Longer-term Yield Decomposition: An Analysis of the Czech Government Yield Curve

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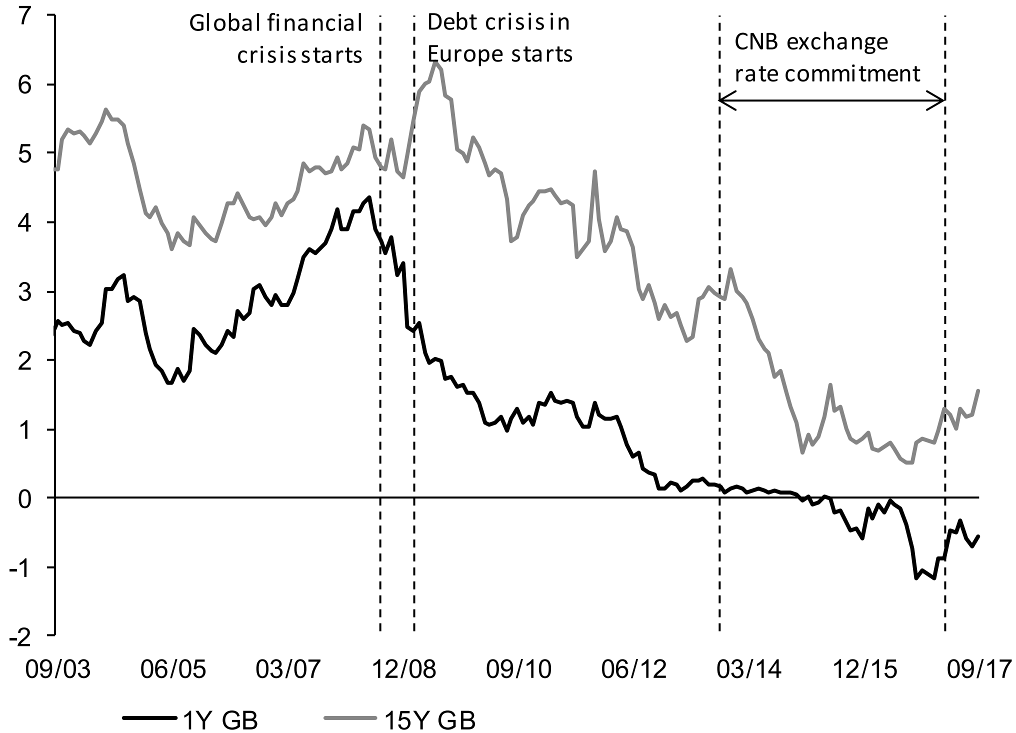
Overview

- Motivation
- Methodology
- Results – Decomposition
- Results – VAR analysis
- Conclusion

Motivation

- **? What are the causes of the yield curve movements ?**
- … important question for investors as well as policymakers (monetary policy expectations and transmission, financial stability)



Source: Bloomberg, PSE, MTS Czech Republic, Thomson Reuters, authors' calculations

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What determines the level of the short yields?

Source: Bloomberg, PSE, MTS Czech Republic, Thomson Reuters, authors' calculations

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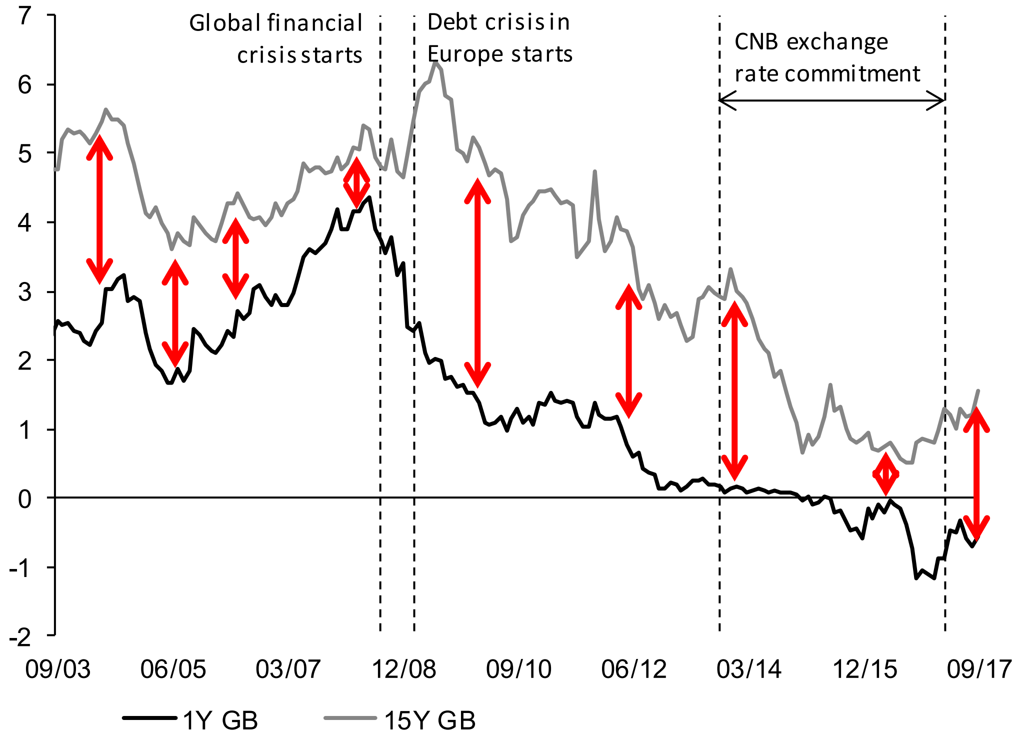
What determines the slope of the yield curve?

