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Dynamics of Linear Forward-looking Structural Macroeconomic Models at the Zero Lower Bound: Do Solution Techniques Matter?

Jan Brůha *

Abstract

Yes, they do matter, sometimes a lot. In this paper, I compare various solution techniques that can be used to solve structural forward-looking macroeconomic models subject to the zero lower bound as the only non-linearity. I use stylized forward-looking models to compare the solution techniques based on impulse responses, on the implications of forward guidance, on the values of fiscal multipliers, and on solution accuracy. I disprove recent claims in the literature that various solution methods yield identical dynamics. The solutions are equivalent only if the zero lower bound constraint binds for no more than one period, otherwise the implied dynamics can be different. Moreover, I find that large effects of forward guidance and large fiscal multipliers at the zero lower bound are found especially when models are solved using 'shadow' shocks. On the other hand, the occasional-binding toolbox and solutions based on a non-linear deterministic solver imply small effects of forward guidance and fiscal multipliers that are not significantly larger at the zero lower bound than during normal times. Moreover, these two types of solutions seem to be the most accurate.

Abstrakt

V tomto článku porovnávám vybrané techniky, které mohou být použity na řešení strukturálních makroekonomických modelů s dopředu hledícími očekáváními, které zahrnují nulovou dolní hranici úrokových sazeb jako jedinou nelinearitu. Používám stylizované teoretické modely s dopředu hledícími očekáváními, abych porovnal různé techniky řešení, co se týče implikací pro impulzní odezvy, pro dopady opatření typu "forward guidance"a pro velikost fiskálních multiplikátorů, a porovnávám také přesnost řešení. Vyvracím nedávná tvrzení v odborné literatuře, že různé metody řešení poskytují identické výsledky. Ukazuji, že různá řešení jsou ekvivalentní, pouze pokud omezení na nulovou dolní hranici sazeb váže jejich trajektorii pro více než jedno období, jinak mohou mít různé techniky řešení rozdílné implikace. Velké efekty forward guidance a vysoké fiskální multiplikátory při nulových úrokových sazbách jsou přítomny zejména pro některá řešení založená na "stínových"šocích. Na druhou stranu, řešení založená na nelineárních deterministických systémech a řešení pomocí příležitostně trvajících omezení poskytují pouze malé efekty "forward guidance"a fiskální multiplikátory, které nejsou v prostředí nulových sazeb výrazně vyšší než v normálních časech. Tyto dva přístupy k řešení se také ukazují jako nejpřesnější.

JEL Codes: C53, E37, E47, E52, E63.
Keywords: Fiscal multiplier, forward guidance, solution methods, zero lower bound.

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Nontechnical Summary

Linear dynamic structural macroeconomic models with model-consistent forward-looking expectations are widely used in central banks and other policy institutions for forecasting and policy analyses. Linearity of applied models is beneficial for practical forecasting, since shock decomposition, ex-post forecast error decomposition, and the application of judgment are more easily dealt with in a linear framework. It is often tacitly assumed that non-linear effects can be either ignored (in normal times), or incorporated into the model-generated forecast by means of expert judgment. After the Great Recession, the zero lower bound started to be a binding constraint. Its existence implies an important non-linearity in the crucial equation – the monetary policy reaction function. This non-linearity typically propagates quickly to the rest of the model equations, affecting both real variables and inflation.

In response to this situation, various 'pragmatic' solutions have been proposed that enable applied linear models to be used in a fast way even under the lower bound constraint. These methods are based on different ideas, such as extending the monetary policy reaction function to include 'shadow shocks' that are designed so that the zero lower bound constraint is satisfied, or basing the simulations on the non-linear perfect foresight counterpart, or exploring the switching nature of the model's reduced form.

In recent literature, one can find claims that in a partially linear framework, i.e., when the lower bound represents the only non-linearity, these approaches yield identical dynamics. If this is really the case, the choice of solution approach is purely a matter of convenience. In this paper I re-examine this claim and find that it holds only under special circumstances, for example, when the lower bound binds for just one period. In a general situation, the solution methods may imply quite different dynamics of inflation and output.

This re-examination is done using stylized forward-looking closed-economy monetary models. However, even using this simple computational laboratory, I show that if shocks are large or persistent enough to drive the policy rate to its lower bound for many periods, the various solution techniques may imply quite different dynamics. The same finding applies for the effects of forward guidance and for fiscal multipliers at the zero lower bound. In fact, it is the 'shadow' shock approach that delivers large effects of forward guidance and large fiscal multipliers. It should be stressed that the results in this paper are not meant to represent simulations relevant to any actual economy. The research is purely theoretical.

When simulating the implications of the solution methods for the fiscal multiplier, one can observe that the fiscal multiplier is especially large for unproductive fiscal spending. If fiscal spending is assumed to increase productivity, the value of the fiscal multiplier decreases under the shadow-shock solution. I explain this feature in terms of New Keynesian economics. The effect of fiscal spending on output at the zero lower bound works in New Keynesian models through manipulation of agents' intertemporal plans. Unproductive fiscal spending is a kind of negative technology shock that drives up inflation and, given nominal interest rates stacked at the lower bound, reduces the real interest rate. This real-interest-rate effect forces agents to reconsider their intertemporal plans and hence output (consumption) is boosted. A productive fiscal shock increases technology and reduces inflation, which mitigates the positive effect on consumption/output. If it were productive enough, fiscal spending would even have an adverse effect on output. One can view this implication of New Keynesian models as rather disturbing, as it casts doubt on the reliability of policy advice based on New Keynesian economics at the zero lower bound.

It should be stressed that this finding does not apply in the case of small fiscal multipliers in a low-interest-rate environment. It is only an indication that New Keynesian views should be considered with caution.

Finally, I evaluate the accuracy of the approaches investigated. The shadow-shock approach is dominated by other techniques. The most accurate solution techniques in the partially linear framework seem to be those proposed by Cai et al. (2015) and Guerrieri and Iacoviello (2015).

1. Introduction

Dynamic structural macroeconomic models with model-consistent forward-looking expectations, such as New Keynesian dynamic stochastic equilibrium models, represent a triumph of contemporary macroeconomics in the sense that they not only occupy a large part of the academic research agenda in modern macroeconomics, but are also widely used in central banks and other policy institutions for forecasting and policy analyses. These applied forecasting models are growing in size in order to capture various dimensions of reality that are considered important for real-world forecasting. Unlike their academic counterparts, therefore, they are typically linear, i.e., their behavioral equations are typically (log-)linearized. Linearity of forecasting models is beneficial for practical forecasting, since shock decomposition, ex-post forecast error decomposition, and the application of judgment are more easily dealt with in a linear framework. It is often tacitly assumed that non-linear effects can be either ignored (in normal times), or incorporated into the model-generated forecast by means of expert judgment.

