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Credit Risk and Bank Lending in the Czech Republic

Narcisa Kadlčáková[*], Joerg Keplinger[**]

Abstract

This project undertakes an empirical analysis in credit risk modeling using a data sample representative of bank lending to the Czech corporate sector. A rating system is constructed using a proprietary database (Creditreform) that provides a solvency index for a large number of Czech firms. Several methods for the calibration and validation of a rating system are described and tested in practice. On the basis of a representative portfolio for Czech industries, systemic predictions of regulatory and economic capital are obtained and compared. The methodologies formulated by the latest Consultative Document of the NBCA (April 2003) and by the Credit Metrics and CreditRisk+ models are applied. The main contributions of this project can be briefly summarized as follows: (a) it shows in an applied manner that input data problems in credit risk modeling can be overcome, (b) it sheds light on regulatory issues that are gaining increasing relevance, and (c) it outlines the most important features of two credit risk models.

JEL Codes: G21, G28, G23.
Keywords: Credit Risk, Economic Capital, Exchange Rate Exposure, Rating System.

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Nontechnical Summary

The banking sector worldwide faces an increasing need to address and put in practice modern practices in the credit risk area. Banks are concerned with credit risk management techniques partly because of the new regulations of the Basel Committee. At the same time, increased competition is forcing banks to develop and implement internal processes in order to find the optimal mix between taking risks, maximizing returns and creating their own capital provisions. This need will be felt even more strongly in countries with transition economies where the implementation of credit risk management procedures is at an incipient phase and where the lack of input data is in many cases severe.

An essential requirement in credit risk management is the creation of a rating (scoring) system. While rating (scoring) systems may prove useful for a large array of bank activities, they are becoming increasingly relevant for regulatory and economic capital provisioning. In this paper a rating system is constructed for the Czech corporate sector using a proprietary database (Creditreform) that provides a solvency index for a large number of Czech firms. Although the methodology for constructing the solvency index and the main features of the Creditreform data set are briefly presented, the main emphasis is put upon the construction and validation of the rating system. The reliability of the method used to construct the rating system is tested through a set of statistical measures of the power of the model (power curves and Gini coefficients) and of the predictive power of the model (Alpha- and Beta-errors, accuracy ratios and information entropy ratios).

A natural extension of the paper is to compare regulatory and economic capital estimations according to different credit risk modeling approaches. We apply two credit risk models (CreditMetrics, CreditRisk+) and the latest Consultative Document of the NBCA (April 2003). These capital estimations reflect a “macro” lending view in the sense that all loans granted by banks active in the Czech Republic are aggregated at the industry level and all other required risk inputs are estimated at this level.

Our results can be seen as an overall empirical assessment of the New Basel Capital Accord and of several credit risk models analyzing the bank credit conditions in the Czech economy. The quantitative results of the paper can be briefly summarized:

- Several validating tests show that our rating system displays a similar performance to rating systems constructed on the basis of Creditreform data in Austria or Germany.
- The regulatory capital estimated according to the IRB approach of the New Basel Accord is in the range estimated by the credit risk models at a 95% confidence level. Among the credit risk models implemented, the CreditMetrics model predicted the lowest economic capital values. However, this outcome is due to several simplifications made in order to circumvent the non-availability of input data into this model.

1. Introduction

The successful application of the New Basel Capital Accord and credit risk models is significantly dependent on the availability of the required input data. Although ratings are fundamental inputs, the empirical estimation of other elements is equally important for the practical implementation of these methodologies. In this paper we construct a rating system using Czech corporate data and provide other estimates of the primary inputs needed in credit risk modeling. Subsequently, the implications of the constructed rating system for the estimation of regulatory and economic capital are examined.

Recent regulatory norms that are contained in Basel documents view ratings as good quantifiers of bank clients' default risk and as an essential tool in estimating banks' regulatory capital. For expositional purposes, ratings provided by well-known rating agencies (Standard and Poor's, Moody's – KMV) are used in these documents. The major drawback of this approach is that these ratings cover an insignificant share of the market, especially in the corporate sector of the developing countries. To avoid this problem, regulators purport to allow banks to build their own models for constructing internal rating systems. In principle, these models are scoring-based and employ client-specific accounting and payment default information, with model-specific default probabilities being assigned to scoring groups. Although banks' internal rating systems can overcome the problem of non-availability of ratings, other types of problems may arise. Regulators will have to adopt and test the eligibility of a large variety of modeling approaches. Moreover, the outcomes of these models will not necessarily provide a consistent view of the default trends of the corporate sector as a whole.

For this reason, we would like to undertake a quantitative analysis that offers a fundamental, even incipient, macro perspective of bank lending to the corporate sector in the Czech Republic. Our analysis is supported by data obtained from an external agency (Creditreform), which has monitored the Czech corporate sector over the transition period. The Creditreform dataset depicts the general trends in default behavior within the non-financial corporate sector and is a reliable starting point for the construction of a rating system for Czech non-financial firms. Further on, we restrict our attention to Czech industries (according to the NACE classification) by estimating the credit risk-required inputs at the industry level.

A natural extension of the paper is to compare regulatory and economic capital estimations according to different credit risk modeling approaches. In this sense we apply two credit risk models (CreditMetrics, CreditRisk+) and the latest Consultative Document of the NBCA (April 2003). The primary goal is to shed light on the practical implementation of these methodologies and on several theoretical constructs facilitating the estimation of the required input data. By analyzing a loan portfolio that reflects the macro structure of bank loans in the Czech Republic at the end of 2002, we are able to answer questions that are relevant from the supervisory point of view:

- What is the rapport between regulatory and model-based estimations of default risk capital?
- Is there an acceptable confidence level in the VaR-based methodology where economic capital approaches regulatory capital?

- Does the application of the NBCA look likely to be perceived as burdensome from the Czech banks' perspective in terms of regulatory capital provisioning?
- Is the estimated economic capital likely to differ significantly depending on the chosen credit risk model?

Since these questions are answered from a "macro" lending view, their generalization at the individual bank level may be meaningless. Portfolio composition effects and the peculiarities of estimating the risk inputs into credit risk modeling by different banks may induce significant differences from our results for individual banks.

The paper is organized as follows. Chapter 2 presents a brief literature review. Chapter 3 describes the Creditreform agency and the dataset used. Chapter 4 contains the construction, calibration and validation of the rating system for Czech firms. The estimation of the remaining input prerequisites and risk capital according to different methodologies is contained in Chapter 5. Chapter 6 presents the main conclusions. The detailed quantitative results are contained in the Appendix.

2. Literature Review

This project draws intensively on similar studies conducted using Creditreform data in Austria. Schwaiger (2003) describes several techniques for the construction of a rating system in this context, such as the cohort, logit/probit and Bayesian approaches. In all these cases specific methods are proposed to group the firm population into separate classes (according to the Creditreform solvency index) and estimate the class-specific probabilities of default (PDs). Schwaiger also examines several analytical tools able to assess the performance of a model, such as power curves, Gini coefficients, Alpha- and Beta-errors, accuracy ratios, etc. We include a brief account of these performance indicators when validating our rating system.

General guidelines and techniques for the validation of a rating system are predominantly researched by rating agencies. Moody's – KMV, for example, makes available a wealth of documents dealing with this topic. Stein (2002) and Sobehart et al. (2000) examine the two basic dimensions of a model validation process – power and calibration. Power reveals the ability of a model to discriminate among good (non-defaulted) and bad (defaulted) firms. Calibration indicates how well a model's predicted PDs correspond to the real outcomes. It is also shown in these papers how the performance statistics of a rating system are affected by the sample used in the model's construction. These authors recommend a walk-forward approach that combines out-of-time and out-of-sample testing (the data is pooled from year to year and the model is adjusted step by step) or bootstrapping techniques that consist in numerous re-samplings from the sample under investigation. These general principles of rating system validation are tested in practice and are described in a series of documents emphasizing the performance of the Moody's – KMV model for rating private firms (RiskCalc) in several European countries.

Migration matrices are important inputs into credit risk modeling. These matrices assess the degree of mobility among the rating classes of a rating system over a selected time period (the common assumption is one year). In most cases, the estimation of these matrices rests upon historical frequencies of default and rating migrations that are averaged class by class over a reasonably long time period. The aim is to capture an entire business cycle in order to isolate the influence of particular phases in the business cycle on firms' default behavior. Besides the difficulty of estimating migration matrices dependent on the particular phase of the business cycle, it is the procyclicality argument that justifies the use of average migration matrices. For example, if higher migration probabilities of downgrading are used during a recession, the resulting bank tendency to restrict credit may push the recession even further. There are several studies that examine the procyclical effect of the business cycle on rating migrations (see Bangia et al., Corcostegiu et al., and Nickell et al.).

In general, rating systems are assumed to display Markov properties, meaning that the distribution of ratings of an obligor evolves between the consecutive moments *t* and *t+1* according to the rule $R(t+1) = R(t) \cdot M$. Here $R(t)$ is the ratings distribution of an obligor at time t and M is the migration matrix. In general, M is assumed to be time homogeneous or constant over time. The time homogeneous property of the migration matrices is tested in several studies on account of matrix norms and metrics (see Jafry and Schuermann, Schuermann and Jafry, Bangia et al.) that rely on the eigenvalues and eigenvectors of the migration matrices. Additionally, these studies propose a counterpart to the cohort way of estimating migration matrices. If rating migrations are available in continuous time, i.e. through the year and not only at the beginning and end of the year, it is possible to estimate migration matrices using homogeneous and non-homogeneous duration methods (see Lando and Skodeberg and Schuermann and Jafry). However, the lack of data makes the applicability of these methods redundant in our case.

Another essential input into credit risk modeling, especially in the case of mark-to-market credit risk models like CreditMetrics, is the discount factors used in loan valuation. The basic assumption made here is that the valuation of different loans has to account for the risk characteristics of different bank obligors and for the time-value of money. In general, the risk premia are extracted from market prices of traded debt instruments (bonds) and rely on explicit pricing formulas of these instruments (see Arvantis et al., Jarrow et al.). In this paper the estimation of the term structure of credit spreads is the outcome of empirical application of the Markov-based methodology of Jarrow, Lando and Turnbull (1997).

The regulatory guidelines of the New Basel Capital Accord (NBCA) and their subsequent amendments are contained in a series of Consultative Documents and Quantitative Impact Studies released by the Basel Committee between 2001 and 2003. In terms of credit risk modeling, the most detailed descriptions of the relevant models are contained in the technical documents accompanying the release of these models. A comparative illustration of the most prominent credit risk models is provided in Crouhy et al. (2000) and Derviz and Kadlcakova (2001). In this paper we briefly touch upon the most important features of the Basel II regulations and credit risk models and put a particular emphasis on their practical implementation.

3. The Empirical Project

3.1 Creditreform and its Solvency Index

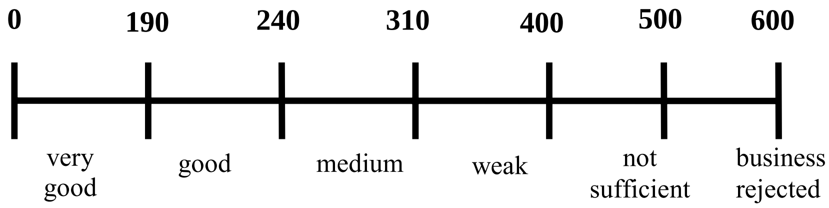
Creditreform is a business information service and debt collection organization with 176 agencies throughout Europe. Creditreform was present in the Czech Republic from 1890 to 1948 and was re-established in 1991. It has been operating for more then ten years in the Czech Republic and provides a stable history of its solvency index for a large number of Czech enterprises.

We had been searching for a partner with sufficient empirical data for the project to help predict the impact of Basel II and current credit risk models on regulatory and economic capital. The Creditreform solvency index makes it possible to estimate the capital adequacy changes under the New Basel Capital Accord affecting future credit conditions upon implementation in 2006.

Creditreform was chosen as a partner for this project because it provides a stable data history of more than five years for its solvency index. This solvency index represents an independent sample of Czech credit ratings that could not have been achieved by pooling the data of banks. Banking systems today lack information about the future development of companies whose credit applications have been refused in the past, especially if they have not been watched as clients. This information is needed for validating the prognostic value of the rating system used.

3.2 Construction of the Solvency Index

Based on a model created in Germany which has been successfully applied in several countries all over Europe, the solvency index was adjusted for Czech enterprises. The Creditreform solvency index can be described on a scale from 100 to 600 risk points. The value 100 is the best, while the value 600 represents the state of legal default. In the intermediate range, the solvency index is defined as follows:



- From 100 to 190 risk points - very good creditworthiness
- From 191 to 240 risk points - good creditworthiness
- From 241 to 310 risk points - medium creditworthiness
- Around 400 risk points - weak creditworthiness, liquidity problems
- 500 risk points - insufficient creditworthiness or payment behavior
- 600 risk points - business connections refused, bankruptcy or legal default

Creditreform obtains information from debt collection, supplier information and its own research and forms an opinion about the payment behavior of a company. Balance sheet information and research similar to a credit application fill the financial and structural criteria. Subsequently, a credit opinion and the final solvency index are calculated. Each of the fifteen criteria is evaluated individually and assigned risk points in one of the six classes. A private limited company, for example, is always evaluated with 16 risk points in the fourth class because of its limited liability.

The algorithm used to calculate the solvency index is weighted in the following way:

1. Financial data 25%
2. Payment behavior 20%
3. Credit opinion 25%
4. Structural data 15%
5. Industry and size 15%

With a weight of 25% the credit opinion and financial data are the strongest factors in this kind of rating system. These factors can be weighted differently by company size, legal form or industry.

The way in which the Creditreform solvency index is calculated is shown in the following example for two public limited companies.

Example A

Legal form:	s.r.o.
Line of business:	Construction
Age:	12 years
Business development:	constant (class 3)
Order book:	good (class 2)

Payment behavior: within agreed targets (class 2)
Credit opinion: business connections acceptable (class 2)

This is a Czech construction company with an annual turnover of around CZK 30 mil., with 15 employees and with the s.r.o. legal form (similar to ltd.). The payment behavior of the company is good. In previous years there has been only one case of debt collection with this enterprise, meaning that the debt was paid after the first reminder.

Example B:

Legal form: s.r.o.
Line of business: Construction
Age: 12 years
Business development: constant (class 3)
Order book: stagnating (class 4)
Payment behavior: partly out of agreed targets (class 4)
Credit opinion: business connections not denied (class 4)

This is also a construction company that has a turnover of around CZK 250 mil. and 150 employees. The researcher knows that the company often pays its debt only after several reminders are sent to the company.