What determines the level of the short yields?



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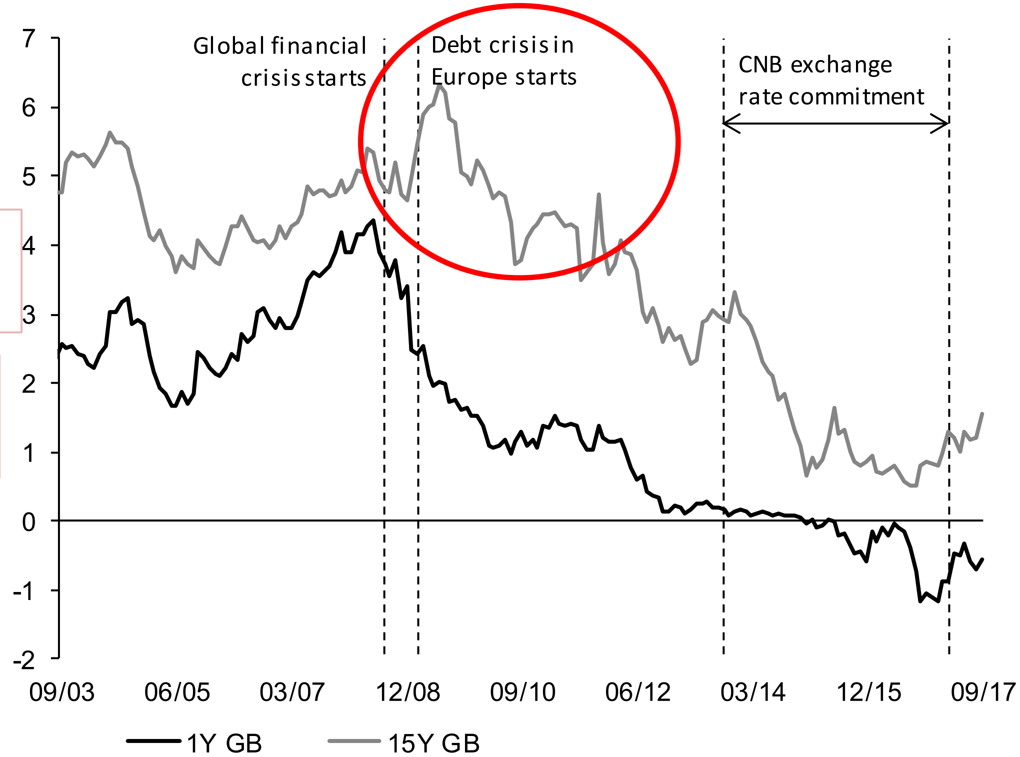
How did the debt crisis affect the yields?

What determines the slope of the yield curve?

What determines the level of the short yields?



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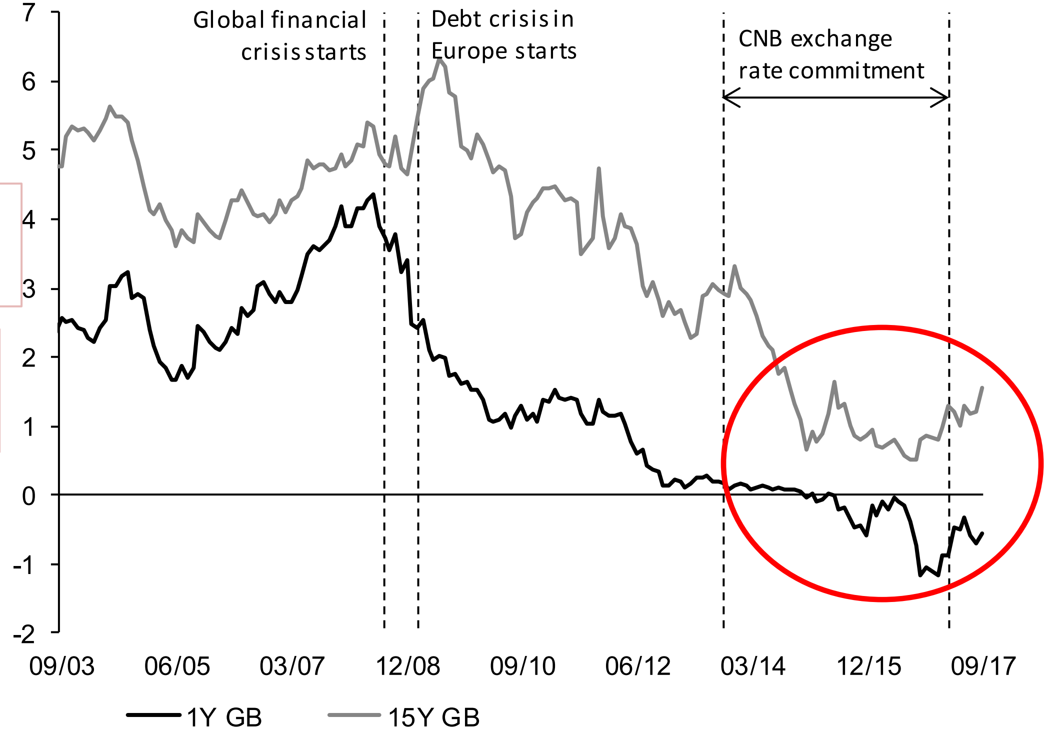
What happened during the exchange rate commitment?



