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# **Eigenvalue Decomposition of Time Series with Application to the Czech Business Cycle**

Jaromír Beneš\*, David Vávra†

## **Abstract**

We follow a Beveridge-Nelson like time series decomposition method (into trend, business cycle and irregular components), and examine a stylized model of price inflation determination using the Czech data. We characterize the estimated components of CPI, IPPI and import inflations, together with the real production wage and real output, and survey their basic correlation properties; furthermore we compute structural innovations imposing restrictions on their long-run effects, draw the impulse responses, and test the results by means of bootstrap simulation. We conclude that major room for further refinement of the research is found in two areas: First, from an economist's perspective, in the construction of real marginal cost indicators, and second, from a statistician's perspective, in further investigation of the robustness of the method.

**JEL Codes:** C32, E32

**Keywords:** bootstrap, business cycle, inflation, structural VAR, time series decomposition

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## **Nontechnical Summary**

This working paper develops a methodology for time series decomposition à la Beveridge and Nelson (BN), and applies it to the Czech data. It adds to the existing empirical literature that has so far, especially as concerns the Czech data, seemed to prefer the unobserved component (UC) methodology. BN decompositions have the advantage of imposing less economic structure on the underlying series; the uncovered cycles and trends are therefore more driven by the data themselves. This methodology then appears a useful complement to the UC techniques based on an a priori theoretical model. Cross-checking the results from the two types of methods can lead to a more robust assessment of the cyclical position of the economy to the benefit of monetary policy making. In addition, as this working paper demonstrates, the application of BN methodology can give additional insight into the workings of the economy at specific moments of its history.

The standard BN methodology decomposes a time series (or a set of them) into its trend and transitory component by identifying permanent and temporary effects of random innovations. The actual time series is made up of the two and, as a consequence, the trend and temporary components are (typically) negatively correlated. This stands in contrast to more common methods of business cycle identification, which identify the innovations themselves, and makes the empirical comparison of the two methods cumbersome.

In addition, the transitory component contains (apart from the business cycle) also irregular fluctuations. In order to facilitate the interpretation, we then further decompose the transitory element of the BN decomposition into its business-cycle and irregular fluctuations by imposing eigenvalue restrictions corresponding the typical business-cycle frequencies.

However, the business-cycle fluctuations (and the corresponding trend components) extracted by the eigenvalue decomposition have the property of showing pressures on the variable in the long run in the opposite directions, hence they must be warily interpreted. In other words, a cyclical peak is an indicator of an innovation or a cumulation of innovations that cause an ultimate decline in the respective variable over the long run. This might be in sharp contrast to the intuition gained from traditional UC models.

We apply the multivariate eigenvalue decomposition to extract the trend, business-cycle and irregular components from a set of measures of Czech inflation and real economic activity in order to investigate the workings of the economy typified by stylized Phillips curve mechanics. To this end, we combine net CPI inflation, industrial PPI inflation, import price inflation, the real production wage in the business sector, and real gross domestic product. The last two measures are to capture movements in the real marginal cost of domestic value added. The robustness of the results is checked by bootstrap estimation.

The analysis yields several important conclusions. Overall, we find evidence favouring the hypothesis of a) faster import price and exchange rate pass-through into domestic producer prices than directly into those of domestic consumers, b) of the nominal wage being more sticky than

the nominal price, as real wage changes are induced by movements in the underlying price indices. This is especially illuminating in the period of 2000 and 2001, when import prices were rapidly changing. The experience from this period hints at substantial technology or labour supply (intratemporal preference) shocks. Importantly, a Phillips curve type of relationship is detected, as the behavior of the permanent components fails to exhibit strong signs of superneutrality, with disinflation having negative effects on real variables. However, the real marginal cost concept hypothesized in the Phillips curve in our working paper yields counterintuitive results, namely an inverse effect of real output on the one hand and the real wage on the other hand into the CPI and IPPI, and needs to be further refined.

Last, the overall statistical properties of eigenvalue decomposition are subject to further thorough research, as they display a significant skewness.

## 1. Introduction

Increasing attention has been paid to extracting the cyclical and trend components from observed time series at central banks in the last decade as many of them have moved towards more or less explicit inflation targeting regimes. This type of monetary policy raises demands for a deep understanding of business cycle mechanisms and for robust knowledge of the current business cycle position of the economy, particularly with regard to cyclical fluctuations and interactions between inflation and real economic activity. This also explains the recent preoccupation of many central bank researchers with multivariate techniques that explore elementary economic relationships, while being simultaneously more immune to common problems of many simple filters, such as those of end-points.

Two strands of multivariate filtering approaches have been developed for these purposes within the time domain<sup>1</sup>: first, unobserved-components models (UC), and second, more classical Beveridge and Nelson (1981) (BN) types of decompositions (BN). A typical association has the former rely on a prior about the structure of the economy, while the latter observe more the intrinsic features of the data series. However, as noted by Morley, Nelson and Zivot (2003), both methods are in fact theoretically equivalent. Yet, they usually yield starkly different results. The authors show this fact can be ascribed to fundamental differences made usually in the assumptions preceding the empirical analyses. Most UC models compute projections of components that are orthogonal by assumption, i.e. recover processes driven by uncorrelated innovations (typically random walk plus stationary autoregression), while BN models in fact identify the permanent and temporary effects of the same innovation, or a set of them, on the observed time series, which is in turn consistent with a strong (mainly negative) correlation of these components.

Recently, various central banks have favoured extensive research in the area of UC models, see e.g. research at the CNB: Benes, Vavra, Vlcek (2002), Coats, Laxton and Rose (2003) or Benes and N'Diaye (2004). As a product, a suite of structural multivariate models have been created to support forecasting and policy analysis systems, particularly in association with issues regarding the cycle in real output, the real exchange rate, the real interest rate, and the real marginal cost of producers. On the other hand, the data-driven decompositions of the BN type have been relatively neglected.

This neglect is also a characteristic of the modeling research of the Czech National Bank, which otherwise prides itself on being a front-user of UC filtering techniques. Against this backdrop, this working paper is a first attempt to explore the potential of the BN approach, both in its theory and in its application to the Czech data. We examine the business-cycle components and basic properties of consumer price inflation (as the target variable of monetary policy) and its main medium-term supply-side determinants in a small open economy, i.e. imported inflation and the marginal cost of domestic production.

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<sup>1</sup>Band-pass filters are then a third strand of research in this area, coming from the frequency domain, see Baxter and King (1995) or Christiano and Fitzgerald (1999).

The working paper is organized as follows. In Section 2 we review the classical Beveridge-Nelson concept and relying upon Casals, Jerez and Sotoca (2002) we introduce the canonical state-space representation of multivariate time series models as a convenient device for computation of the BN decomposition into trend and transitory components. We naturally extend this concept in Section 3 to a general decomposition or extraction based upon the specific required properties of the underlying (or carrying) eigenvalues. In Section 4 we discuss the structural identification of time series models in their canonical state-space representation, i.e. computation of structural orthonormal innovations that meet specific impulse-response restrictions. Next, we apply the multivariate decomposition method and structural identification scheme to a set of measures of Czech inflation and real economic activity and examine their cyclical properties in Sections 5 and 6. Section 7 briefly presents bootstrap-simulation results that test the estimator for robustness, and Section 8 concludes.

