CHAPTER 5

A Multivariate Filter for Measuring Potential Output and the NAIRU

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INTRODUCTION

The model described in Chapter 4 embodies the idea that inflation dynamics are influenced by the combined effects of aggregate demand and supply, through a measure of excess demand in the system. Moreover, an important mechanism for monetary influence on the inflation process is characterized by a link between a monetary instrument and excess demand, operating both directly and through the exchange rate. Thus, the output gap-the extent to which output exceeds potential output-plays an important role in the model.

Since the model provides a key input into the policy decisions taken to respect the inflation targets, and since the credibility of the policy framework is of such importance to the overall process of conditioning inflation expectations, it is crucial that the underlying measures of excess demand be clearly understood and accepted as reasonable by both those involved in the policy process and others.

Although there is no explicit model of wage dynamics in this version of the model, there is still good reason to take account of the state of excess demand in the labour market, since such information can help identify the state of excess demand in the product market. Indeed, this point is extremely important in the methodology we report here.[2]

The idea that inflation dynamics are driven by *excess* demand has strong theoretical and intuitive appeal. The first lesson in an economics course about a market economy will feature the idea that when demand exceeds supply, the price will tend to rise. Unfortunately for modellers and policymakers alike, we can never observe the state of excess demand in a market directly. What we see are the signs of excess demand, such as rising rates of capacity utilisation, inventory shortages, delivery lags, hiring difficulties and so on, as well as upward pressure on wages and prices. Thus, to implement the ideas of the model, and to facilitate discussion of monetary policy issues, we must infer measures of excess demand from the available data. How this is done in the FPAS is the topic of this chapter.

We can characterize the problem as one of how to measure potential output-the level of output that can be produced and sold without creating pressures for the rate of inflation to rise or fall-and the NAIRU-the level of unemployment at which there is no pressure for inflation to rise or fall. In a growing economy, potential output is not static. Rather, it rises as the available input resources grow, as more capital is put in place through investment, and as productivity gains are realized. So our task is to measure the path of potential output over time. The NAIRU is a rate, and can, in principle, be constant in a growing economy, but in an economy undergoing major structural change, it is important to allow for the possibility of short-term trend movements in the NAIRU.

Over the years, many methods have been proposed for measuring potential output and the NAIRU.[3] In the case of potential output, one idea that always lies close to the surface is that there is some production function that links output to available inputs of labour, capital and raw materials, given the current technology, and that we can think of the current level of potential output as what would emerge from the production function, given the current levels of fixed inputs and sustainable levels of variable inputs. Although this idea is useful in a general sense, and indeed motivates the idea that there is some link between conditions in labour markets and conditions in product markets, it has been found that, in practice, not much is added by the structure of the production function; the inherent uncertainty in pinning down potential output is simply transferred into uncertainty about total factor productivity.

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[2] It is also useful for the discussion of policy options to have a measure of the current and anticipated state of excess demand in the labour market, even if such a measure is not used to explain anything in the formal model.

[3] See, for example, The Concept, Policy Use and Measurement of Structural Unemployment: Estimating a Time Varying NAIRU Across 21 OECD Countries. OECD Economics Department Working Paper No. 250, 2000.

The modern standard methodology for measuring potential output and the NAIRU is to use some variant of filtering. Time-series techniques are used to fit trend lines through the data, and these trends provide the measures of the underlying "equilibrium" values.[4] It is important to stress that in referring to these values as equilibrium values, we use the perspective of the effect on inflation. The methodology defines trend lines that are used to define "gaps"-deviations of actual observed vales from these trends-that are, in turn, used to describe the dynamics of inflation and the policy control process. The measures are determined, at least in part, by their ability to represent these processes.[5]

Filtering methodologies are many and varied. One economist summarized the early methodology as using "a long and flexible ruler" to draw a bendy line through the data on a graph.[6] In modern methodology, the long and flexible ruler has been replaced by numerical methods that do the same thing on a computer, with more or less complexity.

In the simplest variants, which are called univariate filters, only the data for the series itself are used to fit the trend. A popular example is the Hodrick-Prescott (HP) filter.[7] In the HP filter, and all other similar filters, the user must supply some judgment as to how smooth the trend should be. In other words, just how flexible should that ruler be? Should it be very stiff so that the trend does not move much with actual cycles in the data, or should it be more flexible and follow the data more closely? The methodology itself cannot provide this choice; the user must impose it or infer it from other information or criteria (such as embedding it in a broader estimation problem, where some other criterion will effectively determine the degree of smoothing).

The issue of the degree of smoothing to use in a filter has a direct link to the issue of the nature of the shocks to the economy. If the shocks to the economy are primarily shocks to aggregate demand, with supply conditions largely unaffected, then potential output does not move closely with the data, and it is appropriate to use a high level of smoothing in the filter. If, on the other hand, there is a high proportion of supply shocks, then potential output is indeed moving with the data, and a lower degree of smoothing is appropriate. Thus, it is important that the judgment of knowledgeable specialists be used to condition what is otherwise a purely mechanical exercise.

One example of a univariate methodology that makes a small step in formalizing the use of judgment is the simple Prior Consistent (PC) filter, which allows some weight to be given to priors on the evolution of the trend through time or its variability relative to the observed data, in the fitting of the trend.[8]

Univariate methodologies all suffer from a number of problems. An important one is that the estimates become relatively imprecise at the end of the sample. In effect, trends are estimated as two-sided moving averages of the data, with future outcomes used to condition estimates of the current trend value. At the end of the sample, where future values are not available, the filter does not have the benefit of hindsight to infer the current trend value. This means that the precision of the trend estimates deteriorates markedly right when those estimates are needed most to prepare a forecast or make judgments as to the appropriate settings of the policy instrument.

[4] We use the phrase "trend lines" to describe the series we identify as potential output, the NAIRU, and so on. They are not necessarily "straight" lines.

[5] In particular, the trend values are not intended to represent equilibrium in the sense, for example, of a production possibility frontier, where all potential productivity gains have been realized and all resources are optimally allocated and fully employed.

[6] See the review of the literature in Laxton and Tetlow (1992).

[7] See Hodrick and Prescott (1981).

[8] We document this procedure later in the chapter and in the appendix to this chapter. See also Box 7, page 30-31 in Laxton and others (1998).

The methodology we use improves on univariate methods by using more information to condition the estimates of potential output and the NAIRU. Our approach is a version of what is called a multivariate filter. The essential idea behind a multivariate system is that in estimating potential output, say, we can profit from considering more than just the data on output. In particular, since we know that there is a link between labour input and output, it may be useful to exploit information about the degree of excess demand in the labour market in forming estimates of the degree of excess demand in the product market. Similarly, if we observe inflation accelerating, it is more likely that we should be assuming that there is excess demand in the product market.[9] Our multivariate methodology treats the filtering problem as a small system, where the estimates of potential output, the NAIRU, and some of the parameters of the dynamic model are determined simultaneously, allowing us to account for interactions among unemployment, output and inflation.

THE MULTIVARIATE SYSTEM

In this section, we describe how potential output and the NAIRU have been estimated. The discussion is complete, but technical details are relegated to an appendix. The relevant equations are shown in Table 1. For this discussion, we use a simplified notation. Many of these equations are model equations and have been documented in Chapter 4. The program that implements the multivariate filter is in GAUSS and can be obtained by contacting one of the authors.

Equations (1), (2), (3), (6) and (8) are model equations, exactly as documented in the discussion of the core model. It is important to note that since we use the same equations in the model that we use for the estimation of these key unobservable series, we can claim the advantage of having model-consistent estimates of these variables.

Equations (1) and (2) are the identities defining the link between the levels of potential output and the NAIRU and the respective gap measures. The scaling in equation (1) is to convert the gap units to percentage (y is the log of real GDP). Note that we define the unemployment gap such that positive values mean excess demand for labour, yielding an expected positive correlation with the output gap and positive coefficients in equations linking these measures.

The rest of the equations are not identities.[10] They represent the simple nexus of output, unemployment and inflation that is used in the larger macro forecasting and simulation model, and three additional equations we need in the system described here for the determination of potential output and the NAIRU.

Equation (3) is the model's equation for the dynamics of inflation, the model's Phillips curve.[11] Equation (6) describes the dynamics of the output gap. Equation (8) is an Okun equation that links the movements in the unemployment gap to those in the output gap. All of these are model equations; their structure and calibration are discussed in Chapter 4.

[9] This may not be true. It could be that special factors are driving inflation up, factors that have nothing to do directly with the state of domestic excess demand, such as an external energy price shock. But, all else equal, an observation that inflation is rising should lead us to give more weight to the possibility that there is excess demand.

[10] This is indicated by the presence in each of a disturbance, ε_t, with a superscript for the variable chosen for the left side of that equation. In general, in a simultaneous system of equations, one can choose any variable for the normalization (the variable placed on the left-hand side), so the linking of equation disturbances and shocks of a particular type is not automatic. However, we are writing this system with a view to making such inferences from the empirical system. Thus, for example, we want to be able to interpret historical residuals in the "potential output equation," $\varepsilon_t^{\bar{y}}$, as measures of the historical supply shocks.

[11] The Phillips curve in the model is a reduced-form equation. For an example of an optimising model of the Czech economy with monopolistic competition and sticky wages and prices see Laxton and Pesenti (2002).

Equations (4), (5) and (7) are not model equations, per se. They describe the dynamic properties of the trends that we assume for the multivariate filter. They are necessary to complete the statistical properties of the system to be estimated, and their form represents part of the judgment we use to condition the estimates of potential output and the NAIRU.

Table 1: Equations of the Multivariate Filter

$$y_t = \bar{y}_t + ygap_t / 100 \tag{1}$$

$$u_t = \bar{u}_t - ugap_t \tag{2}$$

$$\pi_t = a_0 \left[\pi 4_t^M + 100 * \Delta_4 lz_t^{eq} \right] + a_1 E_t \pi 4_t + \left(1 - a_0 - a_1\right) \pi_{t-1} + a_2 ygap_{t-1} + \varepsilon_t^{\pi} \tag{3}$$

$$\bar{y}_t = \bar{y}_{t-1} + \mu_{t-1} - b_0 \Delta \bar{u}_t + \varepsilon_t^{\bar{y}} \tag{4}$$

$$\mu_t = c_0 \mu_{t-1} + \left(1 - c_0\right) \bar{\mu} + \varepsilon_t^{\mu} \tag{5}$$

$$ygap_t = d_0 ygap_{t-1} - d_1 \left[e_0 rr12gap_t + e_1 rr4gap_t + e_2 gr\_rrgap_t \right] - d_2 lzgap_t + \varepsilon_t^{ygap} \tag{6}$$

$$\bar{u}_t = \bar{u}_{t-1} + \varepsilon_t^{\bar{u}} \tag{7}$$

$$ugap_t = f_0 ugap_{t-1} + f_1 ygap_t + \varepsilon_t^{ugap} \tag{8}$$

Equation (4) describes the dynamics of potential output. Variable μ_t is the growth rate of potential output. In equation (5), this growth rate is specified to evolve according to a first-order, stationary autoregressive process, reverting in the long run to a fixed steady-state level, $\bar{\mu}$. Our judgment is that a reasonable value for a sustainable steady-state real growth rate is 3.5 percent per annum. In the quarterly equation, this is divided by 4 to express it at a quarterly rate, giving a value for $\bar{\mu}$ in equation (5) of 0.875. In an economy experiencing large structural change, there is good reason to think that the trend growth rate will not converge quickly to the assumed steady-state level. We have set the parameter c_0 to 0.9, which means that in the absence of shocks, output growth would converge to within 1 percent of the steady-state rate in just over 10 years.

In the absence of changes in the NAIRU, equation (4) describes the evolution of potential output as a random walk, driven by disturbances, $\varepsilon_t^{\bar{y}}$, which are interpreted as supply shocks-shocks to total factor productivity and so on. When the NAIRU is changing, however, there is an additional dynamic effect. The operator Δ is a first (quarter) difference; a rising NAIRU implies a falling level of potential output in this specification. The parameter b_0 is set at 0.6, based on the approximate share of labour income in total income, which would be the right magnitude if the production technology were approximately Cobb-Douglas in form.