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How did the debt crisis affect the yields?	Worries about public debt sustainability
What determines the slope of the yield curve?	Present level and long-term expectations about the monetary policy rate and the premium for uncertainty
What determines the level of the short yields?	Present level and short-term expectations about the monetary policy rate
What happened during the exchange rate commitment?	Government bond as a tool within the foreign speculative schemes

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How did the debt crisis affect the yields?	Worries about public debt sustainability	CREDIT RISK PREMIUM
What determines the slope of the yield curve?	Present level and long-term expectations about the monetary policy rate and the premium for uncertainty	TERM PREMIUM
What determines the level of the short yields?	Present level and short-term expectations about the monetary policy rate	RISK-NEUTRAL YIELD
What happened during the exchange rate commitment?	Government bond as a tool within the foreign speculative schemes	PORTFOLIO EFFECT

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FOUR EMPIRICALLY OBSERVED COMPONENTS

Motivation

- Findings in Literature (U.S. Yields):
  - Macroeconomic factors as important drivers of yields
    (Ang and Piazzesi, 2003; De Pooter et. al, 2010)
  - Inflation uncertainty determines high proportion of the term premium movements (Wright, 2011)
  - Geopolitical risks and fiscal policy are important factors, too
    (Bauer, 2015)

Methodology

- Common approach (U.S. Yields):
  - Term premium (wide) = Term premium (narrow) + Credit risk premium + Portfolio effect
  - U.S. Yields ➔ Portfolio effect stable, Credit risk premium low
  - i.e. no-arbitrage assumptions are satisfied
  - ➔ possible to use Affine model (Duffie and Kan, 1996) to decompose yields (Cochrane and Piazzesi, 2008; Crump et al., 2016)

GOVERNMENT BOND YIELD	=	RISK-NEUTRAL YIELD	+	TERM PREMIUM (wide)

Methodology

- No-arbitrage assumptions hardly satisfied for CZ
  - structural breaks (debt crisis: shift in investors' view on gov. bonds)
  - exchange rate floor: bias by speculative motives

- Our approach:
  - IRS rates as a proxy for yields of bonds satisfying no-arbitrage assumptions
  - Affine model applied on IRS allows to obtain two components

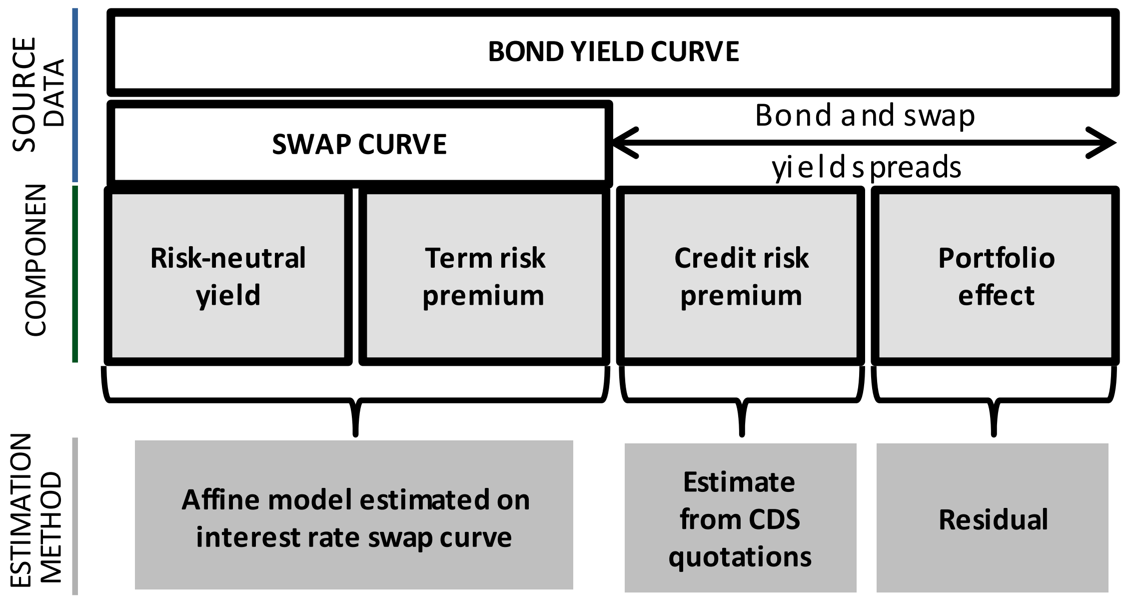
INTEREST RATE SWAP RATE = RISK-NEUTRAL YIELD + TERM PREMIUM (narrow)

  - The difference between gov. bond yields and IRS rates gives the other two

GOVERNMENT BOND YIELD - INTEREST RATE SWAP RATE = CREDIT RISK PREMIUM + PORTFOLIO EFFECT

Methodology

Components of the swap and bond yield curves



Source: Authors

Note: CDS = credit default swap.