## 2. Beveridge-Nelson Decomposition and the Canonical State-Space Form

We consider a class of finite-order VAR processes with possibly integrated or cointegrated elements of maximum order I(1) or CI(1,1), respectively, and with a simple deterministic component. We limit the scope of this note by the following vector error-correction (VEC) representation,

$$A(L)\Delta y_t = \Pi y_{t-1} + d + \varepsilon_t, \quad (1)$$

where  $y_t$  is a  $k \times 1$  vector process with  $\varepsilon_t$  being its i.i.d. prediction innovations distributed as  $WN(0, \Omega)$ , and  $D$  is a deterministic constant vector. The matrix polynomial  $A(L) = I - A_1 L - \dots - A_p L^p$  of finite order  $p$  is assumed stable in that  $|A(z)| = 0$  for  $|z| > 1$  only. Then the rank of  $\Pi$  determines the nature of the process in terms of its integration and cointegration: if  $\Pi$  is a full-rank matrix, then all elements of  $y_t$  are stationary, if  $\Pi$  is zero, then all elements of  $y_t$  are integrated but not cointegrated, and if  $\Pi$  has lower-than-full rank,  $r < k$ , then an  $r$ -dimensional cointegration space exists, or in other words,  $y_t$  is driven by  $k - r$  separate stochastic trends. In the latter case, we may write  $\Pi$  as a product of two  $k \times r$  full-rank matrices,  $\alpha \beta'$ , where the columns of  $\beta$  span the space of cointegration vectors.<sup>2</sup>

Beveridge and Nelson (1981) define the trend component (Beveridge-Nelson trend, BNT or  $y_t^T$ , henceforth), as the limit forecast of the process net of the future projection of its deterministic drift.<sup>3</sup> To be more specific, we first rewrite (1) into its vector moving-average (VMA) representation based upon a modification of the Granger representation theorem by e.g. Johansen (1995),

$$y_t = \tilde{C} \sum_{\tau=1}^t (d + \varepsilon_\tau) + C(L)(d + \varepsilon_t) + P y_0, \quad (2)$$

which consists of a random-walk (unit-root) and a stationary component. The matrix  $\tilde{C} = \beta_\perp [\alpha'_\perp A(1) \beta_\perp]^{-1} \alpha'_\perp$  gives the permanent effect of an innovation on  $y_t$ ,  $C(L)$  is a stable matrix

<sup>2</sup>We exclude seasonal unit roots, i.e. unit eigenvalues with a nonzero imaginary part, or  $-1$  from our considerations in this working paper.

<sup>3</sup>Their verbal interpretation of the trend definition is “current observed value [...] plus all forecastable future changes in the series beyond the mean rate of drift.” Beveridge and Nelson (1981), p. 156.



polynomial, and the matrix  $P = \beta(\beta'\beta)^{-1}$  extracts the least possible information for forming the necessary initial condition. Then the BNT is clearly

$$y_t^T = \tilde{C} \sum_{\tau=1}^t (d + \varepsilon_\tau) + C(1)d + Py_0, \quad (3)$$

whereas

$$y_t^C = C(L)\varepsilon_t$$

may be interpreted as a mean-zero transitory (cyclical) component.

We make use of the exact structural decomposition by Casals, Jerez and Sotoca (2002) and introduce a straightforward convenient representation of the process (1) to deal with this type of decomposition. First, we expand (1) into its VAR representation,

$$B(L)y_t = d + \varepsilon_t,$$

where the matrix polynomial  $B(L) = A(L)(I - IL) - \Pi L = I - B_1L - \dots - B_{p+1}L^{p+1}$  may now possibly contain a number of unit eigenvalues depending on the integration and cointegration within  $y_t$ : more specifically, the number of them equals  $k - r$  where  $k$  is the number of elements of  $y_t$  following an I(1) whereas  $r$  is the dimension of the cointegration space. Second, we write the equivalent first-order VAR representation by introducing substitutions for lagged  $y_t$ 's,

$$F(L)Y_t = D + E\varepsilon_t, \quad (4)$$

where

$$\begin{aligned} Y_t &= [y_t', y_{t-1}', \dots, y_{t-p}']', \\ D &= [d', 0', \dots, 0']', \\ E &= [I, 0', \dots, 0']', \end{aligned}$$

and  $F(L) = I + F_1L$  with

$$F_1 = \begin{bmatrix} B_1 & B_2 & \dots & B_p & B_{p+1} \\ I & 0 & \dots & 0 & 0 \\ & \ddots & & & \\ 0 & 0 & \dots & I & 0 \end{bmatrix}.$$

Finally, we compute the real-form Jordan canonical decomposition,

$$F_1 = PT P^{-1},$$

so that (1) now becomes a state-space model,

$$y_t = Mx_t, \quad (5)$$

$$x_t = Tx_{t-1} + U + V\varepsilon_t, \quad (6)$$

where  $T$  is a block-diagonal transition matrix with blocks of maximum size  $2 \times 2$ ,  $x_t = P^{-1}Y_t$  is a transformed state vector,  $M = [I, 0', \dots, 0']'P$  is a measurement matrix extracting the current-dated realization of  $y_t$  only, and  $U = P^{-1}D$  and  $V = P^{-1}E$ . Eqs. (5)–(6) now constitute what

we refer to as the canonical state-space (CSS) representation of a given process. For the sake of convenience of future references we furthermore assume that the diagonal elements and diagonal blocks of  $T$  are ordered by the modulus of their underlying eigenvalues in descending order, i.e. random walks come first, and introduce the following block decompositions based upon the number of unit eigenvalues,

$$x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}, M = [M_1 \quad M_2], T = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}, U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}, V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

The diagonal elements or diagonal  $2 \times 2$  blocks in  $T$  correspond to the respective real eigenvalues or pairs of conjugate complex eigenvalues all of which lie inside or on the unit circle by the above assumptions; hence the individual elements of  $x_t$  or pairs of them feature decoupled dynamics and have a straightforward interpretation based upon the eigenvalues that carry them. The vector process  $y_t$  is in this way decomposed into a set of canonical autoregressions and random walks, and each element of  $y_t$  is a linear combination of these. We may thus construct the BNT simply by projecting  $y_t$  against those elements of  $x_t$  that are carried by unit eigenvalues and against the unconditional means of the stationary elements,

$$y_t^T = M x_t^T,$$

where  $x_t^T$  is created from  $x_t$  by setting the stationary elements of  $x_t$  to their unconditional means, which can be in turn calculated from (6) as

$$\bar{x}_2 = \mathbb{E}x_{2t} = (\mathbf{I} - T_2)^{-1} U_2.$$

The appropriate mean-zero complementary transitory component is

$$y_t^C = M_2 (x_{2t} - \bar{x}_2). \tag{7}$$

### 3. Eigenvalue Decomposition

Making use of the CSS representation we may extend the Beveridge-Nelson concept of time series decomposition along the lines suggested originally by Casals, Jerez and Sotoca (2002). Eqs. (5)–(6) easily read that we may compute the contribution of any subset of canonical processes, selected on the basis of underlying (carrying) eigenvalues, to  $y_t$ . In general, to compute the contribution of the canonical processes carried by a subset  $\Lambda$  of eigenvalues  $\{\lambda_1, \dots, \lambda_n\}$  we only need to project  $y_t$  against these processes,

$$y_t^\Lambda = M^\Lambda x_t^\Lambda,$$

where matrix  $M^\Lambda$  is created by deleting all columns of  $M$  that correspond to eigenvalues not included in  $\Lambda$ , and  $x_t^\Lambda$  is created by similarly deleting these rows. If  $\Lambda$  is a subset of stable eigenvalues then it is meaningful to subtract the asymptotic means of the processes carried by them, likewise in (7), simply to get mean-zero fluctuations. Moreover, if a well-defined

autocovariance (or autocorrelation) generating function exists for  $x_t^\Lambda$  we may easily transform it to obtain the ACGF for  $y_t^\Lambda$ ,

$$\begin{aligned}\Gamma_x(k+1) &= T^\Lambda \Gamma_x(k), & k = 1, 2, \dots, \\ \Gamma_x(0) &= T^\Lambda \Gamma_x(0) T^{\Lambda'} + V^\Lambda \Omega V^{\Lambda'}, \\ \Gamma_y(k) &= M^\Lambda \Gamma_x(k) M^{\Lambda'}, & k = 0, 1, 2, \dots,\end{aligned}$$

where  $\Gamma_x(k)$  and  $\Gamma_y(k)$  are  $k$ -th order autocovariance matrices of, respectively,  $x_t^\Lambda$  and  $y_t^\Lambda$ ,  $T^\Lambda$  ( $V^\Lambda$ ) are submatrices created by deleting all columns and rows (all rows) that correspond to eigenvalues not included in  $\Lambda$ , and  $\Gamma_x(0)$  may be solved for by vectorization of the second equation, see e.g. Lütkepohl (1993).