After the Great Recession, many central banks have been operating in an environment of low interest rates, or even at the zero lower bound (ZLB). The existence of the ZLB is an important non-linearity in the crucial equation – the monetary policy reaction function. This non-linearity typically propagates quickly to the rest of the model equations, affecting both real variables and inflation. In response to this situation, one would like to work with the exact non-linear solution. For large-scale DSGE models, however, this is either unavailable or impractical. For these reasons, various ‘pragmatic’ solutions have been proposed that enable central banks to simulate and forecast in a fast way and that, in a sense, preserve (at least partially) the linearity of the original model.

In this paper, I investigate the implications of these approaches. I am interested in the following set of questions. **First**, are the implied trajectories of the various proposed solutions different from each other? And if so, how different? This may be important, as if they turned out to imply similar dynamics of key variables, choosing between them would be purely a matter of convenience. Indeed, while some papers claim that various methods yield the same dynamics,[1] my result does not support such statements: I find that the different proposed methods imply the same dynamics under limited circumstances only. Given this result, it makes sense to ask the following questions.

Second, I am interested in how various choices of model solutions influence policy analyses. In particular, I am interested in forward guidance and in fiscal multipliers at the ZLB. Are the effects of forward guidance large (or implausibly large, as some economists would argue) under all solution methods? Is the fiscal multiplier implausibly large at the ZLB under all methods? **Finally**, I am interested in whether any of the methods dominates the others in terms of solution accuracy.

To answer these – I believe – important questions, I use a set of computational experiments with stylized New Keynesian macroeconomic models. The models are in linearized form with the ZLB constraint as the sole non-linearity. Hence, my paper is cast in what is called the ‘partial linear’ framework.[2]

[1] As an example, Guerrieri and Iacoviello (2015) claim on page 27 that their method gives the same result as the method by Holden and Paetz (2012).

[2] To put it differently, I am interested in the partial linear framework only. Although I am aware of results indicating that (log)linearization may be a poor approximation for models where the ZLB binds (e.g. Braun et al., 2012; Fernández-Villaverde et al., 2015), this is irrelevant for this paper: most applied forecasting models are linear anyway. Although the distance of the (log)linearized versions of forecasting models from their (hypothetical) fully non-linear forms is interesting in itself, this issue is outside the scope of this paper.

The rest of the paper is organized as follows. The next section 2 presents the selected solution approaches to the ZLB in the partial linear framework. Section 3 simulates the prototype New Keynesian model under the various solution approaches and compares the responses of the economy to shocks and the effects of forward guidance. Section 4 asks about fiscal multipliers evaluated at the ZLB under the investigated solution techniques. Section 5 evaluates the accuracy of the investigated solution methods. The last section 6 concludes.

2. Methods

As I outlined in the introduction, I am interested in the solution of linear forward-looking structural macroeconomic models under the ZLB constraint. The general framework of this paper is therefore a model of the form:

$$
\begin{aligned}
\mathbf{U}_x x_t &= \mathbf{A}_x \mathbb{E}_t x_{t+1} + \mathbf{B}_x x_{t-1} + \mathbf{D} i_t + \mathbf{C}_x \varepsilon_t, && (2.1)\\
i_t &= \max(\text{ZLB}, \mathbf{A}_{\mathfrak{F}} \mathbb{E}_t x_{t+1} + \sum_j \mathbf{A}_j x_{t-j} + \rho_i i_{t-1} + \mathbf{C}_i \varepsilon_t), && (2.2)
\end{aligned}
$$

where x_t is the vector of model variables except for the policy rate i_t, ε_t is the vector of structural shocks, and $\mathbf{U}_x$, $\mathbf{A}_x$, $\mathbf{B}_x$, $\mathbf{D}$, $\mathbf{C}_x$, $\mathbf{A}_f$, $\mathbf{A}_j$, $\mathbf{C}_i$ are matrices of appropriate dimensions.

The conventional Taylor rule corresponds to the restriction $\mathbf{A}_{\mathfrak{F}} = 0$ and $\mathbf{A}_j = 0$ for $j > 0$, while the inflation-targeting regime would correspond to $\mathbf{A}_j = 0$ and the matrix $\mathbf{A}_{\mathfrak{F}}$ would have a non-zero entry only for inflation. The vector $\mathbf{C}_i$ is a selection vector in a typical situation: it has one at the position corresponding to the monetary policy shock and zeros elsewhere.

It is not trivial to solve large models of type (2.1)–(2.2), even if the ZLB constraint is the sole non-linearity. This is a difficult non-linearity as it is non-smooth and binds only outside the steady state, which means that the usual perturbation methods would not be helpful.

Nevertheless, modelers have proposed a number of 'pragmatic' methods that can be used to solve models like (2.1)–(2.2). In the rest of this section, I give an overview of the methods that are investigated in this paper.

All simulations were done using Matlab. The IRIS toolbox (Beneš et al., 2013) was used to find the reduced forms of the models considered and their impulse responses for the anticipated shocks, which are needed for the shadow-shock solutions. The implementations of the other methods (OccBin, NLCEQ, shadow-shock) were done by the author in his own Matlab codes.

2.1 Naive Simulations

The naive simulation imposes the ZLB 'on the fly': whenever the policy rate falls below the ZLB, it is set to the ZLB. It is very simple to implement this approach.

2.2 Shadow-shock Approaches

Several papers have proposed to enlarge Equation (2.2) with a series of 'shadow' expected shocks that are designed so that the ZLB holds.

Equation (2.2) is then replaced by the following equation:

$$i_t = \mathbf{A}_{\mathfrak{F}} \mathbb{E}_t x_{t+1} + \sum_j \mathbf{A}_j x_{t-j} + \rho_i i_{t-1} + \sum_{k=0}^{K-1} \alpha_k \varepsilon^i_{t+k|t}, \tag{2.3}$$

where α_k is the size of the shock and $\varepsilon^i_{t+k|t}$ are shocks to the monetary-policy rule equation (of unit size 1); these shocks at time t are known to hit the equation at time $t+k$.[3] The coefficients α_k are set so that the ZLB constraint holds.

There are multiple ways of finding coefficients α_k. One well-known approach was introduced by Laséen and Svensson (2011) in the context of evaluating the effects of fixing the interest rate to a pre-specified level. Their approach was used by del Negro et al. (2012) to evaluate the effects of fixing nominal interest rates. Assume that the interest rate is specified to be fixed (e.g. by a policy announcement) to values $[i^c_1, \ldots, i^c_K]$ and let $[i^u_1, \ldots, i^u_K]$ be the unconstrained trajectory. The method finds $[\alpha_0, \ldots, \alpha_{K-1}]$, which means that the coefficients are exactly determined: let $\mathfrak{I}$ denote a matrix whose k-th column represents the impulse responses of the policy rate to a shock $\varepsilon^i_{t+k-1|t}$; then it is obvious that the solution is $[\alpha_0, \ldots, \alpha_{K-1}]^T = \mathfrak{I}^{-1}([i^c_1, \ldots, i^c_K]^T - [i^u_1, \ldots, i^u_K]^T)$.