Methodology: Affine model

Duffie & Kan (1996), notation from Krippner (2015)

Risk–neutral world:

Factor dynamics under Q measure: $\mathrm{d}X_t = \tilde{\kappa}\left(\tilde{\theta} - X_t\right)\mathrm{d}t + \sigma \mathrm{d}\widetilde{W}_t$

Short rate evolution: $r_t = \alpha_0 + \alpha_1 X_t$

- Q measure => risk-averse investors under P-measure are equivalent to risk-neutral investors under Q-measure (the same yield $R_t(\tau)$)
  => risk premia "hidden" in the probability measure
  => expectations hypothesis holds
  => long rates = average expected future short rates:

$$R_t(\tau) = \frac{1}{\tau}\int_0^\tau \left[E_t^{\mathbb{Q}}(r_{t+u}|X_t) - VE(u)\right]\mathrm{d}u$$

- Affine model: Closed-form solution $R_t(\tau) = a_0(\tau) + a_1(\tau)X_t$
- Useful for pricing bonds and interest rate (bond) derivatives, given the factors
- Q measure is not enough for understanding real-world dynamics

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Real world:

- Expectations hypothesis does not hold, investors are risk-averse
- Introduction of the market price of risk: $\lambda_t = \sigma^{-1}(\gamma + \Gamma X_t)$
- It is equivalent to returning from Q-dynamics to P-dynamics:

$$\mathrm{d}X_t = \kappa(\theta - X_t)\mathrm{d}t + \sigma \mathrm{d}W_t$$

where: $\tilde{\kappa} = \kappa + \Gamma$, $\tilde{\theta} = \tilde{\kappa}^{-1}(\kappa\theta - \gamma)$ and $\mathrm{d}\widetilde{W}_t = \mathrm{d}W_t + \lambda_t \mathrm{d}t$

- Then, yields are equal to the real-world expectations about the future short rate (i.e. the pure expectations withouth considering risk, as if investors were risk-neutral => risk-neutral yield) PLUS the term premium:

$$R_t(\tau) = \frac{1}{\tau}\int_0^{\tau}\left[E_t^{\mathbb{P}}(r_{t+u}|X_t) - VE(u)\right]\mathrm{d}u + TP$$

Final model:

- State-Space representation, states filtered with Kalman filter
- Discrete-time equivalent to the continuous-time process for some discrete time step Δt (1 month, since we use monthly observations)

$$R_t(\tau) = a_0(\tau) + a_1(\tau)X_t + e_t(\tau) \quad \text{... yield equation}$$

$$X_t = \theta + exp(-\kappa\Delta t)(X_{t-1} - \theta) + \upsilon_t \quad \text{... real-world state process}$$

Methodology: Affine model

Lower bound proximity:

- Yields cannot be significantly negative (cash as non-negative possibility)
  => yield dynamics close to lower bound not symmetric, not Gaussian
- solution = shadow yields (Black 1995)
  - Unobservable shadow yields evolve as if no lower bound exists

$$rs_t = \alpha_0 + \alpha_1 X_t$$

  - Difference between shadow and observed yields given by an option effect, Krippner (2013) shows a tractable approach to calculate the option effect
  - Consequence = non-linearity in the yield equation

Final model:

- State-Space representation, states filtered with Extended Iterative Kalman filter
- State process remains unchanged

$$R_t(\tau) = F(X_t, \tau) + e_t(\tau) \qquad \text{... yield equation}$$

$$X_t = \theta + exp(-\kappa \Delta t)(X_{t-1} - \theta) + v_t \qquad \text{... real-world state process}$$

Identifying restrictions:

$\tilde{\kappa} = \begin{bmatrix} \tilde{\kappa}_1 & 0 & 0 \\ 0 & \tilde{\kappa}_2 & 0 \\ 0 & 0 & \tilde{\kappa}_3 \end{bmatrix}$, $\tilde{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, σ lower triangular, zero covariance among $e_t(\tau)$ and v_t

Methodology: Components

- Affine model estimated for IRS rates
  - output = risk-neutral yields and term premia
- Spread between government bond yields and IRS rates need to be explained as well
- credit risk premium is related to bonds, but not to IRS (or very limited)
  - obtained from CDS quotations (fitted by Nelson-Siegel function)
- portfolio effect represents the effects on yields beyond the expected future short rates and the risk premia
  - portfolio allocation, flight to/from quality, government bonds as collateral, government bonds as a tool in speculative motives etc.
  - calculated as residuum

Data

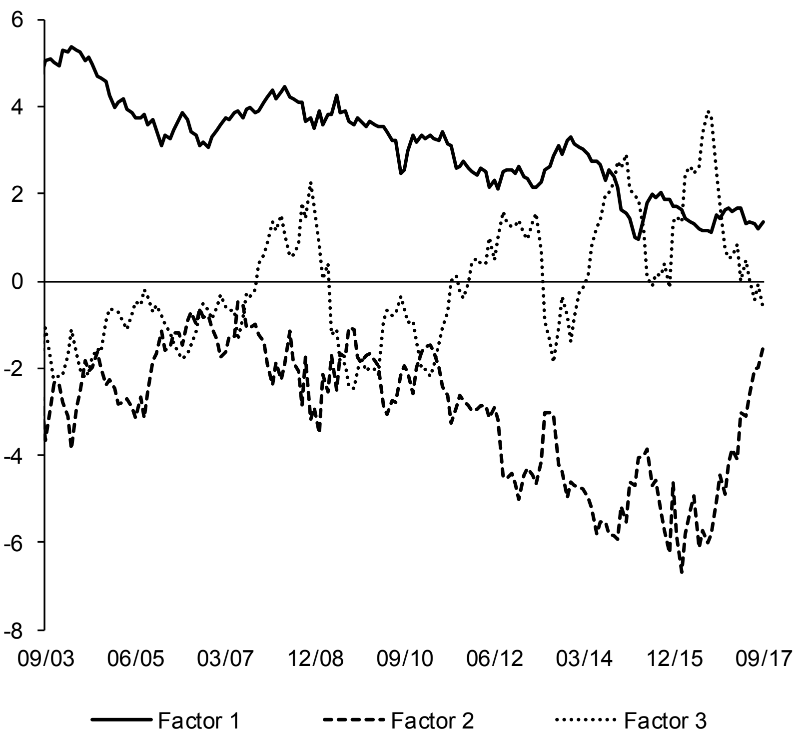
- Monthly observations, period 2003/03-2017/09
- Czech government bond yields and CZK IRS rates
  - maturity 1-15Y
  - zero-coupon curves obtained by Fama-Bliss (1987) bootstrap
- CDS quotations
  - maturity 1-5Y, 10Y, 20Y, 30Y
  - smoothed by 3M moving average