The evolution of the NAIRU is specified in equation (7) as a pure random walk driven by shocks $\varepsilon_t^{\bar{u}}$. Despite the fact that the NAIRU cannot literally follow a random walk, this represents a useful empirical assumption when the NAIRU has a tendency to drift over time in ways that are difficult to explain sensibly on the basis of variation in conventional "structural determinants".[12]

[12] For a further discussion on this point see Boone and others (2003).

The ε_t^{π}, $\varepsilon_t^{\bar{y}}$, ε_t^{μ}, ε_t^{ygap}, $\varepsilon_t^{\bar{u}}$, ε_t^{ugap} variables are random variables that are assumed to be identically, independently normally distributed and to be uncorrelated.

This system is processed using an application of Kalman Filtering. The methodology is described more formally in the Appendix.[13]

Before completing the discussion of the application of this methodology to derive measures of potential output and the NAIRU, we need to establish the methodology and results for certain input variables, and in particular the measures for the components of monetary conditions.

A METHODOLOGY FOR PRE-FILTERING: THE PC FILTER

A number of variables that are endogenous in the full model are treated as exogenous in estimating potential output and the NAIRU. In particular, we need to specify values for the four contributors to real monetary conditions, including the truly exogenous influence of German interest rates. The methodology used to establish these values is described in detail in the appendix. We describe, below, a simplified version. The method is called the Prior Consistent (PC) filter, because it permits the imposition of certain "priors" on the properties of the measures.[14] The same methodology, essentially, provides us with initial estimates for the endogenous values that ultimately emerge from the multivariate Kalman filter.

Consider, as an example, the problem of inferring a measure of trend equilibrium real interest rates. Calling r_t the real interest rate, $\bar{r}_t$ its trend equilibrium values, and $rgap_t$ the deviation of r_t from its equilibrium values, the measurement equation that links r_t to the two state variables $\{\bar{r}_t, rgap_t\}$ is given by:

$$r_t = \bar{r}_t + rgap_t\,. \tag{9}$$

The transition equations that summarize the dynamics of the state variables are,

$$\bar{r}_t = \bar{r}_{t-1} + \varepsilon_t^{\bar{r}} \tag{10}$$

$$rgap_t = \varepsilon_t^{rgap} \tag{11}$$

The covariance matrix of the error terms in equations (10) and (11) is:

$$Q = \begin{pmatrix} \sigma^2_{\varepsilon^{\bar{r}}} & 0 \\ 0 & \sigma^2_{\varepsilon^{rgap}} \end{pmatrix}, \text{ or equivalently}$$

$$Q = \begin{pmatrix} \dfrac{\sigma^2_{\varepsilon^{\bar{r}}}}{\sigma^2_{\varepsilon^{rgap}}} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\lambda} & 0 \\ 0 & 1 \end{pmatrix} \tag{12}$$

[13] For further details on this methodology see Hamilton (1994), and Harvey (1989).

[14] It can be shown that the equations we present can be derived from minimizing the value of the following objective function: $\sum_{t=1}^{T}(r_t-\bar{r}_t)^2 + \lambda\sum_{t=2}^{T}\left[(\bar{r}_t-\bar{r}_{t-1})-\Delta\bar{r}_t^*\right]^2$, where $\Delta\bar{r}_t^*$ is the steady-state change in the equilibrium value, which is set to zero here, except for the real exchange rate. Thus, we trade off fitting the data (first term) against penalizing the change in the trend estimate, with a relative weight, λ, on the latter. The higher is λ, the smoother will be the estimates of $\bar{r}_t$. See Box 7, page 30-31 in Laxton and others (1998).

The measurement equation is an identity that states that the variable r_t is the sum of an equilibrium value and a gap. The first transition equation says that the equilibrium values of r_t follow a random walk.[15] The second transition equation says that r_t deviates from its equilibrium level by a random disturbance.[16] The error terms $\left\{\varepsilon_t^{\bar{r}}, \varepsilon_t^{rgap}\right\}$ are assumed to be identically, independently and normally distributed.

Assumptions on the Initial State vector

To apply the Kalman Filter to the system (9) - (12) we need to make some assumptions on the initial values of the state variables, their covariance matrix, and the value of the parameter λ.

The parameter λ has been fixed to 25 in all applications. It is easiest to think of the intuition for this choice in terms of the standard deviations. We judge that if a "large" deviation for the trend were 1, say, then the corresponding measure in gap terms would have a large value at 5. This assumption has been found useful in applications elsewhere.[17]

The initial value of $\bar{r}_t$ is set to the value of the first observation of r_t (the initial gap is set to zero). The initial covariance matrix of the state variables is diagonal with each variance set at 10. This high value denotes the degree of uncertainty on the initial values of the state vector; assuming a diffuse prior is a standard procedure.[18]

ESTIMATES OF REAL MONETARY CONDITIONS

Recall that the output gap is modelled in equation (6) as responding to interest rates and the exchange rate. There are four gap terms used for this: *rr*12*gap* is the deviation of the 3-year (12-quarter) real interest rate from its equilibrium trend level, *rr*4*gap* is the deviation of the 1-year (4-quarter) real interest rate from its trend level, *gr_rrgap* is the deviation of the German quarterly real interest rate from its trend level, and *lzgap* is the deviation of the real exchange rate from its trend level, where this is defined such that a positive value means that the real exchange rate is appreciated relative to its equilibrium level. We sometimes refer to the combined effect of these terms, including the parameters, as the index of real monetary conditions.

We use the simple PC filter to provide historical measures for these four components of real monetary conditions. The same methodology is applied, in turn, to each of the variables, and to the short-term (1-quarter) rate, as well as certain other exogenous variables that we wish to exploit in gap form. In each case, we derive an estimate of both the trend equilibrium level, and the corresponding gap.

15 In this case, we do not allow for any permanent trend change in the equilibrium value. The real interest rate is presumed to be constant in a steady state, and movements in the sample are interpreted, statistically, as the result of a sequence of random shocks. For the real exchange rate, we do allow for a trend change in a steady state. The model for this case is more complicated.

16 It may seem odd that we assume, statistically, that the gap measure has no persistence, when our economic stories always feature persistence in macro cycles. It would be interesting to investigate the sensitivity of the results to this particular assumption. However, for now we stick to the simplest possible specification, for two reasons. First, the two equations interact to give reasonable persistence properties in gap measures, so we do not have to introduce a more complicated statistical assumption to get reasonable output. Second, it has been found that the system we use has reasonable updating properties, that is, as new data arrive, the estimates from the filter change in a sensible manner. See Boone and others (2003).We do not know what would happen, in this regard, in a more complex model. This is a topic for future work.

17 See Box 7, page 30-31 in Laxton and others (1998).for a discussion of this point, and an application to measuring the NAIRU in a number of countries.

18 The assumptions made for the case of the real exchange rate are slightly different. The main point is that we set the initial gap variance term to zero, effectively constraining the first observation of the equilibrium real exchange rate to be very close to the actual measure in the first period.

The results are shown in Figures 1 - 5. In Figure 1, we show the results for the German 90-day real interest rate. There is not much movement, and the value for the forecast horizon is just under 2.5 percent per annum. In Figure 2, we have results for the equilibrium real exchange rate, in log form. Note is that there is a clear trend in the equilibrium rate. For this application of the PC filter, we set the trend real appreciation to the historical mean, which we calculate to be 1.26 percent per annum over this sample. The main story of the cyclical variation around the trend line was presented in Chapter 1. In brief, there was what appears, ex post, to have been an unsustainable appreciation in the period leading up to the exchange crisis in 1997. The abrupt depreciation at that time removed the disequilibrium, but there was not much overshooting until later in the year. The main feature of the subsequent period is the large excess appreciation in 1998, which we have attributed primarily to the high domestic interest rates held well into the recession. Our measure shows that by 2000, the actual rate was not far from its equilibrium.

Figures 3, 4 and 5 show the results across the term structure for the three domestic measures of real interest rates. Figure 3 shows the results for the real 90-day rate. According to our results, the trend increase in domestic real rates from the first part of the sample has been reversed, and the risk premium is beginning to fall. Indeed, the estimated real equilibrium rate has come down from a peak of about 4 percent to about 2 percent, per annum, by the end of the sample. The spike from the period of the exchange crisis dominates the picture. The historical issue, which we have reviewed in Chapter 1, is why this rate was held up as long as it was. In any case, the monetary response to the developing recession is evident, as the rate passes below its equilibrium by the second quarter of 1998 and moves increasingly below for the next year.

Figure 4 shows the estimates for the one-year rate. Again, our results show the pattern of a rise in the equilibrium in the first part of the sample, which is then reversed. The initial rise is not as dramatic as for the 90-day rate, and the end-point is about the same, at about 2 percent. Figure 5 shows a flatter profile for the three-year rate, and no strong evidence of a decline in the risk premium over recent periods.

Figures 4 and 5 show that the effects of the emergency hike in the 90-day rate in 1997 lingered even longer in the one-year and three-year rates. Thereafter, the movements in the three-year rate, which is measured from the government bond rate, are similar, though larger in magnitude than those for the one-year rates.

Figure 1: Actual and Trend Equilibrium German 90-dayReal Interest Rates

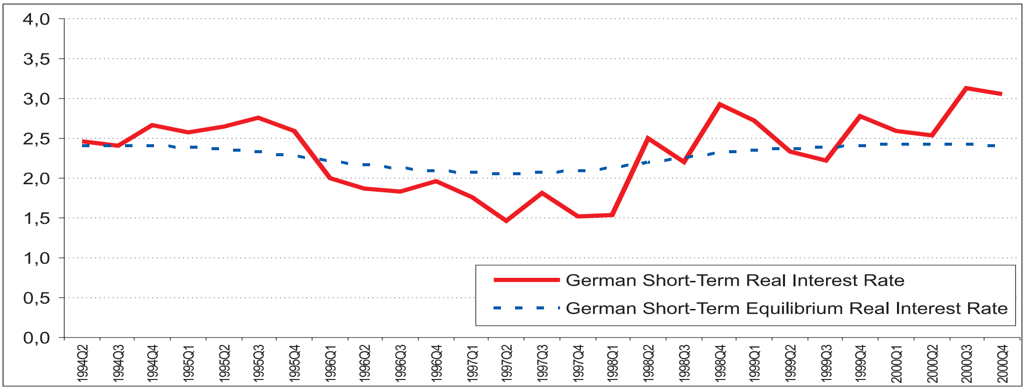


Figure 2: Actual and Trend Equilibrium Real Exchange Rates (CZK/DM)