The approach can easily be adapted to stochastic simulations that respect the ZLB constraint. Let $\{i^u_t,\}$ be the unconstrained trajectory of policy rates that violates the ZLB approach for time $t = 1, \ldots, K$. Then the vector $[\alpha_0, \ldots, \alpha_{K-1}]^T = \mathfrak{I}^{-1}(\max(ZLB, [i^u_1, \ldots, i^u_K]^T) - [i^u_1, \ldots, i^u_K]^T)$ will impose the ZLB on the interest rate trajectory.

Holden and Paetz (2012) propose another interesting approach which finds the shadow shocks by means of the quadratic programming approach. I find that the two shadow-shock approaches lead often to the same solution (or very close solutions). Nevertheless, in some interesting cases (for example, in the case of forward guidance binding for many periods, or when the economy is hit by a series of disinflation shocks), I found that the quadratic programming approach sometimes fails to have a solution. Therefore, in this paper I use the former, i.e., Laséen and Svensson (2011), approach.

2.3 NLCEQ

Recently, Cai et al. (2015) proposed a powerful technique that can be used to globally approximate large non-linear stochastic models. The method is called NLCEQ (NON-LINEAR CERTAINTY EQUIVALENT APPROXIMATION METHOD). NLCEQ solves for a global non-linear decision rule for the non-stochastic problem. This non-linear decision rule obtained for a chosen set of states (nodes) is then applied to the original stochastic model. The decision rule for any state is then based on interpolation of the nodes; in the partial linear framework, the natural choice is piece-wise linearization.[4]

Although this is not discussed in the original article by Cai et al. (2015), this method is easily extended to time-varying models. This is beneficial for the simulation of perfectly credible forward guidance, which can be represented as a temporary change to the structural form of the monetary policy rule.

[3] Obviously, $\varepsilon^i_{t|t}$ is the standard unexpected monetary policy shock.

[4] I am grateful to Jevgenijs Steinbuks for pointing this out to me in an email.

2.4 OccBin Approach

The last method I compare is one called OccBin, which was introduced by Guerrieri and Iacoviello (2015). This method is based on the derivation of the time-varying reduced form from a sequence of two structural forms: one for the normal regime and the second for the constrained regime. The sequence of the structural form should respect the values of the policy rate.

As in the case of the NLCEQ method, the OccBin approach can easily be adapted to models with time-varying structural equations, hence it can be used for simulation of perfectly credible forward guidance.

3. Simulations with a New Keynesian Model

This section presents the results for the standard three-equation New Keynesian model. The simplest model reads as follows (for details see, for example, Gali, 2008):

$$\begin{aligned}
y_t &= \mathbb{E}_t y_{t+1} - 1/\sigma_0 \mathbb{E}_t(i_t - \pi_{t+1}) + \varepsilon_t^y, && (3.4)\\
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa_y y_t + \omega_t, && (3.5)\\
\omega_t &= \rho_\omega \omega_{t-1} + \varepsilon_t^\omega, && (3.6)\\
i_t &= \rho_i i_{t-1} + (1-\rho_i)(\theta \pi_t + \chi y_t) + \varepsilon_t^i. && (3.7)
\end{aligned}$$

The first equation (3.4) is the dynamic IS curve, which relates the current 'output gap' y_t to its next-period expected value $\mathbb{E}_t y_{t+1}$ and to the real interest rate $i_t - \pi_{t+1}$, which is given by the nominal policy rate i_t minus expected inflation $\mathbb{E}_t \pi_{t+1}$. The second equation (3.5) is the Phillips curve, which is subject to a cost-push shock ω_t that follows an exogenous autoregressive process (3.6). The last equation (3.7) is the monetary policy rule. ε_t^y is a 'demand' shock, ε_t^ω is the cost-push shock, and ε_t^i is a monetary policy shock.

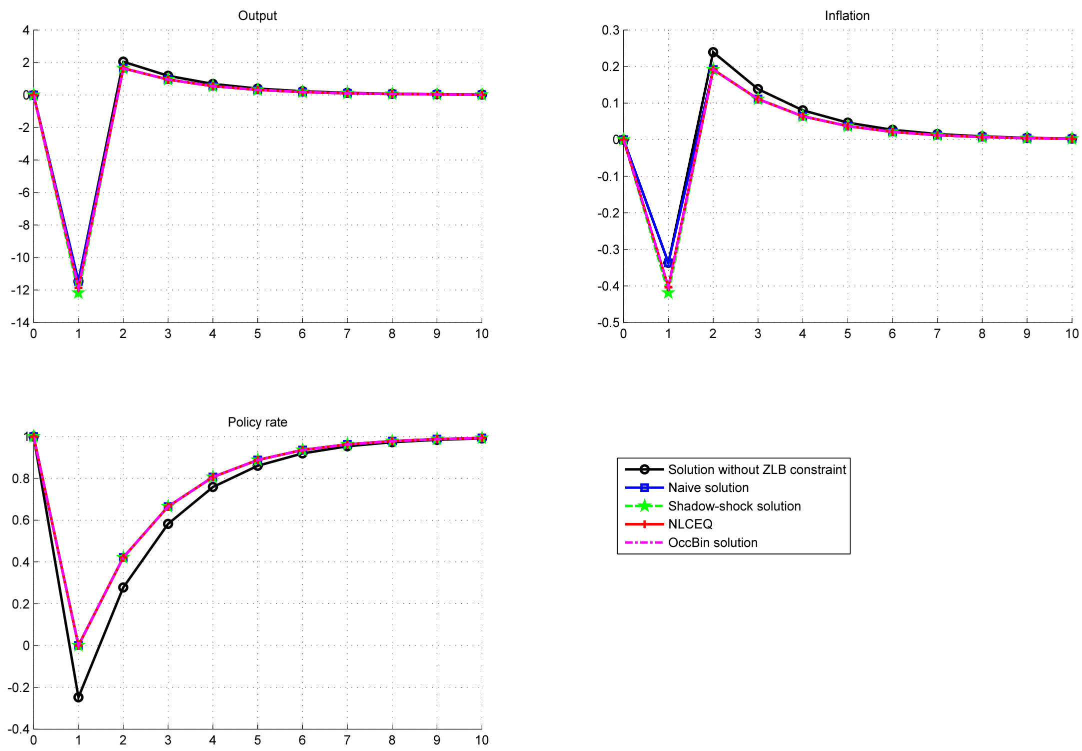
The variables in the system (3.4)–(3.7) are for simplicity presented as deviations from their steady states. Henceforth, I assume – without loss of generality – that the steady state for the policy rate is 1 and that the lower bound on the interest rate is zero.

For the numerical experiments, I employ the parameter values used by Blake (2012). These are: $\beta = 1$, $\sigma = 1$, $\kappa = 0.05$, $\rho_i = 0.8$, $\rho_\omega = 0.8$, $\theta = 1.5$, $\chi = 0.5$. These are standard textbook values and the monetary policy rule (3.7) satisfies the Taylor principle.

