-30 we use the R-project package "ycinterextra" by Moudiki (2013). The package allows us to extrapolate the term structure using the UFR calculated from Equation 2 for every yield curve over the 1985-2015 period and for every value of p. We thus extrapolate U.S. Treasury yields for maturities 21 to 30 using both the Nelson-Siegel and Svensson models.

4.3 Construction of a Loss Function

The last step in constructing our framework is to join the UFR setting and the extrapolation of yields using the two yield curve models into a single statistic for each value of p. In particular, we take into account how stable the UFR set in subsection 4.1 is over the entire observed time period and how close the extrapolated yields using that particular UFR are to the yields at maturities 21-30 extracted from Gurkaynak et al. (2007). We call this aggregate statistic a loss function, as it penalises frequent changes in the UFR and the distance of the extrapolated yields from the actual yields at maturities 21-30. We calculate the loss function for every value of p, which expresses the distance between the average long-term growth of nominal GDP and the regulatory UFR from the previous period, over the 1985-2015 period.

Our proposed loss function has the following form:

$$Loss_p = w_{Prec} \times MSE_p + w_{Stab} \times \left(\min_{t \epsilon T}\left(MSE_{p,t}\right) + k \times \left(\max_{t \epsilon T}\left(MSE_{p,t}\right) - \min_{t \epsilon T}\left(MSE_{p,t}\right)\right)\right) \quad (7)$$
$$T = \langle 1985; 2015 \rangle,$$

where T is the set of years from the observed time period, p is the distance between the long-term growth rate of nominal U.S. GDP at time t and the UFR at time *t-1*, k is the number of UFR changes over the total number of years in the observed period of 1985-2015 for the corresponding value of p, and w_{Prec}, w_{Stab} are the weights of the two loss function components. They can take values from 0 to 1 and express the regulator's relative preference for extrapolation precision versus UFR stability. It needs to hold that $w_{Prec} + w_{Stab} = 1$. Therefore, the weights set to 0.5 would indicate there is no preference for either precision of extrapolation or UFR stability, since the two components are weighted equally in the loss function. MSE_t, the mean square error, is a standard statistical concept that measures the average of the squares of the errors between the yields at maturities 21-30 obtained from extrapolation using the Nelson-Siegel and Svensson models for the chosen regulatory UFR, and the yields from Gurkaynak et al. (2007) at these maturities. For each value of p we calculate the corresponding average mean square error MSE_p over the observed time period defined as follows:

$$MSE_p = \frac{1}{31} \times \sum_{t=1985}^{2015} MSE_{p,t} = \frac{1}{31} \times \frac{1}{10} \times \sum_{t=1985}^{2015} \sum_{i=21}^{30} \left(\widehat{y_{i,t}} - y_{i,t}\right)^2, \tag{8}$$

where i takes values of maturities 21 to 30, t indicates a year in the 1985-2015 period and $\widehat{y_{i,t}}$ stands for an estimate of the yield at maturity *i* and year *t* obtained by extrapolation from either the Nelson-Siegel or Svensson model, while $y_{i,t}$ is the Gurkaynak et al. (2007) yield at maturity *i* in year *t*.

As for the second component of the loss function, the stability of the UFR over the observed period, we approximate it with the ratio of the number of UFR changes for the corresponding *p* over the number of years in the 1985-2015 period, i.e. 31 years. We also rescale this ratio to correspond numerically to the first component of the loss function MSE_p, as shown in Equation 7.

We are interested in the value of p that minimises the loss over the 1985-2015 period for the regulator's preferences regarding extrapolation precision and UFR stability. Such a value of *p* would reveal how much the long-term nominal GDP growth rate in a given year should deviate from the regulatory UFR from the previous year to have the UFR reset to the value given by Equation 2. The loss-minimising value of p depends on the regulator's relative preference for precision versus UFR stability.

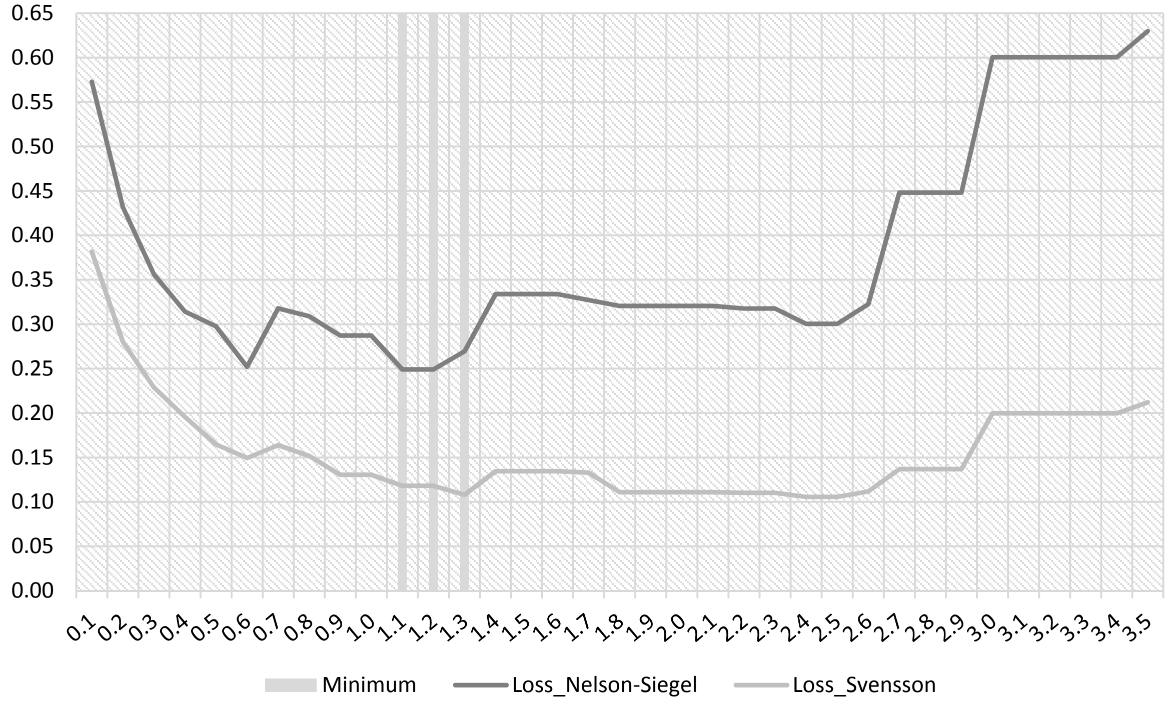
Table A1, presenting the UFRs calculated in each year over the 1985-2015 period for the values of p that minimise the loss under different regulator preferences, is available in the Appendix. The next section presents the results of the calculation of the loss for the overall period across different values of p, using the Nelson-Siegel and Svensson models and different regulator preferences.