The Creditreform researcher evaluates the solvency index in the following way (simplified):

Table 1: Creditreform Solvency Index Calculation

Example A:

		Classes					
Risk factors	**Weight %**	**1**	**2**	**3**	**4**	**5**	**6**
Mode of payments	20		40				
Credit opinion	25		50				
Business development	8			24			
Order book	7		14				
Legal form	4				16		
Line of business	4			12			
Age of company	4		8				
Turnover/employee	4			12			
Equity	4		8				
Capital turnover	4			12			
Payment behavior of the company	4		8				
Shareholder structure	4			12			
Customers' payment behavior	4		8				
Number of employees	2				8		
Turnover	2			6			
Total	100		**136**	**78**	**24**		
Solvency index	**238**						

Example B:

	Classes						
Risk factors	**Weight %**	**1**	**2**	**3**	**4**	**5**	**6**
Mode of payments	20				80		
Credit opinion	25				100		
Business development	8			24			
Order book	7				28		
Legal form	4				16		
Line of business	4			12			
Age of company	4			12			
Turnover/employee	4			12			
Equity	4		8				
Capital turnover	4			12			
Payment conduct of the company	4				16		
Shareholder structure	4			12			
Customers' payment behavior	4		8				
Number of employees	2		4				
Turnover	2	2					
Total	100	**2**	**20**	**84**	**240**		
Solvency index	**346**						

3.3 Data Description

Creditreform currently contains specific information on 77,000 Czech companies in its database. This database is enriched with information collected about every newly registered company in the Czech commercial register (where about 320,000 companies are now represented). When selecting the firm sample we applied the criterion of a turnover of at least three million Czech Koruna to avoid small businesses and trade licenses. After removing inactive companies, this coverage closely represented the active Czech corporate sector.

After a serious data cleaning of blank fields, double entries, defaults at the beginning and new companies at the end of the time scale with the aim of getting the most accurate credit information, the final sample size of Czech corporations for the period 1997–2002 with a Creditreform solvency index in 2002 was 25,735. In the end we got the following picture of the Creditreform solvency index over 6 years and nearly 70,000 observations.

Table 2: Creditreform Solvency Index Data Records 1997–2002

Year / Solvency index	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**Total observations**
100–199	627	607	574	678	491	438	**3,415**
200–299	5,929	6,003	7,744	10,024	6,553	4,522	**40,775**
300–399	3,474	3,003	4,206	5,305	3,258	1,970	**21,216**
400–499	121	72	102	101	35	14	**445**
500	24	184	233	234	193	115	**983**
600	2	219	633	764	719	815	**3,152**
Total	**10,177**	**10,088**	**13,492**	**17,106**	**11,249**	**7,874**	**69,986**

This provides us with a data set of the solvency index between 1997 and 2002, from which we will derive a default structure over five transition periods as a base for our rating system.

Default Definition

Default was defined as the state in which a company received a solvency index from Creditreform of 500 or 600. This definition is not entirely consistent with the default definition used by banks. In banks' models of default risk, liquidity plays the central role. Default is thus triggered by the incapacity or unwillingness of firms to honor their debt obligations within a well-defined time period. Even if liquidity and the payment discipline of a firm were important factors for the construction of the Creditreform index, they played only a partial role. It was not possible to disentangle the role of liquidity from the aggregate index. Thus, our definition of default emphasized the firms' failure to redress their economic fundamentals rather than strictly focusing on the risk of cash shortfalls. A default definition fully consistent with the one used by banks could have been achieved by recourse to the Credit Register of the Czech Republic. However, this represented an equally weak alternative due to the short time history of the data available in the Czech Credit Register.

By our definition, an annual default event occurred if the solvency index of the company migrated from a value strictly below 500 to a value of 500 or 600 in a one-year period. Table 3 shows the number of enterprises and the annual default structure as evidenced in the Creditreform database.

Table 3: Number of Companies and Annual Default Structure

Transition period	1997/98	1998/99	1999/00	2000/01	2001/02	Total
Non-defaulted companies	5,470	7,816	9,778	7,420	6,380	36,864
No. of Defaults	297	472	315	371	360	1,815
No. Of companies	5,767	8,288	10,093	7,791	6,740	38,679
Default rate	5.15%	5.69%	3.12%	4.76%	5.34%	4,69%

4. The Creation of a Rating System for Czech Corporations

4.1 The Construction of the Rating System

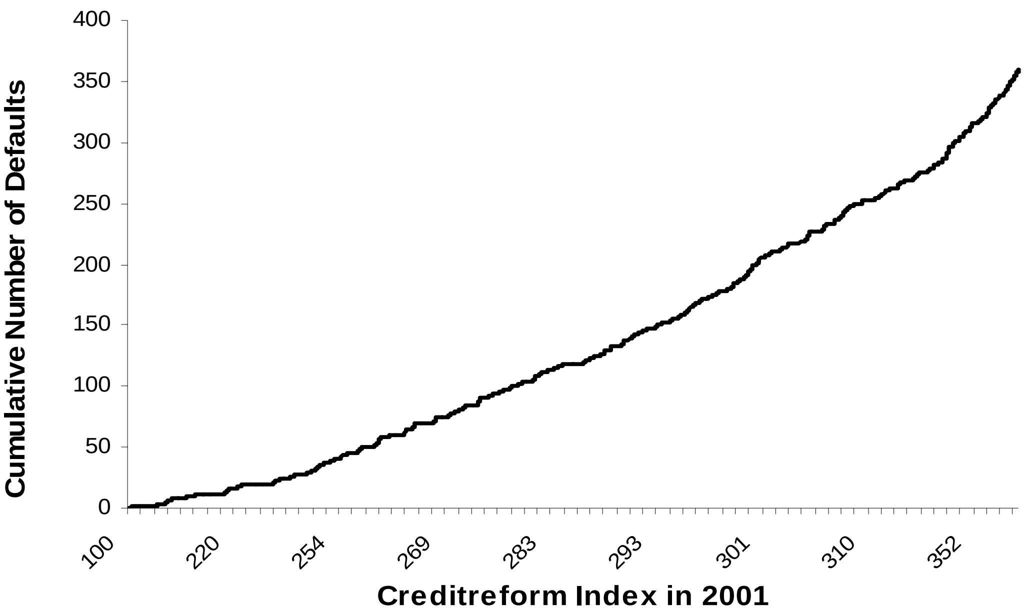
In our understanding a rating system consists of classes of homogenous obligors in terms of default expectations and a set of default probabilities associated with these classes. To devise a rating system, we were looking for the threshold values of the Creditreform index that, for each pair of consecutive years, optimally separated the pool of firms in distinct classes. In terms of the previous discussion, optimality meant that the threshold selection method satisfied the conditions of power (with the default probabilities clearly distinguishing among the classes) and model calibration (to have predictive power).

We had to keep track of several basic requirements in devising the rating classes:

1. the PDs should follow an increasing order as one moves from the best to the worst class,
2. the PD structure should display an exponential shape, thus increasing disproportionately faster within the worst classes, and
3. high concentrations of firms in a single class should be avoided.

In practice it was difficult to satisfy all these criteria. The distribution of the cumulative default for a representative[2] pair of years (see Figure 1) shows that defaults were rather linearly distributed over the entire index range and defaults started at rather low values of the index. Therefore, from the outset one can expect that the class-specific PDs will not reach the exponential shape characteristic of other standard rating systems (S&P, Moody's – KMV). This figure also suggests that compared with some standard rating systems ours would confer significantly higher PDs to the good classes.

Figure 1: Creditreform Solvency Index Versus Cumulative Annual Default from 2001 to 2002



These facts are partially explained by the notion of default employed. Our definition of default was closer to bankruptcy rather than default on specific payments and focused more on the ability of the issuers to honor their overall debt and not particular financial debt instruments. While bankruptcy is to a great extent influenced by financial and economic factors (high leverage, low liquidity, low profitability), factors that are not linked with economic fundamentals might also be strong in transition economies in influencing bankruptcy. It is likely that these external factors limited the ability of the Creditreform index to place defaults predominantly at the lower end of the index scale.

[2] For the other pairs of years the cumulative default curves were very similar to the one considered here.

The algorithm used to find the threshold values of the index defining the classes had to optimally exploit (according to points 1 to 3 above) the given degree of "convexity" of the cumulative curve. We considered the minimum number of classes requested by the NBCA (seven for non-default and one for default). In the first class were included the best firms (with solvency index values roughly in the 100–150 range). The threshold value defining the first class was the lowest value of the index defining a firm sample that excluded any default event in the following year. Thus, by construction the first class was assigned a PD of zero. The thresholds defining the remaining six non-default classes were estimated in such a way that (a) the PD of a given class was reasonably higher than the PD of the previous class and (b) the PD of that class over the remaining threshold scale was minimized. The eighth class contained defaulted firms and was associated with a PD of one.

To define the rating classes the index scale was divided as follows. First, all firms were ordered increasingly according to the Creditreform index in the first period. Let us denote by n_{k-1} and n_k the index values determining the k^{th} class, with *k* ranging from two to six. For each pair of years the threshold value n_1 defining the first class was the index value strictly lower than the value associated with the first default. The remaining threshold values were determined recursively. Let us introduce the following additional notations:

- D_{ij} – the number of defaults in the firm sample defined by the i^{th} and j^{th} values of the index, i, j = 100, 499
- N_i – the maximum firm rank corresponding to the i^{th} value of the index.

Having determined the index value for class *k-1* (n_{k-1}), the index value for class *k* (n_k) corresponded to the minimum default frequency

$$\min_{i>n_{k-1}} \left\{ P_k \mid P_k = \frac{D_{n_{k-1}+1,i}}{N_i - N_{n_{k-1}} + 1} \right\}.$$

We followed the cohort approach in estimating the class-specific annual default probabilities. This means that the probability of default characterizing a certain class was determined as the number of defaults occurring during the subsequent one-year period divided by the total number of firms in that class at the beginning of the period.

This "unconstrained" algorithm of constructing rating classes produced a mixed result in the sense that the default probabilities and the threshold values defining the classes significantly varied from year to year. A convenient way to homogenize the results was to pool the data from all years and then to apply the algorithm for defining classes to this pooled dataset (thus evaluating a homogenized PD structure). Subsequently, for each individual pair of years we looked for threshold values defining classes that minimized the distance between the class-specific PDs and the homogenized PD structure. More precisely, denoting by PD_i^n the default probability of class *i* determined on the basis of the pooled data (with i=2,7), the class-specific threshold values and class-specific PDs were estimated recursively for each pair of years by finding the minimum

$$\min\left|PD_i - PD_i^h\right|.$$

Figure 2 summarizes the main features of the resulting rating classification (class-specific PDs and firm concentration in each class) between 2001 and 2002. This outcome is representative for the other pairs of years, as illustrated by Table 4. Table 4 contains annual default probabilities for the non-defaulted rating classes for the entire six-year period.

Figure 2: Class-specific PD Structure and Firm Concentration in each Class (2001–2002)

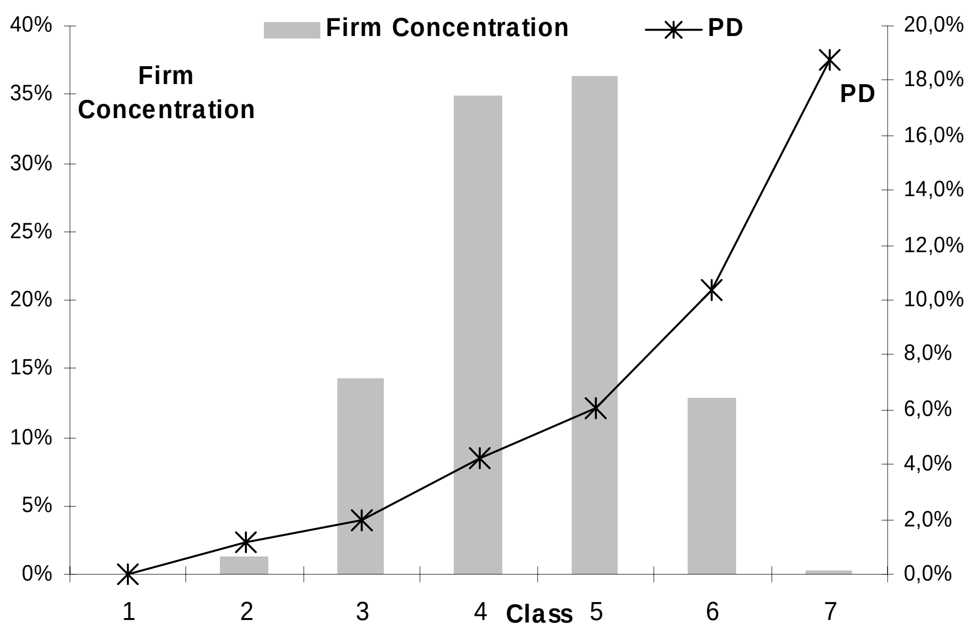
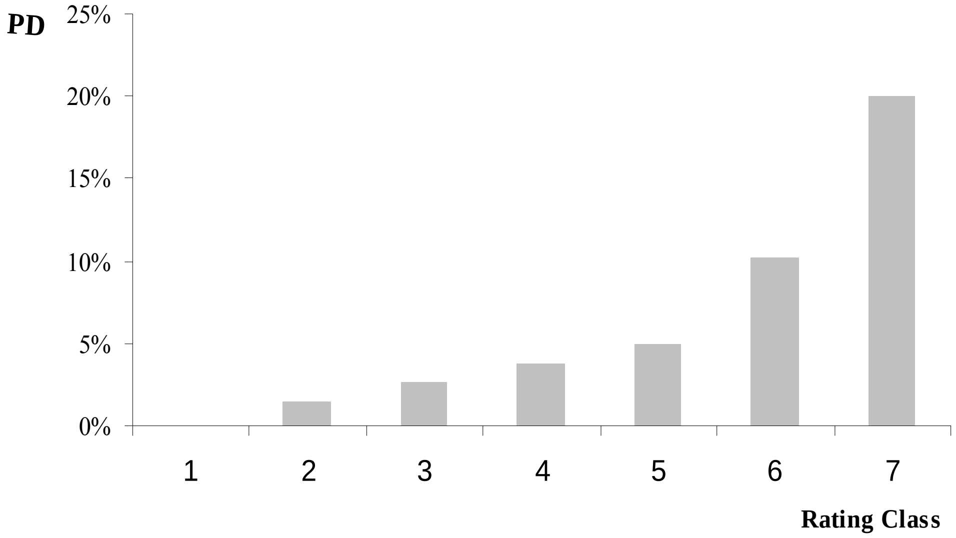


Table 4: PDs over all Transition Periods (%)

Rating Class	1997–98	1998–99	1999–00	2000–01	2001–02
1	0.00	0.00	0.00	0.00	0.00
2	2.94	1.02	0.73	1.33	1.14
3	3.46	3.72	2.39	2.04	1.97
4	3.90	4.71	2.50	3.25	4.21
5	5.31	5.09	3.00	5.52	6.04
6	8.89	13.20	9.38	9.10	10.38
7	16.67	23.53	14.81	25.81	18.75

This eight-class rating system respects the Basel II requirements. The main objective was to devise a rating system with increasing probabilities of default from the best to the worst class, and this objective was fulfilled. The PD structure started from zero in the first class and, as a rule, doubled when switching between adjacent rating classes. The firm distribution in the individual classes also avoided strong concentrations of firms in single classes. The PD structure obtained by averaging the class-specific annual PDs over the entire six-year period is depicted in Figure 3.