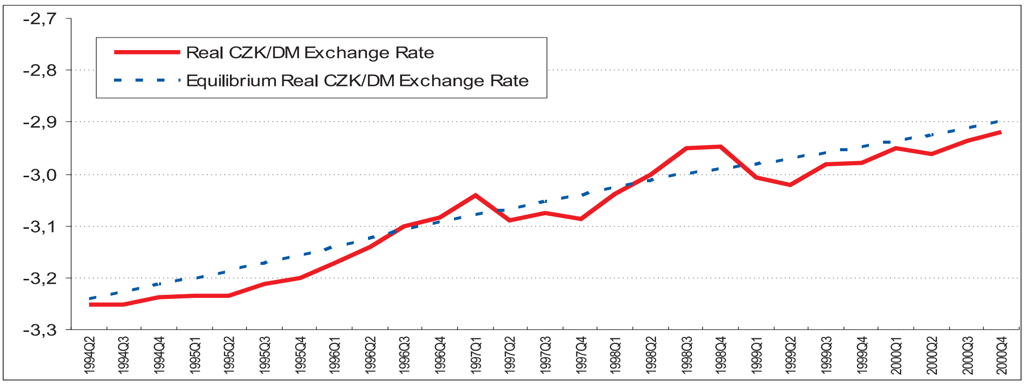


Figure 3: Actual and Trend Equilibrium Czech 90-day Real Interest Rates

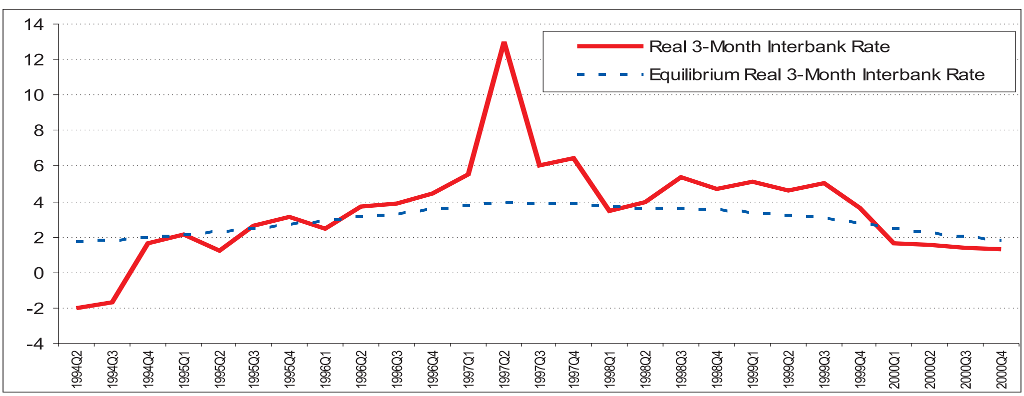


Figure 4: Actual and Trend Equilibrium Czech One-Year Real Interest Rates

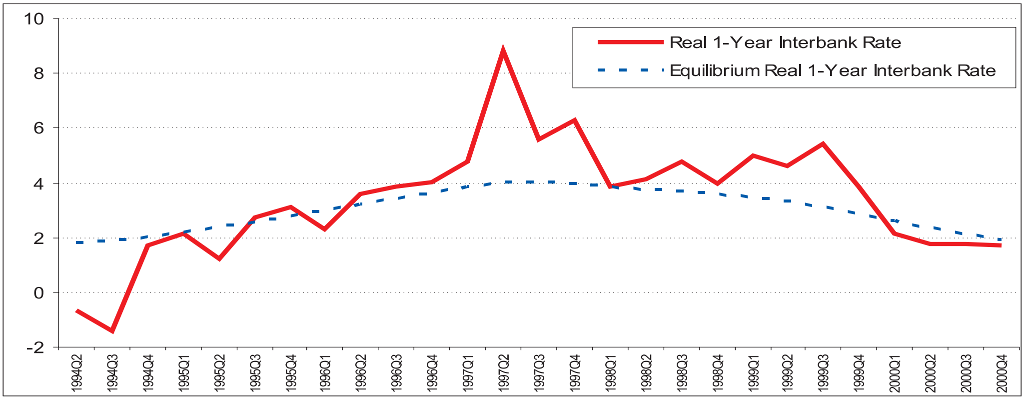


Figure 5: Actual and Trend Equilibrium Czech Three-Year Real Interest Rates

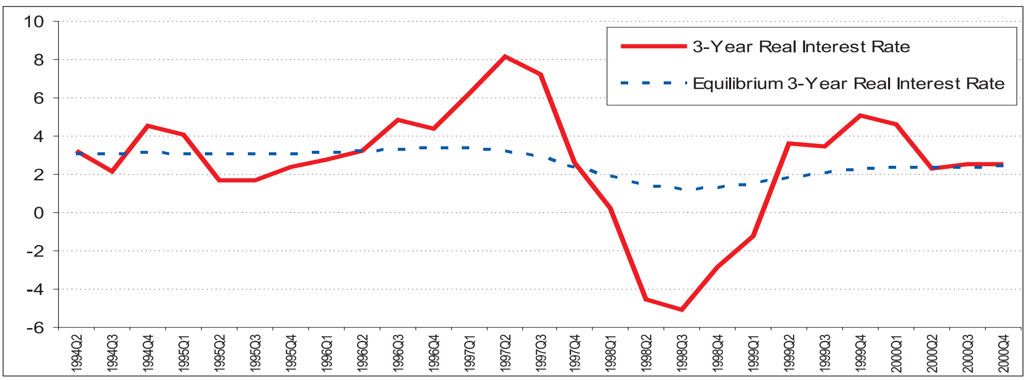
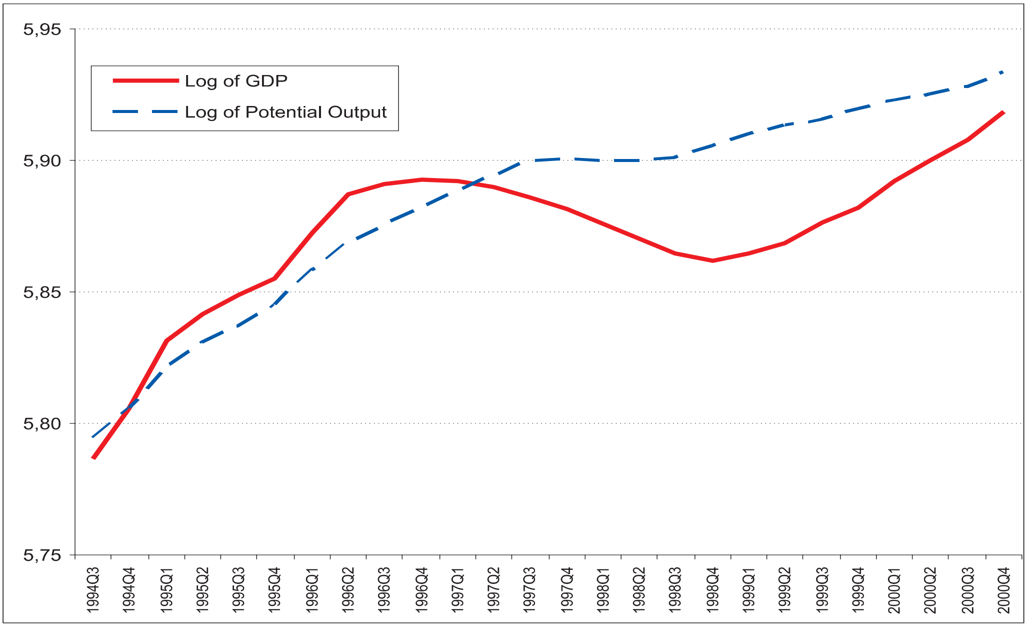


Figure 6: Estimates of the NAIRU and Potential Output



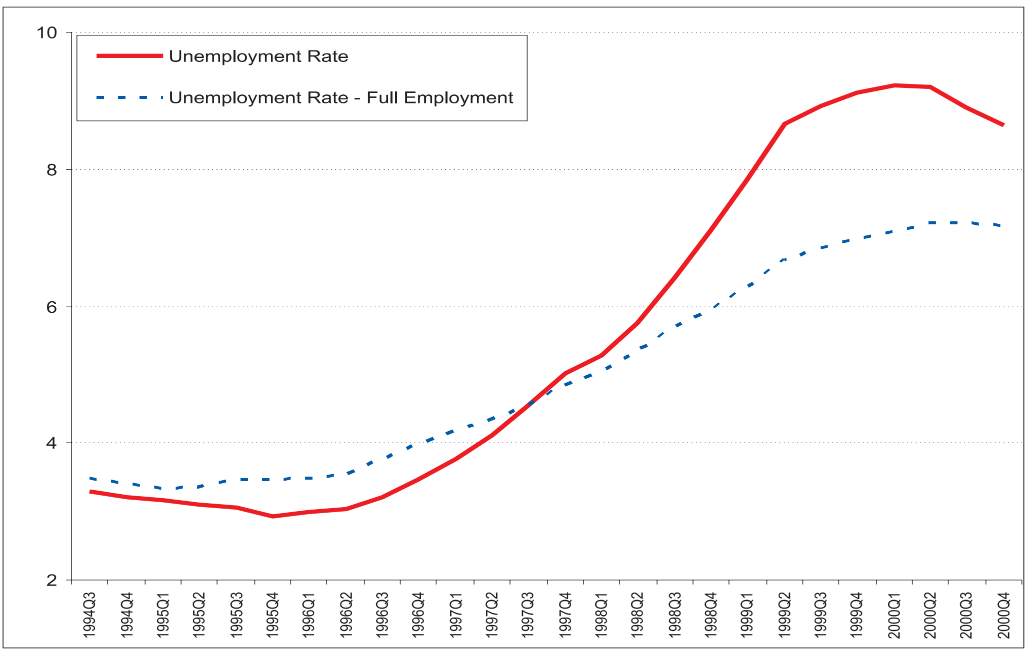
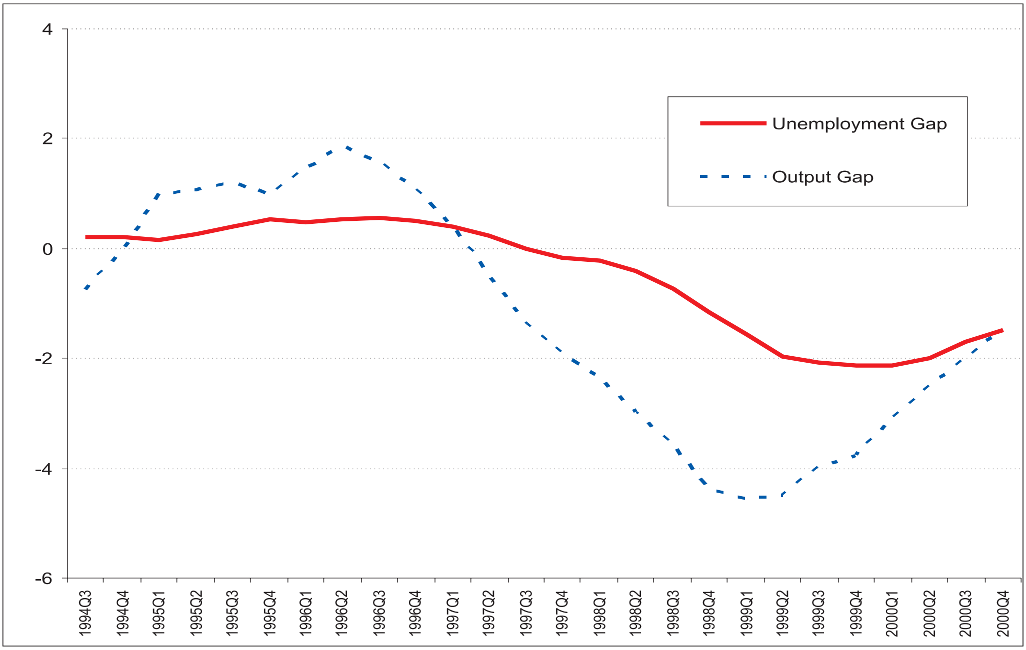