A straightforward extension to the BN decomposition is then a further decomposition of the transitory component,  $y_t^C$ , into its business-cycle and irregular fluctuations. For this we need to define a business cycle and impose appropriate qualification constraints on the eigenvalues (or the canonical processes carried by them) to be recognized as the business cycle contribution. We employ the classical business cycle definition that dates back to Burns and Mitchell (1946) and refers to fluctuations with a periodicity of 6 to 32 quarters. In line with this definition we consider two types of business-cycle qualifications for eigenvalues

1. Modulus qualification. We require that at least 5 per cent of an innovation survives after 5 quarters in the response of the canonical process, and at least 90 per cent dies out within 32 quarters. This is consistent with an approximate range of (0.55, 0.91) for the modulus of the eigenvalues.
2. Phase angle qualification. We require that the phase angle of pairs of conjugate complex eigenvalues implies a periodicity of 6 or more quarters. This is clearly consistent with a range of  $\langle -\frac{\pi}{3}, \frac{\pi}{3} \rangle$  for the phase angle of complex eigenvalues.

#### 4. Structural Identification

In this section, we use the CSS representation to turn the reduced-form innovations,  $\varepsilon_t$ , into their orthonormal structural counterparts,  $u_t = S^{-1} \varepsilon_t$ , such that the matrix of the asymptotic effect of  $u_t$  on the level of  $y_t$  is (lower) triangular, i.e. the long-run effect of these innovations has a recursive structure. This gives rise to an impulse response function with possibly meaningful interpretations. Within the VEC representation, a computationally efficient way for this type of identification scheme relying upon the QR factorization has been worked out by Hoffmann (2001). Indeed, we closely follow his procedure.

Ignoring the case where all elements of  $y_t$  are I(0) we will consider separately two cases:

1. Absence of cointegration vectors. Then  $y_t$  is driven by  $k$  separate stochastic trends (random walks) and we can identify a full number of  $k$  orthonormal structural innovations

with a generally nonzero asymptotic effect on  $y_t$ . Substituting  $Su_t$  for  $\varepsilon_t$  in the state equation (6) yields a system in which the asymptotic effect of  $u_t$  into  $y_t$  is a  $k \times k$  matrix

$$\Psi = M_1 V_1 S.$$

To recover  $S$  we proceed in two stages. First, we compute any type of orthonormal structural innovations,  $v_t$ , such that

$$\varepsilon_t = S_F v_t, \quad \mathbb{E} v_t v_t' = I \text{ and hence } S_F S_F' = \Omega,$$

which is easily performed by e.g. Cholesky decomposition of  $\Omega$ . Second, we find the QR factorization of the matrix of asymptotic effects of first-stage innovations on  $y_t$ , i.e.

$$\Psi_F = \Psi_F' = (M_1 V_1 S_F)' = R' Q',$$

such that  $Q$  is unitary,  $Q Q' = I$ , and  $R$  is upper triangular (or  $R'$  is lower triangular). Now we may set  $S = S_F Q$  so that  $\Psi = \Psi_F Q$  and

$$\Psi = \Psi_F Q = R' Q' Q = R'$$

is lower triangular, and

$$S S' = S_F Q Q' S_F' = S_F S_F' = \Omega$$

holds as required.

2. Presence of cointegration vectors. The existence of  $0 < r < k$  cointegration vectors diminishes the number of underlying stochastic trends (and therefore the number of unit eigenvalues) to  $k - r$ . Hence, out of  $k$  structural innovations only  $k - r$  will have a possibly nonzero asymptotic effect on  $y_t$ , and the rest of them will be of transitory nature only. This introduces one more stage into the identification of  $u_t$ , namely orthogonalizing the permanent and transitory innovations.

As before, we first compute any type of orthonormal structural innovations via a transformation matrix  $S_0$ , e.g. using again Cholesky decomposition of  $\Omega$ . Second, we find the second-stage innovations,  $w_t = S_1^{-1} v_t = S_1^{-1} S_0^{-1} \varepsilon_t$ , such that only  $k - r$  of them have an immediate, and hence also permanent, effect on  $x_t^1$ , and the rest of them only affect  $x_t^2$ . This can be accomplished by the QR decomposition of the transposed  $k \times k$  upper block  $\Phi_0^\dagger$  of the matrix of instantaneous effects of first-stage innovations  $v_t$ .

$$\Phi_F = V S_F = \begin{bmatrix} \Phi_F^\dagger \\ \Phi_0^\dagger \end{bmatrix},$$

so that

$$\Phi_F^\dagger = R' Q'.$$

Setting  $S_1 = Q$  immediately guarantees that the  $k \times k$  upper block  $\Phi_1^\dagger$  of the matrix of instantaneous effects of second-stage innovations  $w_t$ ,

$$\Phi_S = V S_F S_S = \begin{bmatrix} \Phi_S^\dagger \\ \Phi_S^\dagger \end{bmatrix},$$

is lower triangular, or that only zeros appear in the last  $r$  columns within the first  $k - r$  rows. This is what we claim as it implies that there is neither an instantaneous nor a permanent effect of last  $r$  orthonormal innovations  $w_t$  on the first  $k - r$  elements of  $x_t$ , i.e. on  $x_t^1$ .

Third, we set up the  $k \times (k - r)$  matrix of asymptotic effects of second-stage innovations on  $y_t$ ,

$$\Psi_S = M_1 V_1 S_F S_S,$$

and by another QR decomposition of its transpose (which is not square any longer) we yield the transformation matrix for the third-stage structural innovations, namely  $Q$  from

$$\Psi_S = R' Q'.$$

Setting  $S = S_F S_S Q$  clearly gives rise to a lower triangular matrix of asymptotic effects of the first  $k - r$  third-stage orthonormal innovations,  $u_t = S^{-1} \varepsilon_t$ , on  $y_t$ .

Note that the asymptotic recursiveness of these first  $k - r$  structural innovations is independent of the exact ordering of the stationary elements within  $x_t^2$ .

## 5. Application to Czech Inflation and Real Economic Activity

We apply the multivariate eigenvalue decomposition to extract the trend, business-cycle and irregular components from a set of measures of Czech inflation and real economic activity, and consecutively to draw elementary conclusions about their cyclical properties. The primary economic motivation rests in a stylized final-price Phillips curve based upon staggered price-setting arising due to nominal rigidities. The microeconomic foundations for optimal pricing behavior under these restrictions give rise to the following log-linearized relationship,

$$\pi_t = \omega \pi_{t-1} + (1 - \omega) \pi_{t+1}^e + \widehat{rmc}_t,$$

where  $\pi_t$  is the final price inflation, superscript  $e$  refers to expectations, and  $\widehat{rmc}_t$  is the deviation of real marginal cost from its flexible-price level; see e.g. Galí and Gertler (2000), or Christiano, Eichenbaum and Evans (2001) for more background theory.

