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METHODOLOGICAL PROBLEMS OF QUANTITATIVE CREDIT RISK MODELING IN THE CZECH ECONOMY

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Abstract

This paper reviews the guidelines of "The New Basle Capital Accord" (NBCA) and four internal models of credit risk assessment. We treat them from the point of view of their underlying concepts, the institutional pre-conditions of their implementation and data requirements. We specially focus on the possibilities, difficulties and consequences of their application to the banking sector in the Czech Republic. The description of each model focuses on the underlying assumptions, characteristics and theoretical approaches and a brief discussion of their main advantages and limitations. Comparisons among models aim at identifying their common features and points of departure. Last but not least, we try to assess the potential impact that the use of these models could have on credit allocation in the Czech economy, and, consequently, on the resulting environment for the conduct of monetary policy. A number of preliminary recommendations for the models' implementation in this country are formulated.

Contents

1 Introduction

The development of commercial banking in Czechoslovakia and the Czech Republic in the course of market reform since 1990 has coincided with a wave of financial deregulation worldwide, as well as the globalization of banking business. Potentially, the emerging Czech banking sector could have benefited a great deal from this innovative process by tuning its own development to new international standards and expertise. In practice, however, it has taken the whole decade in the Czech Republic for this industry to overcome the infant diseases of mismanagement, connected lending practices and other typical emerging market shortcomings. At the end of this trail-and-error learning process, the Czech commercial banking market is now almost entirely occupied by entities under the control of parent companies from EU countries, as well as branches and subsidiaries of internationally active foreign banks.

This situation creates two types of challenges for the Czech National Bank. On the one hand, as the banking sector regulator, it will be henceforth confronted with risk management practices coming from banks with a long history, experience and an established record. So far, it has had only limited opportunities for acquiring reliable knowledge of specific consequences that the implementation of these practices might have for bank clients used to different treatment. Particularly, valuation of loans to

corporate and private borrowers in the Czech economy might lead to different credit allocation decisions, economic and regulatory capital requirements, compared to established industrial economies. On the other hand, the transmission of monetary policy decisions to the investment, production and consumption behavior of the private sector in the course of adapting to different financing mechanisms may create unexpected effects in the observed price setting and real economic activity. This is why the central bank needs to improve its understanding of how bank balance sheet risks, particularly credit risk, can be measured, managed and regulated in the operation of financial institutions that are its prime partners in the domestic financial market. Specifically, the monetary and regulatory authorities must have an up-to-date, quantifiable picture of both the trends in regulatory treatment of capital requirements for credit risk (developed and disseminated by the BIS), as well as internal procedures and models it is likely to observe in the credit risk management of domestically licensed banks themselves.

The present paper is a step in the direction of systemizing and partially quantifying the economic and regulatory capital procedures related to credit risk that are likely to be relevant for policy making by the Czech National Bank. Particular attention is paid to the methodological problems that can arise when one attempts to implement a specific rule or model in the Czech Republic. The nature of these problems can be both institutional (non-existence, shallowness, and intransparency of capital market segments or pricing distortions in these relevant segments) and technical (lack of necessary data due to inadequate accounting and reporting practices).

The Basle Committee recently released "The New Basle Capital Accord," a document that reassesses the challenges faced lately by banking supervision and regulation. Its inception was aimed at providing a more refined methodology for computing regulatory capital in large, internationally-oriented banks and at bringing the old guidelines of the 1988 Capital Accord in line with the latest developments undertaken by banks and financial markets all over the world. At least three points of criticism of the original 1988 Basle Capital Accord constitute the basis for this revision. First, the methodology for determining the regulatory capital has usually been perceived as too mechanical. Also, it seems that the risk buckets employed by it have made only an approximate distinction among risks faced by different lending sectors. Second, risks

inherent in the banking sector, such as market or concentration risks, have only played a partial role in the old guidelines. Third, banks' increased use of off-balance sheet transactions and regulatory capital arbitrage practices such as securization have rendered the old standards for assessing the capital adequacy of banks insufficient in certain cases.

The main advancement proposed by the New Basle Capital Accord is a shift from rules-based to process-oriented regulatory practices and methods (Karacadag and Taylor, 2000*).* In practice, this means a transfer of emphasis from the strict categories and formulas employed so far to a more refined and flexible approach. Banks will be allowed for example to use credit ratings developed by external agencies in order to assess the quality of their individual obligors. Thus, if under the old guidelines, claims on industrial and commercial enterprises were assigned a 100% risk weight, under the new approach, they could receive a smaller weight, provided they belonged to a risk free country and were assigned a high rating by an external agency. In the foreseeable future, banks will also be allowed to develop internal rating systems and, eventually, to use industry-sponsored models for determining the amount of regulatory capital they must hold.

One potential implication of the change in emphasis from rules-based to process-oriented regulatory practices is that the difference between regulatory and economic capital could diminish over time. Economic capital represents an estimation by the bank's managers of the capital that must be held in order to cover bank's unexpected credit losses. Regulatory capital is a capital measure considered as adequate by regulators to maintain the soundness and stability of the banking system as a whole. In principle, the regulatory capital exceeds the economic capital that banks would like to maintain, unless banks are conservative and risk averse. The margin that makes the difference accounts for the negative externalities that an individual bank failure would impose on the banking system. Potential convergence of regulatory capital to economic capital is thus based on the implicit assumption that the estimations of the internal credit risk models are reliable enough to also account for overall system stability.

During the last decade, effort was directed towards developing models that accurately capture the risk associated with banks' lending activities and make reliable predictions of the economic capital banks have to maintain. Roughly in the second part of the last decade, well-known financial institutions released their credit risk models to the public: JP Morgan's CreditMetrics/Credit Manager, CreditRisk+ of Crédit Suisse Financial Products, the KMV model of the KMV Corporation and, finally, McKinsey's CreditPortfolioView. Even if, in principle, all these models attempt to capture the risk associated with obligors' defaults and, eventually, migrations to a lower credit quality, assessed within a Value at Risk framework, their modeling approaches exhibit a high range of diversity. CreditMetrics and KMV follow a dynamic asset pricing approach and are based on a Merton-type model of the firm's value. CreditRisk+ draws intensively on the actuarial models used in insurance economics. Finally, CreditPortfolioView introduces an econometric approach that links macroeconomic variables to the credit quality of individual obligors.

In this paper, a review of both the internal and regulatory approaches is undertaken. The description of each model is given in the same structure. We name the underlying assumptions of the given model both from the standpoint of the bank and the environment in which it operates, discuss the underlying theoretical approach and give a summary of its main advantages and limitations. Also, whenever possible, comparisons of the models will explore their common and distinct features in order to assess their relative advantages and limitations. Last but not least, attention will be paid to the potential impact that the use of these models could have on credit allocation in the Czech Republic and to possible recommendations for their implementation in this country.

2 Literature review

The new Basle guidelines are easily accessible on the BIS web page in the form of two consultative documents, "The New Basle Capital Accord" and "The Internal Ratings-Based Approach." They were released in January 2001 and were open to comments and feedback from the banking industry roughly until May 2001. Also available is the Basle Committee's assessment of the credit risk models in the form of a document entitled "Credit Risk Modelling: Current Practices and Applications." All these documents express the regulatory opinions with regard to the current state in credit risk measurement together with the perceived challenges left for accomplishment in the future.

Credit risk and the models developed to assess its magnitude have become topics of intense interest since the release of the internal credit risk models. This fact has been reflected in the increasing number of articles published by economic journals centered on this topic. Nevertheless, the most detailed description of each model is available in the technical document that accompanies its release. Two such documents, CreditMetrics and CreditRisk+, are freely available to the public while the other two are accessible only as a part of the whole package sold to interested customers.