Figure 7: Output, Unemployment and Inflation



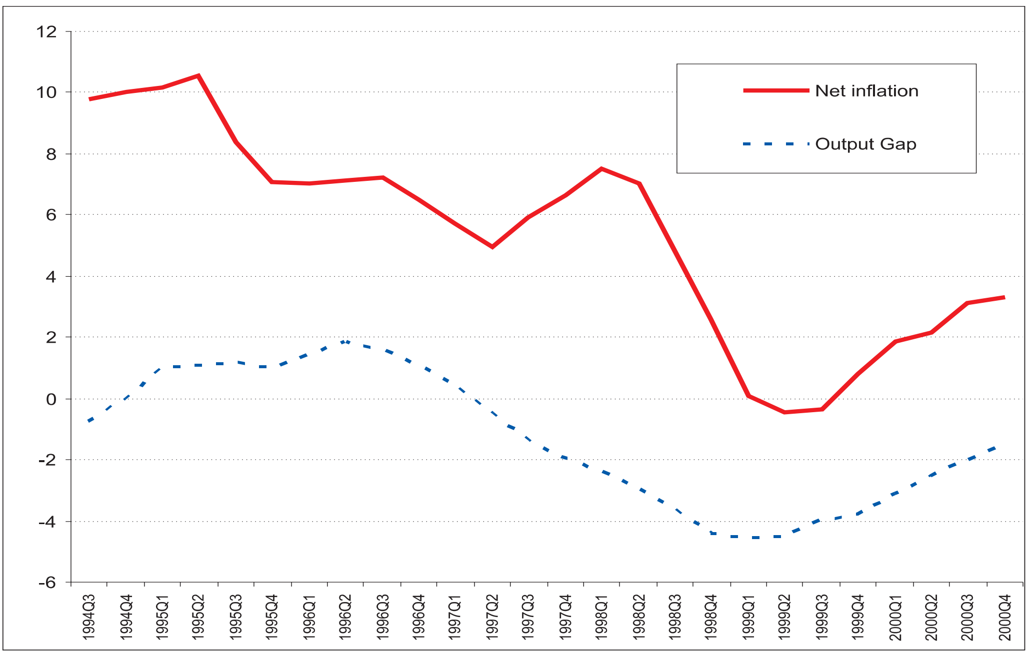


Figure 8: Real-Time Updating Test for the Estimates of the NAIRU

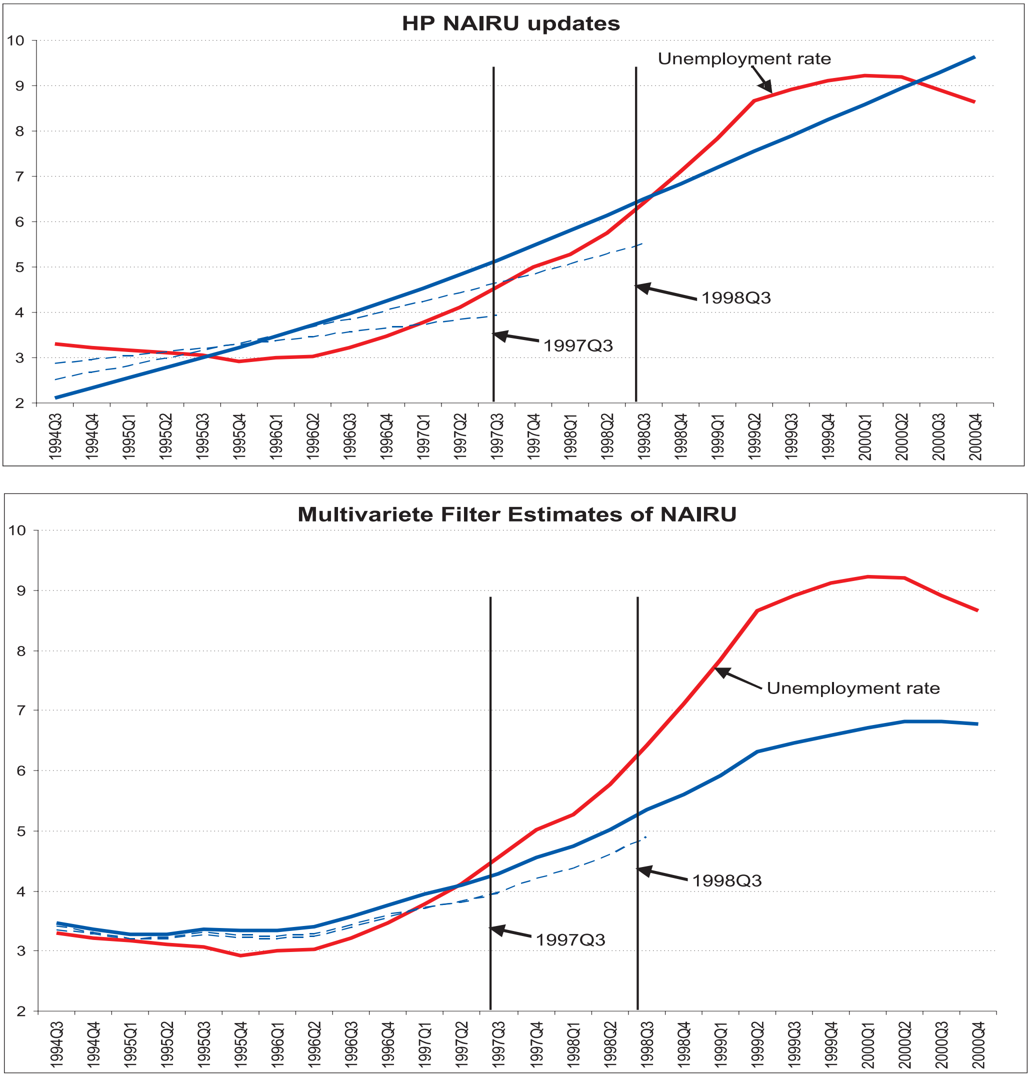
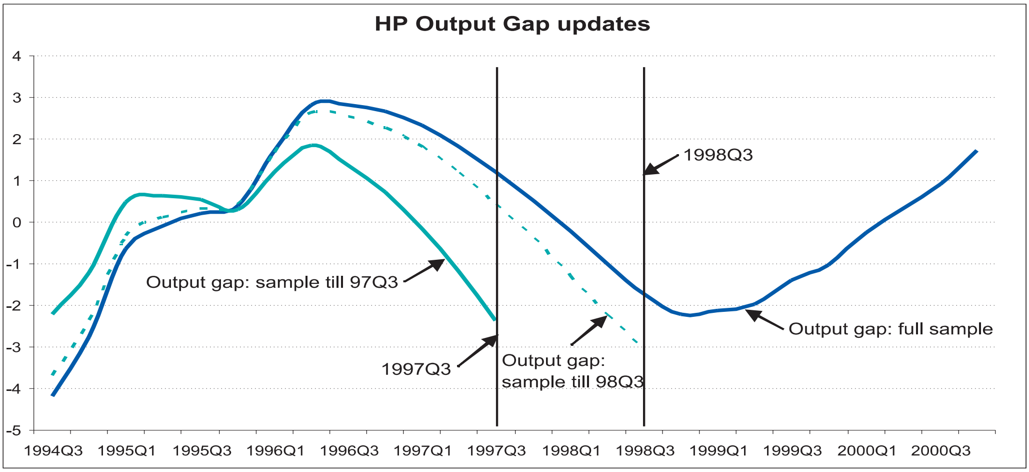
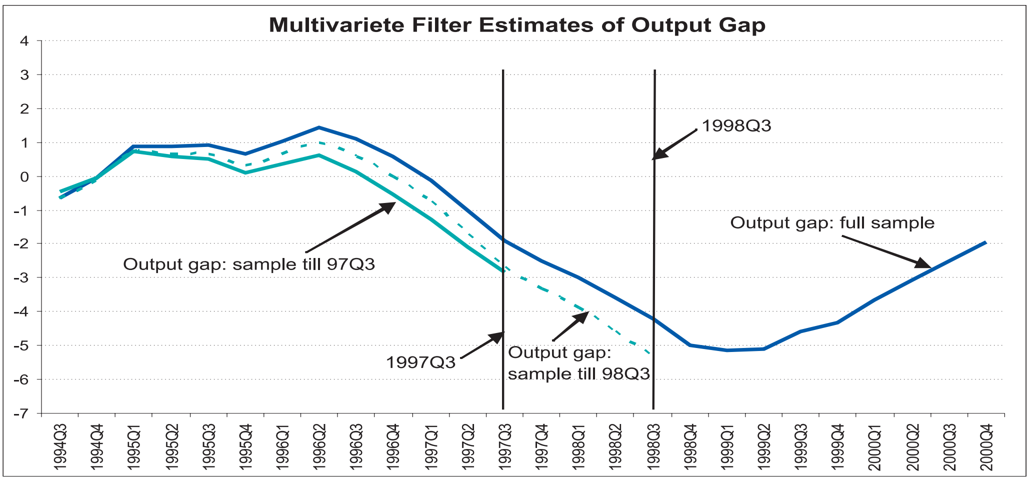


Figure 9: Real-Time Updating Test for the Estimates of the Potential Output





ESTIMATES OF OUTPUT AND UNEMPLOYMENT GAPS

Our estimates of the historical output and unemployment gaps are shown in Figures 6 and 7. These estimates are discussed in some detail in Chapter 1; the reader is referred to that discussion for interpretation of these results and their usefulness in describing the recent macroeconomic history of Czech Republic. Further information is provided in the comparison below.

COMPARISON WITH GAPS DERIVED FROM THE HP FILTER

We have argued that the estimates of potential output and the NAIRU from our multivariate system have some strong advantages: that using more information, and in particular information from the macro system, will help identify the equilibrium measures better; and that updating properties of the multivariate estimates are better.[19] In this section, we explore these claims by comparing our full-sample estimates with the results of the HP filter, and by performing an experiment where we look at the updating properties of the two methods during a critical period of history.

Recall that the HP filter uses only the data of the series itself to identify a trend line. The nature of the results depends a lot on the choice of the smoothing parameter. A low value will produce trend estimates that follow the data closely; a very high value will produce a straight-line trend. We use the standard assumption, coming from the original Hodrick-Prescott paper, setting the smoothing parameter at 1600. Harvey and Jeager (1983) argue that this is an optimal choice for deriving estimates of potential output for the United States using the HP filter.[20]

The charts in Figure 8 compare the resulting HP estimates of the NAIRU with our multivariate-filter estimates. The solid lines extending to the end of the sample are the two full-sample estimates of the NAIRU. The difference between the two is dramatic. The HP estimates put a line through the data, and, in particular, the actual unemployment rate towards the end of the sample. Most of the rise in actual unemployment is identified as a rise in the NAIRU, with an end-of-sample estimate of the NAIRU at well over 9 percent. Indeed, the HP estimates show the labour market in excess demand in 2000. This contrasts markedly with our results, which allocate only about half of the increase in unemployment to the NAIRU, and show a large measure of excess supply in 2000. We think that the latter is a much better reflection of reality, and a much better starting assumption for forecasting inflation.

Figure 9 repeats the comparison for potential output; the results are shown in terms of the output gap that emerges from the two approaches. The same sharp contrast emerges from the estimates in the last part of the sample. The HP results show a much smaller recession, starting significantly later, and a return to excess demand by 2000. Our estimates show a deeper trough, and one that continues through the end of the sample. Again, we believe that the multivariate results characterize the situation at the end of the sample much better and provide a much better base for a forecast of inflation.

Figures 8 and 9 also contain the results for our “real time” illustration of the updating properties of the two methods. For both methods, we estimate the NAIRU and potential output using data up to 1997, Q3; we then repeat this, adding another year of data and estimating up to 1998, Q3.

[19] See also, Boone and others (2003).

[20] The Harvey-Jeager “optimality” argument does not necessarily carry over to an application to data for the Czech Republic, but we think that our choice is reasonable. It has been used in many applications of the HP filter in many countries.

In Figure 8, the lowest dashed lines, which end in 1997, Q3, show the results for the two methods on the first sample. Note, first that the difference between the HP estimates from the short sample and the HP estimates for that same period from the full sample are much farther apart than are the two sets of estimates from the multivariate filter. For 1997 Q3, the end-point of the short sample, the difference for the multivariate filter is less than 0.4 percentage points, while for the HP filter it is over a full percentage point. Now compare what happens when we add another year of data, estimating to 1998, Q3. The dashed line showing the HP results has moved up sharply, about half way towards the final estimates. The multivariate filter estimates also rise, but by much less. The HP results are both more volatile, and from the perspective of the final estimates, much less accurate than the MV results through this period.

Figure 9 shows that the same basic messages emerge from the application to output. Here we see what may be a more striking point, however. The HP results for the short samples are indeed volatile, but their levels and the stories they tell, especially from the shortest sample, are much more like those from the MV filter during this period than they will end up being in the full sample results. It may be comforting to know that had the HP approach been used in 1997, the results would have been reasonable, from an ex post perspective. However, it is hard to take too much comfort from this result, when the story is virtually revised away within a year, and totally reversed, eventually.

APPENDIX: THE MULTIVARIATE KALMAN FILTER

In this appendix, we present the details of the multivariate filter used to provide our measures of potential output and the NAIRU. In the first section, we explain how the system of equations described in the text is transformed in order to obtain a state-space representation that allows us to apply the Kalman Filter procedure.[21] Then, we show how the same procedure can be used to obtain results for an HP filter.

STATE SPACE REPRESENTATION OF THE SYSTEM

The system of equations (1) - (8) from Chapter 4 can be represented by three measurement equations that link the current values of output, unemployment rate, and inflation rate to seven state variables $[\bar{y}_t, ygap_t, \mu_t, \bar{u}_t, ugap_t, \pi_t, \bar{\mu}]$.

$$\begin{bmatrix} y_t \\ u_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ ygap_t \\ \mu_t \\ \bar{u}_t \\ ugap_t \\ \pi_t \\ \bar{\mu} \end{bmatrix} \tag{A1}$$

Note that owing to the presence of lagged endogenous variables, the third measurement equation has been written as an identity that states that the sixth state variable is equal to the current observed values of inflation. When forecasting the next n-step-ahead values of inflation this allows us to take into account the errors arising from the use of predicted values.