Results: Affine Model

- Estimated factors are not strictily Level, Slope and Curvature, however first factor is level-like
- Implied shadow yields reach -2 %
- Measurement error is small

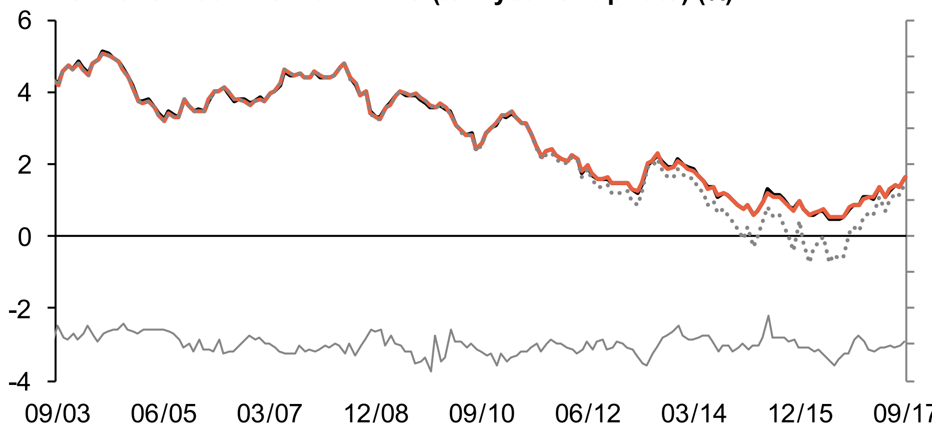
Affine model factors

(values of factors, multiplied by 100)

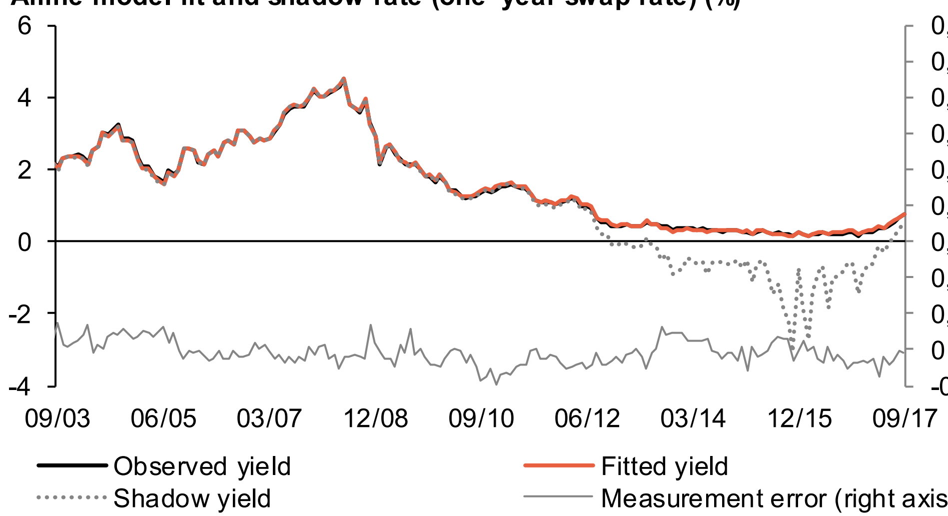


Source: authors' calculations

Affine model fit and shadow rate (ten–year swap rate) (%)



Affine model fit and shadow rate (one–year swap rate) (%)