Departing from the strict technical dissection of the models, the academic sphere has extended the research in several directions. One of the most natural questions to pose was whether significant differences exist among the models' theoretical frameworks and also among their estimations of economic capital. One evident dividing line on the conceptual level is pointed out, e.g. in Frey and McNeil, 2001. They distinguish between *latent variable models*, represented by CreditMetrics and KMV, and *mixture models*, including CreditRisk+ and CreditPortfolioView. In the first group, an obligor's default is triggered by an unobserved variable crossing a given threshold, and dependencies among defaults are caused by the existence of common risk factors governing the latent variables. In the second group, defaults of individual obligors are conditionally independent given the values of a set of economic factors. Despite this formal difference, commonalities between representatives of both groups were identified as well. Gordy, 2000, focused on the "comparative anatomy" of CreditMetrics and CreditRisk+ just to assess that, beside differences in functional forms and distributional assumptions, these two models were built on a common underlying structure. Their simulations revealed that the models performed similarly when applied to loan portfolios of average quality and that both models were particularly sensitive to default correlations and the distributions of the systematic risk factors. Koylouglu and Hickman, 1998, compared CreditMetrics, CreditRisk+ and CreditPortfolioView and also concluded that these models were built on common grounds. In the view of these two authors, the main factor that led to result differentiation was the parameterization of the joint-default behavior among different assets. Other studies, i.e. Crouhy, Galai and Mark, 2000, or Phelan and Alexander, 1999, undertook a detailed examination of the models without trying to assess the mapping procedure from one model to another. All these papers offer useful insights into the concepts and methodologies employed by different models.

This paper proceeds with a survey of the four already mentioned models used by major banks in credit risk measurement from the common implementation methodology viewpoint. Emphasis is put on the particular conditions that may either encourage or thwart the implementation of these models in the Czech Republic. The remaining text is organized as follows. Section 3 gives a brief review of the new guidelines of the Basle Committee and confronts it with the VaR approach of industry-internal models of credit risk measurement. Section 4 covers, consecutively,

the CreditMetrics, KMV, CreditRisk+ and CreditPortfolioView models individually. Section 5 offers a schematic comparison of the four models. Section 6 discusses the potential application impact of these models on credit allocation in the Czech economy. Section 7 concludes. Examples of credit loss calculations according to individual models are collected in the Appendix.

3 The regulatory and internal model-based approaches to credit risk

3.1 The BIS approach (regulatory capital)

Departing from the 1988 Basle Accord, the guidelines of the New Basle Capital Accord explicitly distinguish between market, operational and credit risks. Regulatory capital for market risks covers unexpected changes in the portfolio's value due to underlying changes in the market conditions (changes in interest rates, exchange rates, etc.). Operational risk originates in unexpected adverse effects due to flaws in the internal operation of the bank. Credit risk is associated with the likelihood of individual obligors not paying back in due time the interest or principal on their contractual obligations.

To comply with the regulatory rules, each bank has to maintain the ratio of regulatory capital to a weighted-sum of capital requirements for market and operational risks and the risk weighted assets for credit risks above the benchmark value of 8%. The weight applied to the capital requirements for market and operational risks is 12.5%.

The New Basle Capital Accord proposes two main methodologies for determining regulatory capital in the case of credit risk: a standardized approach and an approach based on banks' internal rating systems.

The *standardized approach* assigns risk weights to individual claims based on standard credit rating systems proposed by external agencies. The main asset categories recognized by this approach are: sovereigns and their central banks, non-central government public sector entities, multilateral development banks, other banks, securities firms, corporates and off-balance sheet assets.

Under the *Internal Ratings Based approach* (IRB) each asset category is characterized by specific risk inputs, rules for determining the risk weights and minimum requirements for eligibility. Banks are allowed to develop their own credit rating methods subject to supervisory approval and control. The main elements that enter the calculation of the risk weights are:

A. Probability of Default (PD) of the obligor within the risk horizon
B. Exposure at Default (EAD) – the total amount the bank is exposed to if the borrower defaults
C. Loss Given Default (LGD) – the percentage of the exposure lost if a default event occurs (also equal to one minus recovery rate).
D. Maturity of exposures
E. Granularity – the concentration of a bank's exposure to a single borrower or to a group of closely related borrowers.

Furthermore, to obtain estimates for these inputs the IRB approach differentiates between:

- **The foundation approach** in which banks can use internal estimates only for the PD of an exposure while all other elements are estimated based on supervisory rules

- **The advanced approach** in which banks are allowed to use internal estimates for all the inputs that enter the risk weight formulae. In order to become eligible for employing it, banks must satisfy a set of stricter rules compared to the foundation approach.

Under the internal ratings methodology, banks are obliged to classify all their exposures into one of the following classes: corporate, retail, sovereign, inter-bank, project finance and equity. The next section exemplifies the way in which the risk weights and the inputs to these functions are derived in the case of corporate exposures.

3.2 Risk weights for corporate exposures

The derivation of the risk weights uses PD, LGD and M as inputs, and it is based on the formula:

$$RW_C = \min\left\{ \frac{\textbf{LGD}}{\textbf{50}} \times \textbf{BRW}_\textbf{C}(\textbf{PD}) \times [\textbf{1} + \textbf{b}(\textbf{PD}) \times (\textbf{M} - \textbf{3})] \ , \ \textbf{12.5} \times \textbf{LGD} \right\}$$

Here, $b(PD)$ is a maturity adjustment and BRW_C is a corporate benchmark risk weight associated with the exposure's PD. The dependence of BRW_C on PD is shown in Fig. 1.

Figure 1

The benchmark risk dependence on the probability of default
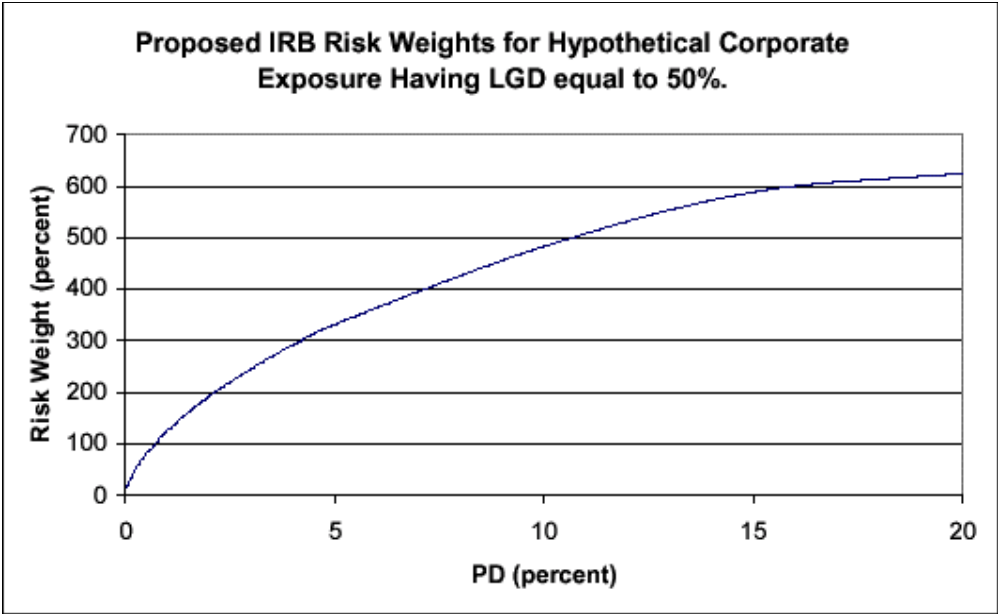
3.3 Inputs to the risk-weighting functions

Guidelines are provided that support the derivation of the PD, LGD and EAD both in the foundation and the advanced case. In the foundation approach, the PD equals the maximum of two values: 0.03% and the PD associated with the internal grade of the exposure. If credit risk mitigation is available in the form of guarantees or credit derivatives, the PD is adjusted to take into account this effect. EAD is defined as the nominal value of the exposure for the on-balance sheet items and as the committed, but not yet drawn, line multiplied by a conversion factor for the off-balance sheet items. LGD is determined based on the seniority of the claim and the availability of collateral.

The advanced IRB methodology allows banks to use internal-based estimates for the PD, LGD and EAD provided that they meet the specific minimum requirements characteristic for each of these inputs and for corporate exposures in general.

The methodology used for computing the risk weights and inputs for other types of bank exposures is similar to the one presented here for corporate exposure. The only difference arises in the case of retail exposure where the IRB does not provide a foundation approach for the derivation of the risk inputs.