5. Results

In this section we present the results of the loss calculation over the 1985-2015 period and different values of p using both the Nelson-Siegel and Svensson models and different preferences, i.e. weighting schemes.

First, we assume that the regulator places equal weight on UFR stability and extrapolation precision. In this case, the following condition holds for the weights in Equation 7: $\mathrm{w}_{\mathrm{Prec}} = \mathrm{w}_{\mathrm{Stab}} = 0.5$.

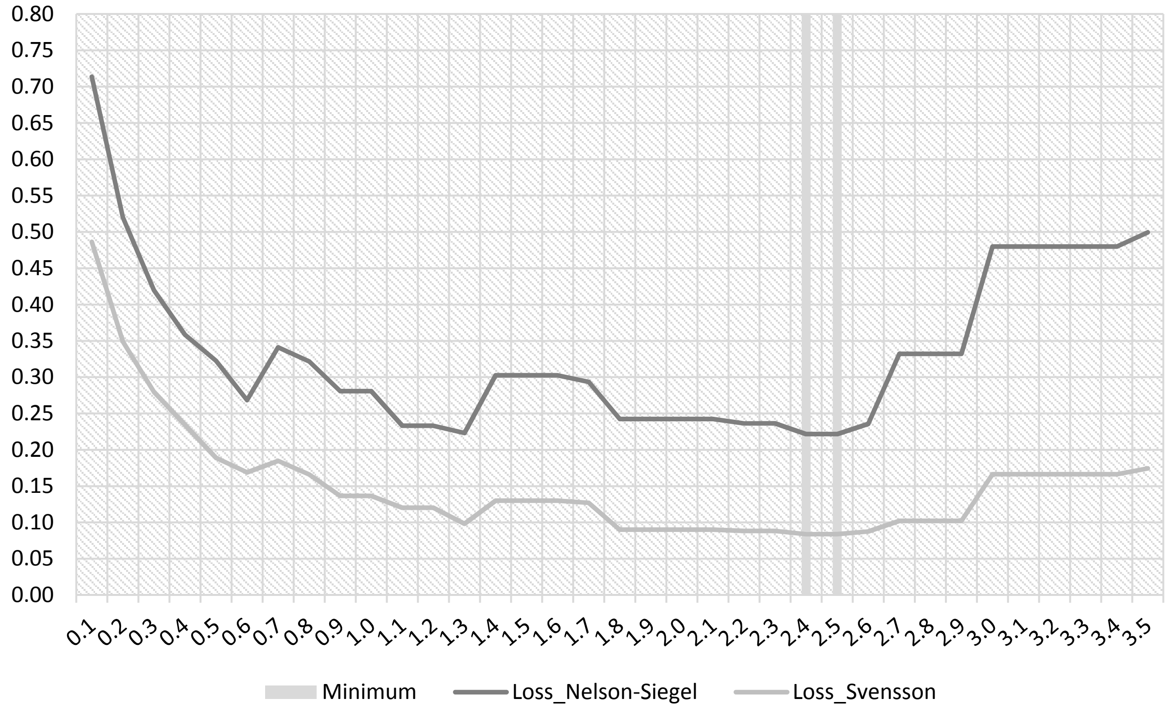
Figure 2: Loss for Different Values of p (weights: 0.50, 0.50)



Notes: The dark grey line shows the loss over the 1985-2015 period for different values of *p* (on the horizontal axis) calculated from the Nelson-Siegel model, while the light grey line depicts the loss from the Svensson model over the same period. The light grey bars highlight those values of p that minimise the loss function for both models. The vertical axis indicates the magnitude of the loss. The calculation uses equal weighting for both interpolation models.

We can observe from Figure 2 that a value of *p* equal to both 1.1% and 1.2% minimises the loss over the 1985-2015 period when yields are extrapolated using the Nelson-Siegel model. For Svensson, the loss-minimising value of *p* equals 1.3%. Next, we turn to alternative weighting schemes for the case where the regulator considers stability of the UFR over time more important than how closely the model can extrapolate long-term yields to their observed values (yields on Treasury securities for maturities of 21-30), and vice versa.

Figure 3: Loss for Various Values of p (weights: 0.33, 0.67)