First, I simulated the impulse responses to negative demand and cost-push shocks designed to drive the policy rate below the ZLB for one period. Figure 1 shows the results for the negative demand shock. For both shocks, all the solution methods considered give virtually the same result: output and inflation fall, while inflation falls rather more than in the situation without the ZLB, so that the expected real interest rate is unchanged. The dynamics are very similar both qualitatively and quantitatively. The slight exception is the naive solution, where the drop in inflation in the first period is slightly shallower, but because of the nature of the naive solution this does not propagate to the output dynamics.

This equivalence of the solution methods breaks down for shocks large enough to drive the policy rate below the ZLB for more than one period. Likewise, the effects of policy announcements (forward guidance) differ according to the solution methods. To illustrate this, assume that the monetary authority announces forward guidance at the same time as the negative demand shock hits the

Figure 1: The Negative Demand Shock in the Basic New Keynesian Model



economy. This forward guidance regime pre-commits it to fixing interest rates at zero for the next four periods.

The results are visible in Figures 2 and 3. Figure 2 shows the trajectories for a demand shock of the same size as in Figure 1 but under forward guidance. Figure 3 shows the partial effect of forward guidance: it is the difference between the outcome of the demand shock with forward guidance and the outcome without this policy (i.e., between the trajectories in Figures 3 and 1). Now the results are quite different: the shadow-shock solutions find a very strong positive effect of forward-guidance on output, while the effect found by the NLCEQ and OccBin approaches is weaker.

Using various simulations (with other types of shocks or shocks of different magnitudes), I found that this is a general tendency: the shadow-shock approaches tend to find larger effects of forward guidance than the OccBin and NLCEQ approaches.

The model (3.4)–(3.7) is a forward-looking model without much backward-lookingness. One might ask whether the differences between solutions would disappear (or at least become less pronounced) in a more realistic model where shocks have more persistent effects. For this reason, I employ the empirical model by Lindé (2005) for the next set of experiments.

The Lindé (2005) model is characterized by three equations: the IS curve, the Phillips curve, and the monetary policy rule. The IS curve now reads as:

$$y_t = (1-\sigma_1)y_{t-1} + \sigma_1 \mathbb{E}_t y_{t+1} - 1/\sigma_0(i_t - \pi_{t+1}) + \varepsilon_t^y, \tag{3.8}$$

Figure 2: The Negative Demand Shock in the Basic New Keynesian Model with Simultaneous Introduction of Forward Guidance

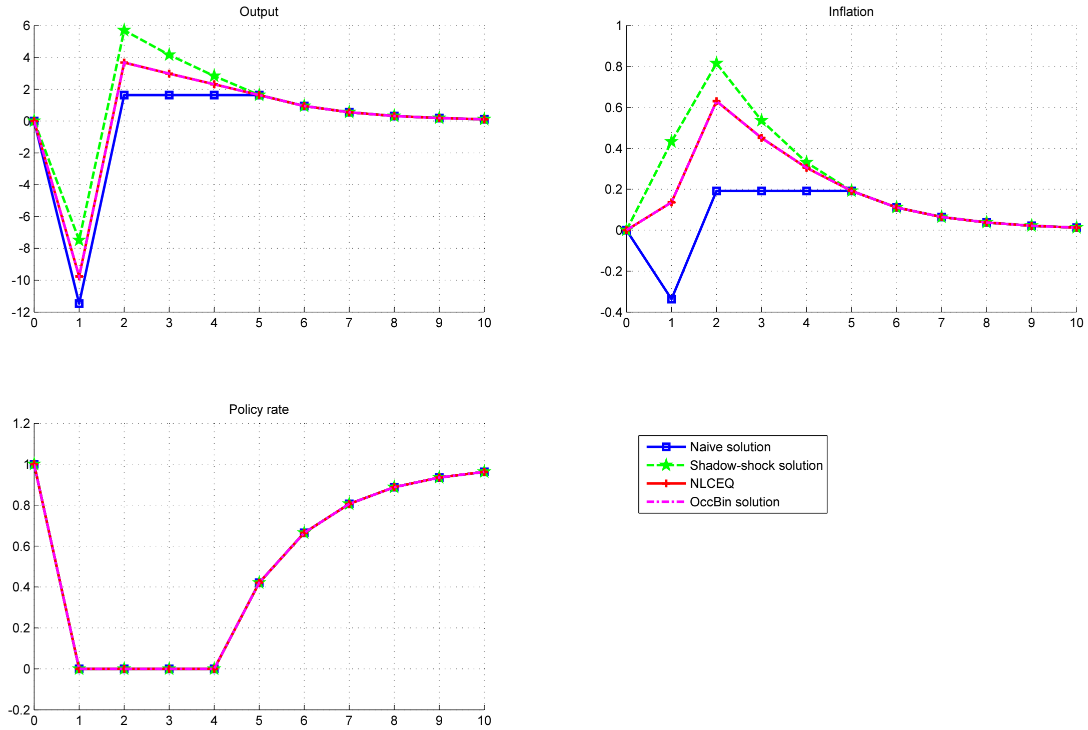
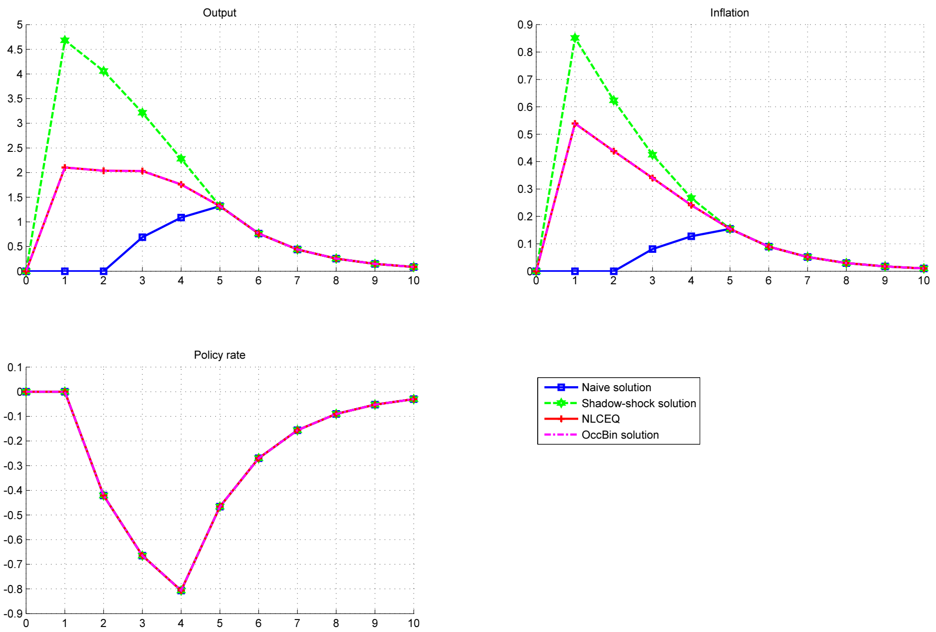