Granularity adjustment is an additional adjustment of the risk weights in order to account for concentration risk. For a bank with a finer than typical granularity, it is assumed that it has removed a part of the risk through diversification. Thus, the risk weights are scaled downwards. On the contrary, a bank exposed to only a few obligors or to a set of closely related obligors is assumed to face a higher idiosyncratic risk and thus it is obliged to maintain more regulatory capital.

3.4 The Value at Risk framework for credit risk (economic capital)

The value of a bank's assets susceptible to adverse future changes can be viewed as a random variable. The standard risk management convention associates the part

of the bank's stock of liquidity assigned to cover *expected (mean) losses* with reserves, whereas a major portion of possible credit losses in excess of the mean is supposed to be protected by *economic capital*. If V_0 is the current value of the risky portfolio and V_T is its value at the end of the chosen risk horizon T, then the loss is given by $L=V_0-V_T$. Economic capital is the value determined by the probability properties of L (equivalently, V_T).

The estimation of economic capital follows a Value-at-Risk (VaR) paradigm analogous to the one employed by banks to measure market risk: economic capital is determined in such a way as to make the probability of unexpected credit losses exceeding the economic capital less than a target value (for example 1 % or 5 %). In general, the risk of a bank's operations could be quantified either by the standard deviation of the loss variable or by the expected loss exceeding the chosen loss quintile. The loss distribution derived in the case of market risk closely resembles the normal distribution. For instance, standard deviation is a good measure of the risk associated with banks' proprietary trade desk operations. However, in the case of credit risk, the correlations among obligors make the loss distribution associated with credit operations look asymmetric and with fat tails, which imposes a departure from the normal distribution. Due to this fact, the standard deviation of the loss (value) variable is an unreliable measure of credit risk, and this leaves the loss quintile calculation as the only available alternative.

For the given target probability level p, the loss (value) p-quintile is the value L^p ($v^p=V_0-L^p$) such that

$$\Pr\left\{L \geq L^p\right\} = \Pr\left\{V_T \leq V_0 - L^p\right\} = \Pr\left\{V_T \leq v^p\right\} = p.$$

It represents the value above or below which the portfolio's value or, respectively, loss would fall with probability p. In other words, it is the confidence level selected by the bank's credit risk analysts. Depending upon which distribution is estimated by the model (the portfolio loss or portfolio value, at the end of risk horizon), *economic capital* is then computed as the difference between the distribution's expected value and a p-quintile: $EC=L^p-E(L)$ or $EC=E(V_T)-v^p$. (symbol $E(.)$ stands for the expected value).

Fig. 2 (portfolio loss estimation) and Fig. 3 (portfolio value estimation) illustrate the procedure. In Fig. 2, the area under the probability density function (pdf) of L and to the right of L^p represents the probability that extreme losses will occur in the future. More precisely, their value will be higher than L^p. The economic capital needed to cover extremely high losses equals the difference between L^p and $E(L)$. Similarly, Fig. 3 highlights the case when the p-quintile is taken for the portfolio value distribution itself. In this case, the area under the pdf of V_T and to the left of v^p equals the probability that the portfolio value reaches a low level, namely less than or equal to v^p. As before, the economic capital estimated to cover losses within a 1-p% confidence interval equals the difference between the expected value $E(V_T)$ and v^p. Different credit risk models presented in this paper prefer to work with either the portfolio loss or the portfolio value distributions.

Figure 2

   Economic capital in models that estimate the portfolio loss distribution
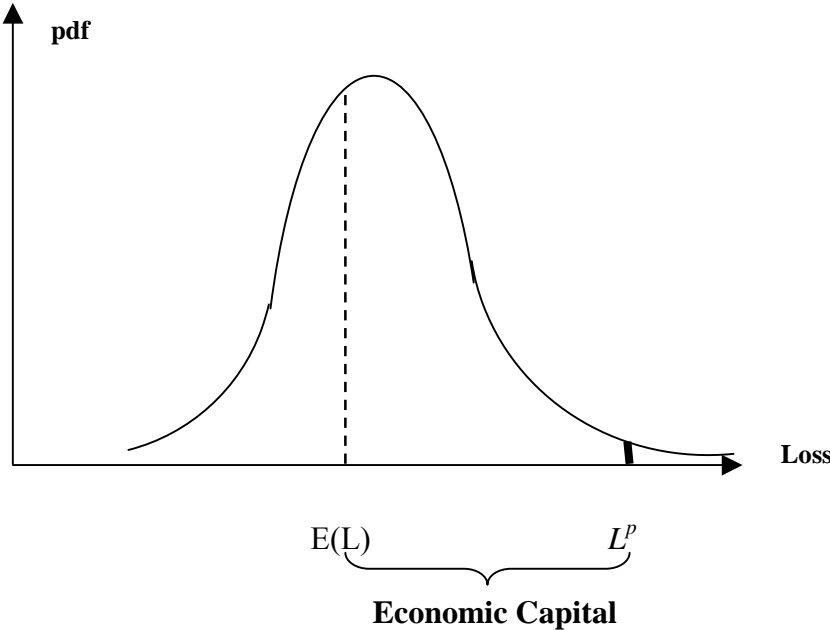
Figure 3

Economic capital in models that estimate the portfolio value distribution
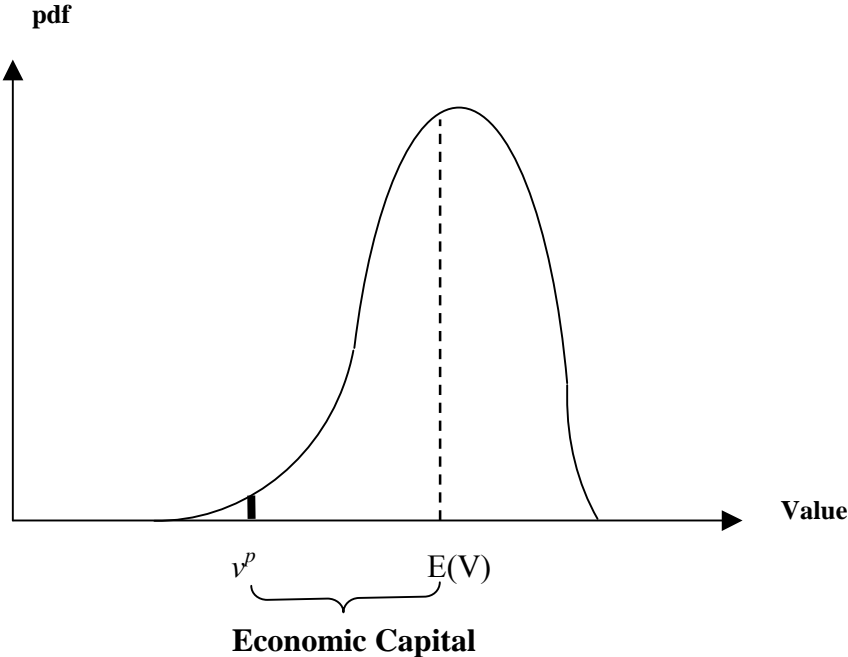
4 Internal credit risk models

Derivation of the economic capital level in a formal model presumes the existence of a natural logic in the behavior of banks with respect to expected credit losses. These losses are a part of the overall risk found on the asset side of a bank's balance sheet (others are market, liquidity and operational risks). Therefore, the economic analysis of capital requirements is faced with the question of whether one can find a choice-theoretic reason for a financial intermediary to maintain a positive cushion against credit losses voluntarily. Only when the answer is positive can internal credit risk models be a sustainable alternative to risk management procedures imposed by the regulator. Otherwise, there would be no rationale for economic capital apart from supervisory pressures. On the other hand, decentralized economic capital allocation can be socially suboptimal when the lender population (the banking sector) is severely heterogeneous, since asymmetrically exposed lenders may insufficiently protect themselves against important systemic risks. The role of regulator should be, therefore, more pronounced when the said asymmetries in the financial sector as a whole are large.

A line of corporate finance research has emerged that tries to understand the conditions under which economic capital is being generated endogenously. The basic paradigm is concentrated around a financial institution that can invest in four

types of assets. These are: x^a – regularly traded real and financial assets, x^b – non-traded assets of a corporate loan type, x^h – derivative instruments that hedge against risks contained in x^a and x^b, and x^0 – a cash buffer against losses that can occur in x^a and x^b. One can assume that x^0 has its origin in the economic capital provided by the shareholders, as introduced in the preceding section.