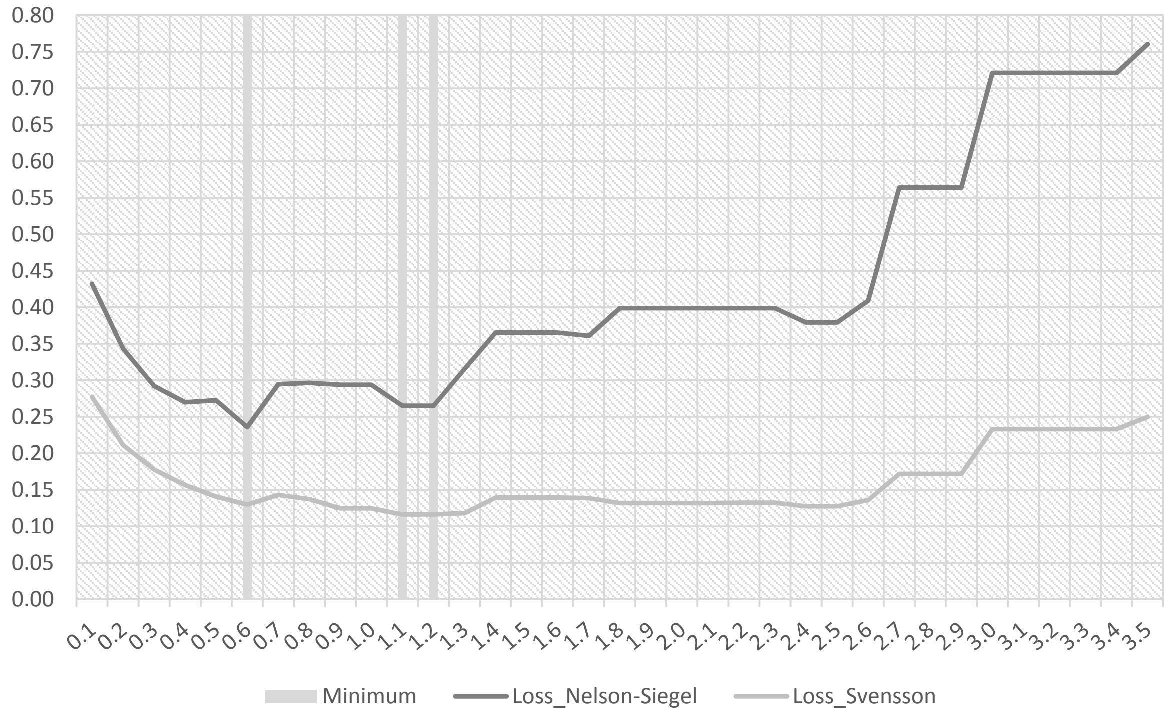
Notes: The dark grey line shows the loss over the 1985-2015 period for different values of *p* (on the horizontal axis) calculated from the Nelson-Siegel model, while the light grey line depicts the loss from the Svensson model over the same period. The light grey bars highlight those values of p that minimise the loss function for both models. The vertical axis indicates the magnitude of the loss. A weight of 33% is placed on extrapolation precision while 67% is placed on UFR stability.

Figure 3 shows that the loss-minimising value of *p* when a weight of 33% is put on extrapolation precision and a weight of double that is placed on UFR stability increases to 2.4% and 2.5%, which is approximately double the value of *p* that minimises the loss under equal weighting. All in all, the loss is minimised at p=2.4% and p=2.5% for both models under the given preferences.

Next, we choose to favour extrapolation precision over UFR stability in our calculation. We put a weight of 67% on the MSE component of the loss function in Equation 5 and half that weight on how stable the UFR is over time.

In this case, the loss-minimising value of *p* drops to 0.6% when extrapolation is performed using the Nelson-Siegel model. As for Svensson, the loss-minimising *p* equals 1.1% and 1.2% under these preferences, which is quite close to the optimal value of *p* under equal weighting. Figure 4 presents the results.

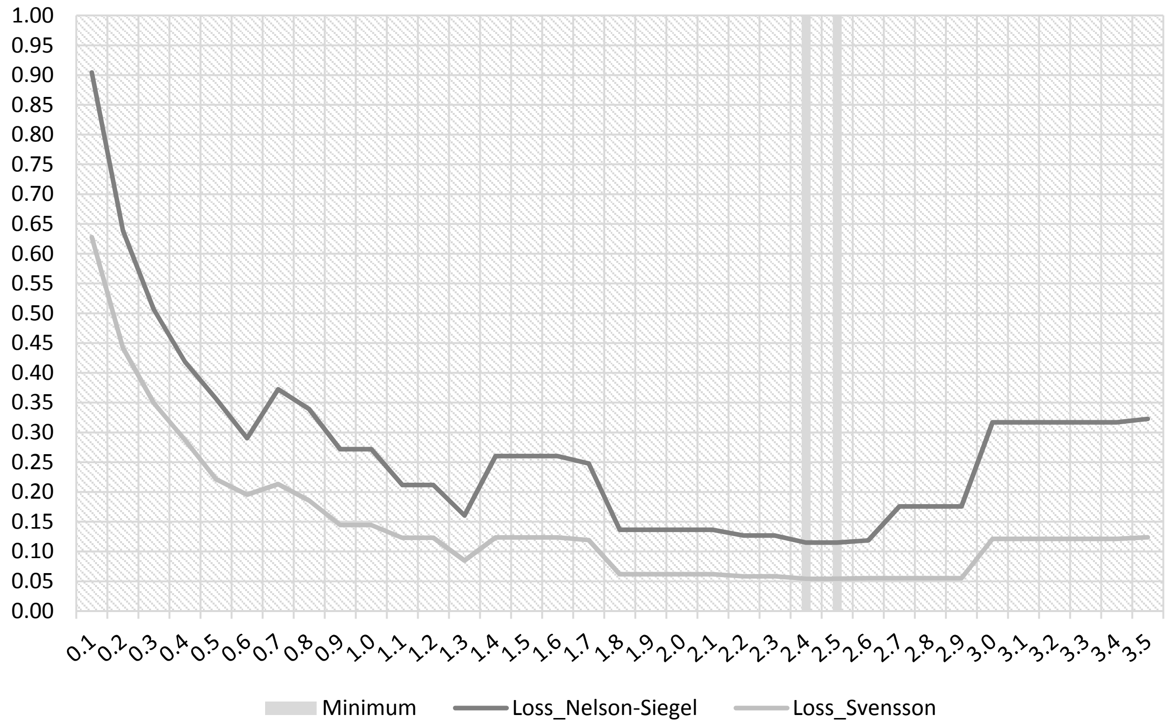
Figure 4: Loss for Various Values of p (weights: 0.67, 0.33)



Notes: The dark grey line shows the loss over the 1985-2015 period for different values of *p* (on the horizontal axis) calculated from the Nelson-Siegel model, while the light grey line depicts the loss from the Svensson model over the same period. The light grey bars highlight those values of p that minimise the loss function for both models. The vertical axis indicates the magnitude of the loss. A weight of 67% is placed on extrapolation precision while 33% is placed on UFR stability.