We combine the following (seasonally adjusted) time series<sup>4</sup> in a VAR model: net CPI inflation<sup>5</sup>, industrial PPI inflation, import price inflation, the real production wage in the business sector, and real gross domestic product. The last two variables on the list are to capture movements in the real marginal cost of domestic value added<sup>6</sup>, whereas the effect of import price inflation is twofold: first, a direct impact on the CPI via directly consumed imports, and second, an indirect impact via material imports (and hence the real marginal cost of domestic gross production) passed through into producer prices and consecutively consumer prices. Next, we impose restrictions on these series in terms of their integration and cointegration:

<sup>4</sup>One-quarter inflation rates are always implicitly assumed throughout the rest of this working paper.

<sup>5</sup>Net CPI stands for the consumer price index net of administered prices.

<sup>6</sup>Fluctuations in real marginal cost are determined by fluctuations in factor prices and in the level of production, once the production function features diminishing returns to scale (some factor, typically capital, being fixed over the short run).

- All series as they stand above are assumed  $I(1)$ .
- CPI inflation, PPI inflation, and import price inflation share one common stochastic trend. This restriction is easily imposed by the following basis of their cointegration space:  $[1, -1, 0]'$  and  $[0, 1, -1]'$ .

We consider integration of inflation series of order  $I(1)$  to be a useful shortcut for reduced-form within-sample modeling of the disinflation episode of the Czech economy in the 1990s. However, from the economics perspective it is clearly inconsistent with the existence of a monetary authority in itself and hence an utterly inappropriate basis for any out-of-sample conclusions. With these  $I(1)$  inflations, the respective price levels themselves follow  $I(2)$  but cointegration of them makes the relative prices  $I(1)$  again; in other words, we allow for stochastic trends in relative prices, which might be thought of as a result of e.g. Balassa-Samuelson type effects (relative import price) or systematic changes in producers' markups due to changes in the market structure (relative CPI vs. PPI price).

Regarding the identification of structural innovations our set of five  $I(1)$  variables with a 2-dimensional cointegration space is underlied by three stochastic trends. Therefore we may obtain three innovations with permanent effect and two acting only temporarily. Furthermore, keeping the ordering as earlier in this section (cointegrated inflations come first) the matrix of asymptotic effects,  $\Psi$ , must have its  $3 \times 4$  top-right block filled with zeros (since the permanent effect of any innovation must be the same for all inflations).

We estimate the model of second order in its VEC representation, (1), using multivariate LS on a data sample stretching over 1995Q1 through 2003Q1, then we turn it into its CSS, (5)–(6), and compute the individual components and their elementary properties. This is discussed in the subsequent section. Nonetheless, as the qualification of eigenvalues (and the respective canonical processes) is a qualitative choice it is impossible to analytically derive the statistical properties of this type of estimator of components, and on top of this we may suspect it to be rather fragile or unrobust given the small sample of our data. We therefore perform a bootstrap examination of the estimator using 1,000 random draws from estimated innovations, see e.g. Lutkepohl (2000) for more on bootstrapping in cointegrated models, and comment on results of this simulation too.

## 6. Estimation and Bootstrap-Simulation Results

In this section we review the estimation results and accompany them with issues that remain open or unclear.

As illustrated in Figure 1, four eigenvalues (or the canonical processes carried by them) qualify as contributors to business-cycle fluctuations. However, when interpreting these business-cycle fluctuations and the corresponding trend components (plotted in Figures 2 and 3 along with the irregular high-frequency complements) extracted by eigenvalue decomposition we must always

read a cyclical peak as an indicator of an innovation or a cumulation of innovations that cause an ultimate decline in the respective variable over the long-run horizon. In other words, a positive transitory deviation means that the observed variable will be pushed downwards over the long run. This might be in sharp contrast to the intuition gained from traditional UC model setups. Figure 2 also gives a simple eyeballmetric for the inspection of the relative signal-to-noise ratio in each examined series. Here we can learn that in line with our intuition both of the real variables (output and wage) together with CPI inflation are much less noisy than import price inflation or IPPI inflation. This may be viewed as a basis for our selection of robust measures of the business-cycle position of the economy.

Considering the identification across wider economic evidence, we find an interesting period in late 2000 and early 2001. A sudden drop in inflation trends was accompanied by a simultaneous decline in real domestic output and a rise in the real wage. The underlying hypothesis is that a fall in the relative import price also occurred about this time (since the relative price trends themselves are considered  $I(1)$ ) and intratemporal goods substitution took place, cutting the demand for domestic goods. This is confirmed by the bottom-right panel in Figure 3. As IPPI inflation almost immediately followed the new trend we may furthermore hypothesize a sharper or faster development in material and intermediate imports rather than those for final consumption; the CPI then reacted fully only in 2002. On the production side, substitution occurred between domestic value added (mainly labor) and imports, potentially pushing the marginal product of labor up. The fundamental source for these movements may lie in a shift of households' marginal rate of substitution between consumption and leisure, or as an observational equivalent, in institutional or bargaining changes in the labor market.

The overall business-cycle autocorrelations are reported in Table 1. First, we can check that there is a faster pass-through of import inflation into IPPI inflation than into the CPI. This may indicate either a high content of material or intermediate imports in gross domestic production and a relatively high elasticity of substitution between these imports and domestic value added, i.e. real output (which is also indirectly supported by a rather large negative correlation of real output and current-dated and lagged import prices) or a fundamentally different nature of price contracts in the wholesale and retail sectors.<sup>7</sup> Next, the reported overall correlation patterns fail to support the sketched Phillips curve in the preceding section as a systematic tie of inflation and real economic activity. Opposite signs on the correlations between IPPI inflation and the real wage versus real output over the whole reported range of lags leave room for future refinements of the current concept of measuring the real marginal cost. Finally, Table 1 gives strong evidence in favor of countercyclical behavior of real wages, which is congruent with the traditional Keynesian interpretation of the business cycle conditioned upon nominal wage stickiness: however we can only discover a higher degree of stickiness in the nominal wage relative to the CPI from the negative correlations between the real wage and CPI inflation, not relative to the PPI.

The impulse responses to structural innovations computed as subject to long-run recursiveness

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<sup>7</sup>E.g. significantly shorter average duration of a typical wholesale contract.

are summarized in Table 2 and Figures 4 and 5. Depending on the sign of the long-run effect of permanent innovations (the first three out of five) on individual variables, see Table 2, we may attempt to attach a more or less deep structural interpretation to all of them. The first permanent innovation turns out to be disinflationary (being the only one permanently passing through into both rates of inflation), simultaneously affecting also real output (negatively) and the real production wage (positively). This is a breakdown of monetary superneutrality, simply because we have not imposed it in the model at all, and arises evidently as a consequence of the disinflation being run at real costs in the 1990s. The second permanent innovation influences solely the real variables, namely in the same direction, whereas the third one does the same with an opposite effect on each variable. They are thus candidates for, respectively, technology (productivity) and intratemporal preference innovations.

The response profiles are then depicted in the subsequent Figures 4 and 5. As noted in the preceding paragraph the disinflationary innovation incurs permanent real cost, although primarily the drop in CPI inflation is led by faster and more pronounced changes in import inflation—obviously in the nominal exchange rate indeed—and in domestic IPPI inflation, with both of them jumping below or to the new steady-state level instantaneously. In all the reported shocks we can again find evidence favoring the hypothesis of the nominal wage being more sticky than the nominal price, as the real wage changes are markedly induced by movements in the underlying price indices. This is especially the case with the second permanent shock (compare the immediate profile of IPPI inflation and the real wage).