Figure 3: An Increasing PD Structure



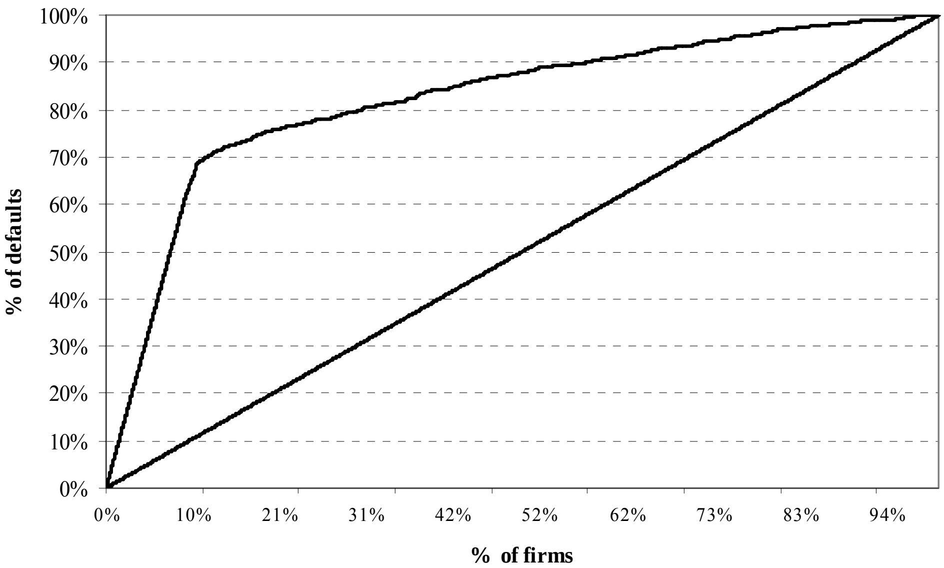
4.2 Validation of the Rating System

The two dimensions of a rating system validation are its power and its ability to match predicted and real default events. The tools used in the validation are particularly relevant in a comparative context since, in general, numerous methods of constructing a rating system are available and the credit analyst has to select the optimal one. In this subsection we describe a set of measures that quantify the "goodness" of the model utilized to create a rating system and present their estimates when applied to our specific case. We first discuss the notions of power curves and Gini coefficients. These are statistical measures of the power of a model. The discussion is accomplished with an account of the measures used to assess the predictive power of a model, such as Alpha- and Beta-errors, accuracy ratios and information entropy ratios.

In statistical terms, the power of a model is assessed by means of power curves and Gini coefficients. A power curve is a graphical representation showing on the horizontal axis the percentage of firms (x) ranked from the worst to the best scoring (rating) and on the vertical axis the percentage of default events "produced" by the firm sample determined by 0 and x over the considered risk horizon. The logic behind such a representation is that likely candidates for default in the future are firms that have a bad scoring (rating) today and that the likelihood of getting new defaults decreases as one moves closer to the sample of good firms. At one limit stands the random model that uniformly distributes defaults over the entire sample. The power curve of the random model is the first diagonal. At the opposite limit stands the perfect model that includes all the defaults within an infinitesimal move away from zero on the horizontal axis. A strong model displays a power curve strongly biased towards the northwestern corner of the figure.

The power of our rating system relied upon and was constrained by the power of the Creditreform index. In other words, we could not outperform Creditreform in distinguish among good and bad firms. To exemplify, a representative power curve is displayed in Figure 4 below.

Figure 4: A Representative Power Curve (2001–2002)



Gini Coefficients

A Gini coefficient is a measure closely related to the notion of the power curve. It is defined as the ratio between the area under the power curve of a model and the area under the power curve of the perfect model. According to this definition, Gini coefficients take values between 50% and 100%. A Gini coefficient of 50% describes the random model and a Gini coefficient of 100% is representative of the perfect model. Table 5 at the end of this section contains the Gini coefficients and the Conditional Information Entropy Ratio values for the individual pairs of years of our data. The relative medium values of the Gini coefficients suggest that our model's performance in terms of power belongs to the middle range.

Conditional Information Entropy Ratio

The information entropy is defined as the amount of additional information a lender would require to determine whether a certain obligor will default or not. In mathematical form it is equal to (see Sobehart et al., 2000):

$$H_0 = -[PD \cdot \log PD + (1 - PD) \cdot \log(1 - PD)],$$

where PD denotes the probability of default of the obligor and $\log(\cdot)$ is the logarithm in any base[1]. This function reaches its maximum, i.e. the bank would need the largest amount of additional information about the obligor when deciding whether to approve the loan or not, when the probability is $1/2$. This case represents a state of absolute ignorance, since both possibilities are equally likely for the bank.

[1] Conventionally, the natural logarithm is used, though the logarithm in base 2 is more convenient to work with because in this case the information entropy lies between 0 and 1. The choice of base does not affect the final result.

In our model we have a set of rating classes $R = \{1,2,...,7\}$ corresponding to the Creditreform solvency index. The conditional entropy measures the information about the probabilities of default and non-default for a specific class R_j in the following manner:

$$H(R_j) = -\lfloor PD(R_j) \cdot \log PD(R_j) + (1 - PD(R_j)) \cdot \log(1 - PD(R_j)) \rfloor,$$

where $PD(R_j)$ is the probability that the issuer defaults given that the rating class is R_j.

The Conditional Information Entropy is defined as an average over all possible risk classes:

$$H_C = \sum_{i=1} w(R_i) \sum_{j=1} P(R_j \mid R_i) \cdot H(R_j),$$

where $w(R_i)$ is the frequency of rating class R_i and $P(R_j | R_i)$ is the migration probability of moving into class R_j conditional on current class R_i.

The Conditional Information Entropy Ratio (CIER) compares the information entropy related to the overall default rate in the sample to the information entropy after we introduce a model, i.e. split the issuers into a number of rating classes and derive the probability of default for each of them. That is, we measure the uncertainty associated with the default frequency in the firm sample and compare it to the uncertainty left over after taking into account the predictive power of the model. The CIER is one minus the ratio of the latter to the former, that is:

$$CIER = 1 - H_C / H_0$$

If CIER is 0 the model has no relevance. In our case, for the pooled dataset we got a CIER value of 29%, which means that the rating system we created clarifies 71% of the information compared to the case in which no rating system would exist. Table 5 compares the values of the Gini coefficients and the CIERs over the entire period studied.

Table 5: Gini Coefficients and CEIRs over all Transitions (%)

Transition period / Ratio	1997–98	1998–99	1999–00	2000–01	2001–02
Gini Coefficient	60.3	60.1	60.1	63.2	61.9
Information Entropy Ratio	80.5	78.0	84.3	81.6	79.8
Conditional Information Entropy Ratio	19.5	22.0	16.7	19.4	20.2

Alpha and Beta Errors

Alpha and Beta errors are the equivalent of the Type I and Type II errors often used in statistical and econometric tests. In the context of a rating system validation, an Alpha error is defined as the proportion of firms that were ex ante classified as good but defaulted, relative to the number of ex post non-defaulted firms. A Beta error is the ratio of firms that were ex ante classified as bad but did not default, relative to all firms that ex post defaulted. This logic can be extended to particular rating classes. Figure 5 shows the distribution of solvent and insolvent firms ex post.

Figure 5: Ex Post Analysis of (In)solvent Enterprises

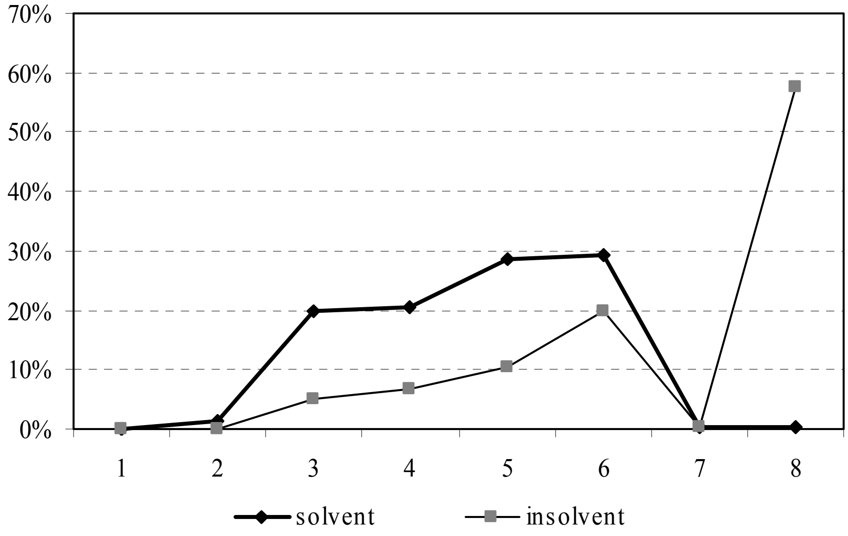


Figure 6 separates the good firms from the bad ones. It shows that most of the solvent enterprises belonged to the positive rating classes while insolvent companies were predominantly in the default class. To identify the Alpha and Beta errors in the rating system it is necessary to compare the number of firms predicted by the model as good (in rating classes 1 to 7) with the real defaults in the next period. This represents the Alpha Error, which is a measure of the default risk. Analogously, one can consider all enterprises classified as bad by the model and compare them to the real outcome of defaulted firms. All those who did not default represent the Beta error. The Beta error is divided over all non-defaulting companies, resulting in a relatively small number, which can be seen in the following figure.

Figure 6: Classification Error over Rating Classes

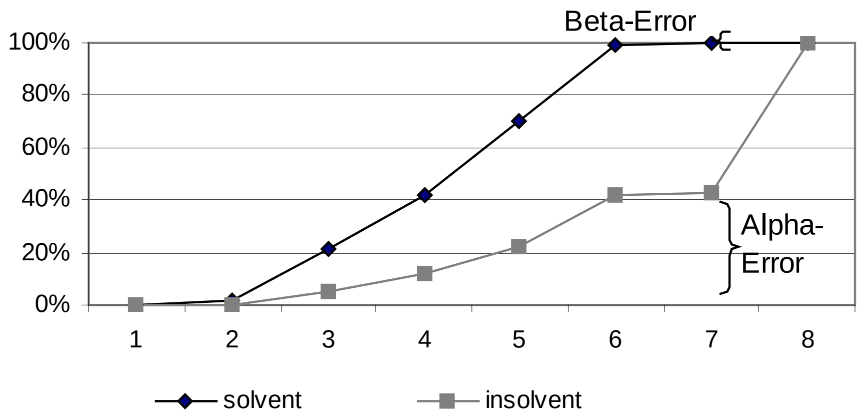


Table 6 shows the distribution of Alpha and Beta errors over the entire period. Like the Gini coefficient it shows the discrimination power of the rating system, which is visualized as the area between both figures. The Alpha error for the insolvent companies is around 42%. These companies were previously rated as good and subsequently defaulted. If they had been given credit this would have resulted in a loss. The Beta error (0.27%) represents only administration costs or lost business, since it refers to those companies which were refused credit in the past but would have otherwise been creditworthy. The first period 1997/98 cannot be taken into account as it is the first transition period and does not include previous defaults for verification. The Alpha error is decreasing over time overall, which is positive for identifying default risk.

Table 6: Alpha / Beta Errors over transitions

Transition period	Alpha Error	Beta Error
1998/1999	47.16%	0.27%
1999/2000	32.52%	0.25%
2000/2001	39.56%	0.22%
2001/2002	31.21%	0.40%

In terms of Alpha / Beta errors, our model displayed a slightly higher Alpha error (42% compared to 39%) and a significantly lower Beta error (0.27% compared to 9%) than in Austria (see Table 7). However, this comparison is only a rough benchmark for the quality of our model, since the input data were slightly different. Not ignoring the fact that the data had a longer history and better quality in Austria, we can nevertheless conclude that the rating system built upon the Czech version of the Creditreform data is close to international standards.

Table 7: Comparison of Creditreform rating system: Czech Republic vs. Austria (%)

	Creditreform – rating system Czech Republic	Creditreform – rating system Austria	Moody's – rating system APPLIED IN Austria
Gini Coefficient	63	81	79
Information Entropy Ratio	79	83	88

4.3 Migration Matrices

The next step is to construct the one-year migration matrices. The elements situated on the rows of these matrices represent the percentage of firms that, starting from a certain rating class at the beginning of the period, migrate to another rating class at the end of the one-year period. The assumption made by the cohort approach is that the historical frequencies of migration are good approximations of the implicit migration probabilities. Appendix A1 contains the migration matrices associated with our rating system. In general, the probability mass of the migration is concentrated on the first diagonal, meaning that migrations become less likely as the distance between classes becomes higher. Appendix A1 also contains the average of the five migration matrices (cell-by-cell average) over the six-year period.

In the literature, rating systems are typically assumed to display Markov (stochastic) properties. In this sense the probability distribution R(t) of the credit ratings of an obligor is assumed to follow a Markov process, i.e. the history of R(t) is described by the relation $R(t+1) = R(t) \cdot M$, where M is the migration matrix among the rating classes. Under the Markov assumption, migration matrices are supposed to be time homogeneous (constant in time). The time homogeneous assumption is an extremely useful property, since, if it holds, it allows the computation of multi-period migration matrices as the power of the one-year migration matrix. The extent to which migration matrices satisfy the time homogeneity property in our rating system is evaluated by means of several matrix metrics that assess the discrepancy among matrices. These metrics are called mobility indices. They have often been referred to in the literature in connection with the time homogeneity property of the migration matrices of a rating system[2].