For the last two weighting schemes, we suppose that the regulator cares very little about one component of the loss function, either MSE or UFR stability, while the other aspect is found to be crucial.

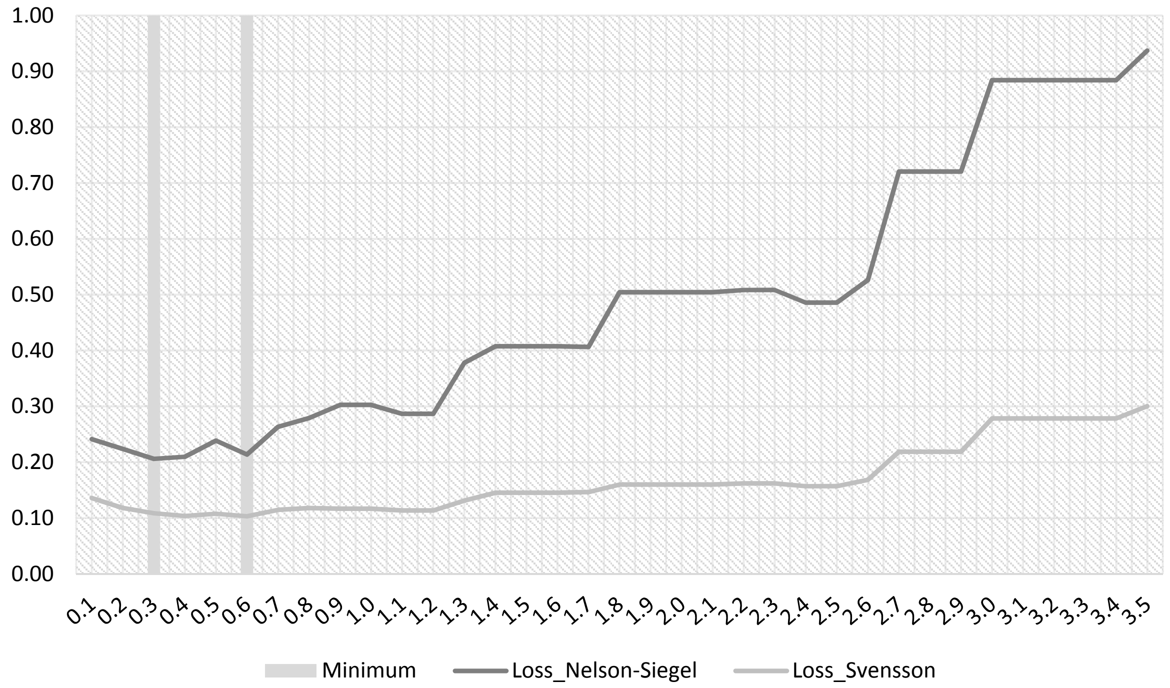
Figure 5: Loss for Various Values of p (weights: 0.10, 0.90)



Notes: The dark grey line shows the loss over the 1985-2015 period for different values of *p* (on the horizontal axis) calculated from the Nelson-Siegel model, while the light grey line depicts the loss from the Svensson model over the same period. The light grey bars highlight those values of p that minimise the loss function for both models. The vertical axis indicates the magnitude of the loss. A weight of 10% is placed on extrapolation precision while 90% is placed on UFR stability.

Figure 5 presents the loss-minimising values of *p* when a weight of only 10% is placed on extrapolation precision as opposed to a weight of 90% put on stability of the UFR. For yield extrapolation by both the Nelson-Siegel and Svensson models, the optimal value of *p* is 2.4% and 2.5%, which is the same as under the weighting scheme of 33% placed on extrapolation precision and 67% placed on UFR stability.

Figure 6: Loss for Various Values of p (weights: 0.90, 0.10)



Notes: The dark grey line shows the loss over the 1985-2015 period for different values of *p* (on the horizontal axis) calculated from the Nelson-Siegel model, while the light grey line depicts the loss from the Svensson model over the same period. The light grey bars highlight those values of p that minimise the loss function for both models. The vertical axis indicates the magnitude of the loss. A weight of 90% is placed on extrapolation precision while 10% is placed on UFR stability.

In the case of the reversed weighting of 90% for the MSE component of the loss function and 10% for UFR stability, the value of *p* that minimises the loss under Nelson-Siegel extrapolation drops to 0.3% while *p* equal to 0.6% is optimal for the Svensson model, as shown in Figure 6.

All in all, it appears that if the difference between the long-term rate measured by average 20-year growth of nominal GDP at time *t* and the UFR valid in period *t-1* exceeds 1.2% when the Nelson-Siegel model is used for extrapolation, the UFR at time *t* should be adjusted to reflect long-term average growth of nominal GDP at time *t*. This difference slightly increases to 1.3% for the Svensson model also under equal regulator preferences. The optimal value of *p* equal to 1.1% and 1.2% for the Nelson-Siegel model amounts to a total of three UFR adjustments over the 1985-2015 period, while the optimal p=1.3% for the Svensson model implies only two adjustments, as shown in Table A1.

The loss-minimising value of *p* either rises or drops in response to changing regulator preferences. With the regulator at least two thirds in favour of UFR stability over time compared to the MSE component, the distance indicative of resetting the UFR increases to 2.5%. On the other hand, a regulator caring very little about UFR stability would lean towards more frequent revisions of the UFR. This is reflected by the optimal distance between economic fundamentals and the regulatory UFR being as small as 0.3% and 0.6% under the Nelson-Siegel and Svensson models respectively.

Next, we discuss the UFR in the context of equilibrium rates in the world economy, the benefits that can be derived from implementing a time-varying UFR and the implications of accommodative monetary policy for the UFR. We then present an insurer's hypothetical portfolio of liabilities to demonstrate the valuation effects of changes in the UFR.