[21] For further details on this methodology see Hamilton (1994) and Harvey (1989).

The dynamics of the state variables are summarized by the following transition equations.

$$\begin{bmatrix} \bar{y}_t \\ ygap_t \\ \mu_t \\ \bar{u}_t \\ ugap_t \\ \pi_t \\ \bar{\mu} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & d_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_0 & 0 & 0 & 0 & (1-c_0) \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & f_1 d_0 & 0 & 0 & f_0 & 0 & 0 \\ 0 & a_2 & 0 & 0 & 0 & (1-a_0-a_1) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_{t-1} \\ ygap_{t-1} \\ \mu_{t-1} \\ \bar{u}_{t-1} \\ ugap_{t-1} \\ \pi_{-1} \\ \bar{\mu} \end{bmatrix} + \begin{bmatrix} 0 \\ \kappa_{2,t} \\ 0 \\ 0 \\ \kappa_{5,t} \\ \kappa_{6,t} \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\bar{y}} - b_0 \varepsilon_t^{\bar{u}} \\ \varepsilon_t^{ygap} \\ \varepsilon_t^{\mu} \\ \varepsilon_t^{\bar{u}} \\ \varepsilon_t^{ugap} + f_1 \varepsilon_t^{ygap} \\ \varepsilon_t^{\pi} \\ 0 \end{bmatrix} \quad \text{(A2)}$$

where

$$\kappa_{2,t} = -d_1 \left[e_0 rr12gap + e_1 rr4gap + e_2 gr\_rrgap \right] - d_2 lzgap_t$$

$$k_{5,t} = f_1 \kappa_{2,t}$$

$$\kappa_{6,t} = a_0 \left[\pi 4_t^M + 100 * \Delta_4 lz_t^{eq} \right] + a_1 E0\pi 4_t$$

The covariance matrix of the residuals of the transition equations is as follows:

$$Q = \begin{bmatrix} \sigma^2_{\varepsilon^{\bar{y}}} + b_0^2 \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & -b_0 \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & 0 \\ 0 & \sigma^2_{\varepsilon^{ygap}} & 0 & 0 & f_1 \sigma^2_{\varepsilon^{ygap}} & 0 & 0 \\ 0 & 0 & \sigma^2_{\varepsilon^{\mu}} & 0 & 0 & 0 & 0 \\ -b_0 \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & 0 \\ 0 & f_1 \sigma^2_{\varepsilon^{ygap}} & 0 & 0 & f_1^2 \sigma^2_{\varepsilon^{ygap}} + \sigma^2_{\varepsilon^{ugap}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\varepsilon^{\pi}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A3)}$$

Once the values of the parameters have been set, and given initial values of the state variables and their corresponding covariance matrix, optimal estimates of the potential output, output gap, NAIRU and unemployment gap based on the information available at time t (referred to as filtered estimates) and on information available from the full sample of observations to time T (referred to as smoothed estimates) are obtained from the Kalman Filter. The calculations are done in GAUSS. The precise code will be made available on the IMF's web site.

A SPECIAL CASE: THE HP FILTER

In the past, the HP filter has been widely used in policymaking institutions to measure the potential output and the NAIRU. The popularity of this univariate filter resides in its simplicity and its ability to fit quite well, at least for some countries, the historical variations of inflation when the estimated unemployment gap or output gap is included in a Phillips curve.[22]

[22] For a discussion of this point see Boone and others (2003). For examples, see Coe and McDermott (1997), Bank of England (1999) and Cozier and Wilkinson (1990).

HP filter estimates of the potential output and the NAIRU can be obtained from the Kalman filter procedure when the state space representation is as follows. Calling x_t the output or the unemployment rate, $\bar{x}_t$ the potential output or the NAIRU, and $xgap_t$ the output gap or the unemployment gap, the measurement equation is given by,

$$x_t = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \bar{x}_t \\ \bar{x}_{t-1} \\ xgap_t \end{bmatrix} \qquad \text{(A4)}$$

with the transition equations, which summarize the dynamics of the state variables,

$$\begin{bmatrix} \bar{x}_t \\ \bar{x}_{t-1} \\ xgap_t \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_{t-1} \\ \bar{x}_{t-2} \\ xgap_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\bar{x}} \\ 0 \\ \varepsilon_t^{xgap} \end{bmatrix} \qquad \text{(A5)}$$

The variance covariance matrix of the two shocks is:

$$Q = \begin{pmatrix} \sigma^2_{\varepsilon^{\bar{x}}} & 0 \\ 0 & \sigma^2_{\varepsilon^{xgap}} \end{pmatrix} \text{, or equivalently}$$

$$Q = \begin{pmatrix} \frac{\sigma^2_{\varepsilon^{\bar{x}}}}{\sigma^2_{\varepsilon^{xgap}}} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\lambda} & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{(A6)}$$

The smoothed estimates obtained from the Kalman filter correspond to the HP filter estimates. The degree of volatility of the estimates depends on the value of the smoothness parameter λ. The higher λ, the less volatile the trend, as λ tends to infinity (zero) the trend tends to be deterministic (highly volatile). The value of this parameter determines how much the trend should fit the data. More specifically it determines the weight given to the past observations relative to the last observations. Small values (high values) of λ correspond to small weight (high weight) on the past observations.

A METHODOLOGY FOR PRE-FILTERING: THE PC FILTER

We do not use the HP filter in our work, except for illustrative purposes. For applications where a univariate approach is judged appropriate, we use a filter called the Prior Consistent (PC) filter.

State-Space Representation

Calling x_t the variable we wish to filter, $\bar{x}_t$ the equilibrium values of x_t, $\bar{\bar{x}}_t$ the growth rate of x_t, and $xgap_t$ the deviation of x_t to its equilibrium values, the measurement equation which links x_t to the three state variables $\{\bar{x}_t, \bar{\bar{x}}_t, xgap_t\}$ is given by:

$$x_t = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_t \\ \bar{\bar{x}}_t \\ xgap_t \end{bmatrix} \qquad \text{(U1)}$$

The transition equations which summarize the dynamics of the state variables is,

$$\begin{bmatrix} \bar{x}_t \\ \bar{\bar{x}}_t \\ xgap_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_{t-1} \\ \bar{\bar{x}}_{t-1} \\ xgap_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\bar{x}} \\ 0 \\ \varepsilon_t^{xgap} \end{bmatrix} \qquad \text{(U2)}$$

The matrix of variance covariance of the error terms in (U2) is

$$Q = \begin{pmatrix} \sigma^2_{\varepsilon^{\bar{x}}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\varepsilon^{xgap}} \end{pmatrix}, \text{ or equivalently}$$

$$Q = \begin{pmatrix} \dfrac{\sigma^2_{\varepsilon^{\bar{x}}}}{\sigma^2_{\varepsilon^{xgap}}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\lambda} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{(U3)}$$

The measurement equation is an identity that states that the variable x_t is the sum of an equilibrium value and a gap. The first transition equation states that the equilibrium values of x_t follow a random walk plus drift. The drift term, $\bar{\bar{x}}_t$, is assumed to be constant as described in the second equation of (U2). When this constant term is assumed to be equal to zero, as is the case for most applications, the main exception being the real exchange rate, the number of state variables reduces to two. The last equation of (U2) states that x_t deviates from its equilibrium level according to a random disturbance. The error terms $\left\{\varepsilon_t^{\bar{x}}, \varepsilon_t^{xgap}\right\}$ are assumed to be identically, independently and normally distributed.

Assumptions on the Initial State vector

In order to apply the Kalman Filter to the system (U1)-(U2) we need to make some assumptions on the initial values of the state variables, their matrix of variance covariance, and the value of the parameter λ.

The parameter λ has been fixed to 25 for all the values of x. It is easiest to think of the intuition for this prior in terms of the standard deviations. We judge that if a "large" deviation for the trend were 1, say, then the corresponding measure in gap terms would have a large value at 5. This assumption has been found useful in applications elsewhere.[23]

[23] See Box 7, page 30-31 in Laxton and others (1998).

The initial value of $\bar{x}_t$ is set to the value of the first observation of x_t (the initial gap is set to zero). In applications to reduced systems (two state variables), the initial covariance matrix of the state variables is diagonal with each element of the diagonal set at 10. This high value denotes the degree of uncertainty on the initial values of the state vector. Assuming a diffuse prior is a standard procedure. In expanded (three state variables) systems, the mean of the growth rate is set at a calculated historical average, or some number set by judgment in the light of the historical value. The initial variance of this variable is fixed at zero. In this case, we also set the initial variance of $\bar{x}_t$ at zero. The zero initial variance of the growth rate, taken with the other assumptions on its dynamics, allows us to treat $\bar{\bar{x}}_t$ as a constant term during the prediction-updating process of the Kalman filter. The zero initial variance on $\bar{x}_t$ reduces the uncertainty due to its initial value in the filtering process. The filtered and smoothed initial equilibrium levels will be close to the first observation of x_t in this case.

CHAPTER 6

Risk Analysis and Confidence Bands for the Forecast

By Jaromír Beneš, David Vávra and Jan Vlček [1]

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INTRODUCTION

The crafting of the Staff's baseline forecast for the main macroeconomic variables is the first step in providing the information necessary for policy makers to think about the appropriate setting for the monetary instrument. The baseline scenario will contain a path for the instrument variable that is consistent with respecting the inflation targets, conditional on all the assumptions and judgment agreed to in creating that scenario, and conditional on the reaction function built into the model. The model plays a role in this, but as we have stressed repeatedly, the baseline forecast comes primarily from the Staff, not the model.

In providing information regarding risks and uncertainty, the model plays a lead role. We have illustrated the model's use in risk analysis through the discussions of its properties in Chapter 4. Here, we continue with a specific example of risk analysis that complements our discussion of the forecast process in Chapter 2. Then we turn to the important issue of creating confidence bands for a forecast.

ASSESSMENT OF RISKS

The „Issues" discussion will have highlighted some risks that the team thinks need explicit attention. The focus of these ideas will have been sharpened through the process, and in the final meeting on the baseline forecast the Board will have mandated certain risks to be analysed in the forecast documents. If there is a formal alternative forecast, this is a major piece of risk analysis that will already have been completed for inclusion in the forecast documents.

Most risk analysis exploits the shock-control properties of the model. The model can be used quickly and easily to simulate the consequences of a number of types of risk. All of the simulations reported in the discussion of model properties can be thought of as examples of very simple risk analysis. There, the risk is characterized as a single shock. Some risk analysis done to support a forecast will be not much more complicated than that, though normally the shock of interest will be more persistent, and it may be appropriate to formulate it as a compound shock, where more than one disturbance is perturbed.

In some cases, risks may be characterized as uncertainties about model properties. One common example concerns dynamic properties, such as the speed at which specific shocks might pass through into inflation. Another is the uncertainty about key estimated or calibrated elasticities. It is appealing to think of the forecast as dealing with how policy must respond, given initial conditions and the informed assumptions of the Staff about the near-term outlook, using an agreed representation of the economy. However, especially in the early stages of work with model-based forecasts, it is to be expected that there will be legitimate questions concerning model properties and their impact on the policy scenario.

Certain issues will arise frequently in assessing the risks inherent in any forecast made by the CNB. For instance, there will almost always be concern regarding the external environment. For example, the October 2001 forecast highlighted uncertainty about the extent of the projected foreign growth recession. In the end, the discussions gave rise to an alternative scenario with a deeper foreign slowdown accompanied by lower foreign inflation and interest rates and a lower import price of oil. Another typical risk concerns the assumptions about the evolution of the fiscal stance. An alternative scenario was prepared, wherein the baseline fiscal assumptions were replaced with an assumption of weaker government spending.

Other examples of risks that could arise for an inflation scenario include: a change in indirect taxes that would have a direct impact on prices; a change in regulated prices that would affect the headline CPI and have spill-over effects on other prices; a change in world oil prices (or other raw materials prices, or indeed any shock that changes the terms of trade) that would have an immediate effect on import prices, the exchange rate and the CPI; and, a shock to the exchange rate itself from other sources.