We shall need the notions of financial wealth (excluding economic capital) depending on the risks included. Specifically, let $w^0 = x^a + x^b$ be the bank's wealth if no hedging instruments are used, $w^h = x^a + x^b + x^h$ its wealth if hedging is employed and, finally, let w^a be a hypothetical wealth attained on condition that credit risk does not exist, i.e. the return on x^b is deterministic.

First, let us assume that all risks resulting from lending activities, i.e. investment in x^b ("credit risk"), can be fully diversified by means of traded assets x^a and hedging instruments x^h. This can be true in at least two cases. One is when all borrowing companies are public, with traded equities being a part of x^a and fully reflecting the uncertainty in future cash flows. The other occurs when the borrowing companies' output, productive assets and other collateral can be costlessly appropriated in the event of default, whereas their risks can be fully laid off in the derivatives market.

Can economic capital x^0 be redundant under such circumstances? We shall make this question more precise by introducing the following notations. Let α be the preferred risk tolerance level (e.g. 1% or 5%) and T the studied risk horizon. For $z=a, h, 0$ define

$$k^z = \min\left[k \left| \Pr\{w_0^z - w_T^z \geq k\} \leq \alpha\right.\right].$$

In other words, k^a, k^h and k^0 are value (or wealth) at risk levels for the wealth definitions w^a, w^h and w^0, respectively, as given above. For instance, k^h is the value at risk when hedging instruments are included in the investment portfolio.

Under normal circumstances, the inequalities

$$0 < k^a < k^h < k^0$$

must be valid, meaning that hedging reduces the level of capital at risk originating from credit exposures, pushing it down in the direction of the minimal rationally attained level k^a (optimizing investors would not go lower than the latter because it

would be suboptimal to lay off all traded risks). Redundancy of economic capital corresponds to the equality $k^a = k^h$: there is no need to maintain non-zero x^0 when derivative contracts offer a 100% replacement of this buffer against x^b-losses. However, can this situation be considered in any sense typical?

In a simple three-period model of bank investment, Froot and Stein, 1998, demonstrate that the zero economic capital outcome is possible in equilibrium, provided that hedging is costless. Namely, they stress the difference between an individual non-bank investor, whose access costs to the derivative markets are high, and a bank. The latter can enter derivative contracts nearly costlessly. For a bank-type investor understood in this way, a trade-off between x^b-risks and x^b-returns is not an issue as long as the former can be laid off in the derivative market without the need to sacrifice any of the latter. Accordingly, in the Froot and Stein model, a perfect hedging opportunity results in zero economic capital being held by the optimizing bank.

A different situation arises when some risks in x^b are non-traded. This is characteristic of a loan granted to a producer company or a debt-financed new project. The non-traded risks are associated with unobserved entrepreneur effort, latent project value and other factors that give rise to asymmetric information phenomena. In the notations introduced above, non-diversifiable components of x^b mean

1. A relative reduction of the distance between k^h and k^0, i.e. a reduction of benefits from hedging

2. A positive lower band for the distance between k^a and k^h, i.e. regardless of the choice of the hedging portfolio, the capital at risk cannot be reduced to the "first-best" level k^a.

The latter outcome also means that the optimally set economic capital (reserve level) x^0 will be positive. Beside that, Froot and Stein demonstrate that non-diversifiable lending risks are responsible for the equilibrium level of RAROC (risk-adjusted return on capital) – the required rate of return on non-traded credit.

In banking practice, as well as in more realistic financial intermediation models than the benchmark model of Froot and Stein, asset market frictions of either informational, inventory or participation cost nature are ubiquitous. Therefore, the

extreme result of Froot and Stein is unlikely to hold precisely. Nevertheless, one still suspects that the individually optimal level of economic capital might be an increasing function of the distance between k^a and k^h, i.e. the optimal required buffer x^0 might be reduced considerably when market hedging opportunities improve.

The above discussion highlights the connection between capital market completeness and the endogenous need for economic capital. To summarize, economic capital is being optimally maintained at a positive level due to

(a) The non-diversifiable/idiosyncratic nature of the uncertainties accompanying the debt service by obligors

(b) Frictions in the markets for risk sharing.

From the regulatory perspective, a policy on economic capital, if it is to pass the social welfare optimality test, ought to be consistent with the intrinsic capital value preferred by the financial intermediaries themselves.

From the bank credit risk management perspective, market incompleteness poses a serious problem as regards applicability of credit risk models based on dynamic asset pricing techniques, such as KMV. Namely, it is unclear in what sense shall one understand debt valuation statements (e.g. about the distance to default, expected default frequency, etc.) when they rely on the notion of risk-neutral probability. The latter is well defined as a unique equivalent martingale measure only when the markets are complete. More will be said on this issue in Subsection 4.2. For now, we shall agree that the background factors behind borrowers' defaults (downward shocks to the value of x^b) are naturally split into systemic, X, reflecting the general conditions in the economy or capital market, and *idiosyncratic*, ε, referring to the situation of the given firm. Statistics of the latter usually cannot be spanned by traded financial instruments, i.e. are non-diversifiable.

The financial industry-sponsored models or credit risk are used to estimate the economic capital needed to cover the risk associated with a bank's credit activities. Each of them makes a specific assumption about the statistics of security return processes. There are two basic paradigms used in defining credit losses and thus in quantifying credit risk: the default-mode paradigm and the mark-to-market paradigm. In the first case, the obligor can be in only two states at the end of the risk horizon,

namely default or non-default. Thus, a credit loss occurs only if the obligor defaults within the contractual period. The second paradigm also assigns risk to migrations to lower credit ratings. Default is defined as the event when the firm does not honor one of its intermediate obligations (coupons, interest rates) and/or the principal at the maturity date of these obligations.

Next, we discuss two models representing the dynamic asset pricing approach to credit risk, namely, the CreditMetrics model developed by J.P. Morgan (Subsection 4.1) and the KMV[1] model (Subsection 4.2)[2]. The actuarial method represented by the CreditRisk+ model (Crédit Suisse Financial Products) is covered in Subsection 4.3. Finally, the econometric method is illustrated in Subsection 4.4 with the CreditPortfolioView model of McKinsey.

4.1 CreditMetrics

CreditMetrics belongs to the "mark-to-market" category of models by associating risk with obligors' potential migrations from current grades to lower ones (downgrades) as well as to default. It consists of an analytical part and a Monte Carlo simulation part. The analytical part includes calculations of stand-alone risk, marginal risk contributions, the portfolio's expected value and standard deviation. Monte Carlo simulation (see 4.1.4 below) derives the distribution of the portfolio value at the end of the risk horizon. Consequently, this distribution can be used to assess the need for economic capital within the Value at Risk framework. Both parts are illustrated in this paper. The risk exposures considered by CreditMetrics are trade credits, bonds and loans, loan commitments, financial letters of credit and market-driven instruments such as swaps and forwards with default-admitting, i.e. mostly corporate counterparts. As inputs, CreditMetrics requires a rating system, characteristics of individual assets, equity returns for each obligor and a set of country and industry equity indices.

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[1] KMV is a trademark of KMV Corporation founded in 1989 by Stephen Kealhofer, John McQuown and Oldrich Vasicek.
[2] Our exposition of the dynamic asset pricing method draws extensively on the analysis given in Crouhy, Galai and Mark, 2000, where some other models of this class are also mentioned.

4.1.1 Definitions and assumptions

A number of assumptions are employed relating to the bank view of its obligors. First, it is assumed that all obligors can be assigned to rating classes. This assignment identically affects the pricing of all assets with a given grade as long as class-specific discount factors are used in their valuation. Another strong assumption made is that all obligors in a given rating class have the same migration and default probabilities. CreditMetrics explains changes in assets' returns by changes in systematic risk factors, which are represented by equity indices specific to different countries and industries, and to idiosyncratic (obligor specific) risk factors. To compute asset return correlations, CreditMetrics assumes that the obligors' equity returns approximate their asset returns well, and thus the former are used as a proxy for the latter. Finally, the time horizon over which the risk is assessed is usually taken to be one year, even if other interval lengths could be used.