6. Policy Implications and Discussion

First, in line with EIOPA's approach to suggesting different UFRs across countries conditionally on the magnitude of their expected inflation, our framework also leads to different country-specific UFRs. By extension, one can argue that different UFRs across economies imply differences in their long-run growth. Indeed, the literature on economic growth documents the importance of initial conditions in terms of distribution of income for the evolution of economies and their steady-state behaviour. In this spirit, the club convergence hypothesis allows for countries that are similar in their structural characteristics and their initial level of output per capita but that differ in their initial distribution of income, to cluster around different steady-state equilibria (e.g. Galor and Zeira, 1993).

Second, a time-varying UFR and the adoption of a transparent UFR-updating mechanism would be beneficial for insurance firms to better reflect the slow-burning risk stemming from duration mismatch and the high level of guarantees beyond the last liquid point. Under our proposed framework, the data needed for UFR benchmark construction are publicly available, as are Treasury yield curves. In addition, the methodology provides clear guidance for updating the UFR based on whether the threshold for the distance between the outstanding UFR and economic fundamentals is exceeded. Under the updated UFR, the present value of an insurer's long-term liabilities will be recalculated, and if necessary the regulatory capital should be increased. Therefore, the time-varying UFR serves as an automatic macroprudential tool for the insurance sector.

Third, the accommodative monetary policy pursued in recent years with the objective of boosting economic growth has resulted in an ongoing low-yield environment, as manifested by shrinking term premia at the long end of the yield curve. Recently, Hanson and Stein (2015) have documented the impact of changes in the stance of monetary policy on distant forward real interest rates through the resulting movements in term premia. In the light of these findings, the methodology presented in this paper accounts for growth and inflationary developments within the construction of the UFR benchmark which led to the UFR being gradually revised downward over the sample period, thus reflecting changes in the term structure. Consequently, our framework should provide a more appropriate discount rate for valuation of long-term liabilities and highlight a potential need for capitalisation to adequately reflect risk fundamentals.

The proposed framework is calibrated for the U.S. economy, which offers long time series. However, it could be used for any economy/currency providing sufficiently long time series to arrive at a robust calibration. For those cases where sufficiently long historical time series are not available, the calibration for their peers could be used.

Next, we turn to a demonstration of the valuation effects resulting from UFR changes on an insurer's hypothetical portfolio of liabilities.

Under a low-yield regime, a decline in benchmark interest rates translates into a reduced discount rate applied in the overall valuation of an insurer's liabilities. This in turn leads to a steeper increase in the present value of liabilities over assets, eroding the insurer's surplus and exacerbating the insolvency risk of insurance entities. While actual market interest rates are applied in the valuation of liabilities with short maturities, the ultimate forward rate is used for discounting liabilities with long maturities. In line with our assumption that the LLP is set to 20 years in accordance with the EIOPA Technical Standards (March 2016), changes in the UFR only affect the value of those liabilities with durations greater than 20 years.[2]

As some countries cannot boast debt instruments with very long durations denominated in the country's currency that are available in the market, we also consider the impact that a change in the UFR would have on the valuation of such liabilities. In particular, we focus on the duration brackets of 16 and 18 years.

Primarily, we illustrate on long-term liabilities of different duration within a hypothetical insurer's portfolio how their present value changes in response to changes in the long-term interest rate (UFR). We take long-term U.S. nominal GDP growth in 2005 as the UFR benchmark value. We assume that the UFR has been constant since then, i.e. that it is fixed at 5.39% in 2015. We calculate alternative UFRs in 2015 from the formula given in Equation 2. We choose those UFRs which correspond to a loss-minimising value of p under the different regulatory preferences given in Section 5.

Table 1 shows the changes in the present value of long-term liabilities of different duration within the hypothetical portfolio given different regulatory preferences about the UFR setting, and using both the Nelson-Siegel and Svensson models. We calculate the change in the present value of an insurer's long-term liabilities due to changes in the UFR for average long-term durations of 21, 22, 25, 28 and 30 years using the standard definition of modified duration:

$$\begin{gathered}\Delta PV_\tau = -\Delta IR_\tau \times MD \\ \tau = \{21,22,25,28,30\},\end{gathered} \tag{9}$$

where ΔPV_τ indicates the change in the present value of liabilities with average duration τ, ΔIR_τ expresses the change in the spot yields of liabilities with average duration τ with respect to the spot yields of liabilities with the same duration under the benchmark UFR, and *MD* stands for modified duration, i.e. the corresponding duration bracket from the set τ.

[2] This is in line with the EIOPA Technical Standards (March 2016).

Table 1: Impact of Different Regulatory Preferences on the Long-term Liabilities within a Portfolio

NELSON-SIEGEL EXTRAPOLATION

PREFERENCES	*benchmark*	w_{UFR}=0.10	w_{UFR}=0.33	w_{UFR}=0.5	w_{UFR}=0.67
UFR VALUE IN 2015	**5.39%**	4.22%	4.56%	4.07%	5.39%
AVERAGE MODIFIED DURATION OF LIABILITIES (IN YEARS)	16	4.65%	3.30%	5.25%	0%
	18	6.81%	4.83%	7.68%	0%
	21	10.18%	7.22%	11.48%	0%
	22	11.32%	8.03%	12.77%	0%
	25	14.79%	10.49%	16.68%	0%
	28	18.28%	12.97%	20.62%	0%
	30	20.61%	14.62%	23.25%	0%

SVENSSON EXTRAPOLATION

PREFERENCES	*benchmark*	w_{UFR}=0.10	w_{UFR}=0.33	w_{UFR}=0.5	w_{UFR}=0.67
UFR VALUE IN 2015	**5.39%**	4.22%	4.56%	4.07%	5.39%
AVERAGE MODIFIED DURATION OF LIABILITIES (IN YEARS)	16	0.14%	0.03%	0.18%	0%
	18	0.97%	0.57%	1.14%	0%
	21	2.72%	1.74%	3.14%	0%
	22	3.42%	2.22%	3.94%	0%
	25	5.82%	3.86%	6.66%	0%
	28	8.55%	5.77%	9.77%	0%
	30	10.52%	7.14%	11.99%	0%