## **7. Robustness of the Results**

It remains to verify our results against the bootstrap simulation. To summarize this exercise, the point estimates of the business-cycle component are rather indicators for the upper (if positive) or lower (if negative) bands of the empirical distribution. The simulated distributions are asymmetric and skewed towards zero (evidently seen particularly with import inflation). Moreover, we also detect bimodal distributions peaking at zero and near to the point estimate. This is documented by the 0.10 and 0.90 percentiles attached to the actual estimates of the business-cycle component, see Figure 6, and by example profiles of empirical distributions plotted at the points of maximum deviation of the respective variable from zero, see Figure 7. The sources of these distortions are subject to further research: they may be attributed either to small sample biases, or to the qualitative (discrete) nature of eigenvalue qualification. In the latter case, some kind of fuzziness in the categorization of the eigenvalues might improve the robustness and overall properties of the estimator.

## **8. Concluding Remarks**

In this working paper we follow the Beveridge-Nelson type of multivariate time series decomposition (as a complement to the more frequently encountered unobserved-components type models) into their trend, business-cycle and irregular components. We use this concept to ex-



amine a stylized model of supply-side inflation determination with nominal rigidities and a particular real marginal cost composite measure for a small open economy using the Czech data. We characterize the estimated components of CPI, IPPI and import inflations, together with the real production wage and real output, and survey their basic correlation properties; furthermore we compute structural innovations imposing restrictions on their long-run effects, draw the impulse responses, and test the results by means of bootstrap simulation. The conclusions we make on this basis regard the speed of import price and exchange rate pass-through, the basic pro- or counter-cyclicality with implications for the degree of stickiness in prices and wages, and the relevance of the real marginal cost measure used in the model.

We conclude that major room for further refinement of our research is found in two areas: First, from an economic perspective, in the construction and further refinement of the real marginal cost indicators relevant to producer and consumer inflations, and second, from a statistical perspective, in further investigation of the robustness of the method, in particular because of the evidence for towards-zero skewness of the component estimator.

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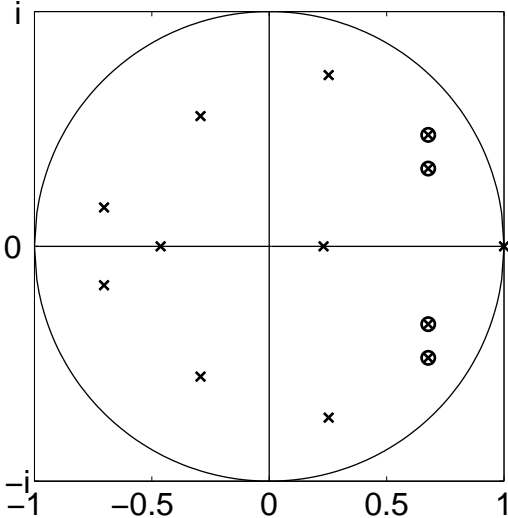
**Table 1: Estimates of autocorrelations of business-cycle components.**

| Autocorrelation of order 0 |       |       |       |       |       |
|----------------------------|-------|-------|-------|-------|-------|
| Net CPI inflation          | 1.00  | 0.56  | -0.21 | -0.49 | 0.43  |
| IPPI inflation             | 0.56  | 1.00  | 0.43  | 0.25  | -0.26 |
| Import inflation           | -0.21 | 0.43  | 1.00  | 0.65  | -0.55 |
| Real wage                  | -0.49 | 0.25  | 0.65  | 1.00  | -0.23 |
| Real output                | 0.43  | -0.26 | -0.55 | -0.23 | 1.00  |
| Autocorrelation of order 1 |       |       |       |       |       |
| Net CPI inflation          | 0.66  | 0.57  | 0.32  | -0.10 | 0.09  |
| IPPI inflation             | 0.24  | 0.79  | 0.74  | 0.54  | -0.32 |
| Import inflation           | -0.66 | 0.08  | 0.83  | 0.78  | -0.63 |
| Real wage                  | -0.51 | -0.11 | 0.37  | 0.68  | -0.04 |
| Real output                | 0.46  | -0.25 | -0.36 | -0.38 | 0.78  |
| Autocorrelation of order 2 |       |       |       |       |       |
| Net CPI inflation          | 0.16  | 0.42  | 0.62  | 0.27  | -0.23 |
| IPPI inflation             | -0.18 | 0.45  | 0.84  | 0.72  | -0.40 |
| Import inflation           | -0.78 | -0.18 | 0.45  | 0.64  | -0.48 |
| Real wage                  | -0.45 | -0.34 | 0.07  | 0.36  | 0.09  |
| Real output                | 0.30  | -0.22 | -0.16 | -0.34 | 0.45  |
| Autocorrelation of order 3 |       |       |       |       |       |
| Net CPI inflation          | -0.27 | 0.19  | 0.66  | 0.48  | -0.41 |
| IPPI inflation             | -0.51 | 0.11  | 0.70  | 0.73  | -0.41 |
| Import inflation           | -0.63 | -0.32 | 0.03  | 0.36  | -0.21 |
| Real wage                  | -0.31 | -0.43 | -0.18 | 0.06  | 0.20  |
| Real output                | 0.11  | -0.19 | -0.05 | -0.23 | 0.17  |
| Autocorrelation of order 4 |       |       |       |       |       |
| Net CPI inflation          | -0.51 | -0.02 | 0.48  | 0.50  | -0.41 |
| IPPI inflation             | -0.63 | -0.16 | 0.40  | 0.58  | -0.31 |
| Import inflation           | -0.31 | -0.32 | -0.28 | 0.04  | 0.07  |
| Real wage                  | -0.11 | -0.39 | -0.35 | -0.17 | 0.27  |
| Real output                | -0.01 | -0.15 | -0.03 | -0.15 | 0.01  |

*Table 2: Estimates of asymptotic (long-run) effects of recursive orthonormal innovations.*

|                   | Permanent |       |       | Temporary |      |
|-------------------|-----------|-------|-------|-----------|------|
|                   | #1        | #2    | #3    | #1        | #2   |
| Net CPI inflation | -1.07     | 0.00  | 0.00  | 0.00      | 0.00 |
| IPPI inflation    | -1.07     | 0.00  | 0.00  | 0.00      | 0.00 |
| Import inflation  | -1.07     | 0.00  | 0.00  | 0.00      | 0.00 |
| Real wage         | 0.55      | -0.23 | -0.98 | 0.00      | 0.00 |
| Real output       | -0.35     | -0.87 | 0.10  | 0.00      | 0.00 |

Figure 1: Eigenvalues of estimated VAR (circled are business-cycle qualified eigenvalues).