From the external (environment) standpoint, CreditMetrics assumes that all interest rates and forward curves are deterministic. Therefore, the model is not sensitive to spread fluctuations arising from changes in the capital markets. The general economic conditions are not directly captured in the model. The move in asset returns is linked to systematic factors, but these are industry indices that might only partially reflect the economic trends.

In accordance with the named principles, the methodology proposed by CreditMetrics strongly relies on the chosen rating system. Therefore, we start with a brief description of possible ratings. In principle, any of the widely accepted rating systems (Standard & Poors (S&P), Moody's, etc.) as well as banks' internal systems could be used. The rating system assigns credit ratings to each obligor and contains probabilities of migration from one rating class to another (including default) over a chosen time horizon. The migration probabilities represent average annual frequencies of migration among credit classes. They are made available by some rating agencies in a matrix format such that each row of the matrix contains the migration probabilities from a given grade (excluding default) to any other credit class (including default). To obtain these transition matrices, rating agencies normally use

data sets containing long time series (15–20 years) and a large number of firms (4–5 thousand). For example, the 90.81% value in the uppermost left cell of Table A3 in the appendix shows that the average frequency with which an initially AAA-rated asset remained AAA-rated in one-year's time is 90.81%.

4.1.2 An individual asset value distribution at the end of the risk horizon and the default probability of an obligor

In accordance with the usual understanding in finance, asset riskiness is identified with the degree of dispersion of its future value around the expected value. According to this logic, the standard deviation of the present value of the future cash flows becomes a good measure of the risk embodied in that asset. CreditMetrics is based on this principle. Therefore, to assess the risk embodied in individual assets, CreditMetrics derives the distributions of their present value at the end of the risk horizon. The value (price) of an asset is derived from the present value of its future cash flows (cf. the introduction to this section). his distribution contains the values of a particular asset assuming potential migrations to all rating classes and the probabilities associated with these migrations. For example, assume that a bond with the principal/face value F and the annual coupon C matures in T years. Let its rating in one year's time be equal to $g \in G$, where G is the set of possible ratings. Since one needs to discount the cash flows of the bond between years 1 and T with respect to time $t=1$ and not the present time moment $t=0$, it is necessary to do this with the help of the properly selected forward rates (the ones that apply to loans and/or deposits *between times $t=1$ and $t=2,…,T$*). In addition, since implied forward rates rarely exist for maturities longer than one year (i.e. for $t>2$), the appropriate *zero coupon forward rates* (or simply *zero forward rates*) must be extracted from prices of existing market instruments. Moreover, CreditMetrics recognizes that the forward zero rates applied to instruments with different ratings must differ, too (we will return to this point in the context of the KMV model in Subsection 4.2).

Let the annualized zero forward rates for the period between year 1 and year t, applicable to class *g*-obligors, be denoted by $f_t^{\,g}$. Then the value of this bond at the end of the first year is

$$V_1^g = C + \sum_{t=2}^{T-1} \frac{C}{\left(1+f_t^g\right)^{t-1}} + \frac{C+F}{\left(1+f_T^g\right)^{T-1}} \, . \tag{1}$$

In the case of default, the valuation represents just the product between the face value of the bond and a recovery rate. These recovery rates represent a percentage of the bank exposures recovered if some obligors default. They depend on the seniority of the bonds, and their estimation is based on historical data. The valuation of assets other than bonds (especially loans) is similar to the one presented in the bond case.

The expected value at time zero of the asset at the end of the risk horizon can then be computed using the obvious formula

$$E[V_1] = \sum_g V_1^g \pi(g),$$

where V_1^g is the asset value if a migration to the gth grade had taken place and $\pi(g)$ is the probability associated with this migration. The variance of the future asset value is given by

$$\sigma_0[V_1] = \sum_g \left(V_1^g - E[V_1]\right)^2 \pi(g)$$

As was mentioned before, standard deviation (the square root of the variance) represents the risk measure used in the case of individual assets.

4.1.3 Portfolio loss distribution

The translation from individual assets' risk calculations to the portfolio risk measure rests largely upon an aggregation technique that considers all the technical details associated with this aggregation. In principle, a usual risk measure such as the portfolio standard deviation could be applied. For a portfolio containing n assets,

- The mean equals the sum of the individual assets' mean values, and
- The variance is obtained on the basis of the variances of sub-portfolios containing one and two assets (the proof is straightforward):

$$\sigma_p^2 = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sigma_{ij}^2 - (n-2)\sum_{i=1}^{n} \sigma_i^2$$

In this formula, σ_p^2 represents the portfolio's variance, σ_i^2 represents the individual assets' variances and σ_{ij}^2 represents the two-asset sub-portfolios' variances. It is clear that the derivation of the portfolio standard deviation in the analytical case rests largely upon the computation of all variances of sub-portfolios containing two assets. In order to do this, we need the joint migration probabilities for all pairs of assets (the probabilities that two assets together migrate from their initial ratings to another pair of rating grades, for example from (AA, B) to (A, BB)). CreditMetrics derives asset return correlations as an intermediate step in determining the joint migration probabilities. Asset return correlations and joint migration probabilities form the topic of discussion of the following two sub-chapters.

4.1.3.1 The correlation (ρ) in the case of two obligors

Although the model is primarily interested in the debt value distribution at the end of the risk horizon, its calculations are simplified by viewing the value V of an obligor's assets as a stochastic process in continuous time. Its evolution law – usually invoked in connection with Merton's option pricing model and its generalizations, as well as in the modern semimartingale-based dynamic asset pricing theory at large – is conventionally written as

$$\frac{dV}{V} = \mu dt + \sigma dZ,$$

where μ is the mean gross return on the firm's assets, σ is the instantaneous row vector-valued volatility of this return and Z is a column vector of mutually independent standard Wiener processes. The widespread case of constant μ and σ corresponds to the log-normally distributed V (a geometric Brownian motion). Then, the asset value at time t can be written out explicitly as

$$V_t = \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}Z_t\right\}.$$

CreditMetrics further employs the normalized log-returns or standardized returns R, which are standard normal random variables defined as:

$$R = \frac{\ln\left(\frac{V(t)}{V_0}\right) - \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad .$$

The model uses the correlation between firms' equity returns as a proxy for the correlation between asset returns. To determine the correlation ρ between asset (actually equity) returns, it is assumed that the firms represented in the portfolio face a set of systematic or common risks and a residual defined as a firm-specific or idiosyncratic risk. The systematic risks are captured through a set of country- and industry-specific equity indices whose volatilities and pairwise correlations are known. Next, each obligor's industry participation is determined, which pins down its exposure to the systemic risk. Thus, the standardized returns (standard normal random variables) for two particular assets can be decomposed in the following manner:

$$R = \sum_i w_i x_i + w\varepsilon , \quad R' = \sum_i w_i' x_i + w'\varepsilon'. \tag{2}$$

Here $\{x_i\}_i$ is a set of standardized returns on some predetermined country and industry equity indices with known pairwise correlations; w_i, w_j' are firm-specific weights. They are associated with the common factors; ε, ε' are idiosyncratic factors assumed as standard, normally distributed and uncorrelated with the common factors and the other firms' idiosyncratic risk factors. The idiosyncratic factors' weights w and w' are determined in such a way as to make the variances of the equity returns R and R' equal to one. The correlation ρ between two obligors' asset returns can then be easily computed as

$$\rho(R,R') = \sum_i \sum_j w_i w_j' corr(x_i, x_j)$$

4.1.3.2 The joint migration probabilities for a pair of obligors

Their estimation is based on a generalization of the Merton's option pricing model[3]. In other words, it is assumed that changes in the firms' asset values ultimately generate changes in ratings and defaults. Thus, if the asset value falls below the liability value,

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[3] Focused primarily on the assumptions and the derivation technique of the option prices, the theoretical constructs of Merton's model can be easily translated to the pricing of many other option-like contracts. In the case of a firm, its equity is considered as a call option on the firm's assets with the strike price equal to the face value of the firm's debt and the same maturity as the firm's debt. If the firm's assets at maturity exceed the debt, then the equity holders pay this debt and keep the firm. Otherwise, the firm defaults on its obligations. For details, see the original model of Black and Cox, 1976.