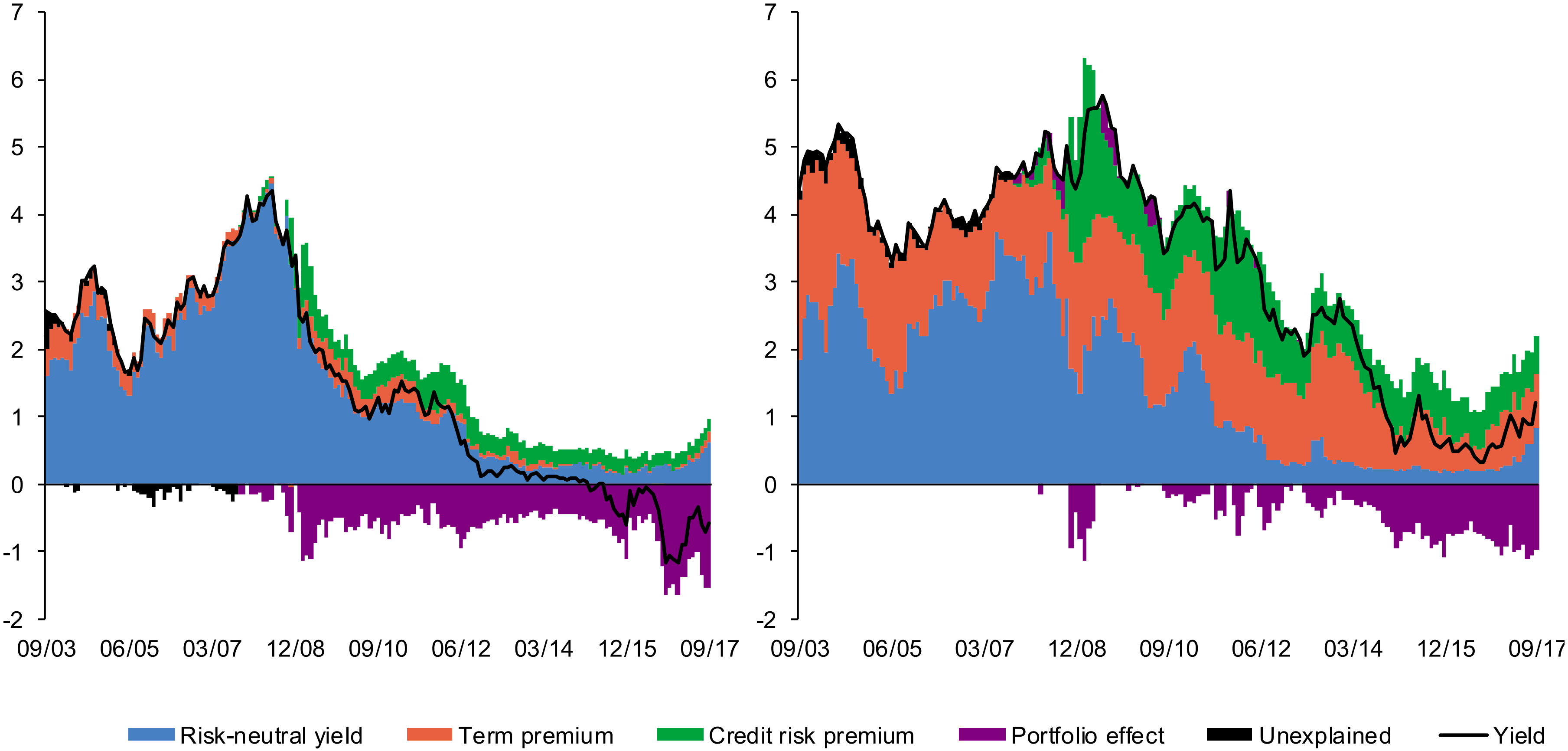


Source: Bloomberg, Prague Stock Exchange, MTS Czech Republic, Thomson Reuters, authors' calculations

Results: Components

Decomposition of the 1Y zero-coupon bond yield (%)

Decomposition of the 10Y zero-coupon bond yield (%)



Note: Reliable data on CDS quotations are not available until 2008. As a result, the difference between the bond yield and the swap rate could not be decomposed and is reported as Unexplained.

Source: Bloomberg, PSE, MTS Czech Republic, Thomson Reuters, authors' calculations

Results: Components

- Short yields governed by the risk-neutral yield
- Term premium matters for longer yields
- Credit risk premium significant since EA debt crisis outbreak
- Portfolio effect causes the most recent yield drop

Results: Components

- Correlations of monthly changes roughly confirm interpretation of the components (10Y yields):

Type of variable	Name of variable	Risk-neutral yield	Term premium	Credit risk premium	Portfolio effect
Macroeconomic	Inflation (CPI)	0.08	0.05	-0.21 (**)	0.15
	GDP growth	-0.04	0.06	-0.20 (**)	0.01
	CZK/EUR exchange rate	-0.07	0.05	0.21 (**)	-0.13
Short interest rates and market expectations	CZEONIA index	0.01	0.10	-0.06	-0.05
	CNB 2W repo rate (current)	0.18 (**)	-0.08	-0.28 (***)	0.22 (**)
	3M PRIBOR	0.40 (***)	-0.31 (***)	-0.13	0.14
	3M OIS in CZK	0.24 (***)	-0.22 (***)	-0.20 (**)	0.20 (**)
	CNB 2W repo rate (1-year expectations)	0.37 (***)	-0.17 (**)	-0.56 (***)	0.29 (***)
	Inflation (1-year expectations)	0.05	-0.04	-0.14	-0.02
Fluctuations in short interest rates and market uncertainty	Variability# of inflation	-0.20 (**)	0.13 (*)	0.17 (*)	-0.06
	Variability# of 1-year inflation expectations	0.06	-0.05	-0.07	0.19 (**)
	Variability# of 1-year expectations about CNB 2-week repo rate	-0.16 (**)	0.13	0.15	-0.14
	VIX volatility index	-0.07	-0.01	0.24 (***)	0.01
Credit risk of Czech state and Czech interbank market	Czech GBs issued/GDP	-0.23 (**)	-0.08	0.02	0.17
	5-year CDS spread for Czech GB	0.06	-0.16 (*)	0.54 (***)	-0.11
	Spread between 3-month PRIBOR and 3-month OIS	0.09	-0.02	0.15	-0.13
	Spread between Czech and German 5-year GB yields	0.21 (***)	-0.22 (***)	0.26 (***)	0.28 (***)
Investment flows	Czech GB trading volume	0.08	-0.04	-0.06	0.13
	Proportion of foreign holders of Czech GBs	0.09	0.06	0.05	0.08
	Profit on hedged investment in Czech GBs##	-0.14 (*)	0.21 (***)	-0.21 (**)	0.15
	Net portfolio and other investment in balance of payments	0.02	-0.15 (*)	0.07	-0.06