Given two migration matrices A and B whose elements $a_{i,j}$ and $b_{i,j}$ $i, j = 1, n$ sum to one for each *i* (stochastic matrices), the following metrics can be computed:

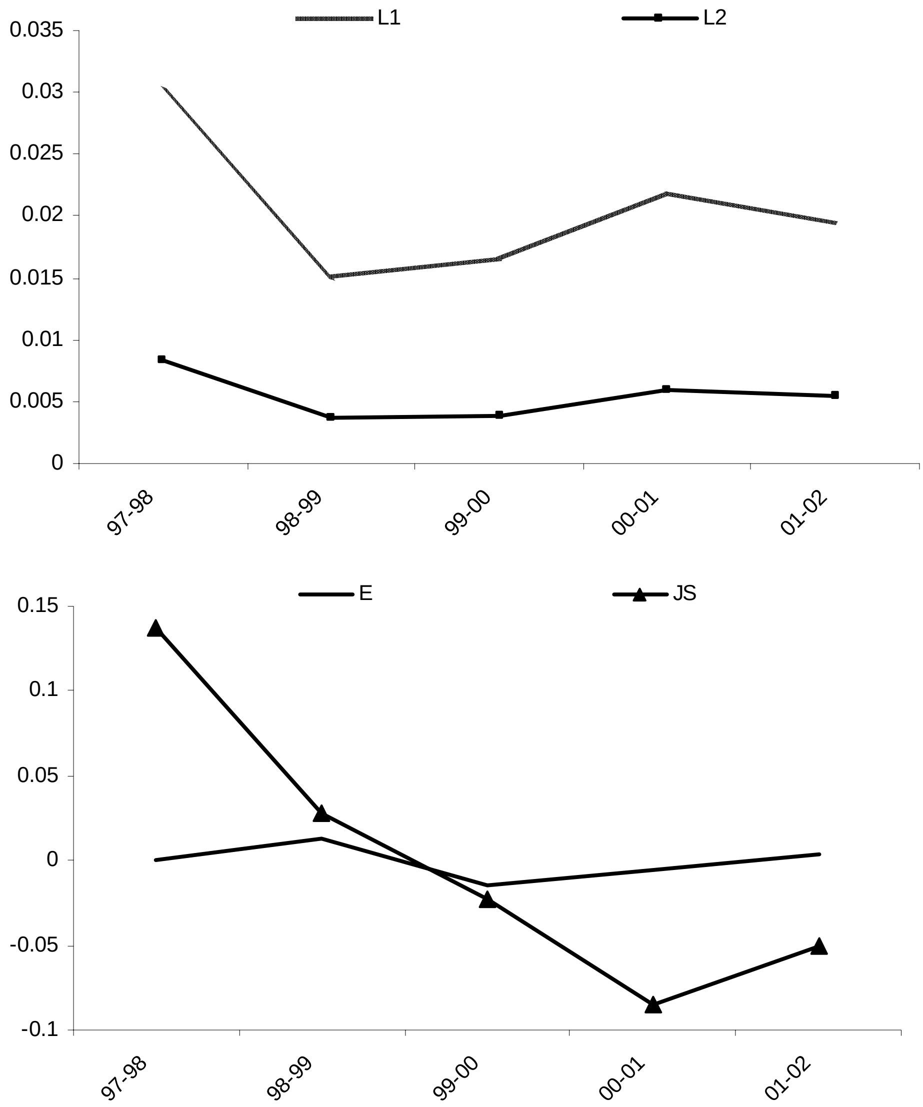
1. $L^1(A,B) = \dfrac{\sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{abs}(a_{i,j} - b_{i,j})}{n^2}$,
2. $L^2(A,B) = \dfrac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} (a_{i,j} - b_{i,j})^2}}{n^2}$,
3. $E(A,B) = \lambda_2(B) - \lambda_2(A)$, where $\lambda_2(\cdot)$ is the largest eigenvalue below one,
4. $JS(A,B) = \dfrac{\sum_{i=1}^{n} \sqrt{\lambda_i(\tilde{A}'\tilde{A})}}{n} - \dfrac{\sum_{i=1}^{n} \sqrt{\lambda_i(\tilde{B}'\tilde{B})}}{n}$.

The first two metrics are the equivalent of the standard Euclidian distances defined in the R^n space. They aggregate into an overall measure the cell-by-cell distances between the elements of the two matrices. The third measure captures differences in the convergence rates towards the steady states of the probability distributions R governed by the two transition matrices. The fourth measure reflects differences in the average probability of migration as defined by Jafry and Schuermann (2003). Their matrix norm is constructed as the average of the singular values of the mobility matrix $\tilde{A}'\tilde{A}$ ($\tilde{A} = A - I$, $\tilde{A}' = \mathrm{transpose}(\tilde{A})$, A = the migration matrix, I = the identity matrix of order *n*). The singular values represent the square root of the eigenvalues of the matrix $\tilde{A}'\tilde{A}$. This norm describes an "average" propensity of migration of the rating system from the current rating classes to different rating classes within the considered period. The closer to zero these metrics are, the more likely it is that the time homogeneity assumption is valid.

Computing the values of these metrics it is possible to compare the "distance" between annual migration matrices and their period average. Figure 7 depicts the results.

[2] However, to our knowledge these mobility indices do not constitute a formal test statistic by themselves. Further research in this direction is required.

Figure 7: Comparison between Annual Migration Matrices and their Period Average



Based on visual inspection, the migration matrices are similar according to the L1 and L2 metrics, with the migration matrix from 1997 to 1998 being an outlier. The E metric reveals that differences in convergence rates between individual migration matrices and their period average are insignificant (the E line has small fluctuations around zero). The degree of mobility of the annual migration matrices relative to their period average decreased between 1997 and 2001 but increased slightly between 2001 and 2002. Thus, the average tendency of ratings to change over a one-year period (rather than remain in the current state) lost pace from 1997 to 2001 but changed trend between 2001 and 2002.

5. Credit Risk Modeling for Czech Loans

5.1 Introduction to Credit Risk Modeling

The release of a series of consultative documents and quantitative impact studies by the Basel Committee and several commercial credit risk models by renowned financial institutions has reinforced the awareness of the banking sector of the necessity to measure and control the risk associated with banks' lending operations. The publication in 2001 of the New Basel Capital Accord and three consequent impact studies triggered an intensive dialog among regulators and bankers worldwide. The aim is to formulate an optimal set of norms and regulations meant to become standard practice in bank capital provisioning against credit risk by 2006. At the same time, credit risk models have raised constant interest within the banking industry because these models allow sensitive measurement of default risk at the portfolio level. The most prominent credit risk models developed and to a certain extent already applied by the banking industry are Credit Metrics (JP Morgan), CreditRisk+ (Credit Suisse Financial Products), KMV (Moody's – KMV) and CreditPortfolioView (McKinsey's & Company).

In what follows, the methodologies applied in this paper are briefly presented: the Internal Ratings Based (IRB) approach as formulated by the latest Consultative Document of the NBCA (April 2003), and the Credit Metrics and CreditRisk+ models. The main goal of the paper is to reflect on the applicability of these methodologies and not to offer a comprehensive theoretical description. For this reason the remaining part of this section sketches the most important steps that are essential in estimating the risk capital in each considered case.

5.1.1 The New Basel Capital Accord (NBCA)

The regulatory guidelines referring to credit risk assessment and capital budgeting are exposed in the NBCA under two main headings, the standardized approach and the Internal Ratings Based (IRB) approach. Irrespective of the approach selected, banks are supposed to categorize all their exposures within a well-defined range of categories and to apply category-specific regulatory rules[3]. The contribution of credit risk-related regulatory capital to an overall minimum capital requirement (additionally incorporating capital for market and operational risk) is identical under the two approaches.

The standardized approach relies on rating systems provided by external agencies (Moody's – KMV, S&P) and on risk weights that are calibrated to the rating classes of these rating systems. The IRB approach specifies concrete regulatory capital formulas permitting banks to use their own estimations of the required input data, including among them banks' internal ratings. If a bank applies the IRB approach, it has to estimate the following risk inputs at the obligor (asset) level: the probability of default (PD), an estimate of the loss incurred if default occurs (Loss

[3] The following claim categories are considered under the standardized approach: sovereigns, non-central government public sector entities (PSEs), multilateral development banks, banks, securities firms and corporates. Considered under the IRB approach are sovereign, bank, corporate, retail, equity and purchased receivables exposures.

Given Default or LGD), the loan exposure (EAD) and an effective maturity (M). Specific eligibility criteria are provided in the regulatory documents that allow banks to estimate these risk inputs based on an internal process. Additionally, under the IRB approach two alternative procedures are mentioned, the foundation approach and the advanced approach. Under the foundation approach, banks may derive their own estimates only for the PDs[4], with all other risk-input estimations conforming to the regulatory rules. Under the advanced approach, banks can use their own estimates for all the required risk inputs.

The initial IRB methodology for corporate exposure (January 2001) was significantly modified in the recent Consultative Document of the NBCA (April 2003). The current formulation entails the application of the following algorithm when computing a risk-adjusted value of a bank asset:

- Computing a correlation coefficient (R):

$$R = 0.12 \cdot \frac{1 - e^{(-50+PD)}}{1 - e^{-50}} + 0.24 \cdot \left[1 - \frac{1 - e^{(-50+PD)}}{1 - e^{-50}}\right],$$

- Computing a maturity adjustment coefficient (b):

$$b = \left(0.08451 - 0.05898 \cdot \ln(PD)\right)^2,$$

- Computing a capital requirement coefficient (K)[5]:

$$K = LGD \cdot N\left[\frac{G(PD)}{\sqrt{1-R}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] + \frac{1 + (M - 2.5) \cdot b(PD)}{1 - 1.5 \cdot b(PD)},$$

- Estimating the risk-weighted asset value:

$$RW = 12.5 \cdot K \cdot EAD.$$

The regulatory capital represents 8% of the sum of the risk-weighted assets. Our computations conformed to the foundation approach of the IRB by employing our own estimates for the PDs and determining all other risk inputs according to the regulatory rules. We used an LGD value of 45% (as required for senior claims on corporates), an effective maturity of 2.5 years and an EAD equal to the face value of the loans (the amount legally owed to the bank).

[4] The only restriction that applies in this case is that the PDs are bounded from below by the 0.03% value, meaning that PDs below this value (according to the bank's internal rating system) must be replaced by the 0.03% value for regulatory capital estimation purposes.

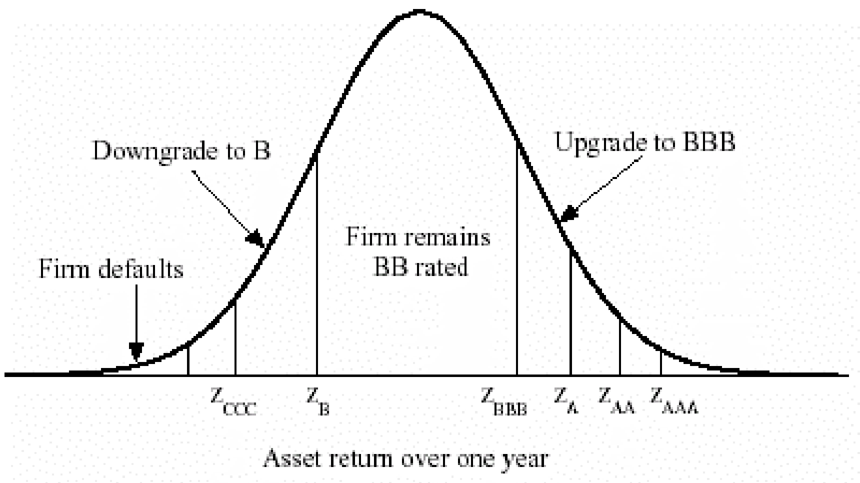
[5] N(x) and G(x) denote the cumulative distribution function for a standard normal distribution and its inverse, respectively. It should also be mentioned that the PD value must be expressed as whole numbers in the computation of R. For example, a PD of 2.5% should enter the R formula as 2.5 and not as 0.025. On the contrary, in computing the coefficients b and K one must use PD values in the traditional sense, thus as numbers between 0 and 1.

5.1.2 CreditMetrics

CreditMetrics is a typical mark-to-market model in which changes in asset value are induced by credit migrations and defaults taking place over the risk horizon. Therefore, two elements play a crucial role in the theoretical construction and practical implementation of this model: a loan valuation method dependent on credit ratings changes and a tool that generates random changes in credit quality over the risk horizon. While the second target is fulfilled by recourse to Monte Carlo simulations, the first one draws insights from a Merton-type model of the firm's value and, more generally, from the dynamic asset pricing theory.

Based on Merton's theory, the arrival time of a default event is defined as the first time when the borrowing firm's asset value falls below the outstanding debt obligations of the firm[6]. This principle is extended to accommodate changes in rating quality, by assuming that certain asset threshold values can be defined that mark the change from one rating class to another, once the firm's asset value falls below the corresponding threshold values. To simplify the analysis, CreditMetrics links the calculation of these threshold values to the migration matrices made available by rating agencies. The way in which CreditMetrics operates this mapping is based on the observation that the probability mass of the migration matrices is usually distributed on the first diagonal, and dies out at a high speed as one moves towards more distant ratings. This means that the most likely credit event for the firm is to preserve the current rating over the risk horizon. The adjoining rating migrations are less likely and the likelihood of further changes decreases further. Then the idea is to link the probabilities of realization of these credit quality changes to the standard normal distribution. This is shown in Figure 8 below.

Figure 8: The Partition of the Standard Normal Distribution and the Threshold Values



Source: CreditMetrics Technical Document.

[6] More precisely, CreditMetrics works with percentage changes in asset values or asset returns.

The figure shows the partition of the region under the standard normal distribution in distinct zones, whose areas equal the migration probabilities of the BBB rating class according to the S&P rating system. The middle zone characterized by the largest area reflects the probability of preserving the BBB rating. The next step is to observe that these zones are well delimited and can be mapped in a one-to-one manner into a set of real numbers (the Z values on the *x* axis on the figure). These Z values are called threshold values and are computed with the help of the inverse of the standard normal cumulative function.

The calculation of the threshold values represents the key tool in implementing Monte Carlo simulations. The idea is to draw random numbers from the standard normal distribution (one for each asset), to compare them with the rating class-specific threshold values and to assess what rating migrations these numbers would suggest. This procedure is complicated to some extent in the portfolio context, since rating migrations for different obligors are, as a rule, correlated. Performing independent random draws from the normal distribution would make no sense in this context. The problem, however, can be easily solved on the basis of Cholesky factorization or singular value decomposition methods that are usually available in the current statistical software programs.

An important aspect of the CreditMetrics model is the loan valuation. Depending on the credit migration occurring in the case of non-default, a loan can be valued according to the formula:

$$V_g^{g'} = \sum_{t=1}^{T_i-1} \frac{r}{\left(1+d_t^{g'}\right)^t} + \frac{r+F}{\left(1+d_{T_i}^{g'}\right)^{T_i}}, \tag{1}$$

where *r* and *F* are the loan interest and face value, and $d_t^{g'}$ are the discount factors for the years 1 to *T*, applicable to the rating class *g′* (here *T* is the maturity of the loan). In this specification it is assumed that the present rating changes from *g* to *g′* over the one-year period. In the case of default the present value of the loan is computed as the product of the face value of the loan and the recovery rate. The portfolio value is the sum of the individual loans' valuations.