the firm will default. If the asset value decrease reaches a certain threshold, the firm will downgrade to the CCC class and so on. In this way a series of thresholds $(Z_g, g \in \{ \text{Default}, \ldots, \text{AA} \})$ is determined for each rating class, such that a given asset return value in one year's time is associated with a certain credit rating at that time. These thresholds are adapted to a standard normal distribution of the asset return, i.e. one with unit variance. The mapping procedure is better emphasized by the following representation:

Table 1
Migration probabilities for
the initially BBB-rated bond

Grade	Migration Probabilities
AAA	0,02
AA	0,33
A	5,95
BBB	86,93
BB	5,30
B	1,17
CCC	0,12
Default	0,18

Figure 4
Standard normal distribution and the
threshold values

$$Z^{\text{Def}} \ Z^{\text{CCC}} Z^{\text{B}} Z^{\text{BB}} \qquad Z^{\text{BBB}} \quad Z^{\text{A}} \ Z^{\text{AA}}$$
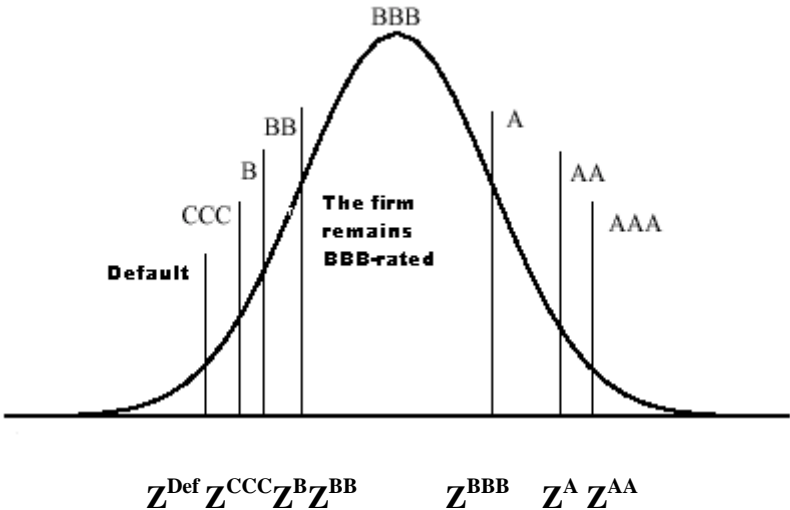
The table on the left displays the probabilities with which an initially BBB-rated bond, for example, will migrate to a new grade over a one-year period. The graph on the right displays the standard normal distribution together with the threshold values. These threshold values are obtained in the following way: Z_{DEF} represents a real number such that the area under the pdf and to the left of the vertical line designated by Z_{DEF} is equal to 0.18 (the default probability). Z_{CCC} is a real number such that the area under the pdf and between the two vertical lines represented by Z_{DEF} and Z_{CCC} equals 0.12 (the probability of migration to the CCC grade). This procedure continues until the entire area under the pdf is delimited into a finite number of non-overlapping bands such that the area of each band equals a migration probability given in the table. The threshold values are class-specific but may vary from country to country.

The joint distribution of standardized returns (R, R') for a pair of assets is assumed to follow a bivariate normal distribution with a correlation ρ. Then the probability of joint credit migrations in one year's time from a pair of rating classes (g, γ) to (h, χ) is given by:

$$P(h, \chi) = P\left(Z_g^{h-1} < R < Z_g^h, Z_\gamma^{\chi-1} < R' < Z_\gamma^{\chi-1}\right) = \int_{Z_g^{h-1}}^{Z_g^h} \int_{Z_\gamma^{\chi-1}}^{Z_\gamma^{\chi-1}} f(u, u', \rho) \, du \, du',$$

where $f(u, u', \rho) = \dfrac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{\dfrac{-1}{2(1-\rho^2)}\left(u^2 - 2\rho u u' + u'^2\right)\right\}$ is the bivariate normal distribution.

It can be shown that the probability of joint credit migrations does not depend on the variances of the individual asset returns. The only things that these variances determine are the threshold values Zs.

Having derived these probabilities, the risk analysis at the level of two-asset portfolios is completed with the calculation of the corresponding variances and standard deviations. For example, the variance of such a two-asset sub-portfolio can be computed with the formula:

$$\sigma_{ij}^2 = \sum_{h=1}^{G}\sum_{\chi=1}^{G}\left(V_i^h + V_j^\chi\right)^2 P(h, \chi) - \left(\sum_{h=1}^{G}\sum_{\chi=1}^{G}\left(V_i^h + V_j^\chi\right)P(h, \chi)\right)^2,$$

where G represents the number of rating classes and V_i^h and V_j^χ are the values of the assets i and j in states h and χ, respectively (cf.(1)).

The mean and variance are the first two moments of the portfolio value distribution at the risk horizon. If this distribution were normal, then the mean and standard deviation would suffice to analyze the portfolio credit risk and derive an estimation of the economic capital needed. However, the portfolio value, being a sum of many log-normally distributed variables, is not normal. An easy parametric representation for it is hard to find, and, moreover, the standard deviation of the portfolio is not a reliable measure of the portfolio risk. For this reason CreditMetrics employs a Monte Carlo simulation technique that derives the distribution of the portfolio's value at the risk

horizon and then performs the risk calculations at the portfolio level in the Value at Risk framework. However, the mean and variance provide useful information about the loan portfolio and, actually, fulfill two important functions. First, they allow testing of the accuracy of the Monte Carlo estimation once the portfolio value distribution is determined by this method. Second, they are actively used in determining the marginal risk contributions of individual assets. The marginal risk contribution of an asset represents the incremental increase in portfolio risk reached when that particular asset is included in the portfolio. In both cases, the marginal risk contributions are smaller than the stand alone risk measures of the two bonds. This is due to the diversification effect induced when increasing the number of assets in the portfolio. Marginal risk contributions suggest which assets contribute more to the risk profile of the portfolio and thus represent a useful tool in portfolio selection.

4.1.4 The Monte Carlo simulation

Monte Carlo simulation is a numerical method that infers information about a particular random process (characterized by a probability density function) by utilizing randomly drawn real numbers from the interval [0,1]. The logic supporting this method is that, as the number of scenarios increases to infinity, the artificially simulated process approximates with increasing precision the real random process.

For a portfolio containing n assets, the Monte Carlo method generates a simulated distribution of the portfolio value at the risk horizon in the following manner:

-   It specifies the asset return thresholds for each rating class (see 4.1.3.2).
-   It determines the pairwise correlation among individual assets (see 4.1.3.1).
-   It generates a great number of scenarios (say 10000 or 20000). A scenario contains n standardized asset returns, one for each asset, and preserves the correlation structure of the real portfolio
-   Comparing the standardized asset returns and the threshold values, a new credit rating is assigned to each asset. Then the asset is valued in the manner presented in Subsection 4.1.2 (see formula (1)).
-   The portfolio value is the sum of the n valuations derived above.

- With a large number of portfolio values obtained in this way, a histogram can be plotted. Such a histogram is simply a graphical representation that has on the horizontal axis the portfolio values and on the vertical axis the frequencies with which these values are attained. The larger the number of scenarios, the smoother the histogram becomes approaching in the limit the value distribution presented in Figure 3.
- It derives estimates of the risk associated with the portfolio distribution thus obtained based on the Value at Risk methodology presented in Subsection 3.2.

4.1.5 Advantages and limitations

CreditMetrics is a useful and sensitive tool for assessing changes in the portfolio's value due to credit quality migrations. The analytical framework enables the computation of the expected portfolio value and its variance at the risk horizon. The Monte Carlo simulation determines the asset value distribution and thus, based on a Value at Risk rationale, may offer reliable estimates of the risk embodied in the portfolio.