Colour scale for Pearson's correlation coefficient

-1 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 1

Source: Bloomberg, CNB, MTS Czech Republic, PSE, Thomson Datastream, authors' calculations

Methodology: VAR Analysis

- Aim: present empiricel usage of the decomposition in understanding macro-financial linkages
- VARX model

$$V_t = A_0 + A_1 V_{t-1} + A_2 W_t + \epsilon_t$$

V_t: 7 endogenous variables:

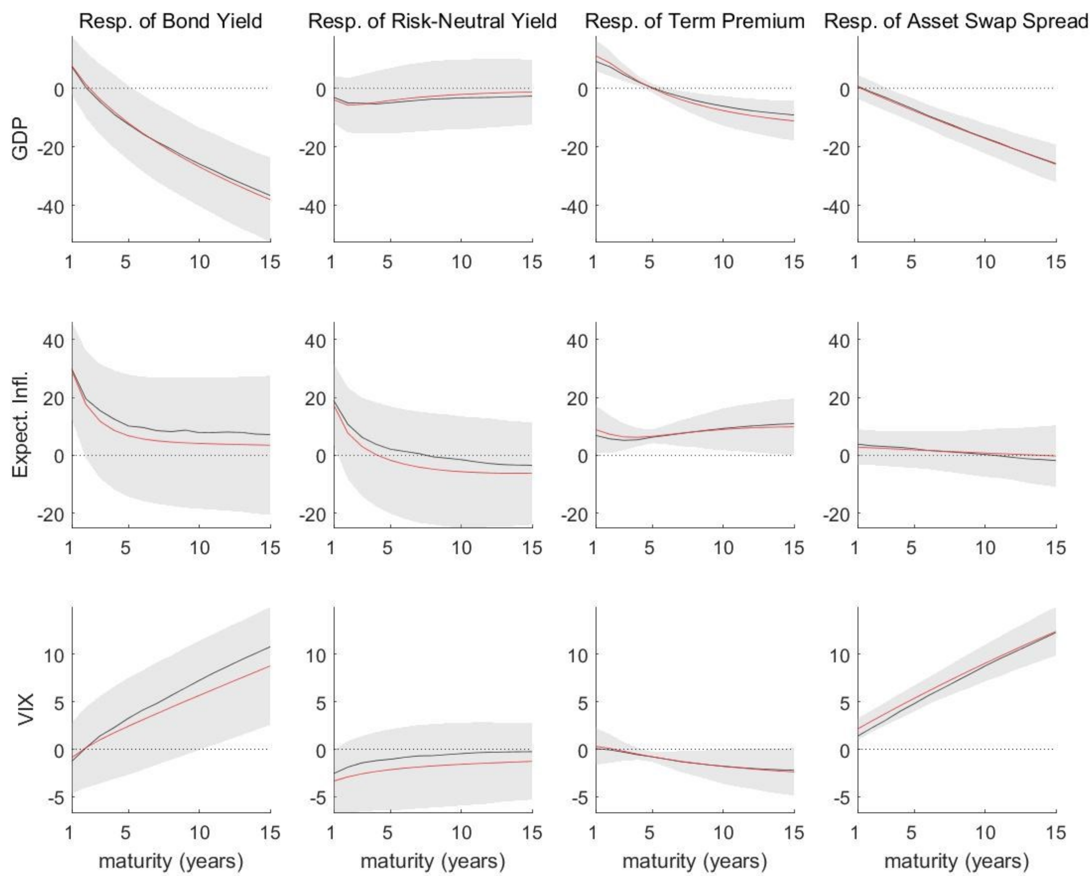
- 2 macroeconomic variables (real activity + expected inflation)
- 3 affine model state variables representing shadow short rate, risk-neutral yield and term premia
- level and slope of asset swap spread representing sum of credit risk premia and portfolio effect

W_t: general market uncertainty proxy (VIX index)

- Choleski decomposition, single lag (parsimony)
- Result: Impulse-Response Functions

Results: VAR Analysis
(Response at 6M horizon)

- Positive real activity shock rotates the curve (reducing slope)
- Expected inflation shock significantly influences the short yields only
- VIX shock raises long yields due to flight from the bond market



Note:
The figure displays the response of the whole Czech GB yield curve (1-15 years) to the shocks after 6 months.