A particular difficulty in (1) is posed by the estimation of the discount factors *d* entering bank loans' valuation. This estimation relies on risk neutral probabilities whose existence (and uniqueness) is conditional on the assumption of complete markets and no-arbitrage conditions. Even if these general assumptions are disregarded in practice, the estimation of the discount factors is still problematic, since it requires information on the price of risk in the loan market. Since loans are non-traded debt instruments, the price of risk can be at most proxied. At least from a theoretical point of view, a few approaches for the estimation of the discount factors have been developed, one of which will be explained in more detail in Subsection 5.2.

Random draws of real numbers replicate random changes in credit quality. Contingent on the rating migration simulated, loans are re-evaluated and the portfolio value is computed. This is the principle of the Monte Carlo simulation. Random drawings of real numbers are performed a large number of times (preferably ten thousand times or more). The corresponding portfolio values generate the empirical portfolio distribution. Economic capital estimations can then be performed in a manner to be presented in Subsection 5.3.

5.1.3 Credit Risk+

CreditRisk+ is a default mode model that borrows intensively from the actuarial models used in insurance economics. Default mode means that the credit standing of a certain obligor over the risk horizon can reach only two states: default and non-default. CreditRisk+ adopted the reduced-form approach to modeling default risk by calibrating random default and loss events to standard statistical distributions. Even if default events at the individual obligor level are not directly modeled, default probabilities represent a compulsory input into the CreditRisk+ model and must be estimated by the credit analyst.

The approach followed by CreditRisk+ is to "homogenize" the pool of risky loans by grouping them into classes ("bands" in the CreditRisk+ terminology) with similar risk characteristics. The deciding factor in performing this classification is the so-called common exposure at the band level. In this sense, credit exposures are scaled down by a selected unit of exposure and obligors with similar exposures (after rescaling and rounding to the nearest integer) are grouped together. Two distributions are relevant in the analysis. The distribution of default events at the band level is modeled as a Poisson distribution:

$$P(\text{number of defaults} = k) = \frac{m^k e^{-m}}{k!}, \qquad k = 1, 2, ...$$

with the expected number of defaults given by m and with a standard deviation $\sqrt{m}$. The second distribution is the one related to the entire portfolio loss, i.e. portfolio losses expressed as multiples of the unit of exposure related to their probabilities of realization. The derivation of both distributions rests upon the construction of a probability generating function and its statistical properties. In the latter case a recursive formula is derived that estimates the probabilities that loss equals multiples of the unit of exposure. The estimation of the economic capital is performed on the basis of the portfolio loss distribution thus derived in a manner to be presented in Subsection 5.3.

CreditRisk+ allows two types of generalizations in the basic set-up, a risk analysis extending over a multi-year period and a risk analysis with variable default rates. In the latter case diversification effects are captured in the model by incorporating the sensitivity of the obligors to systemic risk factors ("sectors" in the CreditRisk+ terminology). The average default rate of a certain sector is assumed to follow a Gamma distribution, which transforms the random variable describing the number of defaults at the sector level into a negative binomial distribution. The same reasoning as in the basic case applies for the derivation of the portfolio loss distribution in this case.

We applied the CreditRisk+ model with a multi-year default structure and with all the exposures assigned to a single sector (the general economy). Default rates at horizons longer than one year were determined by multiplying the one-year migration matrix by itself n times (n is the number of years) and then examining the elements situated in the last column of the resulting matrices.

5.1.4 The Czech Loan Portfolio

The Czech economy has undergone dramatic changes and dynamic development since the 1989 revolution. The corporate sector is now mostly privatized and Czech banks are currently mainly foreign-owned. The banking system has stabilized after a series of bank bankruptcies that took place at the beginning of the transition period. However, even if default rates have been decreasing for years, credit risk still represents a major risk for bank lending operations to businesses. Classified loans (for which payment has been delayed for more than 30 days) as a percentage of the total credit volume fell from 32% in 1999 to 16% in 2002.

In what follows we construct a representative portfolio for bank lending to corporates in the Czech Republic. The assets of the portfolio are represented by bank credits to Czech industries (according to the NACE classification). For this reason the terms assets and industries will be used interchangeably in the rest of the paper. In principle, defining the portfolio is tantamount to estimating all the industry characteristics that represent the necessary inputs in the different credit risk methodologies approached in this paper. A quick reference to the required input data is contained in Table 8.

Table 8: Input Data Required by Different Approaches

NBCA	CREDITMETRICS	CREDITRISK+
• Ratings and PDs	• ratings and PDs/migration matrices	• credit exposures
• Credit exposures	• credit exposures	• PDs
• Maturities	• maturities	• PDs' volatilities
• Recovery rates	• recovery rates	
	• interest rates on loans	
	• asset return correlations	
	• discount factors (credit spreads)	

The remaining part of this section examines our data sources and the methodologies applied to estimate the required model inputs.

5.1.5 Ratings

To assign ratings at the industry level we computed historical default rate frequencies and their period averages at each industry level. The statistical significance of the average default rates was assessed according to the methodology proposed by Cantor and Falkenstein (2001). More precisely, following their notation let us define:

- $n_{i,t}$ as the number of firms in industry *i* at time *t*,
- $d_{i,t}$ as the number of defaults in industry *i* at time *t*,
- $PD_{i,t}$ as the default rate in industry *i* at time *t*,
- $N_i = \sum_t n_{i,t}$ as the number of issuer-years in industry *i*,
- $D_i = \sum_t d_{i,t}$ as the number of defaults-years in industry *i*,
- $PD_i = \frac{D_i}{N_i}$ as the average default rate over the entire period in industry *i*.

Assuming that the underlying (true) default rate p_i of industry *i* is constant in time, Cantor and Falkenstein show that the empirical default rates $PD_{i,t}$ and PD_i are approximately normally distributed with mean p_i and standard deviations $\sqrt{\frac{p_i \cdot (1-p_i)}{n_{i,t}}}$ and $\sqrt{\frac{p_i \cdot (1-p_i)}{N_i}}$, respectively. If the underlying default rate is not constant in time, due either to fluctuating macroeconomic conditions or to idiosyncratic reasons at the industry level, then the average default rate for industry *i* has a standard deviation of $\sqrt{\frac{p_i \cdot (1-p_i)}{N_i} + \frac{\sigma^2}{N_i^2}\sum_t n_{i,t}^2}$, where σ is the standard deviation of a macroeconomic shock affecting the economy.

Standard deviations at the industry level were computed assuming both constant and variable true default rates. In the latter case σ was approximated by the standard deviation of the default rate of the entire sample of firms over the period 1997–2002. In the final portfolio we selected those industries that displayed standard deviations of the default probabilities of less than 2%. Additionally, we eliminated those industries for which the sample contained a very small number of firms.

The results are reported in Appendix A2. The elements of that table contain annual default rates at the industry level, the standard deviations of the average default rates and the rating class assignment of each industry. Designated in bold numbers are the industries whose average default rates seemed reliable and thus were considered as assets of the portfolio. By comparing the default rates at the industry level (the average) with the default rates situated in the last column of the average transition matrix, we assigned the individual industries to rating classes. For example, if the average PD of a given industry was 4.14% (agriculture), the industry was assigned to the rating class 5, since its PD belonged to the interval defined by the representative PDs of the fourth and fifth classes.

5.1.6 Credit Exposure

We aggregated the loan volumes granted by all Czech banks to non-financial firms. These loans are reported as a stock at the end of each month in the SUD database of the Czech National Bank. We sorted the loans according to industry destination, currency denomination (Czech crowns, US dollars and euro) and the maturity classification used by the Czech National Bank (less than 1 year, 1–4 years, 4–5 years, more than 5 years). We aggregated only loan volumes in the classification range from one to four, with non-performing loans (loss loans or loans in the fifth category) being neglected. The aggregate bank credit exposure respecting this structure was estimated at the end of 2002 and is shown in Appendix A3.

5.1.7 Interest Rates

Loan interest rates are not classified according to industry in the databases of the Czech National Bank. For this reason we used data made available by the Ministry of Industry and Trade of the Czech Republic. We computed an implicit interest rate at the NACE level defined as the ratio between interest expenditure and total bank loans at the industry level at the end of 2002.

Unfortunately, these estimations provided only a general indication of the loan interest rates and were not differentiated according to loan maturity. For those industries for which relevant data was missing we used the figures available at the next level of aggregation (for example, data was available as an aggregate over NACE 50, 51 and 52 but not at the level of each of these industries) or the economy-wide average (NACE 1). The results are displayed in Appendix A4.

5.1.8 Recovery Rates

No explicit measure of the loans' recovery rates at the industry level is publicly available. For this reason we assumed a recovery rate of 55% for all industries. This figure is compatible with the 45% LGD value considered in the regulatory case[7].

5.1.9 Asset Return Correlations

Asset returns at the industry level were proxied by the corresponding producer price indices and correlations among these indices were computed. The implicit assumption made was that an adverse shock affecting a certain industry would induce a fall in the corresponding producer prices. In principle, measuring correlations among sectoral equity returns would have been preferable, since equity indices offer a better image of the trends in asset returns at the industry level. The main drawback was that equity indices were not available for some of the industries included in the portfolio. Additionally, the feeble firm representation on the Czech Stock Exchange could have depicted a biased picture of the real productivity trends of particular industries anyway.

Each price index was divided by the PPI to eliminate systemic influences that could have inflated the correlations. Monthly price indices that covered the period January 1995 – September 2003 were obtained from the Czech Statistical Office. Since such price indices were not available for the industries NACE 52, 63 and 73 these industries were not further considered. The industries NACE 65 and 67 represented the financial sector, so they were also ignored in the computations. Correlations among price indices at the industry level are contained in Appendix A5.

5.1.9 Discount Factors (Credit Risk Spreads)

The approach of Jarrow, Lando and Turnbull was implemented to estimate the term structure of credit spreads. Jarrow, Lando and Turnbull formalize changes in rating quality as a Markov chain described by a time-homogeneous migration matrix.

Consider an n-class rating system $R = \{1,2,...,D\}$ (the last class *D* denoting default) and the associated time homogeneous migration matrix *M* whose elements M_{ij} represent the probability of migration between the rating classes *i* and *j*. If the life horizon of the loan [0, T] is divided into *m* intervals [t, t+1], t = 0,…,T-1 over which changes in credit quality may occur, the risk premia at

[7] The recovery rate is 1-LGD.

time *t* are the rescaling factors $\pi_i(t)$ that transform the physical migration probabilities M_{ij} into the corresponding risk-neutral migration probabilities $M^*_{ij}(t, t+1)$:

$$M^*_{ij}(t, t+1) = \pi_i(t) \cdot M_{ij}, \qquad \text{i, j = 1,n, i = j}$$

In Jarrow, Lando and Turnbull the estimation of the risk premia is done recursively. The spot values $\pi_i(0)$ are given by

$$\pi_i(0) = \frac{B^0(0,1) - B(0,1)}{B^0(0,1) \cdot (1-\delta) \cdot M_{iD}}$$

where $B^0(0,1)$ and $B_i(0,1)$ are the "prices" of default-free and risky loans respectively, δ is the recovery rate and M_{iD} is the default rate of the rating class *i*. At the time horizon *t* the risk premia values are the solution of the system

$$M^*(0,t) \cdot \begin{pmatrix} \pi_1(t) \cdot M_{1D} \\ \ldots\ldots \\ \pi_D(t) \cdot M_{DD} \end{pmatrix} = \begin{pmatrix} \dfrac{B^0(0,t+1) - B_1(0,t+1)}{B^0(0,t+1)(1-\delta)} \\ \ldots\ldots \\ \dfrac{B^0(0,t+1) - B_D(0,t+1)}{B^0(0,t+1)(1-\delta)} \end{pmatrix} \tag{2}$$

where $M^*(0, t)$ is the risk neutral migration matrix between time 0 and t defined by the recursive formula $M^*(0,t) = M^*(0,t-1) \cdot [I + \pi(t-1) \cdot (M - I)]$.

Jarrow, Lando and Turnbull also derive a closed form solution for the credit spreads

$$S_i(t) = \ln \frac{1}{1 - (1-\delta) \cdot \pi_i(t) \cdot M_{iD}}. \tag{3}$$

In our case the "prices" of default free and risky loans in (2) were computed as

$$B^0(0,t) = e^{-r_t \cdot t} \qquad \text{and} \qquad B_i(0,t) = e^{-(r_t + \varphi_{it}) \cdot t}, \qquad t = 1, 2, \ldots, 6$$

where r_t was the risk free rate (1-year PRIBOR) and φ_{it} were the interest rate charges on risky loans requested by some Czech banks at the end of 2002. Here *t* represents the maturity of the loans and *i* the rating class.

The credit spread estimations in the Czech market based on (3) are depicted graphically in Appendix A6

The Jarrow, Lando and Turnbull model constitutes a convenient theoretical technique for the estimation of the credit spreads in the Czech bank loan market. The main deficiency of this estimation was related to the quality of the input data. The interest rate charges on risky loans were calibrated to banks' internal rating systems and these rating systems were not consistent with the one we used. Moreover, the probabilities of default characterizing our risky classes (the sixth and seventh classes) were significantly higher than the default probabilities accepted by the Czech banks when granting loans to corporate clients. Since interest charges on loans with such risk characteristics were not available as real data we had had to perform several calibrations[8]. Additionally, since data was obtained from a small number of banks, the possibly of depicting a biased picture is very high.

The estimated model inputs are summarized in Table 9. This represents the 33-asset portfolio on which the aforementioned methodologies were applied.