**Figure 2: Estimates of business-cycle components (—) and irregular high-frequency components (- -).**

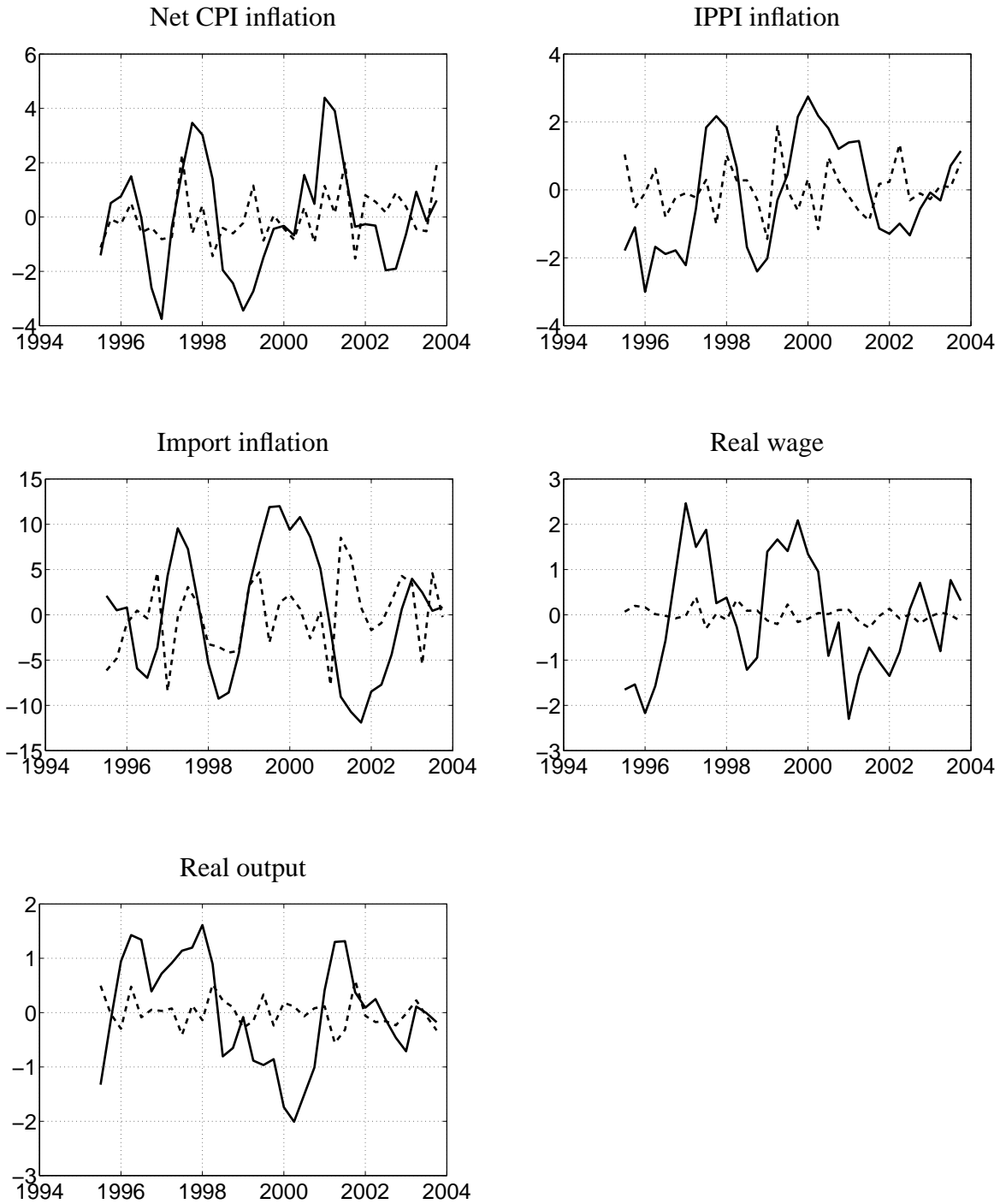


Figure 3: Observed data (—) and estimates of trend components (- -).

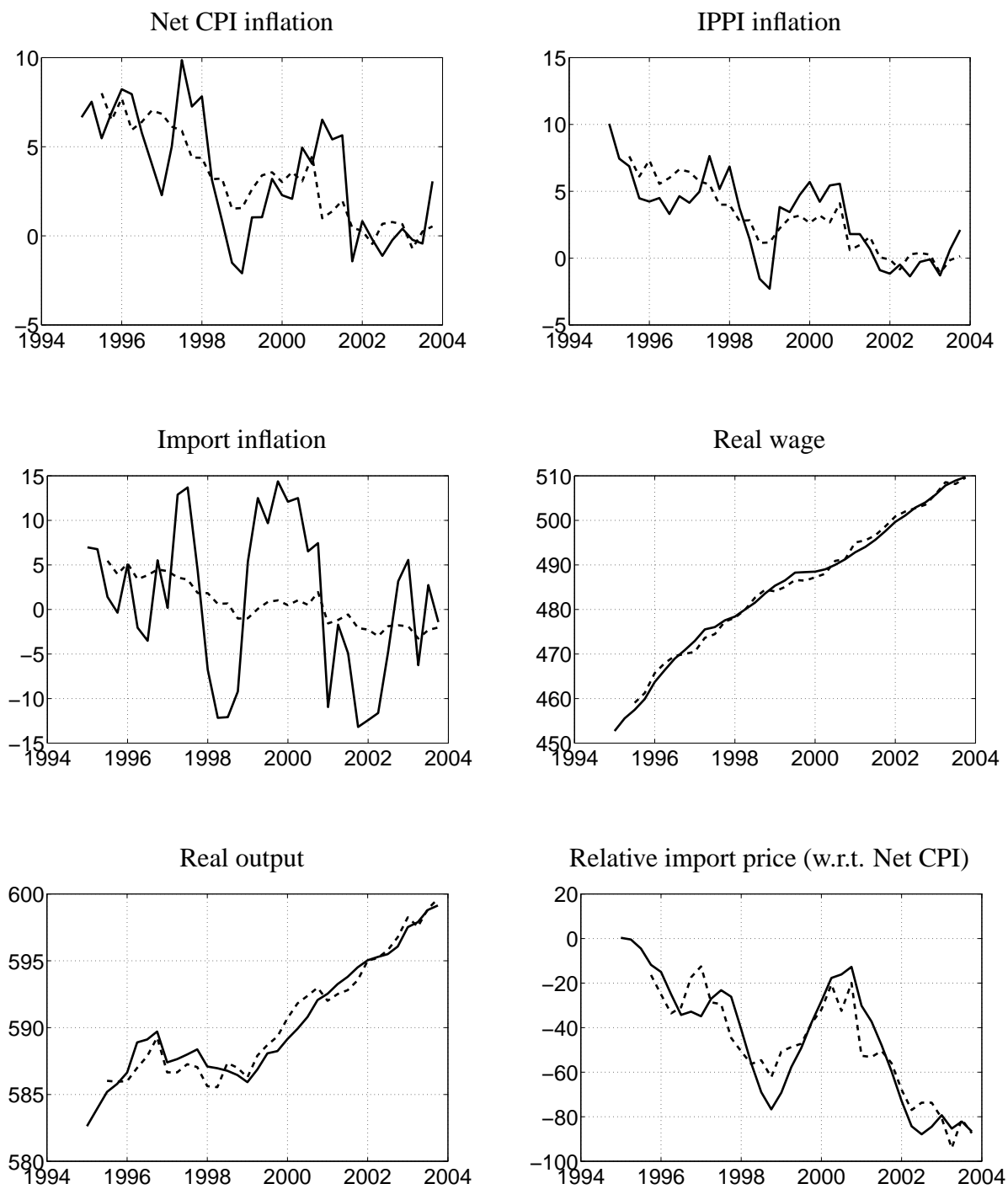


Figure 4: Impulse responses of business-cycle plus trend components (with asymptotes).