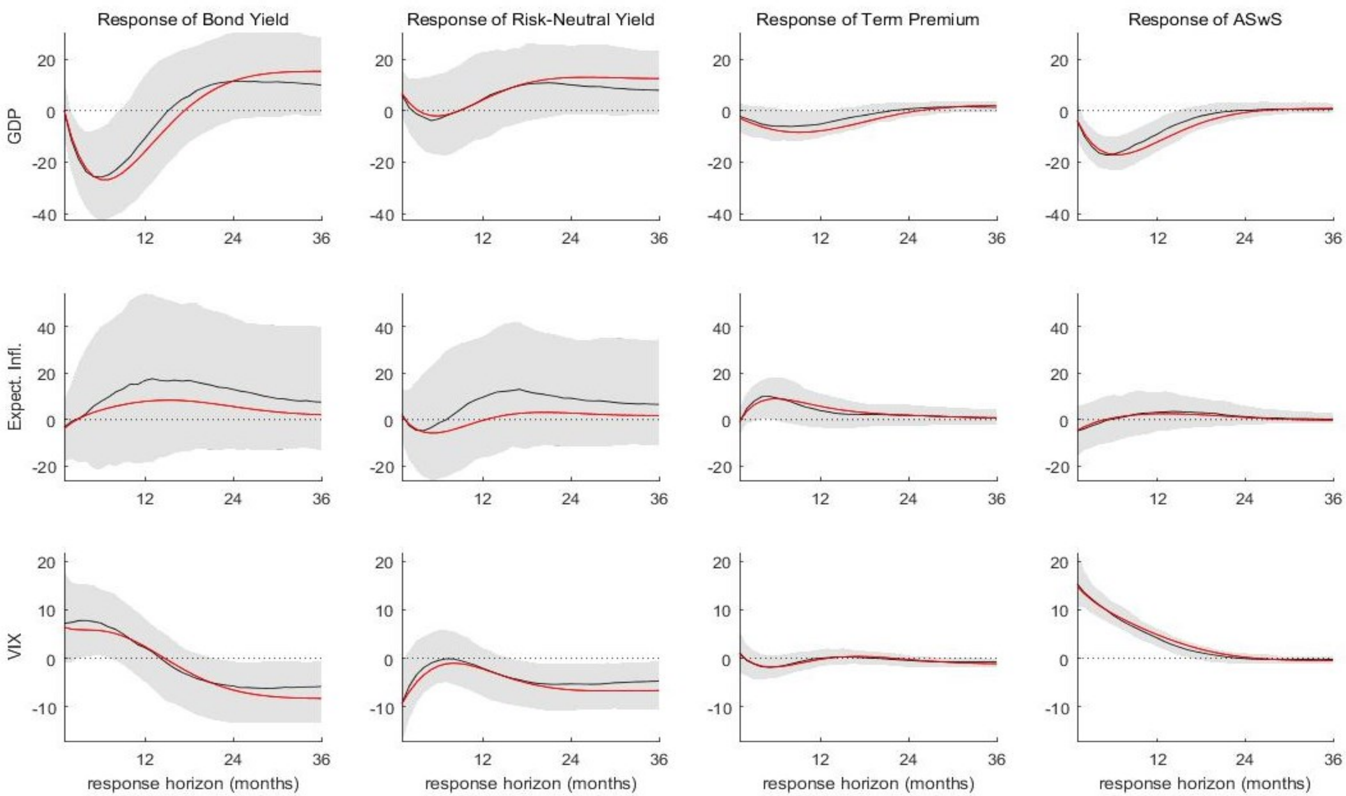
The shocks are defined as a 1pp increase of GDP and expected inflation, and as an increase of VIX index from 12 to 30 points.

The red line displays the estimated response.

The grey area and the grey line display 80% confidence intervals and a median response obtained by bootstrapping.

Results: VAR Analysis
(Response of 10Y bond yield)

- Positive real activity shock temporarily reduces term premia and triggers bond market infow, but increases expected short rates in the long-term
- Expected inflation shock causes slight growth of expected rates and TP
- VIX shock raises yields due to flight from the bond market and pushes expected short rates down



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Results: VAR Analysis

- We presently try to further extend the illustrative analysis
  - Introducing more lags
  - Separation of the ASwS
  - Macroeconomic variables replaced by expectations (Consensus Forecast)
- These elements make the model larger vs
  - vs. small sample
  - We are implementing Bayesing techniques to bridge this shortcoming

Conclusion

- The exctracted four components reflect the nature of factors influencing the yield curve
  - $\Rightarrow$ improved ability to interpret the yield curve movements
- Important for disentangle the recent drop of yields
- Allows to estimate possible future lift-off (=> stress testing)
- IRFs show a global position of Czech GBs as an investment asset rather than a safe haven
- IRFs show mixed behavior of the components to the positive economic shock (Greenspan‘s (2005) conundrum in practice)

Thank you for attention

Backup Slides

- Term premium calculation:
  - Wright (2011)
    - p 1517: „I decompose forward rates into average future expected three-month interest rates and the term premium by fitting an affine term structure model"
    - p 1520-21: „Having estimated the model parameters, the difference between the five- to ten-year forward rate under the Q measure and the average expected three-month interest rate from five to ten years hence under the P measure is the term premium"
  - Bauer et al. (2014)
    - p 3:„We follow Wright in decomposing five-to-ten year forward rates into an expectations component and a forward term premium. The **risk-neutral** forward rate is calculated as the average expected three-month interest rate from five to ten years' horizon."
  - Krippner (2015)
    - p 47: „... risk premiums in the term structure may be gauged by the difference between expectations for the future evolution of the short rate under the P measure, and expectations adjusted for risk under the Q measure. Risk premiums are a combination of the quantities of risk, provided by the standard deviation matrix (sigma), and the market prices for those risks, (Lambda(t)). "

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