Figure 3: The Effect of Forward Guidance Introduced after the Negative Demand Shock (Basic New Keynesian model)



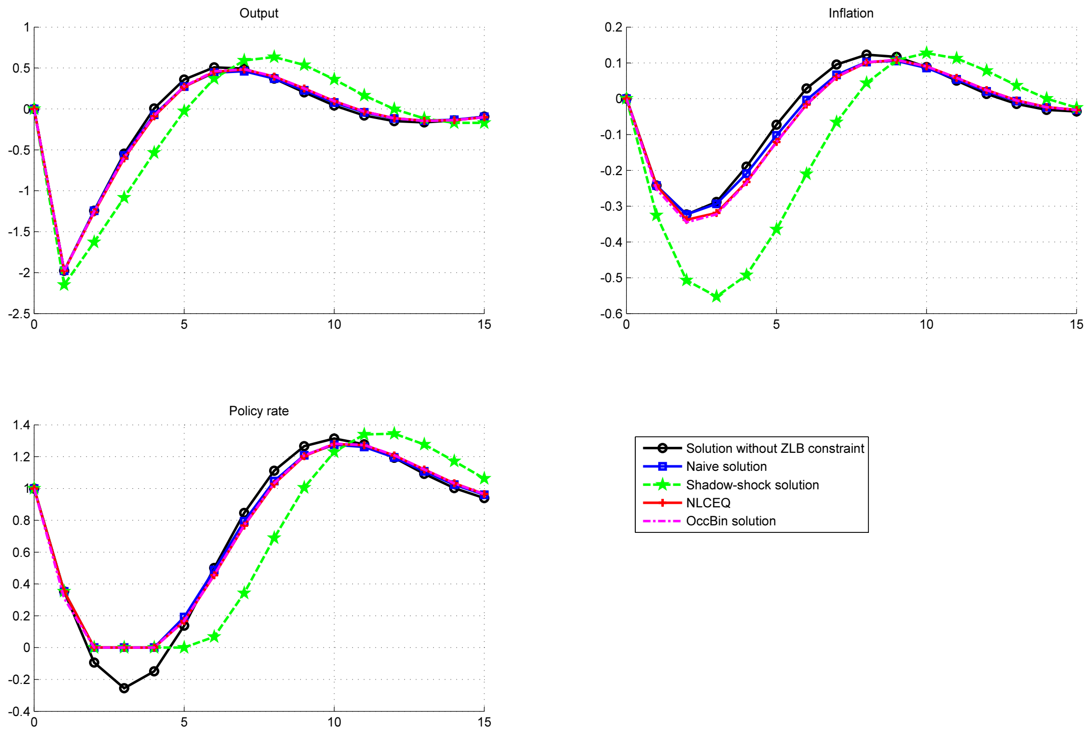
and the Phillips curve as:

$$\pi_t = (1-\beta_1)\pi_{t-1} + \beta_1 \mathbb{E}_t \pi_{t+1} + \kappa y_t + \varepsilon_t^{\pi}, \tag{3.9}$$

where ε_t^{π} is now an i.i.d. cost-push shock. The model is still closed by the monetary policy rule (3.7). I use the estimated parameters reported by Lindé (2005), which are $\sigma_1 = 0.425$, $\beta_1 = 0.457$, $\kappa = 0.048$, and $1/\sigma_0 = 0.156$.

For this model, I simulated the responses for a large negative demand shock that would drive the policy rate below the ZLB for three periods. The results are visible in Figure 4. Now the dynamics implied by the various methods differ. While the NLCEQ and OccBin solution methods yield very close dynamics,[5] the shadow-shock solutions are quite different.

Figure 4: The Negative Demand Shock in the Lindé (2005) Model



I also experimented with forward guidance in the Lindé (2005) model. I simulated the response to the same negative demand shock as in Figure 4 under the assumption that the monetary authority now announces that it is fixing rates for eight periods. The results are displayed in Figures 5 and 6. Again, Figure 5 shows the implied trajectories, while Figure 6 shows the partial effect of forward guidance. The shadow-shock solution implies much larger effects of the policy announcement, but the effect on output is now negative. The effect of forward guidance is again lower for the NLCEQ/OccBin solutions.

Finally, one might ask whether an alternative monetary policy regime would change these conclusions. I simulated both variants of the New Keynesian models under inflation-targeting monetary

[5] Based on simulations with another set of models, I conjecture that although the two methods do not yield *identical* dynamics, they are always close to each other. I failed to find a model or a parametrization that produced qualitatively different reactions to shocks for these two methods.

Figure 5: The Negative Demand Shock in the Lindé (2005) Model with Simultaneous Introduction of Forward Guidance