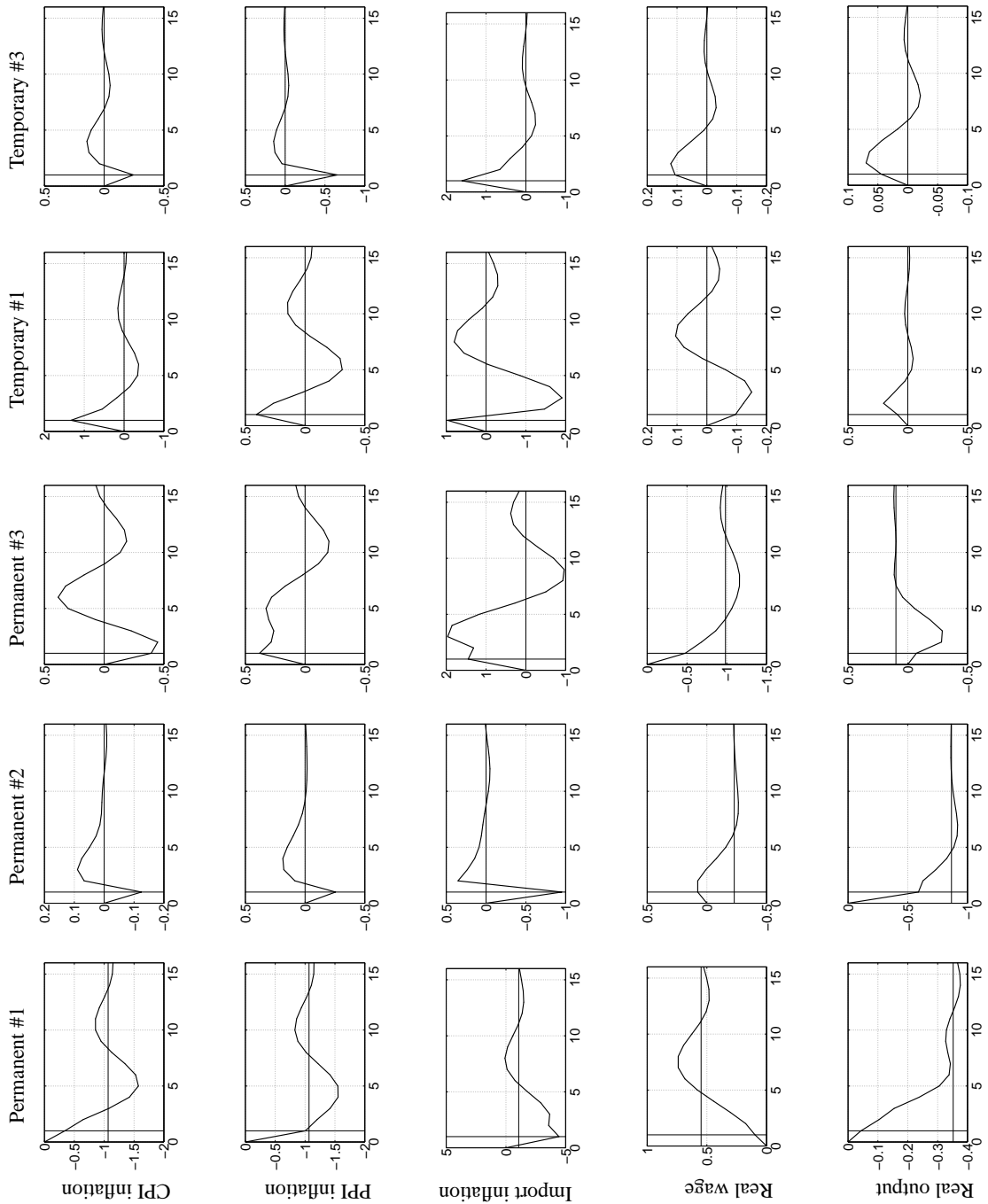




Figure 5: Impulse responses of irregular high-frequency components.