A first limitation rests upon the assumption that firms within a given rating class have the same probability of default and transition to different grades. This assumption and the one that the actual probabilities equal historically determined averages have been questioned by KMV. They found that obligors within a given rating class might exhibit significantly different migration probabilities, while obligors in different rating classes may exhibit identical migration probabilities. The KMV Corporation also argued that actual migration rates might significantly depart from historical estimates. Second, the default free interest rates are assumed to be deterministic, and thus the model is insensitive to market risk and underlying changes in the economic environment. Third, the model proxies asset return correlations by equity return correlations, and this might lead to an imprecise estimation.

4.2 The KMV model

4.2.1 Definitions and assumptions

The model belongs to the default mode category of models by associating risk only with default and estimating explicitly only the default probability (Expected Default Frequency). The probability of default is endogenous and related to the debt and asset structure of the obligor's firm. The firm's capital consists only of equity, short-term debt, long-term debt and convertible and preferred shares. The unobservable intrinsic value of the firm's assets is assumed to follow a random process with log-normally distributed values. The KMV Corporation has concluded that the fall of a firm's market asset value below the value of its total debt is not an accurate measure of the probability of default but rather of the probability of bankruptcy. Based on observations, they decided that a default occurrence is more likely when the firm's asset value reaches a level situated between the value of the short-term debt and half of the long-term debt. Therefore, in the KMV view, firms are likely to default later than their asset value reaches a level conventionally regarded as signaling default.

To derive the loss distribution, KMV assumes that the portfolio is highly diversified – an assumption only partly true in real cases. Departing from the Value at Risk framework, KMV estimates the need of economic capital within its own analytical approach. As a result, tabulated values of the loss variable are provided for different confidence levels.

The model employs deterministic interest rates, i.e. the market interest rate risk is not modeled explicitly, and the same is true for the general economic conditions.

The model estimates three basic elements: the *probability of default* for individual obligors, the *present value of the future cash flows* expected from individual assets and a *loss distribution* based on which the credit risk of the portfolio can be quantified.

4.2.2 Risk calculations at the asset level

Figure 5
Mapping of Distance-to-Default to Expected Default Frequency
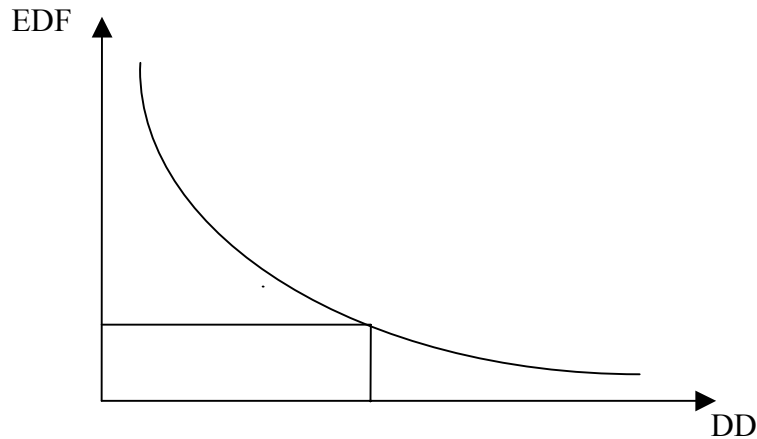
The forecast of the probability of default (named *Expected Default Frequency* or shortly EDF) for an individual obligor follows the steps below:

1. Estimation of the asset value (V^A) and volatility of asset returns (σ^A) as functions of the capital structure of the firm and the risk-free interest rate

2. Estimation of the default point (DP) which is the sum between the value of short-term debt (the debt that must be paid out during the reference period) and half of the long-term debt.

3. Estimation of the Distance to Default (DD) which is the difference between the expected asset value at the end of the risk horizon and the default point. This difference is expressed as a multiple of the standard deviation of the asset distribution. The larger the Distance to Default is, the safer the asset is perceived to be.

4. Estimation of EDF. Based on large cross-sectional data sets, the KMV Corporation has established a mapping procedure from DD to EDF, which is represented in Fig. 5.

Distribution of the debt value and asset pricing problems

KMV computes the expected loan portfolio loss as the difference between the riskless value of the portfolio (the discounted value assuming no default) and the

valuation of the cash flows to be presented below. KMV established that, under some simplifying assumptions, the limiting distribution of the expected loss defined in this way follows a normal inverse distribution[4]. The p-percentiles of this distribution give estimates of the unexpected losses at different confidence levels and thus can be used to assess the need for economic capital.

From the standard asset pricing model point of view, at least two questions need to be clarified before the above loss calculation method can be endorsed:

A. The legitimacy of calculations based on the risk-neutral distribution in environments where market completeness and lack of frictions is not guaranteed

B. Justification of the standard arbitrage-free option pricing approach in derivations of the borrower's asset value used in the distance-to-default calculations.

A quantitative treatment of both problems is easiest within a modeling framework of general equilibrium asset pricing in incomplete markets. Specifically, using the notations introduced at the beginning of this section, we shall look at the bank assets of lending type, x^b, whose uncertainties are different from those contained in the traded assets x^a. Analogously to the benchmark KMV approach, we will admit that the borrowing firms' equities can be traded (belong to x^a). However, remember that, according to standard corporate finance theory, direct investor-firm relations in the presence of unobservable components in many firms' returns are insufficient. That is exactly why deposit-collecting and lending intermediaries (banks) who exercise "delegated monitoring" (Diamond, 1984) are believed to exist in any economy. Therefore, one cannot rely upon a borrower's market equity price being a sufficient statistic of the latent uncertainties in the company's assets and cash flow value.

However, let us for the sake of the argument assume that all the conditions that the KMV model needs in order to conduct calculations exemplified below are satisfied. This means that we are calculating the value of a loan in an economy where

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[4] The normal inverse distribution equals the inverse of the cumulative standard normal distribution. In other words, for a given value p in the interval (0,1) it represents that value α below which the standard normal distribution falls with the probability p. The properties of the normal inverse distribution show that it is skewed and leptokurtic, thus asymmetric and with fat tails. For a given distribution, skewness and kurtosis are measured using respectively the moments of the third and fourth order and the variance of the distribution.

I. The debt itself is traded prior to maturity or default (henceforth: debt contract termination time)

II. The debtor firm's assets are also traded both prior to and at the debt termination time

III. These assets are identical with the collateral that the lender receives in the case of default

IV. Claims contingent on the firm's asset value and maturing at dates up to the debt termination time are priced by means of the same pricing kernel (alternatively: zero-coupon term structure curve) as the debt market instruments.

In short, one must assume an economy where at least the debt and equity markets referring to the obligors in the bank loan portfolio are complete.

In accordance with notations employed in the KMV documents, let V^A be the value of the debtor's assets, and let the debt contract be defined by a sequence of payments (m_t), $t=1,\ldots,T$ agreed upon at time $t=0$. Further, dp will denote the default point in the set of V^A-values. The default at time t occurs (given that it has not occurred before) if and only if $V^A_t \leq dp$. In such a case, the lending bank does not receive either m_t or the remaining payments (if any), but instead, the property rights to the firm's assets are transferred to it.

Considering V^A a random process, we define $\chi^+(V^A - dp)_t$, or simply χ^+_t, to be the indicator function of the set $\bigcap_{n=0}^{t}\{dp < V_n^A\}$ in the event space, i.e. it is equal to unity if there is no default between times 0 and t, and zero otherwise. Analogously, let $\chi^-(V^A - dp)_t$ (shortened to χ^-_t) be the indicator function of the set $\bigcap_{n=0}^{t-1}\{dp < V_n^A\}\bigcap\{V_t^A \leq dp\}$. That is, χ^-_t takes the value of unity if the default happens exactly at date t, and zero otherwise.

The following notations and statements will quantify the assumptions I–IV subsumed by KMV. The firm's assets are traded in the secondary market at price P^A (the debt contract held by the bank is traded at the secondary market price V^B). In order to

allow more freedom in the choice of a default-triggering mechanism, we deliberately distinguish between unobservable V^A governing the default event, and observable P^A, in the same way as KMV distinguishes between dp and m. If the assets generate a stream of cash flows $(y_t)_{t\geq 0}$, then conventional asset pricing models state that their fair bubble-free price is linked to $(y_t)_{t\geq 0}$ by the formula

$$P_t^A = \sum_{s\geq t} E_t\left[K_t^s y_s\right], \tag{3}$$

where E_t is the conditional expectation at time t and K_t^s is a pricing kernel between times t and s (see Duffie and Kan, 1996, for a recent treatment of the pricing kernel, the stochastic discount factor and the term structure of interest rate models of asset pricing in discrete time). Note that, so far, we have not required asset market completeness.