The possibility of terms-of-trade shocks generally, and oil price shocks in particular, is likely to generate regular interest. A typical risk analysis could involve investigating the implications of a shock of some magnitude and duration to this baseline view, perhaps based on the historical distribution of shocks to oil prices (if no specific information were available) or some specific alternative judged pertinent for the particular forecast period.

The hard part in risk analysis is choosing and characterizing the risk. Running the simulations is relatively easy. The output of a risk analysis is shock-control paths for the main macro variables, including the policy instrument, relative to the baseline forecast. When presenting the result of risk analysis, it is often useful to show a comparison of the shock path and the control path in levels, as well as the pure shock-control information.

Figures 1 and 2 illustrate risk analysis of the type considered as part of the October 2001 exercise. In Figure 1, scenario 1 acts as a baseline; scenario 2 has weaker foreign demand, lower German interest rates and a lower world price of oil. The results show that monetary conditions must be looser when there are weaker external conditions for the output gap to close. The lower foreign demand makes the output recovery more sluggish. This puts downward pressure on the initial interest rate settings. The inflation profile is very much the same in the first half of the scenario because the effects of the weaker aggregate demand are offset by lower nominal appreciation, owing to the lower domestic interest rates. The difference in inflation performance in the latter half of the period results from higher nominal appreciation in the scenario 2, owing to lower foreign interest rates.

The fiscal experiment is similar in its qualitative effects on output. The magnitude is much smaller, however, and while there is downward pressure on interest rates, it is rather small. This illustrates the relatively great importance of the external conditions to any inflation scenario in the Czech Republic.

UNDERSTANDING THE POLICY IMPLICATIONS OF UNCERTAINTY

In the end, policy decisions are the responsibility of the Board. It would be wrong to think that the Board should simply do what the baseline scenario says must happen to the policy instrument. The forecast analysis is but one input into the policy decision.

The baseline scenario is derived using a reaction function-a stylised representation of how the instrument might be set, given the information implicit in the macro scenario, such that, in the absence of shocks, an inflation scenario consistent with the targets would be realized over the medium term. The presence of some such reaction function is crucial to the model; without it, no solution to the forward-looking problem would be possible. But it should not be taken literally as a model of behaviour or a prescription for action (see Box 3 in Chapter 2).

The model is not the truth. It is a highly stylised representation of the dynamics of the macro economy. But even within the confines of the economy described by the model, the baseline forecast is simply one plausible outcome. There will be shocks, and the outcome will differ from the baseline scenario. In the next section, we show how the model itself can be used to place confidence intervals around the baseline scenario.[2]

The current version of the CNB's QPM is almost linear. In a linear model, shock-control properties are independent of the control solution. Moreover, in a linear world, if an error is made, say policy is too easy from an ex-post perspective and inflation begins to rise, the costs of correcting this error later, as measured by things like the cumulative output gap that will emerge in the process do not depend much on when or at what pace the correction is made. Moreover, the costs will be roughly symmetric with respect to the sign of the error.

[2] This may understate the uncertainty, because it takes the model as given and ignores the uncertainty regarding the measures of key inputs, such as the equilibrium levels and trends of variables like potential output and the real exchange rate.

In a non-linear world, things are not so simple. If, for example, there is asymmetry in the Phillips curve, such that inflation rises more strongly in the face of excess demand than it falls in the face of excess supply, then the effects of policy errors are far from symmetric. The response necessary to respond to an error that resulted in rising inflation will be greater than the response to falling inflation, and the timing will matter a lot. If there is delay in responding to inflation, the structural asymmetry will generate a compounding problem and necessitate even greater policy response and greater costs of re-anchoring expectations.[3] The original Phillips curve featured this type of asymmetry, and several central banks assume such structure within their QPMs.[4] Our point here is not that this should necessarily be a feature of the CNB's model, but that the risks associated with inflation can be more complicated than captured by deterministic risk analysis or even stochastic analysis based on a presumption of linearity.

An issue that further complicates the discussion is how expectations are formed and respond to information of various kinds. Part of the logic of an IT regime is that the explicit commitment to a target will help anchor expectations. It makes a world of difference if the system is credible and inflation expectations therefore give some weight to the target. If there is no such anchor, then when a positive shock or sequence of shocks is experienced, and inflation drifts up, expectations may respond dramatically, since the risk of a permanent escalation in inflation has palpably[5] increased, and this, in turn, may trigger wage demands and change asset decisions with all the costs these things entail. If, in contrast, there is credibility, the observation of inflation may lead to some short-term predictions of an ongoing problem, owing to the persistence of typical cycles, but the belief that the authorities will respond and respect the medium-term target will keep such effects on expectations relatively muted and concentrated in the short term. This makes the costs of routine forecasting errors much lower, since the central bank's task is limited to doing what is necessary to deal with the shock and not with the potentially more damaging effects of a change in longer-term inflation expectations.

Since the dynamic properties of the system are so sensitive to the credibility of the regime, and a loss of credibility is so costly to reverse, the prudent policy maker may want to give some weight to these concerns in making short-term policy decisions. Even if the presumed model economy is linear, there may be a case for asymmetric policy response.

AN OVERVIEW OF STOCHASTIC SIMULATION AND THE CREATION OF CONFIDENCE BANDS

Up to this point, we have reported experiments using the model in what is called deterministic mode. Forecasts, specific risk analysis and simulations of the model's response to particular shocks are all examples of analysis of solution paths under specific assumptions. In the simplest properties experiments, we assume that all disturbances are zero, except for the specific shock being studied. In the forecast, we are using all the resources at our disposal to pick specific paths for all the disturbances, which, taken together with the model, produce the baseline scenario.

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3 For discussion of these issues see, for example, Laxton, Meredith and Rose (1995), Clark, Laxton and Rose (2001) and Laxton, Rose and Tambakis (1998).

4 Two early examples are Canada and New Zealand. Documentation on their models is cited in our references.

5 For some recent empirical evidence on the importance of credibility and its implications for the short-run unemployment-inflation trade-off see Laxton and N'Diaye (2002).

In stochastic simulation, the model is solved repeatedly using hypothetical drawings from some assumed joint distribution of all the disturbances. Because the model is forward-looking, the solution must cover a much longer horizon than we wish to consider for each forecast. Each replication provides a possible future scenario in a world with shocks for one particular sequence of shocks to all disturbance terms. With enough observations of these possible paths, we can create measures of the distributions of the outcomes for each macro variable at each future date. This process requires extensive computation using sophisticated software, but with modern technology, the methodology has become feasible.

Stochastic simulation of a non-linear forward-looking model is not a trivial problem because the solution today depends on the whole future path, through expectations, the policy reaction process, and so on. The model must be solved for the entire future path given current shocks to get the current point on the solution path; then the next set of shocks is added to the scenario created by the first set of shocks, the model is simulated again over the entire future period, and the second point emerges. Note that we cannot permit the central bank to know the future shocks in advance. At each point in time, the central bank is assumed to respond to the current information, preparing a forecast for the main macro variables over some horizon and to set the policy instrument as determined by the IT reaction function. The process continues into the hypothetical future, until the whole scenario, with repeated shocks, has been created. We can consider this to be the first drawing of a possible alternative future path. Then a new set of shocks is chosen, and the process is repeated until a second possible future path has been created. The process continues until we have enough observations on possible future paths to measure the distributions with reasonable precision.[6]

Because the current CNB model is essentially linear, the problem is greatly simplified. Indeed, we can derive what amounts to an analytical solution for the confidence bands. This means that the computations are easy and rapid.[7]

A key point in understanding the results of stochastic simulations is that for every scenario considered in generating the distributions of possible future outcomes, the reaction function is operating to provide the response of monetary policy to whatever happens. Policy is always acting to try and keep inflation from deviating systematically from the target. Therefore, as part of the output from the stochastic simulations, we obtain a distribution for the possible course of the policy instrument over the forecast horizon. Without the reaction function, there would be no bounds on the confidence intervals for inflation.

CHOOSING THE SHOCKS

Choosing the distribution from which the shocks are drawn is not straightforward. One place to start is to use the information in the historical residuals from the model equations. Table 1 shows the standard deviations for the historical errors from a subset of model equations. In principle, one could also consider autocorrelation and covariance measures in characterizing the historical shocks, but we begin with the simplifying assumption that the drawings are independently distributed, both across time and across variables.[8]

6 There is important work going on to make this process easier and faster. A leader in this work is Michel Juillard. He is developing more reliable algorithms in a program called DYNARE. See his web site, http://www.cepremap.cnrs.fr/dynare.

7 The details of the methodology are provided in the Appendix.

8 Typically, model residuals are autocorrelated. Using autocorrelated shocks complicates the methodology of stochastic simulation, because sequences matter.

Using the historical variance measures directly risks overstating the true degree of uncertainty in the forecast, perhaps considerably. There are a number of reasons for this. We have stressed that the model attempts to capture just the essential core macro dynamics. The forecast contains the detailed considerations of specialists, which implicitly adds to the model a wealth of complexity in dealing with the particular circumstances. If one had a history of staff forecasts, one could use that to compute errors that would reflect much better the dispersion of shocks that will affect the outcomes relative to the baseline projection. While we do not have such information at hand, it is clear that the historical error variances overstate the effective uncertainty in a projection, and perhaps greatly so. Another reason why we might be suspicious of the historical measures is that they reflect a period of relative turbulence. For example, the relative size of the energy-price shocks over the historical period is extremely large. This may provide relevant information for the future, but this is at least questionable. Specialists who provide the baseline may well be able to provide information on reasonable assumptions about shock dispersion given current circumstances.[9] From a policy perspective, there is also reason to doubt the historical measures. In part, they come from an era with a very different policy regime, and also from a period of great internal turmoil in the Czech economy. A model built to reflect an IT world may be expected to have some difficulty replicating a different world.[10]

One particular example of a fundamental change is in the interpretation of shocks to interest rates. Let us ignore the difficulty of interpreting this when moving from a fixed exchange rate regime to a flexible rate regime under IT, and focus on another problem. Part of the history of the Czech case, as elsewhere, involved a transition to *lower* inflation as part of the IT goal of defending (low and) stable inflation. In any case where there must be a transition in the level of inflation, the role of monetary policy is different from what it will be subsequently. To lower the level of inflation expectations, monetary policy must intervene; monetary policy itself is a source of shock to the economy. Thereafter, however, monetary policy becomes more *reactive*; its main role is to respond to shocks arising elsewhere. Thus, it is questionable whether we should allow for shocks to interest rates at all in computing confidence bands.[11]

In the experiments reported below, we begin with analysis based on the historical residuals. We then offer an alternative to illustrate the sensitivity of the results to the stochastic assumptions.

EXAMPLES OF CONFIDENCE BANDS

To illustrate the content of model-based confidence bands, we use the methodology for linear models, where we can obtain an analytical solution. The model is not exactly linear, but the difference is minor and these results should be reliable. We begin by assuming that the shocks are independent, mean-zero random drawings from the normal distribution. In this case, uncertainty can be measured sufficiently using the simple measure of dispersion, the standard deviation. The particular shocks are those listed in Table 1, where we also report the standard deviations of the historical residuals.[12]

[9] Indeed, in creating confidence bands for a projection, the views of the specialists on the risks in individual markets may provide valuable conditioning information.

[10] The record of the expert forecast over the period of disinflation is, in fact, not as good as our simple model. However, this was an unusual period, with a major policy regime change. The recent record of the near-term forecasters, in a more stable period, does support the arguments in the text.

[11] For an example of stochastic analysis where the authors argue that shocks to the policy variable should be excluded see Black, Macklem, and Rose (1997).That study also avoids completely using historical model residuals to characterize shock distributions; rather, an extensive analysis is done using a VAR to provide the stochastic assumptions.

[12] There are two exceptions. We use a standard deviation of 10 for the energy price disturbance, rather than the larger number in Table 1. We have based this on the CNB expert's view as to what the number should be now. Second, we base the standard deviation for non-energy import prices on the historical errors in the Consensus Forecast for the German wholesale price index.