Table 9: Czech Loans Portfolio

NACE	Average Default Rate	Rating Class	Loan Exposure (bil. CZK)	Implicit Loan Interest Rate	Maturity (years)	Recovery Rate
1 – Agriculture	4.14%	5	14.76	16.30%	6	55%
14 – Mining other	2.86%	4	1.47	6.23%	6	55%
15 – Manufacture of food products and beverages	7.51%	6	22.24	8.65%	6	55%
17 – Manufacture of textiles	4.22%	5	6.37	9.27%	6	55%
18 – Manufacture of wearing apparel; dressing and dyeing of fur	4.11%	5	2.49	11.88%	6	55%
20 – Manufacture of wood and of products of wood	7.73%	6	2.05	7.76%	6	55%
21 – Manufacture of pulp, paper and paper products	3.56%	4	7.03	6.79%	6	55%
22 – Publishing, printing and reproduction of recorded media	4.17%	5	4.59	7.81%	6	55%
24 – Manufacture of chemicals and chemical products	2.98%	4	14.15	9.18%	6	55%
25 – Manufacture of rubber and plastic products	3.39%	4	7.88	9.90%	6	55%
26 – Manufacture of other non-metallic mineral products	4.90%	5	14.14	11.49%	6	55%
27 – Manufacture of basic metals	6.07%	6	5.49	10.29%	6	55%
28 – Manufacture of fabricated metal products	4.36%	5	8.21	12.63%	6	55%
29 – Manufacture of machinery and equipment	4.72%	5	12.92	11.14%	6	55%
30 – Manufacture of office machinery and computers	5.41%	6	0.17	1.66%	6	55%
31 – Manufacture of electrical machinery and apparatus	2.59%	3	5.67	15.56%	6	55%
32 – Manufacture of radio, television and communication equipment and apparatus	3.49%	4	0.82	19.12%	6	55%

[8] More precisely, for the last two rating classes (associated with default probability values of 10% and 20%) we assumed interest rate charges over the risk free rate of 50% and 200%, respectively. This was an artificial way to reflect the fact that Czech banks refuse, as a rule, to grant loans to corporate customers with default probabilities in this range. However, these assumptions affected the estimation of the credit spreads in the last two non-defaulted classes. Since credit risk spreads are essential for CreditMetrics, its estimated economic capital values are to some extent inadequate.

33 – Manufacture of medical, precision and optical instruments, watches and clocks	3.65%	4	1.19	8.80%	6	55%
34 – Manufacture of motor vehicles, trailers and semitrailers	1.29%	2	7.33	28.29%	6	55%
36 – Manufacture of furniture	5.12%	6	4.63	7.87%	6	55%
37 – Recycling	3.37%	4	0.64	14.73%	6	55%
40 – Electricity, gas, steam and hot water supply	2.80%	4	24.90	13.96%	6	55%
41 – Collection, purification and distribution of water	0.00%	1	2.75	3.06%	6	55%
45 – Construction	5.58%	6	9.81	9.83%	6	55%
50 – Sale and repair of motor vehicles and cycles and fuel	4.45%	5	9.58	9.63%	6	55%
51– Wholesale trade	5.70%	6	56.53	9.63%	6	55%
55 – Hotels and restaurants	5.79%	6	2.21	7.03%	6	55%
60 – Land transport; transport via pipelines	2.39%	3	16.44	16.30%	6	55%
64 – Post and telecommunications	1.89%	3	10.67	16.30%	6	55%
70 – Real estate	8.99%	6	36.69	11.05%	6	55%
71 – Renting of machinery and equipment	4.80%	5	13.01	11.05%	6	55%
72 – Computer and related activities	1.45%	3	3.04	11.05%	6	55%
74 – Other business activities	5.59%	6	21.47	11.05%	6	55%

5.2 Regulatory and Economic Capital Estimations

Table 10 illustrates the algorithm discussed in 5.1.1 for the estimation of regulatory capital.

Table 10: Regulatory Capital According to the IRB Approach

NACE	PD	EXPOSURE (bil.CZK)	R	B	K	RWA (bil.CZK)
1	4.14%	14.76	0.12	0.07	0.12	22.37
14	2.86%	1.47	0.12	0.09	0.10	1.79
15	7.51%	22.24	0.12	0.06	0.17	47.45
17	4.22%	6.37	0.12	0.07	0.12	9.76
18	4.11%	2.49	0.12	0.07	0.12	3.75
20	7.73%	2.05	0.12	0.06	0.17	4.45
21	3.56%	7.03	0.12	0.08	0.11	9.74
22	4.17%	4.59	0.12	0.07	0.12	6.99
24	2.98%	14.15	0.12	0.09	0.10	17.61
25	3.39%	7.89	0.12	0.08	0.11	10.61
26	4.90%	14.14	0.12	0.07	0.13	23.66
27	6.07%	5.49	0.12	0.06	0.15	10.39
28	4.36%	8.21	0.12	0.07	0.13	12.83
29	4.72%	12.92	0.12	0.07	0.13	21.15
30	5.41%	0.17	0.12	0.07	0.14	0.30
31	2.59%	5.67	0.12	0.09	0.09	6.47
32	3.49%	0.82	0.12	0.08	0.11	1.12
33	3.65%	1.19	0.12	0.08	0.11	1.67

34	1.29%	7.33	0.12	0.12	0.06	5.41
36	5.12%	4.63	0.12	0.07	0.14	7.94
37	3.37%	0.64	0.12	0.08	0.11	0.86
40	2.80%	24.90	0.12	0.09	0.10	29.84
41	0.00%	2.75	0.15	0.49	0.01	0.43
45	5.58%	9.81	0.12	0.06	0.14	17.69
50	4.45%	9.58	0.12	0.07	0.13	15.15
51	5.70%	56.53	0.12	0.06	0.15	103.23
55	5.79%	2.21	0.12	0.06	0.15	4.06
60	2.39%	16.44	0.12	0.09	0.09	17.88
64	1.89%	10.67	0.12	0.10	0.08	10.03
70	8.99%	36.69	0.12	0.05	0.19	86.40
71	4.80%	13.01	0.12	0.07	0.13	21.50
72	1.45%	3.04	0.12	0.11	0.06	2.42
74	5.59%	21.47	0.12	0.06	0.14	38.77
Total RWA						573.73
Regulatory Capital = 0.08 · Total RWA						45.90

The two discussed credit risk models use the Value at Risk (VaR) paradigm to perform economic capital estimations. The logic behind Value at Risk is that economic capital provisioning must cover large but unlikely losses resulting from the joint default (or migration to low states) of a large number of obligors in the portfolio. Despite working with different distributions (portfolio value distribution in CreditMetrics and portfolio loss distribution in CreditRisk+), both models provision for unexpected losses by estimating the magnitude of the so called p-quantiles relative to the expected value (loss) of the portfolio.

In terms of losses, a p-quantile represents the threshold value that the portfolio loss would exceed with a *1-p* probability (usually p is expressed in percentages). In terms of the portfolio value distribution, a p-quantile represents the threshold value below which the portfolio value would fall with a *1-p* probability. Formalizing, the p-quantiles can be expressed as

$$\mathrm{p-quantile} = \mathrm{F}^{-1}(1-\mathrm{p})$$

if the model estimates the portfolio value distribution, and as

$$\mathrm{p-quantile} = \mathrm{F}^{-1}(\mathrm{p})$$

if the loss distribution is estimated. Here F is the cumulative function of the portfolio value (loss) distribution. In other words, the p-quantiles are the values where the area under the left tail of the cumulative distribution (right tail if the loss distribution is modeled) is equal to (1-p)% and p%, respectively.

Economic capital is defined as the difference between the p-quantile and the expected loss in models that construct the portfolio loss distribution. In models that estimate the portfolio value distribution, economic capital is the difference between the average portfolio value and the p-quantile. In both cases, however, one can be confident with a p% probability that the economic capital would cover losses defined by the p-percentile. It is the task of the bank's credit risk analysts to select the p level (the confidence level) that most likely guarantees the absorption of large losses by the maintained capital. The common practice is to focus on values ranging from 95% to 99.9%. As a rule, banks might give preference to the 95% value, while the 99% and higher values represent the more conservative regulatory option.

The two distributions generated by the two credit risk models in our case are depicted in Figures 9 and 10.

Figure 9: Portfolio Value Distribution According to the CreditMetrics Model

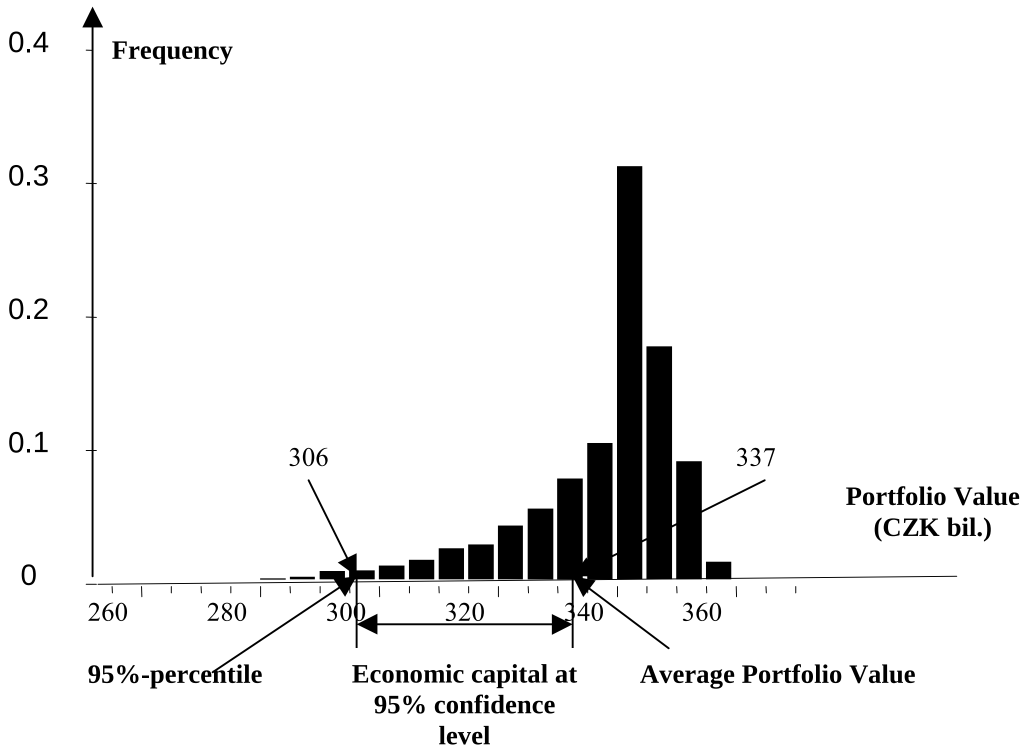


Figure 10: Portfolio Loss Distribution According to the CreditRisk+ Model

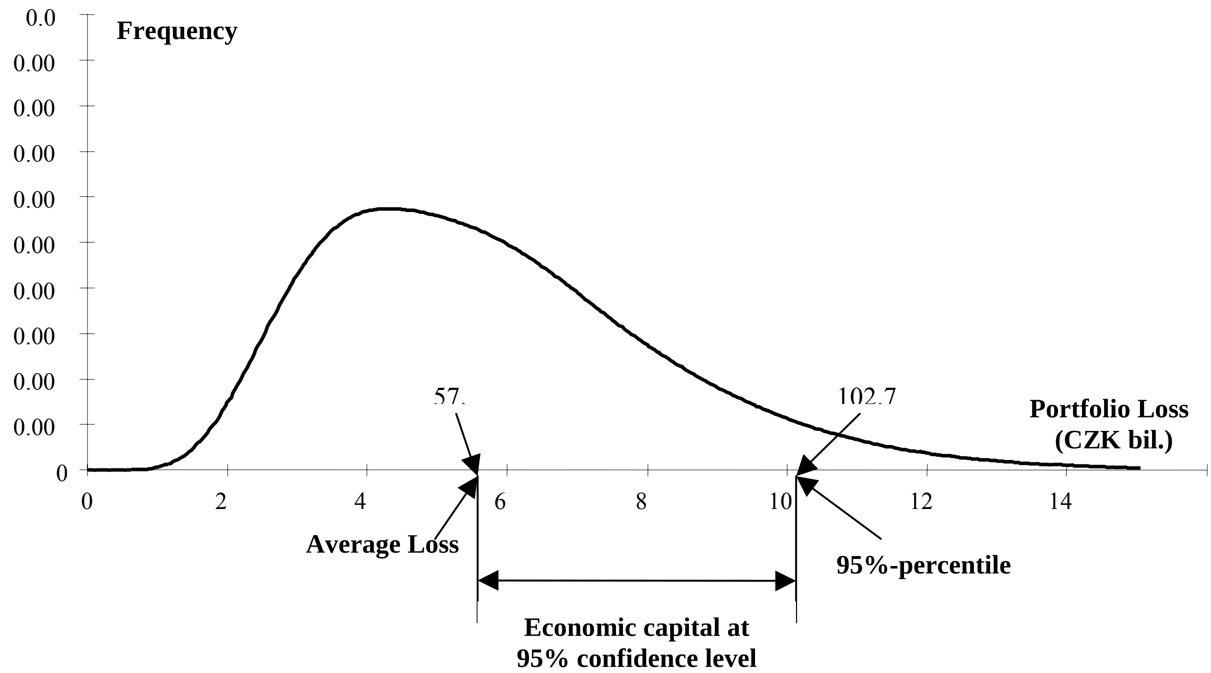


Table 11 summarizes our estimations of regulatory and economic capital.

Table 11: Regulatory and Economic Capital (CZK bil.)

	Regulatory Capital	Economic Capital			
		95%	99%	99.5%	99.9%
NBCA (IRB)	45.90				
CreditMetrics		30.74	49.81	53.96	64.89
CreditRisk+		45.12	70.39	80.32	101.95

Several comments can be made on the basis of these results. First, the predicted economic capital exceeds regulatory capital at the 99% and higher confidence levels, irrespective of the credit risk model selected. It appears that at a systemic level the regulatory guidelines of the NBCA would not impose capital cushions in excess of the predictions of the credit risk models. This result may be explained by the fact that the foundation approach of the IRB homogenized to some extent the regulatory capital computations. In terms of risk characteristics, only the probabilities of default and the exposures at risk were allowed to vary across assets, while the maturities and the recovery rate values were determined according to the regulatory rules and took the same value for all assets. On the other hand, the credit risk models employed additional asset-specific values for the risk inputs required (for example, loan interest rates and discount factors). If the differences among individual assets in terms of their risk characteristics are effectively modeled, the portfolio value is more volatile. Consequently, more risk would call for more capital provisioning.

The fact that the regulatory capital is comparable with the models' predictions at a 95% confidence level seems to support the probable preference of banks for implementing economic capital budgeting at this level. Stricter regulatory rules favoring higher p-values are likely to be perceived as burdensome from banks' viewpoint. However, this conclusion strongly depends on the assumption that the "average" bank portfolio in the Czech market has a structure similar to the macro portfolio that we constructed. In practice this assumption is unlikely to hold. Banks may vary substantially in terms of appetite for risk, with some banks manifesting a particular preference for investment grade and other banks for speculative grade customers. The portfolio composition effect is reinforced if the dependency on systemic risk factors, represented by the Czech industries in our case, has a structure at the bank level different than the one presented here. This fact may have profound consequences for the estimation of asset return and default correlations and implicitly for the default distribution at the portfolio level. An additional departure from our conclusions may be induced by fact that banks have estimation methods for the required risk parameters at the client level that differ significantly from ours. For example, the scoring methods developed by many Czech banks to estimate their clients' probabilities of default take into account different risk factors and risk weights than those that we considered. The direct bank-client relationship is also conducive to a more precise estimation of some client-specific risk characteristics such as loan interest rates, maturities and recovery rates. In this regard we had to make very loose simplifying assumptions.