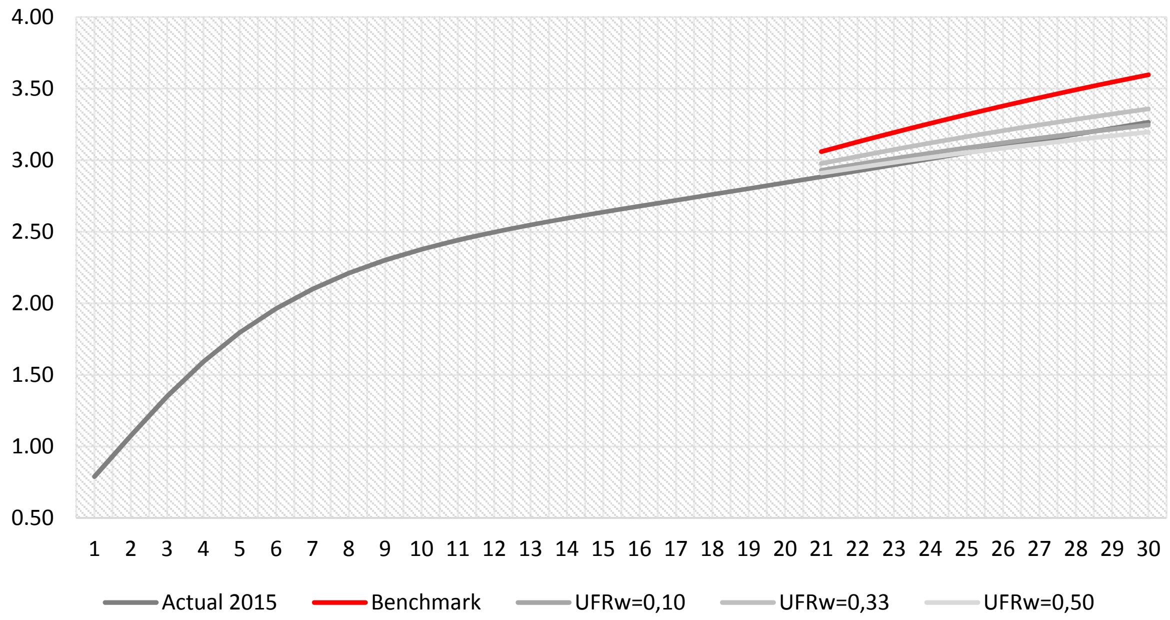
Notes: The impact of deviations of the long-term interest rate from the benchmark given different regulatory preferences on the present value of an insurer's long-term liabilities of different duration. The first row indicates the preference of the regulator for UFR stability. The second row states the corresponding UFR in 2015 calculated from Equation 2.

Overall, we observe greater sensitivity of the present value of liabilities with longer durations to changes in the UFR. The greater the decrease in the UFR with respect to the benchmark, the greater the increase in the present value of liabilities across different average durations. Therefore, for insurance firms whose portfolio consists of very long-term liabilities, such as life-insurers, a relatively small decline in the discount rate of -0.83% to UFR=4.56% would result in an increase in the value of long-term liabilities with an average duration of 30 years of more than 14% under Nelson-Siegel extrapolation and 7% under Svensson. The smaller impact on the present value when the Svensson model is used for extrapolation can be attributed to smaller deviations of the spot yields under different regulatory UFRs from the spot yields under the benchmark UFR compared to the Nelson-Siegel model. As for the valuation of liabilities with shorter average durations of 16 and 18 years, the impact of changes in the UFR is much less pronounced, especially for the Svensson model.

Consequently, for insurers whose portfolios consist of liabilities with shorter average durations, changes in the UFR lead to much smaller revisions in the valuation of their long-term liabilities.

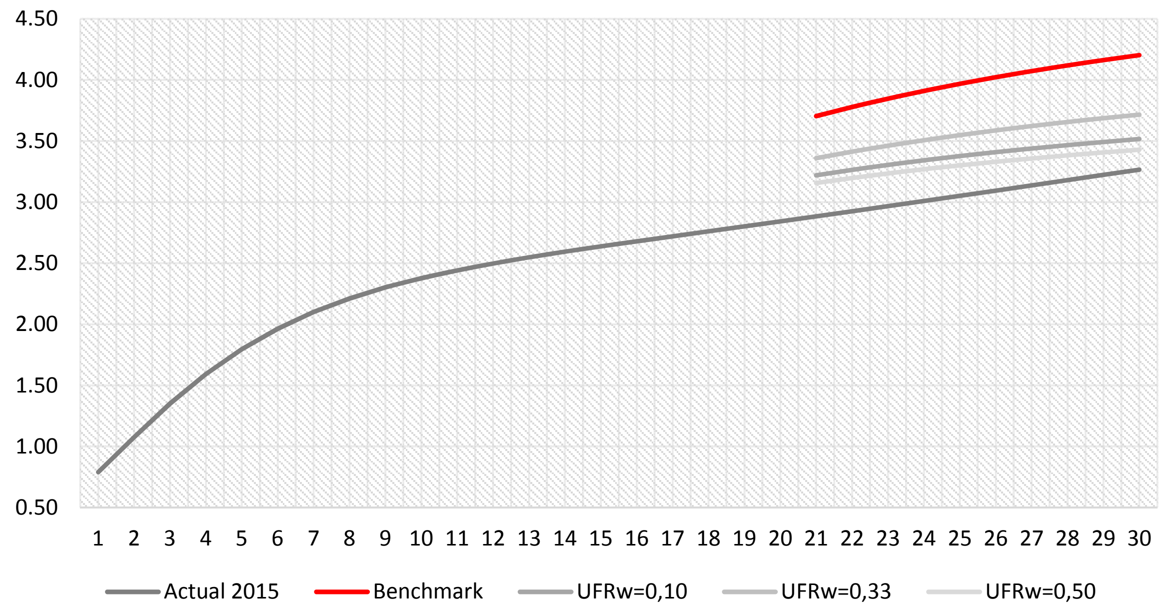
Figures 7 and 8 depict the extrapolated spot yields versus the benchmark for the Nelson-Siegel and Svensson models respectively. The figures show that the extrapolated spot yields under regulatory UFRs with different preferences about UFR stability are lower than the spot yields under the constant UFR scenario (in red) for both models. However, the proximity of the extrapolated yields under different regulatory preferences to the benchmark yields as well as to actual yields is greater for the Svensson model. Therefore, under the Svensson model, changes in the UFR affect the present value of the long-term liabilities in an insurer's portfolio to a lesser degree.