Figures 3, 4 and 5 show what we obtain. We report the results by showing the range encompassed by one- and two-standard deviation bands about the mean. For the normal distribution, one-standard-deviation bands encompass about 66 percent and two-standard-deviation bands about 95 percent of the outcomes. We show the results for up to 5 years into the future; the numbers on the horizontal axes are quarters into the future.

Note that, except for the nominal exchange rate, the confidence bands at first fan out a bit and then stabilize. This reflects the effect of the initial conditions, which provide a fixed point of departure. There is still the chance of very unfortunate shocks in the near term, of course, but there is less chance of a cumulative problem developing. Thus, there is less uncertainty about the outcomes over the first few quarters than about the outcomes several years into the future. The distributions converge relatively rapidly to their limiting forms, but the distribution of the output gap converges to its limiting form a bit more slowly than the distribution for inflation.

In the case of the nominal exchange rate, the distribution is shown around a rising trend. This reflects our view that there will be a trend real appreciation of the koruna of about 4.5 percent per annum in the coming years. Of course, the distributions for nominal levels do not converge. The policy control is on the rate of inflation. If there is a positive shock to inflation, for example, the central bank acts to bring the rate of inflation back to the target, but the cumulative effect on the price level of the temporary deviation of inflation from the target level is ignored. Thus, the bands for nominal variables, including the nominal exchange rate, necessarily diverge over time in stochastic simulations. This shows clearly in Figure 5.

Consider now the details of the results in Figure 3 for the CPI. The confidence bands converge to close their limiting values in about 3 years. The confidence bands are wide. The official bands announced for the CNB's inflation targets encompass +/- 1 percentage point. These results show that if the historical errors are good reflections of the uncertainty from shocks, then there is a relatively high probability that CPI inflation will drift outside the bands. Even the one-standard-deviation confidence bands (66 percent of outcomes) encompass +/- 2 percentage points. The 95 percent confidence bands encompass a bit more than +/- 4 percentage points, in the limit.

An interesting question is why uncertainty is bounded at all in these results. We repeat that the answer is that the central bank is presumed to be doing its job. The operation of the reaction function and the endogenous response of the policy instrument in these simulations are crucial to the results. Otherwise, there would be no bound on the limiting inflation distribution.

For the output gap, it takes about 4 years for the distributions to converge to their limiting forms. Note that based on the dispersion of the historical errors, there is a lot of uncertainty about what will happen to output. The limiting 95 percent confidence interval encompasses almost +/- 4.5 percentage points. This, of course, is a major contributor to the uncertainty for the CPI.

We do not believe that the historical errors provide a good guide to the true uncertainty we now face in forecasting inflation. We have reviewed many of the reasons in the previous section. A major point is that the history we have used for these measures is extremely volatile and contains an exchange crisis, a major change in the policy regime, and large structural changes in the economy. We have too little information to provide a reasoned argument as to what the standard deviations should be. However, to illustrate, we repeat the exercise by cutting all the standard deviations in half. The results are shown in Figures 6 and 7. We do not show the nominal exchange rate again. Now, we have a much greater probability of staying within the IT bands. Yet, to get a high (i.e., 95 percent) probability of staying within the IT bands, we would have to cut the standard errors in half again, that is, to 25 percent of their historical values.

CONCLUSION

It is unlikely that the eventual answer to the question of how the standard deviations should be set will yield proportional rescaling. In particular, with the new FPAS, which combines the expert analysis of the near term with the macro perspective of the medium-term model, and with the period of disinflation behind us, there is a good chance that we will be able to reduce dramatically our forecast errors for inflation. The results are quite highly sensitive to this particular standard error.

The current practice of uncertainty treatment in the CNB's forecasts reflects the discussion above and is organised as follows. Internally, baseline forecast is presented as a point forecast without probability distribution. In the same time, two to four alternative forecasts are always presented to account for the risks discussed during the baseline forecast preparation. The alternatives are not assigned any probability of occurrence by the staff. This is the job done by the Bank Board during its Monetary Policy Meetings. Externally, the internal point forecast is augmented by bands. The inflation forecast band in terms of year to year inflation widens linearly until it reaches 1,4 percentage points at four quarters ahead and stays at this width from then on. The CNB also publishes its year to year GDP forecast where the band expends in a similar fashion until it reaches 1 percentage point width at two years. The fact that the bands stabilise after some period reflects the stabilising role of monetary policy as described earlier in this Chapter. Externally, the bands are used as communication devise to stress the general uncertainty of any baseline forecast. The width of the band is thus constant and does not reflect specific uncertainty of any forecast. As for the alternatives prepared internally, the CNB sometimes refers to them in the Minutes and in those cases they are also shortly described in the Inflation Report.

APPENDIX: THE STOCHASTIC SIMULATION FRAMEWORK FOR COMPUTING CONFIDENCE INTERVALS

The assumptions underlying the stochastic simulation experiments are as follows. The model of macroeconomic behaviour is assumed to consist of the equations in Chapter 4.[13] In each period, the economy experiences five types of exogenous shock. We have not applied shocks to interest rates in these experiments.

1. A shock to the output gap (RES_LYGAP)
2. A shock to the inflation rate (RES_PIE_CORE)
3. A shock to the Risk Premium (RES_PREM)
4. A shock to the Import Price of ENERGY (RES_PIE_M_ENERGY)
5. A shock to the Other Import Prices (RES_PIE_M_XENERGY)

[13] There have been a number of changes to the model since Chapter 4 was written. We do this analysis using the current model.

When the model is linear, as is almost the case here, model responses to shocks (measured as deviations from a baseline simulation) are independent of the level of the variables, so we can do the analysis around a hypothetical steady-state solution.[14] In such a case, the time profile of the model response to a shock of any size is equivalent to the impulse response to a unitary shock multiplied by the actual size of the shock.[15] This means that we can study stochastic properties of the forecast in very much the same way as we did in Chapter 4. We simply perform shock-control simulations for unitary shocks to each of the residuals considered above and store the responses of the variables in question. For instance, say that the variable e(t, lgdp_gap, res_pie_core) measures the response of *lgdp_gap* to a unitary shock in *pie_core t* periods after the shock occurred. Then, if the size of the actual shock to pie_core was res_pie_core(1), the total effect on lgdp_gap in period t after the shock is e(t, lgdp_gap, res_pie_core)*res_pie_core(1).

In each of the stochastic simulations, we create a set of residuals for each of the forecast periods (say 100) by random draws from mean-zero normal distributions. For the first experiment, the standard deviations are set based on historical residuals.[16] The response of a variable to this particular sequence of residuals can then be found using the impulse response function *e(.)* by multiplying these unitary effects by actual shocks that have been realized in this and earlier periods. For instance, the value of the lgdp_gap response in period t of the simulation can be found as:

$$lgdp\_gap(t) = \sum_{i=1}^{t} \sum_{j \in list\_of\_res} e(t-i, lgdp\_gap, j) * j(i),$$

where *list_of_res* is a list of the residuals that are being shocked and index i in j(i) refers to the random realization of the particular residual in the *i*th period of the simulation. Other variables are treated in a similar manner.

The distributions can be generated through repeated calculations of the above sort. The results provide a distribution that converges to stable results in samples of about 70 replications.

Table 1: Historical Error Dispersion

Historical Residuals from	*Standard Deviation (percentage points)*
Output gap	*1.3169*
Unemployment gap	*0.1118*
Core inflation	*2.1269*
Inflation of energy prices	*16.1206*
Exchange rate	*1.8600*
One-year PRIBOR	*0.9901*
Client interest rate	*1.5195*
Risk premium	*0.3045*
Inflation of imported energy prices	*18.7234*
Inflation of imported prices excluding energy	*4.6701*

[14] This method involves a potential error caused by the non-linearity in the model that may make the results dependent on the initial state of the system. To investigate this possibility, we ran the stochastic simulations again, this time beginning from a different steady state. There was no significant difference in the impulse responses, which suggests that the error can probably be ignored.

[15] The only non-linearity in the QPM described in Chapter 4 arises from using aggregate price indices in absolute levels, rather than in logs. Therefore, while all behavioral equations are linear, the headline CPI inflation influencing expectations, is derived from an exponential relationship with respect to the other model variables.

[16] Obviously more structure could be added, if we assumed that the distributions will have non-trivial higher moments (e.g. skewness) and hence, non-normal.

Figure 1: Comparison of Scenarios With Alternative Assumptions of Foreign Demand (Scenario 2 has weaker foreign demand)

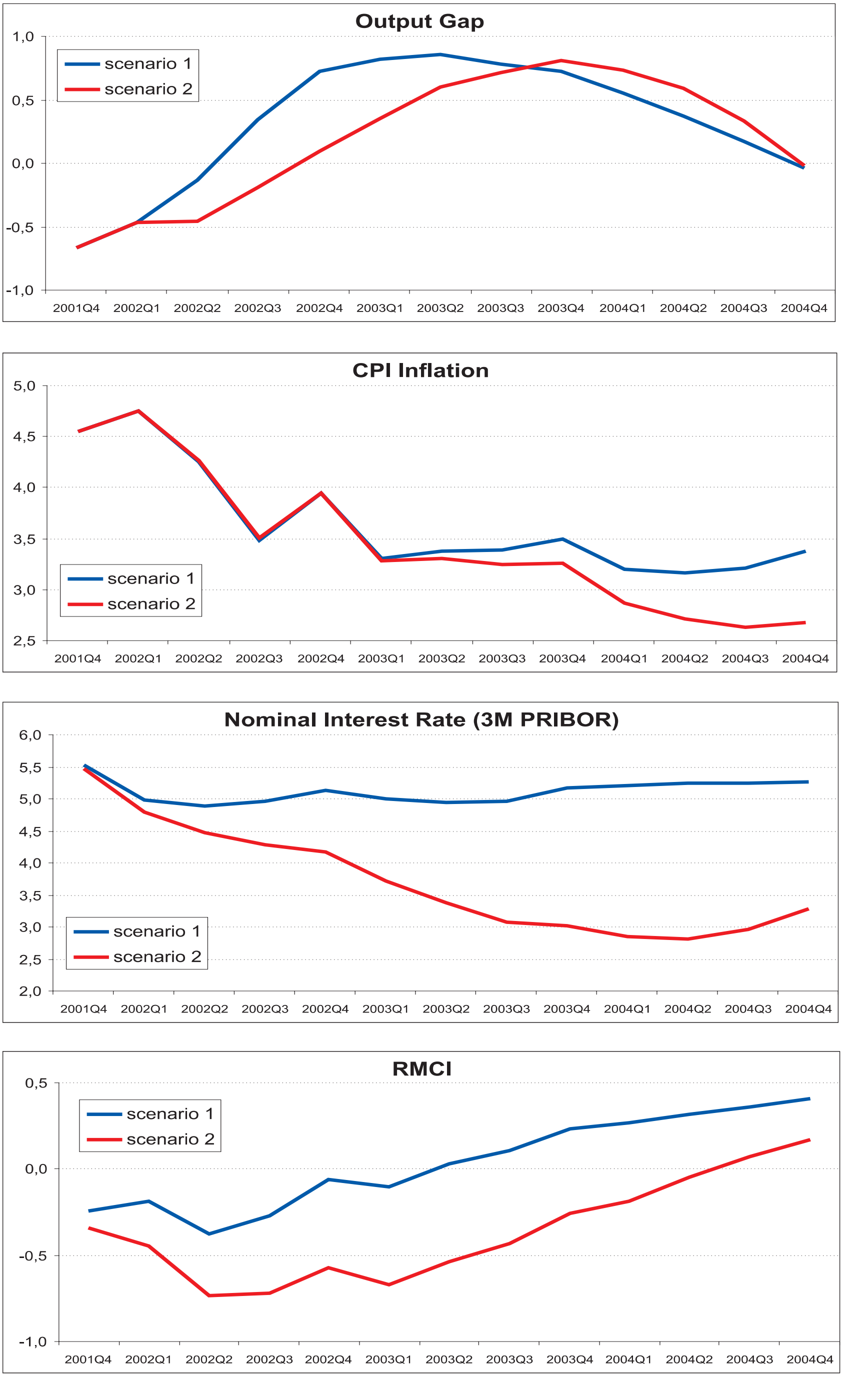


Figure 2: Comparison of Scenarios with Alternative Degrees of Fiscal Stimulus (Scenario 2 has weaker government spending)