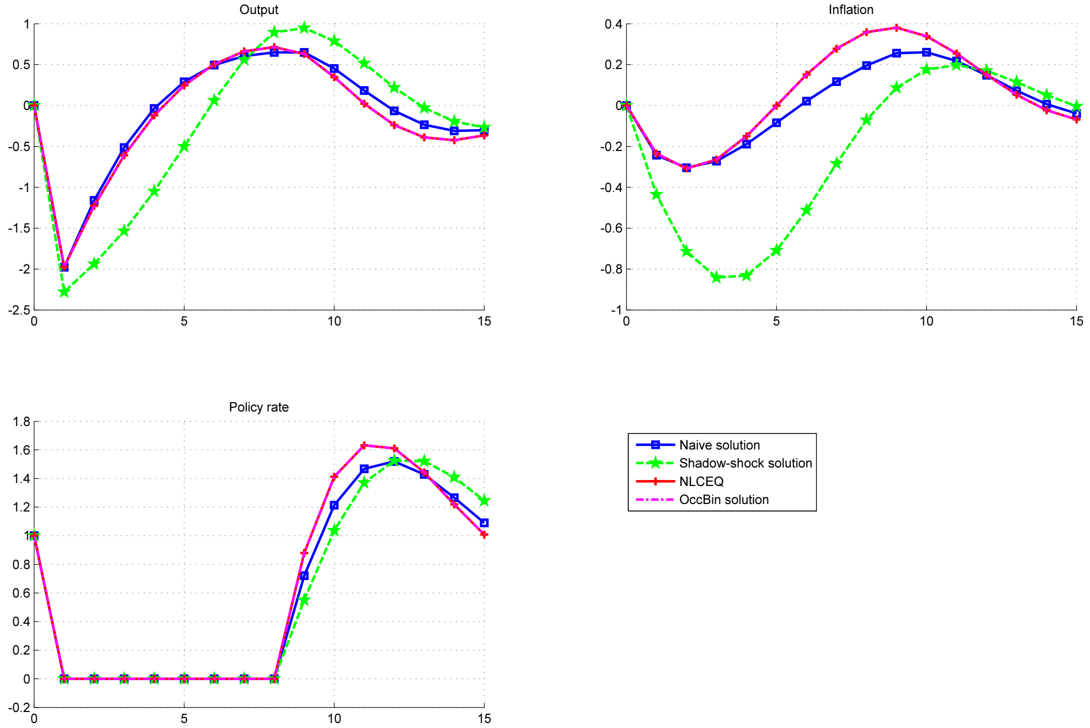
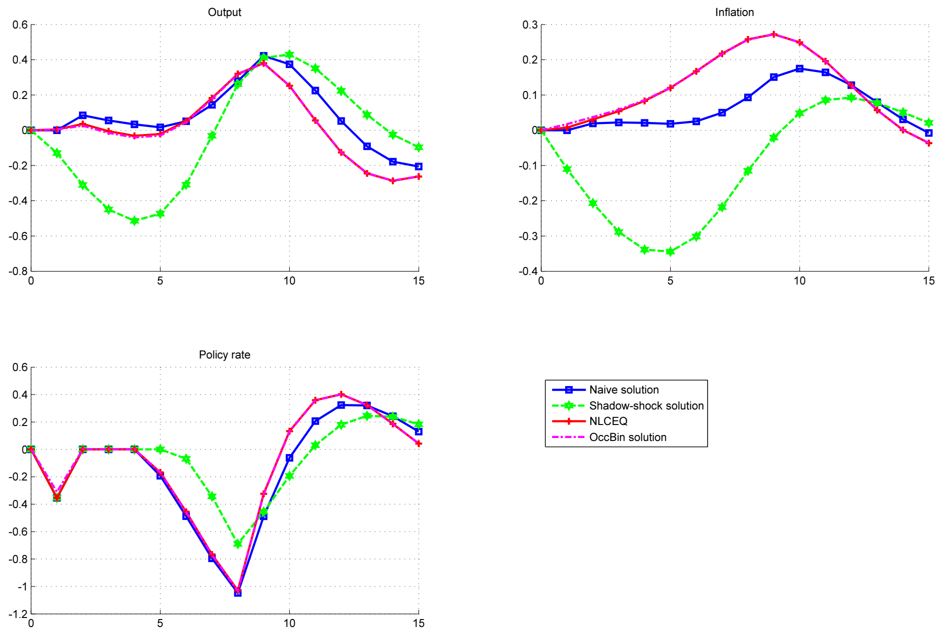


Figure 6: The Effect of Forward Guidance Introduced after the Negative Demand Shock – Lindé (2005) Model



policy, i.e., replacing the monetary policy rule (3.7) with the inflation-targeting rule:

$$i_t = \rho_i i_{t-1} + \kappa_i \mathbb{E}_t \pi_{t+1}, \tag{3.10}$$

with $\kappa_i > 1$. The conclusions continue to hold: the OccBin and NLCEQ solution approaches give very similar results, while the shadow-shock solutions are different, especially for large or persistent shocks and/or under forward guidance. Also, for this modification the effects of forward guidance are much larger for the shadow-shock solutions than for the OccBin and NLCEQ solution approaches.

4. Fiscal Multipliers at the ZLB

Forward guidance is not the only possible policy reaction to economic slack when interest rates are low. Economists, policy makers, and the public alike are interested in the stabilizing effects of fiscal policy in times of macroeconomic distress. In fact, it has been suggested that fiscal policy can play an important role; see Coenen et al. (2012) for model-based assessments. It has been also suggested that fiscal multipliers are especially large at the ZLB; see, for example, Christiano et al. (2011).

Nevertheless, there are widely divergent views on this issue. There are economists such as Stiglitz (2014) who are strong proponents of increasing public spending, while other economists argue against fiscal policy interventions. The difference between these two opposite views can be rephrased as a disagreement on the magnitude of pubic spending multipliers: conservative economists tend to consider the public spending multiplier to be small (or even negative), while the other side would argue that in times of macroeconomic crisis, the multiplier can be large, especially if the government spends money wisely – Stiglitz (2014) explicitly argues for productive public spending (public investment in infrastructure and research) as an effective tool of macroeconomic stabilization policy.

As mentioned above, structural macroeconomic models tend to support the view that fiscal multipliers are large at the ZLB. In this part of the paper, I inquire whether this conclusion depends on the way the model is solved. This inquiry is motivated by Braun et al. (2012), who show that the values of the fiscal multiplier at the ZLB in the log-linearized solution differ significantly from the global non-linear solution of a New Keynesian model. To answer my inquiry, I make a simple extension of the basic New Keynesian model. The model now reads as follows:

$$\begin{aligned}
y_t &= (1-\sigma_1)y_{t-1} + \sigma_1 \mathbb{E}_t y_{t+1} - 1/\sigma_0(i_t - \pi_{t+1}) + \sigma_g g_t + \varepsilon_t^y, &(4.11)\\
\pi_t &= (1-\beta_1)\pi_{t-1} + \beta_1 \mathbb{E}_t \pi_{t+1} + \kappa_y(y_t - \psi a_t) + \varepsilon_t^\pi, &(4.12)\\
i_t &= \rho_i i_{t-1} + (1-\rho_i)(\theta \pi_t + \chi y_t) + \varepsilon_t^i, &(4.13)\\
g_t &= \rho_g g_{t-1} + \varepsilon_t^g, &(4.14)\\
a_t &= \rho_a a_{t-1} + \kappa_g g_t + \varepsilon_t^a, &(4.15)
\end{aligned}$$

where y_t is the output gap, π_t is the deviation of inflation from its target, i_t is the policy rate, g_t is government spending, and a_t is productivity. The first equation (4.11) is the forward-looking IS curve, which explicitly accounts for the positive effect of government spending.[6] The second