Finally, the different credit risk models' estimations of economic capital displayed obvious differences[9]. CreditMetrics estimated the lowest values of economic capital at all confidence levels. We think that this outcome was to a great extent determined by the way in which the discount factors were estimated for the last two rating classes. Although differences among the different credit risk models' estimations were determined, no definitive conclusion should be drawn. Considerably more research in this direction is required, especially aimed at obtaining more robust estimations of the required input data.

6. Results and Conclusions

This project accomplished several goals. The most far reaching one was to put to the test modeling approaches that are gaining increasing relevance in credit risk modeling: the calibration and validation of a rating system and the estimation of economic and regulatory capital. The applied nature of the project had rewarding implications. Besides a better grasp of issues with a demanding theoretical content, we were able to directly compare alternative solutions to questions that do not have an easy answer. At least as important are the incipient model estimations performed with specific Czech data.

One of the primary goals was to construct a rating system for the Czech corporate sector reflecting systemic default trends. This study was conducted using a solvency index provided by a specialized credit information agency (Creditreform). Further, we established links to the bank credit information contained in the databases of the Czech National Bank. The construction of the rating system reinforced the conclusion that a trade-off may be reached between the quality and quantity of the required input data in credit risk modeling. Several validating tests showed that our rating system displayed relatively similar performance parameters compared to the rating systems constructed on the basis of Creditreform data in Austria or Germany.

A natural extension for the constructed rating system was to compare regulatory and economic capital estimations according to two credit risk models (CreditMetrics and CreditRisk+) and the latest Quantitative Impact Study of the New Basel Capital Accord. The majority of these methodologies require estimates of the risk characteristics of bank obligors that are partially a by-product of a rating system. The risk capital estimations suggested that the IRB approach of the New Basel Accord would require capital cushions in the range estimated by the credit risk models at a 95% confidence level. The CreditMetrics model predicted the lowest economic capital values. However, this outcome was due to several simplifications we had to make to circumvent the non-availability of input data into this model.

[9] We also obtained economic capital estimations according to the KMV model: CZK 50.9 bil. at the 95% confidence level, CZK 71.99 bil. at the 99% confidence level, CZK 79.71 bil. at the 99.5% confidence level and CZK 95.63 bil. at the 99.9% confidence level. These estimations are given here only for illustration, because in implementing this model we strongly simplified some estimation techniques that are only briefly discussed in the documents describing the KMV model. In any case, all the credit risk models' estimations of economic capital are comparable insofar as their rapport with regulatory capital is considered.

The systemic focus was accomplished by considering Czech industries and estimating all the required risk inputs at this level. In this sense the paper answered questions from a “macro” lending view. Our results can be seen as an overall empirical assessment of the NBCA and of several credit risk models analyzing the credit conditions in the Czech economy. On the other hand this paper built upon prior research undertaken in the Czech National Bank on credit risk modeling and can be seen as a reference for banks who are going to implement credit risk models at a local level.

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Appendix

Appendix A1 – Migration Matrices

1. 1997–1998

	1	2	3	4	5	6	7	**8-PDs**
1	**33.33%**	16.67%	50.00%	0%	0%	0%	0%	**0%**
2	0%	**52.94%**	26.47%	5.88%	8.82%	2.94%	0%	**2.94%**
3	0.15%	0.23%	**82.39%**	10.08%	1.73%	1.96%	0%	**3.46%**
4	0%	0.05%	9.77%	**76.93%**	3.99%	5.36%	0%	**3.90%**
5	0%	0%	2.32%	15.25%	**72.97%**	4.15%	0%	**5.31%**
6	0%	0%	3.41%	12.96%	7.97%	**66.53%**	0.25%	**8.89%**
7	0%	0%	3.33%	13.33%	3.33%	10.00%	**53.33%**	**16.67%**
8	0%	0%	0%	0%	0%	0%	0%	**100%**

2. 1998–1999

	1	2	3	4	5	6	7	**8-PDs**
1	**70.00%**	20.00%	0%	10.00%	0%	0%	0%	**0%**
2	1.08%	**59.14%**	16.13%	16.13%	3.23%	3.23%	0%	**1.08%**
3	0.08%	1.02%	**74.96%**	13.46%	5.20%	1.42%	0.08%	**3.78%**
4	0%	0.21%	4.71%	**77.83%**	8.83%	3.71%	0.03%	**4.68%**
5	0%	0.13%	0.56%	7.57%	**82.05%**	4.53%	0.07%	**5.09%**
6	0%	0.00%	0.63%	6.44%	15.21%	**63.89%**	0.63%	**13.20%**
7	0%	0.00%	0.00%	6.06%	9.09%	9.09%	**51.52%**	**24.24%**
8	0%	0%	0%	0%	0%	0%	0%	**100%**

3. 1999–2000

	1	2	3	4	5	6	7	**8-PDs**
1	**60.00%**	30.00%	0%	10.00%	0%	0%	0%	**0%**
2	0.18%	**78.17%**	9.91%	8.62%	2.02%	0.37%	0%	**0.73%**
3	0.07%	4.14%	**79.72%**	9.19%	3.93%	0.56%	0%	**2.39%**
4	0%	0.61%	3.06%	**85.74%**	6.45%	1.64%	0%	**2.50%**
5	0%	0.32%	0.91%	7.00%	**86.22%**	2.52%	0.03%	**3.00%**
6	0%	0%	0.53%	6.61%	15.72%	**67.24%**	0.53%	**9.38%**
7	0%	0%	0%	3.70%	11.11%	11.11%	**59.26%**	**14.81%**
8	0%	0%	0%	0%	0%	0%	0%	**100%**

4. 2000–2001

	1	2	3	4	5	6	7	**8-PDs**
1	**50.00%**	50.00%	0%	0%	0%	0%	0%	**0%**
2	0%	**84.67%**	10.00%	3.33%	1.33%	0%	0%	**0.67%**
3	0%	2.04%	**82.94%**	9.22%	2.78%	0.98%	0%	**2.04%**
4	0%	0.33%	4.19%	**83.81%**	7.08%	1.34%	0%	**3.25%**
5	0%	0.07%	1.20%	7.54%	**83.19%**	2.48%	0%	**5.52%**
6	0%	0.27%	0.91%	7.92%	12.83%	**68.88%**	0.09%	**9.10%**
7	0%	0%	0%	0%	9.68%	19.35%	**45.16%**	**25.81%**
8	0%	0%	0%	0%	0%	0%	0%	**100%**

5. 2001–2002

	1	2	3	4	5	6	7	**8-PDs**
1	**80.65%**	6.45%	9.68%	3.23%	0%	0%	0%	**0%**
2	13.79%	**60.34%**	18.97%	3.45%	0.00%	1.72%	0%	**1.72%**
3	0.73%	1.76%	**71.30%**	15.96%	6.42%	1.87%	0%	**1.97%**
4	0.13%	0.17%	5.78%	**78.08%**	8.92%	2.72%	0%	**4.21%**
5	0.04%	0.08%	1.43%	11.72%	**75.95%**	4.70%	0.04%	**6.04%**
6	0%	0%	1.61%	9.80%	15.22%	**62.86%**	0.12%	**10.38%**
7	0%	0%	0%	0%	12.50%	18.75%	**50.00%**	**18.75%**
8	0%	0%	0%	0%	0%	0%	0%	**100%**

6. Period average

	1	2	3	4	5	6	7	**8-PDs**
1	**58.80%**	24.62%	11.94%	4.65%	0%	0%	0%	**0%**
2	3.01%	**67.05%**	16.29%	7.48%	3.08%	1.65%	0%	**1.43%**
3	0.20%	1.84%	**78.26%**	11.58%	4.01%	1.36%	0.02%	**2.73%**
4	0.03%	0.27%	5.50%	**80.48%**	7.05%	2.95%	0.01%	**3.71%**
5	0.01%	0.12%	1.28%	9.81%	**80.08%**	3.67%	0.03%	**4.99%**
6	0%	0.05%	1.42%	8.74%	13.39%	**65.88%**	0.32%	**10.19%**
7	0%	0.0%	0.67%	4.62%	9.14%	13.66%	**51.85%**	**20.06%**
8	0%	0%	0%	0%	0%	0%	0%	**100%**

Appendix A2 – Empirical Default Rates at the Industry Level

	PD										
NACE	**1997–1998**	**1998–1999**	**1999–2000**	**2000–2001**	**2001–2002**	**Average Default Rate**	**N_i**	**D_i**	**StndDev 1**	**StndDev 2**	**Rating Class**
1	**2.41%**	**6.72%**	**2.30%**	**5.77%**	**3.67%**	**4.14%**	**604**	**25**	**0.81%**	**0.00%**	**5**
2	0.00%	0.00%	0.00%	16.67%	10.00%	4.88%	41	2	3.36%	3.40%	
5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	34	0	0.00%	0.54%	
10	0.00%	0.00%	0.00%	16.67%	0.00%	3.13%	32	1	3.08%	3.12%	
11	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	5	0	0.00%	0.49%	
12	-	-	-	-	-	0.00%	0	0	0.00%	0.00%	
13	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	5	0	0.00%	0.49%	
14	**0.00%**	**0.00%**	**0.00%**	**4.17%**	**11.11%**	**2.86%**	**105**	**3**	**1.63%**	**1.70%**	**4**
15	**8.52%**	**6.47%**	**8.78%**	**6.45%**	**7.33%**	**7.51%**	**1331**	**100**	**0.72%**	**0.88%**	**6**
16	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12	0	0.00%	0.50%	
17	**2.92%**	**6.10%**	**3.95%**	**4.14%**	**3.57%**	**4.22%**	**735**	**31**	**0.74%**	**0.89%**	**5**
18	**5.36%**	**2.50%**	**1.04%**	**6.45%**	**8.51%**	**4.11%**	**341**	**14**	**1.07%**	**1.19%**	**5**
19	6.67%	10.00%	3.64%	12.82%	28.57%	10.60%	217	23	2.09%	2.15%	7
20	**4.90%**	**10.29%**	**6.47%**	**6.98%**	**9.76%**	**7.73%**	**660**	**51**	**1.04%**	**1.15%**	**6**
21	**6.12%**	**5.26%**	**0.00%**	**4.69%**	**3.17%**	**3.56%**	**309**	**11**	**1.05%**	**1.16%**	**4**
22	**3.62%**	**6.25%**	**3.00%**	**3.73%**	**4.26%**	**4.17%**	**816**	**34**	**0.70%**	**0.86%**	**5**
23	0.00%	0.00%	20.00%	0.00%	0.00%	4.55%	22	1	4.44%	4.47%	
24	**1.98%**	**1.92%**	**2.20%**	**3.29%**	**6.14%**	**2.98%**	**705**	**21**	**0.64%**	**0.81%**	**4**
25	**3.66%**	**2.58%**	**3.28%**	**3.62%**	**3.98%**	**3.39%**	**1093**	**37**	**0.55%**	**0.74%**	**4**
26	**5.17%**	**7.75%**	**3.73%**	**4.20%**	**3.79%**	**4.90%**	**694**	**34**	**0.82%**	**0.96%**	**5**

27	3.64%	8.70%	2.67%	8.11%	6.85%	6.07%	346	21	1.28%	1.38%	6
28	5.19%	5.06%	2.50%	4.41%	5.37%	4.36%	2131	93	0.44%	0.67%	5
29	5.07%	6.03%	4.40%	2.67%	5.75%	4.72%	1378	65	0.57%	0.76%	5
30	7.14%	5.26%	2.63%	0.00%	15.63%	5.41%	148	8	1.86%	1.93%	6
31	3.33%	1.42%	1.64%	3.55%	3.33%	2.59%	733	19	0.59%	0.77%	3
32	4.65%	2.70%	5.41%	2.11%	2.56%	3.49%	401	14	0.92%	1.05%	4
33	7.50%	0.00%	4.62%	5.56%	1.75%	3.65%	274	10	1.13%	1.24%	4
34	0.00%	1.69%	1.49%	1.52%	1.47%	1.29%	310	4	0.64%	0.81%	2
35	12.50%	52.94%	0.00%	10.00%	9.52%	14.85%	101	15	3.54%	3.57%	
36	2.52%	7.48%	2.27%	5.63%	8.40%	5.12%	703	36	0.83%	0.97%	6
37	0.00%	0.00%	4.55%	6.67%	6.67%	3.37%	89	3	1.91%	1.98%	4
40	0.00%	10.00%	0.00%	0.00%	3.57%	2.80%	143	4	1.38%	1.46%	4
41	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	79	0	0.00%	0.50%	1
45	7.45%	6.32%	2.17%	6.60%	7.93%	5.58%	2744	153	0.44%	0.67%	6
50	4.90%	3.41%	2.34%	8.89%	4.35%	4.45%	742	33	0.76%	0.91%	5
51	6.15%	7.91%	4.20%	4.83%	5.77%	5.70%	7419	423	0.27%	0.56%	6
52	5.61%	6.43%	2.30%	6.53%	5.07%	4.87%	5214	254	0.30%	0.59%	5
55	2.38%	11.59%	1.71%	6.98%	10.00%	5.79%	311	18	1.32%	1.43%	6
60	2.68%	3.29%	1.13%	2.62%	3.06%	2.39%	1420	34	0.41%	0.66%	3
61	0.00%	0.00%	0.00%	0.00%	50.00%	6.67%	15	1	6.44%	6.46%	
62	33.33%	50.00%	0.00%	0.00%	0.00%	22.22%	9	2	13.86%	13.87%	
63	5.13%	1.32%	2.17%	2.82%	0.00%	1.99%	351	7	0.75%	0.90%	3
64	11.11%	0.00%	0.00%	4.00%	0.00%	1.89%	106	2	1.32%	1.42%	3
65	15.22%	12.00%	6.78%	7.69%	10.81%	10.25%	244	25	1.94%	2.00%	7
66	0.00%	0.00%	14.29%	0.00%	12.50%	5.56%	36	2	3.82%	3.85%	
67	13.04%	2.44%	0.00%	4.00%	13.64%	4.91%	163	8	1.69%	1.77%	5
70	7.48%	6.84%	9.35%	8.25%	13.54%	8.99%	556	50	1.21%	1.31%	6
71	13.04%	0.00%	5.88%	4.35%	0.00%	4.80%	125	6	1.91%	1.98%	5
72	1.35%	2.17%	1.08%	1.35%	1.61%	1.45%	898	13	0.40%	0.66%	3
73	0.00%	8.33%	0.00%	0.00%	0.00%	1.41%	71	1	1.40%	1.48%	
74	6.60%	6.03%	4.06%	5.54%	6.60%	5.59%	1843	103	0.54%	0.73%	6