Figure 7: Svensson Extrapolation under Different Preferences



Notes: Figure 7 shows the actual yield curve as of 31 December 2015 over maturities of 1-30 years and the extrapolated spot yields for maturities of 21-30 years under the UFR with different regulatory preferences, and the benchmark using the Svensson model. The benchmark corresponds to extrapolation with the UFR equal to average 20-year U.S. nominal GDP growth in 2005. The vertical axis shows spot yields in percentages, while the horizontal axis indicates maturities in years.

Figure 8: Nelson-Siegel Extrapolation under Different Preferences



Notes: Figure 8 shows the actual yield curve as of 31 December 2015 over maturities of 1-30 years and the extrapolated spot yields for maturities of 21-30 years under the UFR with different regulatory preferences, and the benchmark using the Nelson-Siegel model. The benchmark corresponds to extrapolation with the UFR equal to average 20-year U.S. nominal GDP growth in 2005. The vertical axis shows spot yields in percentages, while the horizontal axis indicates maturities in years.

7. Conclusions

As the liability side of insurance companies' balance sheets is formed of commitments with very long maturities, they need to be discounted by a corresponding long-term interest rate for valuation purposes. However, interest rates over very long maturities are seldom available on the market. As a result, the Ultimate Forward Rate (UFR) needs to be estimated in order to evaluate such long-term contracts. Consequently, changes in the UFR have valuation effects for insurers.

In this paper we propose one possible approach to setting the interest rate for long-term contracts (UFR) in a dynamic way that uses the long-term evolution of economic fundamentals as a benchmark for the UFR. In addition, our approach proposes a loss function that weights two aspects of the UFR – estimation precision and UFR stability.

We propose a UFR-setting algorithm that compares how much long-term economic fundamentals measured by average 20-year nominal GDP growth in a given year differ from the regulatory UFR from the previous year. If this difference is greater than some threshold value *p*, the UFR for this period is set to the value given by economic fundamentals. On the other hand, a difference smaller than the threshold makes the regulatory UFR from the prior year valid in the given year as well.

Next, we extrapolate yields over maturities of 21-30 years using the Nelson-Siegel and Svensson models and compare them with yields from the U.S. Treasury term structure data presented by Gurkaynak et al. (2007) over the period of 1985-2015 using the mean square error (MSE) statistic.

We combine the two aspects – UFR stability (the ratio of changes in the UFR over the observed period) and extrapolation precision (the distance between the actual and extrapolated yields) – into a loss function. The preference for each component of the loss function is expressed by assigned weights.

We search for such *p* (the distance between long-term growth of economic fundamentals and the UFR set in the previous period) that minimises our proposed loss function.

Finally, we find that once the distance between average 20-year nominal GDP growth in a given year and the regulatory UFR from the previous year exceeds 1.2% and 1.3% under preference neutrality for the Nelson-Siegel and Svensson models respectively, the UFR should be adjusted. This result changes in response to the regulator's preferences. When a preference for UFR stability dominates, the distance for resetting the UFR increases, implying fewer changes to the UFR over the period under investigation, and vice versa. Despite the fact that regulators' preferences may differ around the world, we are inclined to propose preference neutrality to balance the stability and precision suggested by economic fundamentals, at least as a starting point for further considerations.

Finally, we illustrate the impact of changes in long-term interest rates on insurance companies by means of a hypothetical portfolio of long-term liabilities. We show that the extrapolated spot yields under regulatory UFRs with different UFR stability preferences are lower than the spot yields generated under the assumption of a constant UFR fixed to average long-term nominal GDP growth levels.

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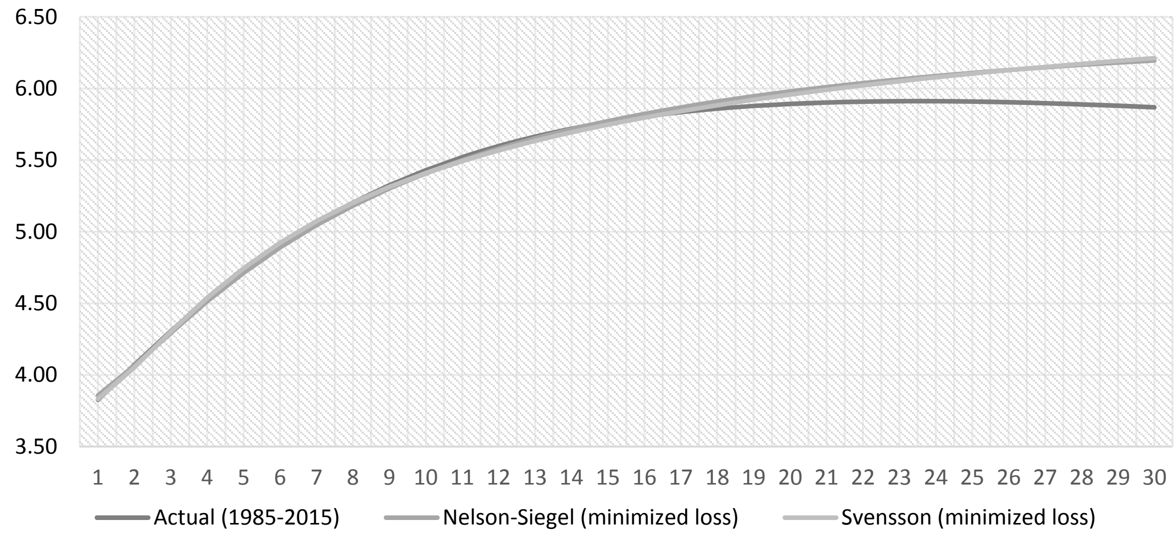
Appendix