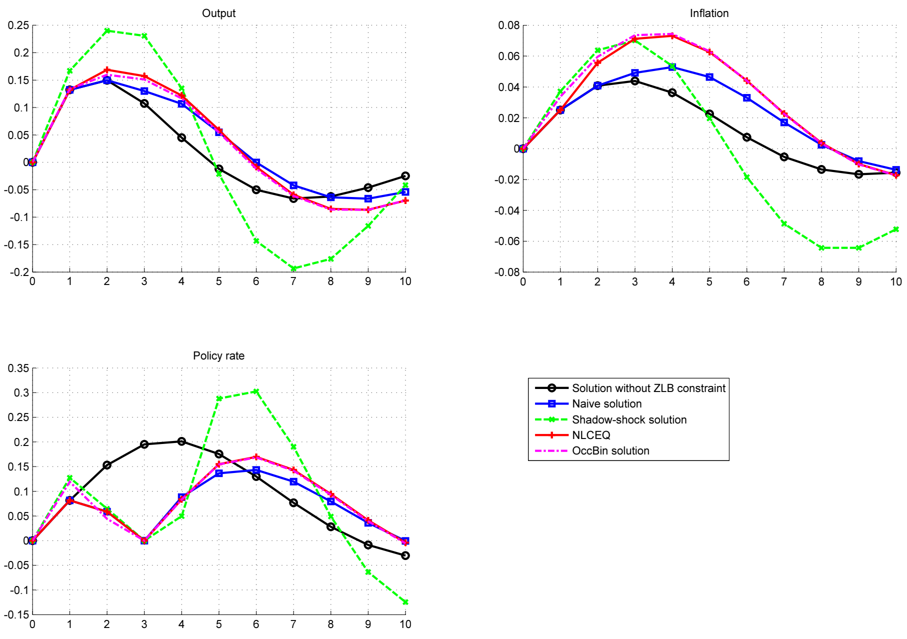
[6] Although the IS curve of this model is derived in a somewhat ad-hoc way, the impulse responses are reasonable. The reader may ask why I have not chosen a model derived from explicit micro-optimization, such as the one by Christiano et al. (2011). The issue is that the well-known model by Christiano et al. (2011) has a particularly unusual feature: if this model is enlarged by productivity shocks, an increase in productivity would cause a decrease

equation (4.12) is the Phillips curve under the assumption that $y_t - \psi a_t$ represents real marginal costs. The third equation is the monetary policy rule. The fourth equation is the (purely exogenous) process for government spending, while the last equation is the law of motion for productivity. The model allows for the possibility that productivity is boosted by government spending.

The benchmark parametrization reflects the parameter values from Lindé (2005). The values of the rest of the parameters are set as follows: $\sigma_g = 0.15$, $\psi = 0.15$, $\kappa_g = 0, \rho_i = 0.8, \rho_g = 0.8$. I then simulate the effect of an increase in government spending during the negative demand shock (the size of this shock is the same as in Figure 4; the government spending shock is normalized so that it would immediately increase output by 0.10 during the time when the ZLB does not bind).

The implied multipliers under the ZLB (which binds due to the negative demand shock) are seen in Figure 7. This figure does not display the trajectories as such, but it does display the *partial effect* of the fiscal shock, i.e., the multiplier. Since the model is linear, the black line (i.e., the solution that is not constrained by the ZLB) is the standard multiplier when the ZLB does not bind. Similarly to the case of forward guidance, it seems that the shadow-shock solutions tend to find large effects of government spending at the ZLB. The OccBin and NLCEQ approaches also imply larger multipliers than in normal times, but quantitatively the rise in the multiplier is more limited.

Figure 7: Fiscal Multipliers at the ZLB ($\kappa_g = 0$)

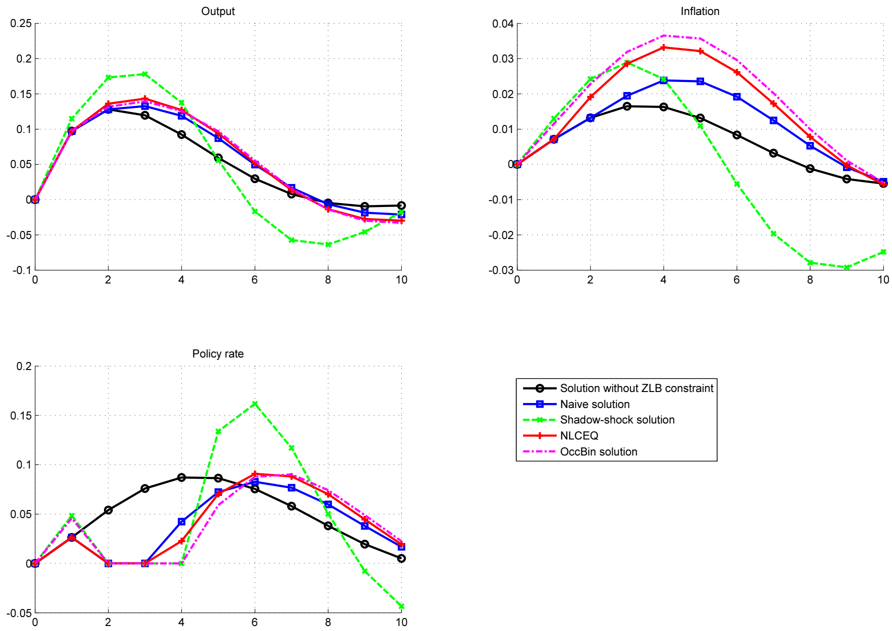


This conclusion – that the value of the multiplier depends on the solution method – suggests that the widely held conclusion of large multipliers at the ZLB is debatable.

in output, consumption, and hours worked (a detailed proof is available on request). The underlying intuition is that under the utility function assumed in their model agents are really willing to substitute consumption with leisure. An increase in productivity enables them to produce output with fewer hours worked, and the ‘perverse’ effect on consumption is caused by the fact that the substitution effect from leisure to consumption outweighs the direct effect of the expansion of the choice set. In other words, in the model of Christiano et al. (2011), consumption is an inferior good relative to leisure. This also explains why they are able to generate a fiscal multiplier larger than one in the benchmark specification even without the ZLB: government spending acts like a negative technology shock, which boosts consumption and output via this unusual substitution effect from leisure to consumption.

The situation is even more involved. Let us now assume a change in the value of parameter κ_g and assume that fiscal spending increases productivity. Figure 8 now displays the partial effect of the fiscal shock for $\kappa_g = 0.5$.[7] Under this assumption, the partial effects of fiscal spending on output at the ZLB are lower than in the model with $\kappa_g = 0$. Going to the model with $\kappa_g = 1$ (not reported but available on request), the positive effect of government spending on output at the ZLB would completely disappear under the shadow-shock solution. Taken literally, this would mean that fiscal spending at the ZLB should be unproductive!

Figure 8: Fiscal Multipliers at the ZLB ($\kappa_g = 0.5$***)***



These results may seem rather alarming in that they show that the New Keynesian conclusions about the value of the fiscal multiplier at the ZLB are very sensitive to the solution method and to the details of the model specification. In the rest of this section, I offer the intuition behind these findings.

Popular discussions supporting the role of fiscal policy are usually cast in the logic of textbook neo-Keynesian economics: fiscal policy is a tool for helping an economy constrained by insufficient demand. This is an attractive option especially when monetary policy is constrained by the ZLB. As we have seen, New Keynesian models also seem to support the notion of positive and large fiscal multipliers in a situation where monetary policy is constrained at the ZLB. *Seemingly*, the New Keynesian and neo-Keynesian views and policy recommendations are consistent. But this consensus is illusory, for at least two reasons.