Appendix A3 – Credit Exposure at the Industry Level (mil. CZK)

NACE	CZK				EURO				USD			
	<1 y	1–4 y	4–5 y	>5 y	<1 y	1–4 y	4–5 y	>5 y	<1 y	1–4 y	4–5 y	>5 y
1	3188.22	2327.18	1507.14	7505.96	3.48	33.58	0.00	193.32	0.01	0.00	0.00	0.00
14	482.11	124.19	235.18	461.57	28.31	61.69	11.38	69.93	0.00	0.00	0.00	0.00
15	11537.33	3426.25	2978.05	3924.17	252.90	3.82	12.59	64.04	36.94	1.51	0.00	0.00
17	2282.79	378.25	200.29	376.48	1001.17	572.72	114.99	1401.99	40.81	0.00	0.00	0.00
18	875.00	645.29	94.82	298.92	306.16	51.20	90.28	101.00	9.20	13.24	0.00	0.00
19	190.09	120.13	73.44	48.89	9.05	0.00	12.27	0.00	0.01	0.00	0.00	0.00
20	476.91	138.66	222.28	468.87	402.73	23.79	72.74	248.09	0.01	0.00	0.00	0.00
21	2599.57	1021.57	72.69	1621.08	327.69	982.76	12.64	369.23	26.52	0.00	0.00	0.00
22	2011.90	347.79	340.56	1709.13	50.79	0.00	55.11	74.81	0.01	0.00	0.00	0.00
24	8450.27	946.53	420.17	1773.87	1013.81	806.53	253.54	429.80	19.87	34.37	0.00	0.00
25	3283.63	636.72	379.76	737.69	1209.62	1070.70	165.30	400.95	0.62	0.00	0.00	0.00
26	6033.56	1090.20	1451.39	1261.81	2701.02	754.29	83.59	688.90	0.01	0.00	0.00	72.69
27	1606.69	758.92	444.10	1939.05	342.02	50.28	36.33	305.51	5.11	3.06	0.00	0.00
28	2857.57	714.36	553.22	1746.32	852.03	266.54	271.10	859.54	80.35	10.05	0.00	0.00
29	4451.32	1350.37	671.98	1306.54	2476.76	541.31	223.80	589.58	1058.21	177.4	70.97	0.00
30	71.39	28.57	10.34	35.00	16.79	0.00	0.00	6.86	0.00	0.00	0.00	0.00
31	2358.56	324.60	166.59	1587.57	321.37	367.47	35.79	327.83	12.42	165.2	0.00	0.00
32	389.38	82.12	12.44	16.48	181.81	55.37	0.00	40.81	2.53	38.68	0.00	0.00
33	619.45	129.83	155.02	198.66	12.70	2.14	4.94	21.92	1.13	40.05	0.00	0.00
34	4601.89	1749.95	57.06	321.46	170.60	123.11	117.98	165.59	22.39	0.20	0.00	0.00
36	1537.35	362.56	227.81	689.11	1104.17	164.84	140.26	169.45	96.18	8.17	5.22	121.46
37	351.58	107.35	15.66	95.73	46.57	19.06	0.00	3.06	0.06	0.00	0.00	0.00
40	3252.47	2668.09	2180.46	13383.21	114.37	215.80	0.00	1241.83	0.00	0.00	0.00	1847.4
41	461.19	180.69	567.56	1458.68	0.01	33.18	35.39	13.20	0.00	0.00	0.00	0.00
45	4709.11	1312.41	521.92	1864.19	41.63	566.21	2.86	775.74	0.03	9.61	4.32	0.00
50	4353.39	1257.93	168.06	3488.94	59.91	8.32	11.29	17.13	187.75	24.82	0.00	0.00
51	30810.67	6530.09	2830.13	9345.83	1778.72	1689.39	485.69	1044.68	461.08	388.0	167.26	1001.7
52	11253.11	3741.20	1650.55	2842.24	171.80	58.79	15.88	223.09	27.68	6.71	6.27	0.00
55	349.93	168.73	139.42	1043.14	80.79	7.15	15.18	401.23	0.11	0.00	0.00	0.00
60	3493.21	1243.34	1618.34	8957.58	92.49	154.05	57.11	825.07	0.01	0.00	0.00	0.00
63	1169.36	80.00	210.96	515.63	24.04	2.50	0.00	1339.33	0.17	0.00	0.00	0.00
64	2227.442	1347.53	72.594	3796.894	1062.95	1308.76	0.00	837.28	20.01	0.00	0.00	0.00
65	2568.00	1450.03	28.06	721.36	172.73	0.00	2.34	0.00	24.22	0.00	0.00	0.00
67	6.59	148.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
70	3335.61	2412.77	428.47	9044.96	1961.42	691.33	657.18	17775.9	21.65	0.00	0.00	361.62
71	3432.69	5758.02	1041.64	1615.74	201.73	517.55	97.67	337.97	2.95	0.00	0.00	0.00
72	1051.75	646.67	186.46	736.68	17.01	0.00	0.00	34.03	371.94	0.00	0.00	0.00
73	90.29	8.33	246.88	21.45	0.01	0.00	0.00	6.73	0.00	0.00	0.00	7.41
74	10511.09	2814.36	582.73	4716.85	1252.92	105.11	165.77	512.69	794.59	12.03	0.00	3.61

Appendix A4 – Loan Interest Rates at the Industry Level

NACE	Implicit Loan Interest rate
1	16.30%
14	6.23%
15	8.65%
17	9.27%
18	11.88%
19	11.72%
20	7.76%
21	6.79%
22	7.81%
24	9.18%
25	9.90%
26	11.49%
27	10.29%
28	12.63%
29	11.14%
30	1.66%
31	15.56%
32	19.12%
33	8.80%
34	28.29%
36	7.87%
37	14.73%
40	13.96%
41	3.06%
45	9.83%
50	9.63%
51	9.63%
52	9.63%
55	7.03%
60	16.30%
63	16.30%
64	16.30%
65	16.30%
67	16.30%
70	11.05%
71	11.05%
72	11.05%
73	11.05%
74	11.05%

Appendix A5 – Asset Return Correlations

NACE	1	14	15	17	18	20	21	22	24	25	26
1	1.00										
14	-0.77	1.00									
15	0.49	0.01	1.00								
17	0.56	-0.60	0.15	1.00							
18	0.06	-0.31	-0.10	0.53	1.00						
20	0.20	-0.10	0.28	0.68	0.55	1.00					
21	0.26	-0.66	-0.59	0.25	0.23	-0.13	1.00				
22	0.61	-0.88	-0.02	0.60	0.62	0.27	0.53	1.00			
24	0.23	-0.57	-0.51	0.05	-0.17	-0.30	0.73	0.35	1.00		
25	0.55	-0.52	0.25	0.88	0.30	0.68	0.14	0.47	0.05	1.00	
26	-0.77	0.90	-0.06	-0.76	-0.25	-0.37	-0.52	-0.76	-0.51	-0.69	1.00
27	0.70	-0.68	0.17	0.89	0.36	0.61	0.38	0.62	0.20	0.77	-0.85
28	-0.21	0.51	0.49	0.17	0.27	0.55	-0.71	-0.26	-0.86	0.26	0.38
29	-0.15	0.28	0.38	0.27	0.41	0.64	-0.66	0.03	-0.64	0.32	0.13
30	0.39	-0.56	-0.05	0.88	0.51	0.64	0.21	0.61	0.17	0.77	-0.72
31	0.43	-0.73	-0.24	0.76	0.47	0.39	0.51	0.73	0.44	0.62	-0.77
32	0.42	-0.38	0.34	0.77	0.47	0.75	-0.21	0.52	-0.14	0.71	-0.59
33	-0.56	0.65	-0.17	-0.01	-0.10	0.15	-0.37	-0.64	-0.43	0.02	0.48
34	0.62	-0.79	0.11	0.82	0.72	0.55	0.36	0.91	0.17	0.66	-0.79
36	0.12	-0.36	0.07	0.42	0.72	0.42	-0.05	0.66	-0.06	0.30	-0.31
37	-0.07	-0.37	-0.45	0.01	0.54	-0.08	0.46	0.57	0.42	-0.24	-0.19
40	-0.56	0.36	-0.36	-0.56	0.05	-0.47	-0.04	-0.27	-0.24	-0.70	0.54
41	-0.77	0.86	-0.13	-0.89	-0.38	-0.48	-0.44	-0.77	-0.35	-0.81	0.96
45	-0.15	0.35	0.32	0.34	0.47	0.73	-0.56	-0.09	-0.59	0.27	0.09
50	-0.65	0.75	-0.18	-0.91	-0.60	-0.66	-0.29	-0.80	-0.13	-0.78	0.86
51	-0.67	0.94	0.10	-0.73	-0.38	-0.28	-0.60	-0.86	-0.55	-0.62	0.94
55	-0.73	0.92	-0.04	-0.76	-0.39	-0.32	-0.50	-0.85	-0.44	-0.61	0.93
60	-0.74	0.91	-0.02	-0.77	-0.35	-0.38	-0.53	-0.82	-0.50	-0.71	0.97
64	-0.73	0.90	-0.14	-0.61	-0.46	-0.21	-0.44	-0.90	-0.34	-0.45	0.82
70	0.24	-0.15	0.31	0.50	0.24	0.53	-0.18	0.21	-0.15	0.54	-0.30
71	0.20	-0.25	0.07	0.34	0.17	0.19	0.11	0.25	0.10	0.31	-0.26
72	-0.57	0.77	0.07	-0.67	-0.40	-0.36	-0.48	-0.72	-0.38	-0.56	0.82
74	-0.30	0.45	0.17	-0.41	-0.25	-0.19	-0.35	-0.39	-0.24	-0.31	0.50

Asset Return Correlations – continue

NACE	27	28	29	30	31	32	33	34	36	37	40
1											
14											
15											
17											
18											
20											
21											
22											
24											
25											
26											
27	1.00										
28	-0.06	1.00									
29	0.05	0.81	1.00								
30	0.76	0.07	0.37	1.00							
31	0.71	-0.25	0.04	0.91	1.00						
32	0.64	0.38	0.67	0.84	0.60	1.00					
33	-0.20	0.48	0.32	0.00	-0.24	-0.09	1.00				
34	0.77	-0.01	0.24	0.81	0.81	0.74	-0.43	1.00			
36	0.23	0.21	0.60	0.59	0.52	0.68	-0.26	0.71	1.00		
37	0.03	-0.43	-0.07	0.20	0.39	0.05	-0.41	0.42	0.60	1.00	
40	-0.60	-0.10	-0.18	-0.51	-0.40	-0.52	0.06	-0.39	-0.12	0.25	1.00
41	-0.91	0.19	-0.02	-0.82	-0.82	-0.69	0.37	-0.86	-0.38	-0.12	0.58
45	0.17	0.75	0.81	0.35	0.02	0.60	0.42	0.22	0.41	-0.11	-0.19
50	-0.87	-0.03	-0.28	-0.87	-0.80	-0.82	0.31	-0.93	-0.59	-0.23	0.46
51	-0.77	0.43	0.12	-0.77	-0.87	-0.57	0.51	-0.84	-0.46	-0.39	0.42
55	-0.82	0.35	0.06	-0.77	-0.84	-0.63	0.52	-0.88	-0.47	-0.35	0.42
60	-0.80	0.33	0.07	-0.76	-0.83	-0.61	0.49	-0.84	-0.42	-0.26	0.53
64	-0.68	0.32	0.03	-0.60	-0.70	-0.53	0.64	-0.87	-0.53	-0.45	0.31
70	0.43	0.34	0.43	0.47	0.28	0.58	0.06	0.38	0.32	-0.15	-0.47
71	0.32	0.01	0.08	0.29	0.26	0.26	-0.05	0.30	0.20	0.04	-0.29
72	-0.69	0.27	0.02	-0.71	-0.76	-0.55	0.37	-0.76	-0.40	-0.31	0.33
74	-0.40	0.21	0.06	-0.45	-0.50	-0.28	0.13	-0.42	-0.18	-0.20	0.10

Asset Return Correlations – continue

NACE	**41**	**45**	**50**	**51**	**55**	**60**	**64**	**70**	**71**	**72**	**74**
1											
14											
15											
17											
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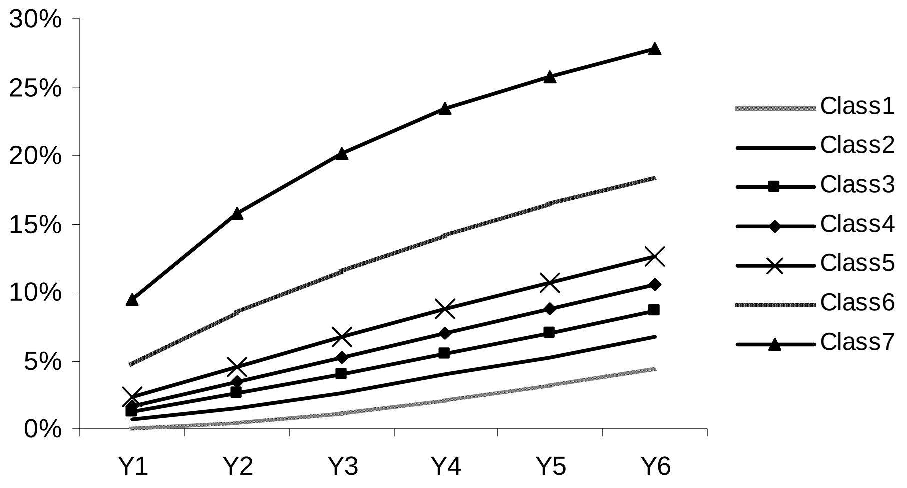
32											
33											
34											
36											
37											
40											
41	1.00										
45	-0.03	1.00									
50	0.93	-0.28	1.00								
51	0.93	0.18	0.86	1.00							
55	0.93	0.09	0.89	0.96	1.00						
60	0.96	0.07	0.88	0.96	0.92	1.00					
64	0.81	0.12	0.79	0.86	0.91	0.81	1.00				
70	-0.40	0.40	-0.47	-0.28	-0.30	-0.32	-0.21	1.00			
71	-0.31	0.06	-0.31	-0.29	-0.29	-0.27	-0.24	0.88	1.00		
72	0.81	0.02	0.78	0.82	0.81	0.82	0.74	0.12	0.24	1.00	
74	0.48	0.02	0.46	0.49	0.48	0.49	0.42	0.49	0.62	0.88	1

Appendix A6 – Rating Class-Specific Credit Spreads



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