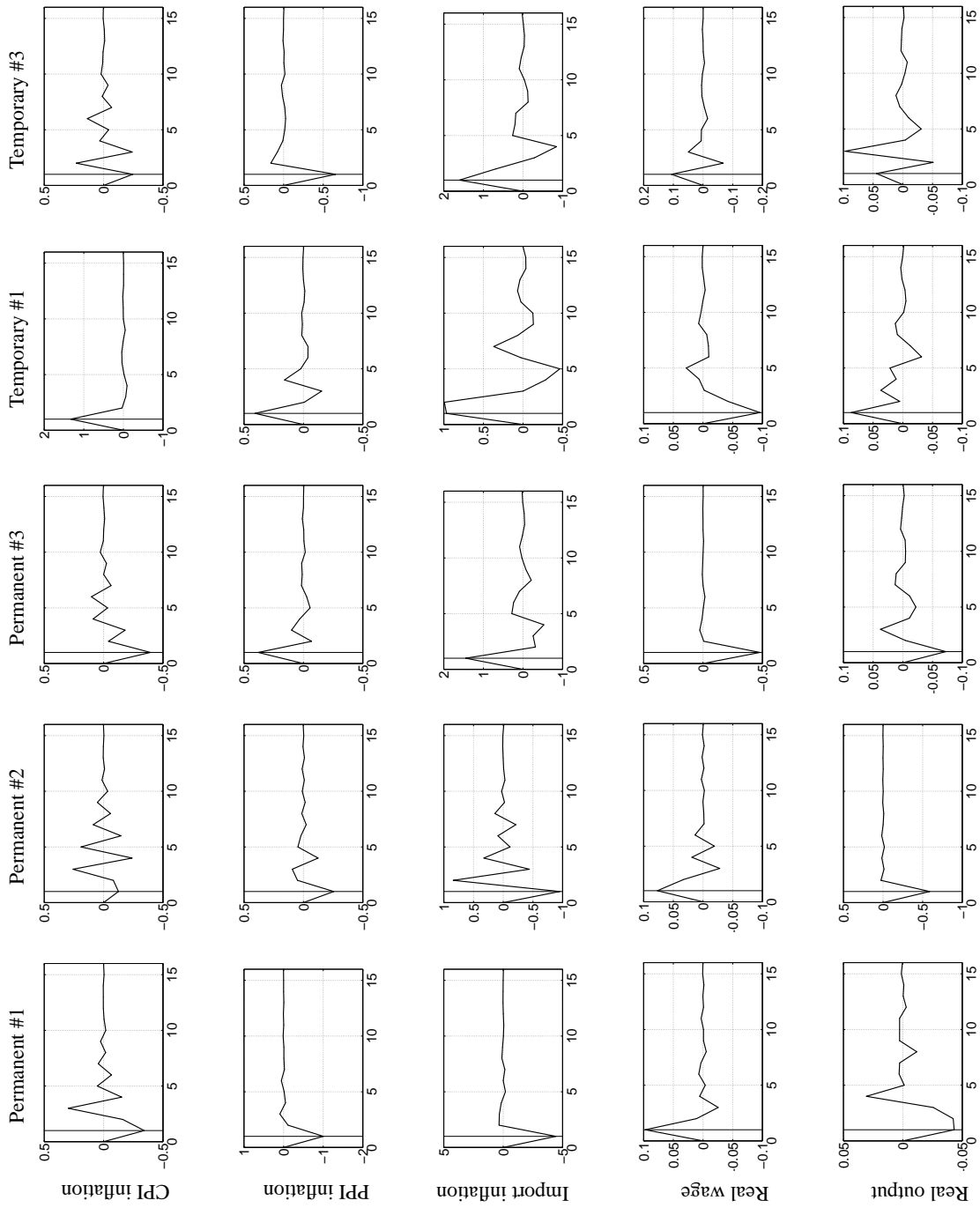
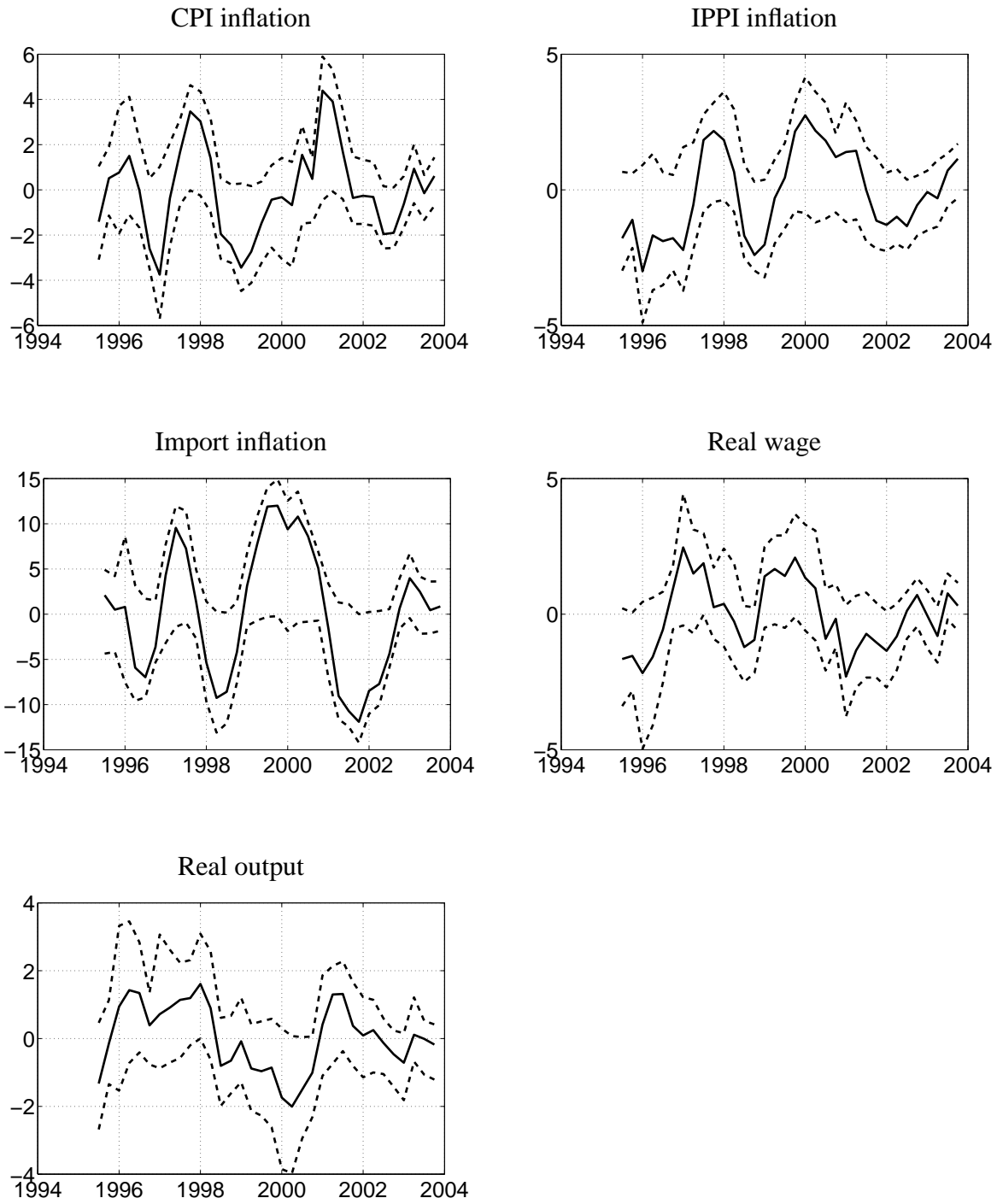
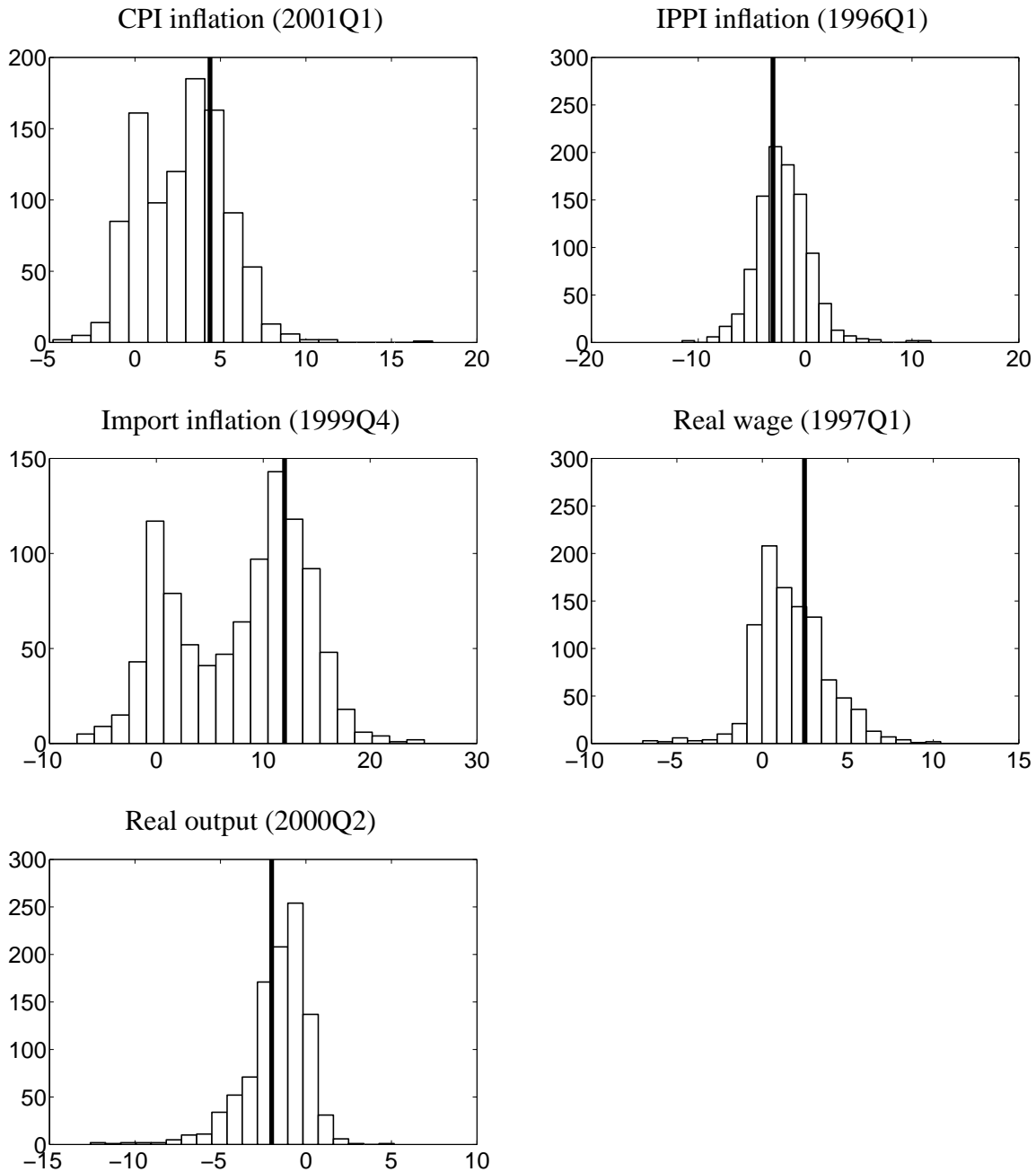


Figure 6: Estimates of business-cycle components (bootstrapped 0.10 and 0.90 percentiles).



**Figure 7: Empirical bootstrap distributions of business-cycle components.**



Profiles drawn at the point of maximum deviation from zero; vertical lines are point estimates.

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