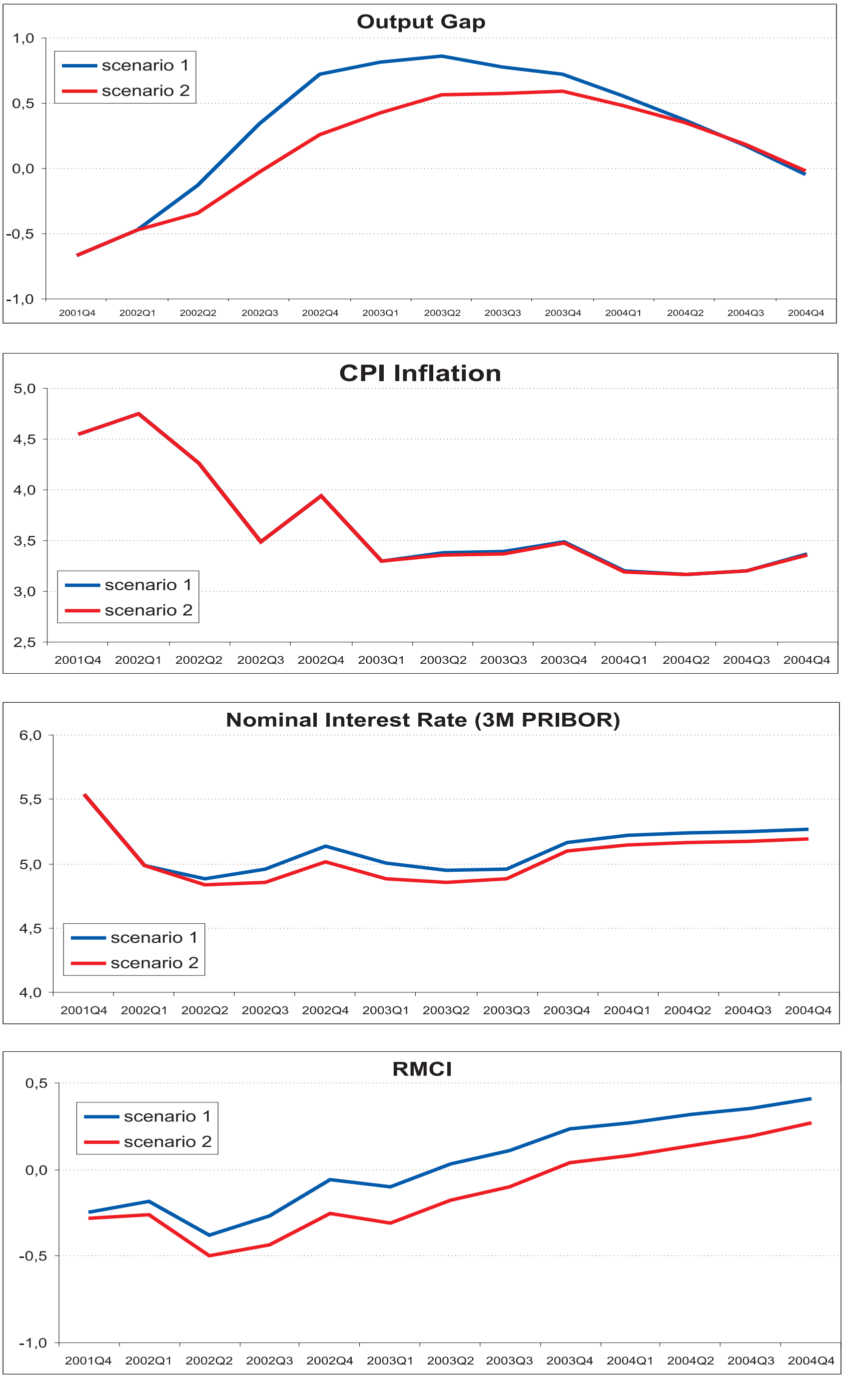


Figure 3
Confidence Bands for CPI Inflation: Historical Errors (percentage points)

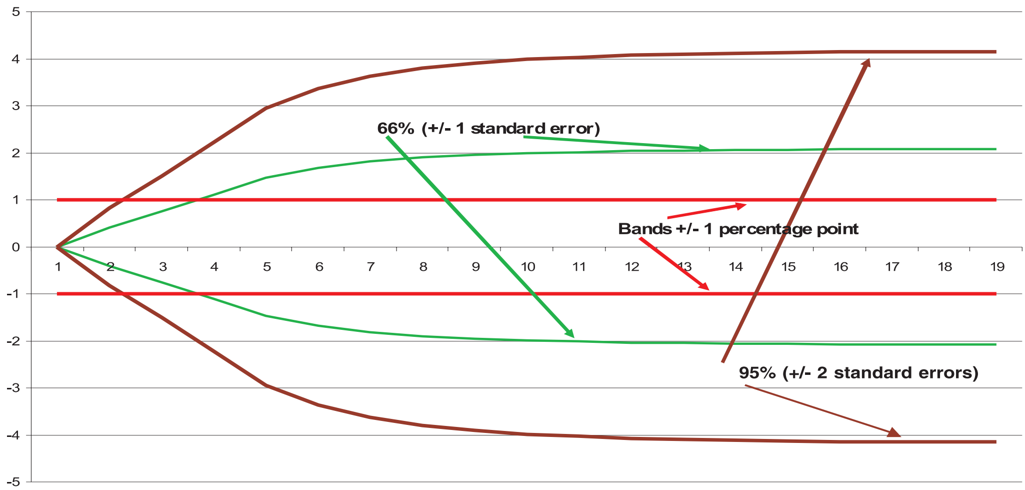


Figure 4
Confidence Bands for Output Gap: Historical Errors (percentage points)

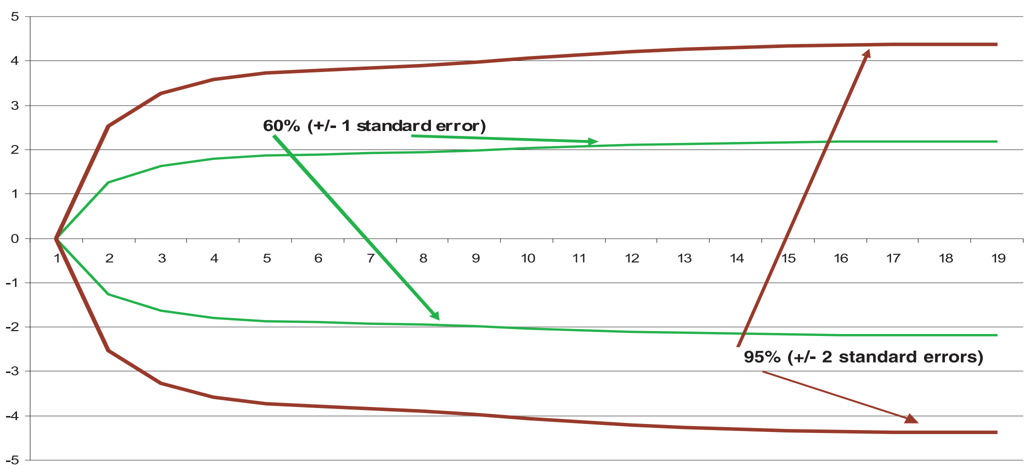


Figure 5
Confidence Bands for Exchange Rate: Historical Errors (per cent)

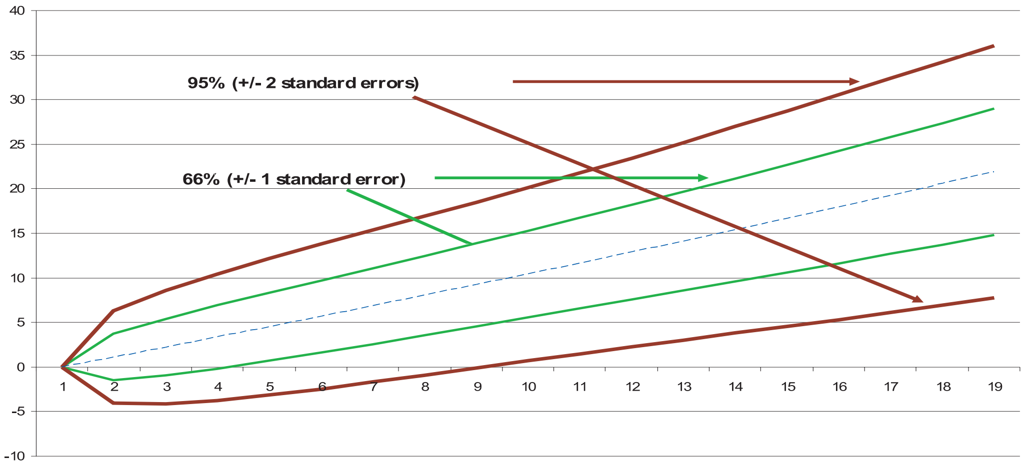


Figure 6
Confidence Bands for CPI Inflation: Rescaled Errors (percentage points)

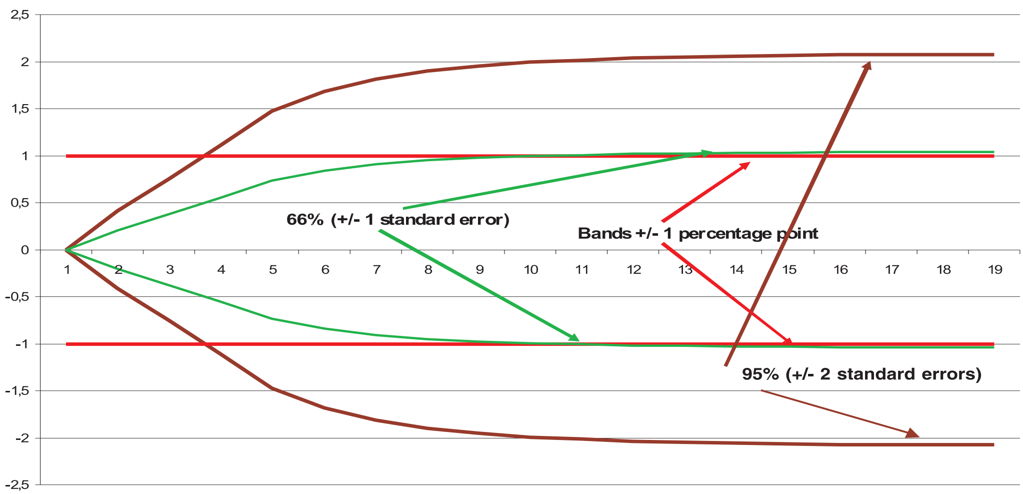
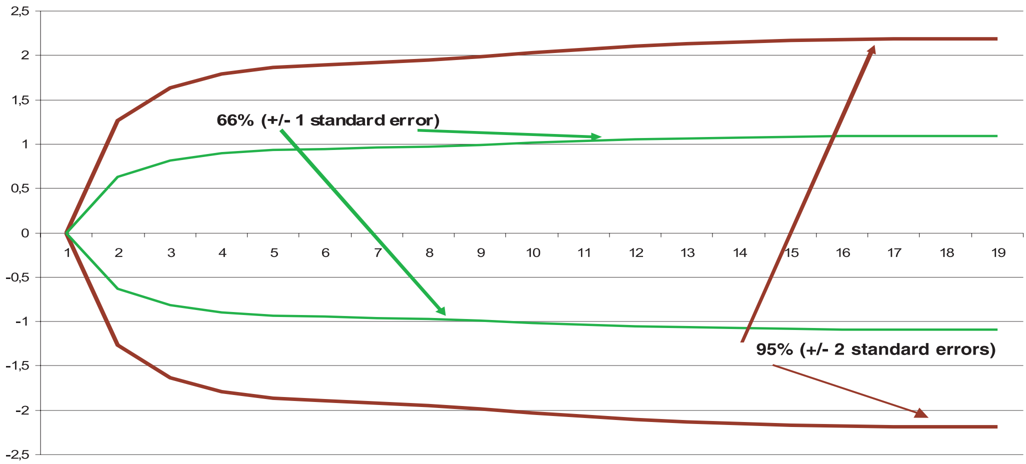


Figure 7
Confidence Bands for Output Gap: Rescaled Errors (percentage points)



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