Table A1

	p	***0.30***	***0.60***	***1.10***	***1.20***	***1.30***	***2.40***	***2.50***
Year	**nomGDP_g (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**
1985	8.77	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1986	8.58	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1987	8.60	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1988	8.52	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1989	8.50	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1990	8.51	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1991	8.25	8.25	8.64	8.00	8.00	8.00	8.00	8.00
1992	8.07	8.25	8.64	8.00	8.00	8.00	8.00	8.00
1993	7.77	7.77	7.77	8.00	8.00	8.00	8.00	8.00
1994	7.67	7.77	7.77	8.00	8.00	8.00	8.00	8.00
1995	7.47	7.47	7.77	8.00	8.00	8.00	8.00	8.00
1996	7.21	7.47	7.77	8.00	8.00	8.00	8.00	8.00
1997	6.98	6.98	6.98	8.00	8.00	8.00	8.00	8.00
1998	6.64	6.64	6.98	6.64	6.64	6.64	8.00	8.00
1999	6.39	6.64	6.98	6.64	6.64	6.64	8.00	8.00
2000	6.28	6.28	6.28	6.64	6.64	6.64	8.00	8.00
2001	5.86	5.86	6.28	6.64	6.64	6.64	8.00	8.00
2002	5.82	5.86	6.28	6.64	6.64	6.64	8.00	8.00
2003	5.64	5.86	5.64	6.64	6.64	6.64	8.00	8.00
2004	5.43	5.43	5.64	5.43	5.43	6.64	5.43	5.43
2005	5.39	5.43	5.64	5.43	5.43	6.64	5.43	5.43
2006	5.40	5.43	5.64	5.43	5.43	6.64	5.43	5.43
2007	5.32	5.43	5.64	5.43	5.43	5.32	5.43	5.43
2008	5.03	5.03	5.03	5.43	5.43	5.32	5.43	5.43
2009	4.56	4.56	5.03	5.43	5.43	5.32	5.43	5.43
2010	4.47	4.56	5.03	5.43	5.43	5.32	5.43	5.43
2011	4.49	4.56	5.03	5.43	5.43	5.32	5.43	5.43
2012	4.40	4.56	4.40	5.43	5.43	5.32	5.43	5.43
2013	4.33	4.56	4.40	5.43	5.43	5.32	5.43	5.43
2014	4.22	4.22	4.40	4.22	4.22	5.32	5.43	5.43
2015	4.07	4.22	4.40	4.22	4.22	5.32	5.43	5.43
No. of UFR changes		11	6	3	3	2	1	1

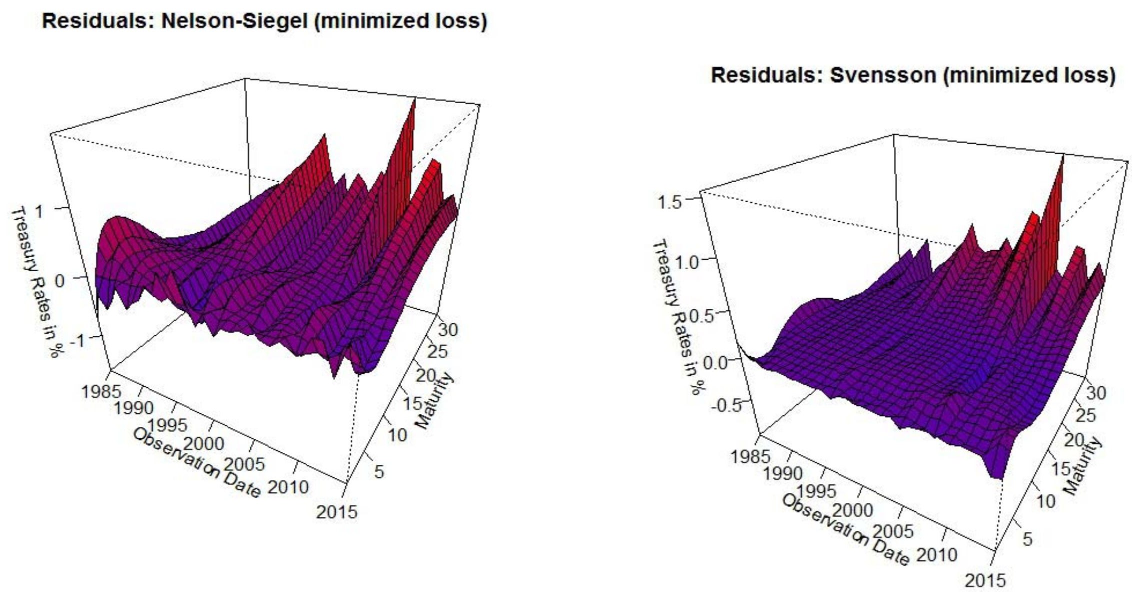
Notes: Table A1 shows UFR vales calculated from Equation 2 for those values of *p* that minimise the loss under individual preference scenarios, and for each year in the 1985-2015 period. The column "nomGDP_g (%)" shows the average 20-year growth of nominal U.S. GDP in each year and serves as the benchmark for the UFR calculation in Equation 2. Table A1 also shows the frequency of UFR changes for selected values of *p* over the observed period.

Figure A1: Average Yield Curves 1985-2015



Notes: Figure A1 depicts the average actual U.S. Treasury yield curve (dark grey), the average yield curve obtained from Nelson-Siegel extrapolation (medium grey) for p=1.1%/1.2% that minimises the loss, and the average yield curve from Svensson extrapolation (light grey) for p=1.3% minimising the loss over the 1985-2015 period. Maturities 1-30 are depicted on the horizontal axis while yields in % are shown on the vertical axis.

Figure A2



Notes: Figure A2 shows the residuals obtained from extrapolation by the Nelson-Siegel and Svensson models over the 1985-2015 period and maturities 1 to 30.

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