First, the concept of insufficient demand is alien to New Keynesian economics: low consumption at the ZLB is an outcome of rational decisions of households about the intertemporal consumption profile rather than a consequence of macroeconomic slack as in the neo-Keynesian world. Fiscal policy is effective in the old- and neo-Keynesian views because it can help alleviate macroeconomic slack, while the same policy is effective in New Keynesian models at the ZLB because it forces

[7] This change in the value of κ_g influences the impulse responses. To enable comparison between Figures 7 and 8, I recalibrate the size of the fiscal shock so that it yields the same effect on output outside the ZLB.

agents to reconsider their intertemporal plans. Putting it differently, consumers living in the traditional Keynesian world are *forced* to reduce consumption during crises times: the markets do not work properly and the government can fix them through wisely chosen fiscal policy. Agents in the New Keynesian world (at the ZLB) instead *choose* to consume little. In principle, they can consume more, but *they do not want to* because the high real interest rate makes this option unattractive.[8]

Second, even if one accepts the logic of New Keynesian models, the analyses support unproductive spending like those with large multipliers. The reason is that for fiscal policy to work at the ZLB in the New Keynesian world, it should primarily reduce real interest rates (which in the ZLB situation means increasing inflation), hence it should resemble a negative technology shock. If fiscal spending increases productivity (for example, by investing in transport infrastructure or education, as Stiglitz (2014) proposes), fiscal policy would not help boost consumption at the ZLB in the New Keynesian world. This goes directly against the recommendation of economists such as Stiglitz who argue for *productive* public spending.

The lesson is that the fiscal policy implications of New Keynesian models at the ZLB should be taken with caution. They should not be interpreted as support for small or negative public spending multipliers at the ZLB. The correct interpretation is that New Keynesian models are probably not very helpful for our understanding of this important question.

5. Accuracy of Solution Methods

In this part of the paper, I investigate the numerical accuracy of the selected methods. All the solution methods imply a law of motion that maps the past state x_{t-1} and the current shock ε_t to the current state x_t:

$$x_t \leftarrow \mathscr{S}(x_{t-1}, \varepsilon_t).$$

The interesting question is to compare the accuracy of the laws of motion implied by the described methods.

I use the following measure of accuracy: for the k-th variable of the structural model (2.1), I define the following criterion:

$$\eta_k(x_0, \varepsilon) = \log_{10} \frac{\left| x_1^k - \left(\tilde{\mathbf{A}}_x^k \widehat{\mathbb{E}(x_2|x_1)} + \tilde{\mathbf{B}}_x^k x_0 + \tilde{\mathbf{C}}_x^k \varepsilon \right) \right|}{\max(1, |x_{ss}^k|)}, \tag{5.16}$$

where x_1^k is the k-th element of the vector $x_1 \equiv \mathscr{S}(x_0, \varepsilon)$, $\tilde{\mathbf{A}}_x^k$ is the k-th row of the matrix $\mathbf{U}_x^{-1}\mathbf{A}_x$ (and analogously for $\tilde{\mathbf{B}}_x^k$ and $\tilde{\mathbf{C}}_x^k$), and x_{ss}^k is the steady state of the k-th variable. The expression $\widehat{\mathbb{E}(x_2|x_1)}$ approximates the conditional expectation of x_2 given the state x_1:

$$\widehat{\mathbb{E}(x_2|x_1)} \cong \frac{1}{M} \sum_{m=1}^{M} \mathscr{S}(x_1, \varepsilon^{(m)}) \equiv \frac{1}{M} \sum_{m=1}^{M} \mathscr{S}\left(\mathscr{S}(x_0, \varepsilon), \varepsilon^{(m)} \right), \tag{5.17}$$

[8] To be fair, this individually rational decision need not be socially optimal, and because of various nominal and real rigidities it is probably not. But this externality is of secondary importance compared to the effect of high real interest rates on the intertemporal profile of consumption.

where $\varepsilon^{(m)}$ are draws from the distribution of shocks.[9] In my experiments, I use $M = 10^5$ in the approximation of the conditional expectation in (5.17).

I evaluate the criterion on the Lindé (2005) model. I simulate the ergodic distribution of endogenous variables for the model (the model equations and parametrization are given in Section 3) under the naive solver. From this ergodic distribution, I randomly draw a set of points $\{x_b\}_{b=1}^{B}$ and for each solution method I compute $\eta_k(x_b, \varepsilon_b)$ for randomly drawn shocks ε_b.

The next table displays three norms of the vectors $[\eta_k(x_1, \varepsilon_1), \dots \eta_k(x_B, \varepsilon_B)]$ for the three endogenous variables. The norms considered are: $\mathscr{L}^1(\eta) = 1/N \sum_{n=1}^{N} |\eta_n|$, $\mathscr{L}^2(\eta) = \sqrt{1/N \sum_{n=1}^{N} \eta_n^2}$, and $\mathscr{L}^\infty(\eta) = \max_n |\eta_n|$.

The comparison is done for the shadow-shock solution and for the OccBin solution (the NLCEQ approach produces virtually the same results as the OccBin solution and is therefore omitted). The OccBin solution clearly outperforms the shadow-shock solution in any norm considered and with the NLCEQ approach is the clear winner of the accuracy contest.

Table 1: Accuracy of Solutions

Variable	Shadow-shock solution			OccBin		
	$\mathscr{L}^1$	$\mathscr{L}^2$	$\mathscr{L}^\infty$	$\mathscr{L}^1$	$\mathscr{L}^2$	$\mathscr{L}^\infty$
Output	1.7090	1.9620	3.9612	0.0074	0.0101	0.0201
Inflation	1.1401	1.5373	3.3983	0.0072	0.0097	0.0190
Policy rate	1.5621	2.3718	7.6677	0.0073	0.0119	0.0277

[9] The numerical accuracy can be increased by considering quasi-random variables. This yields a dramatic improvement in the accuracy of the approximation of the conditional expectation.

6. Conclusion

In this paper, I compared several approaches that can be useful for solving structural forward-looking monetary models in the partially linear framework, i.e., in the framework where the lower bound on the interest rate is the only non-linearity.

The approaches compared are: (i) the naive approach, which achieves the lower bound 'on the fly', (ii) shadow-shock approaches, which extend the monetary policy rule to include unexpected or expected shocks designed so that the lower bound is satisfied, (iii) the non-linear certainty equivalent (NLCEQ) approach by Cai et al. (2015), and (iv) the OccBin approach by Guerrieri and Iacoviello (2015).

There are three main findings in this paper. First, the dynamics implied by the NLCEQ approach and the OccBin approach are close, and sometimes hardly distinguishable from each other. The other approaches may yield significantly different dynamics, especially for large and/or persistent shocks. This disproves some claims in recent literature.

Second, the puzzling behaviors of structural forward-looking models (such as the forward-guidance puzzle and large fiscal multipliers) are present especially under the shadow-shock solutions. The NLCEQ and OccBin approaches yield less puzzling dynamics. Third, it seems that NLCEQ and OccBin outperform other approaches in terms of numerical accuracy. In sum, these two methods seem to be preferable for the partial linear framework investigated in this paper.

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