Equation (3) has a recursive counterpart of the form

$$P_t^A = E_t\left[K_t^{t+1}\left\{y_s + P_{t+1}^A\right\}\right]. \tag{4}$$

Analogously, the debt value can be priced within the same model according to

$$V_t^B = \sum_{s\geq t} E_t\left[K_t^s \chi^+ (V^A - dp)_s m_s\right]. \tag{5}$$

The recursive counterpart of (5) is

$$V_t^B = E_t\left[K_t^{t+1}\left\{\chi_t^+ m_t + \chi_t^- y_t + \chi_t^+ V_{t+1}^B\right\}\right]. \tag{6}$$

The rationale for (6) is as follows: in the event of no default in period t, the debt holder receives the agreed payment m_t and keeps the continuation value of the debt V_{t+1}^B. If the firm defaults at date t, there is no continuation value for the debt contract, but the firm's assets become the lender's property. They pay out the cash "dividend" y_t instead of m_t, after which moment the continuation value of V^A (captured by (4) instead of (6)) is added to the lender's financial wealth.

KMV claims the ability to continue with an explicit calculation of the debt value, departing from an equation like (6), based on the option pricing technique. As noted earlier, such a statement cannot be taken at face value outside the classical Merton model of derivative pricing in continuous time in complete frictionless markets. Therefore, we will next try to formulate the core statements of the KMV approach pertaining to the quantification of (6), by choosing an appropriate interpretation of the

standard derivative pricing techniques and results. The following observations should simplify this task:

(a) Formulae of arbitrage-free option prices are consequences of a general riskless portfolio construction procedure that can be utilized under assumptions I–IV above even when a standard complete frictionless market paradigm is inapplicable;

(b) The so-called risk-neutral probability, appearing in standard option pricing formulae, is a mere tag, under which much more general asset-weighting expressions are concealed; the latter are well defined even when the risk-neutral probability itself is not.

These two points will be made explicit in the example below, which follows the binomial derivative pricing argument in discrete time, originated by Cox, Ross and Rubinstein, 1979.

Example: a binary distribution of the borrower's cash flow

Let us assume that the time t cash flow of the firm, y_t, has just two realizations, y^+ and y^-, corresponding to "good" and "bad" performance. (As is usual in the no-arbitrage derivative pricing models, the exact probability of good performance will prove to be irrelevant.) Accordingly, in "good" times, V_t^A strictly exceeds the default point dp, whereas in "bad" times V_t^A falls short of it. Define by $P_t^{A\pm} = y^\pm + \chi_t^\pm P_{t+1}^A$ the time t realizations of the secondary market price of V^A in good/bad times, and by $V_t^{B+} = m_t + \chi_t^+ V_{t+1}^B$, the time t realization of the secondary market price of debt in good times.

Let us construct a portfolio consisting of the debt plus h shares of V^A (the firm's assets are short-sold if $h<0$). We want to find the *hedge ratio h* such that the realization of the value of this portfolio is independent of y-realization, i.e. is the same regardless of the default event occurrence. Since the good times realization of the portfolio value is equal to

$$m_t + h(y^+ + P_{t+1}^A) + V_{t+1}^B = V_t^{B+} + hP_t^{A+},$$

and the bad times realization to $(1+h)P_t^{A-}$, the value of h which equalizes them is given by

$$h = \frac{P_t^{A-} - V_t^{B+}}{P_t^{A+} - P_t^{A-}}.$$

One can normalize all the uncertainty factors in the economy in such a way that V^A becomes one of the independent fundamental sources of risk in the asset market. Then, by a no-arbitrage argument, the portfolio constructed with the hedge ratio h as calculated above, must earn the riskless rate of return between periods t and $t+1$. We shall denote it by i_{t+1}. Substituting the obtained value of h in the portfolio realized value and equating it with the riskless value increase, we get

$$\left(V_t^B + hP_t^A\right)(1+i_{t+1}) = \frac{\left(P_t^{A+} - V_t^{B+}\right)P_t^{A-}}{P_t^{A+} - P_t^{A-}}.$$

Rearranging, we obtain the following formula for valuation of the debt:

$$V_t^B = \frac{1}{1+i_{t+1}}\left(\alpha_t V_t^{B+} + (1-\alpha_t)P_t^{A-}\right), \quad \alpha_t = \frac{(1+i_{t+1})P_t^A - P_t^{A-}}{P_t^{A+} - P_t^{A-}}. \tag{7}$$

If one defines the time t realizations of the one-period yield on V^A in good/bad times by $\zeta_{t+1}^{\pm} = \frac{P_t^{A\pm}}{P_t^A}$ (the obvious inequalities $\zeta_{t+1}^{-} < i_{t+1} < \zeta_{t+1}^{+}$ must hold), the expression for α can be rewritten as

$$\alpha_t = \frac{i_{t+1} - \zeta_{t+1}^{-}}{\zeta_{t+1}^{+} - \zeta_{t+1}^{-}}.$$

In the binomial derivative pricing literature, α is usually identified with the risk-neutral probability of good times. Clearly, it can be defined and even assigned a numerical value in applications regardless of whether such a probability itself exists or not, since it depends on characteristics of asset V^A alone.

The loss-given-default (LGD) parameter can be identified with the ratio $\lambda = \frac{V_t^{B+} - P_t^{A-}}{V_t^{B+}}$,

and V_t^{B+} itself with the value of exposure. We can think of the default free component as that part of the asset that will not be lost even if the obligor defaults. It is reasonable to approximate it by the product between the recovery rate (or one minus the loss given default) and the exposure. Then the expected return on the default free component equals $(1\text{-LGD}) \times \text{Exposure} \times \alpha + (1\text{-LGD}) \times \text{Exposure} \times (1\text{-}\alpha) =$

(1-LGD) $\times$ Exposure. The risky component, on the contrary, returns nothing if a default occurs (this event happens with the "risk neutral probability" α and the exposure multiplied by the loss given default if default does not occur (this event happens with the "risk-neutral probability" $1-\alpha$). Then, on average the risky component returns $0 \times \alpha + \text{LGD} \times \text{Exposure} \times (1-\alpha) = \text{LGD} \times \text{Exposure} \times (1-\alpha)$. Altogether, we can represent the algebraic procedure described above diagrammatically as in Table 2:

Table 2

Decomposition of a risky asset according to KMV

	Component	State	Risk-neutral Probability	Expected Value
Risky Asset	Default Free Component	Default	α	(1-LGD)$\times$ Exposure
		Non-default	1- α	(1-LGD)$\times$ Exposure
	Risky Component	Default	α	0
		Non-default	1- α	LGD$\times$ Exposure

Equation (7) can be rearranged as follows:

$$V_t^B = \frac{1}{1+i_{t+1}}\left(\alpha_t V_t^{B+} + (1-\alpha_t)P_t^{A-}\right) = \frac{1}{1+i_{t+1}}\left(\frac{P_t^{A-}}{V_t^{B+}}V_t^{B+} + \alpha_t \frac{V_t^{B+}-P_t^{A-}}{V_t^{B+}}V_t^{B+}\right) =$$

$$= \frac{1}{1+i_{t+1}}\left[(1-\lambda)\times\text{Exposure} + \alpha_t \times\lambda\times\text{Exposure}\right] =$$

$$= \frac{1}{1+i_{t+1}}\times(1-\text{LGD})\times\text{Exposure} + \frac{1}{1+i_{t+1}}\times\alpha_t\times\text{LGD}\times\text{Exposure}$$

Let us assume, for example, that the time t realization of the market price of debt in good times equals $100 mil, while the time t realizations of the market price of assets in good and bad times are respectively $120 mil and $80 mil. The market price of assets at the start of date t is $110 mil. Also assume that the riskless rate of return is 4%. Then the LGD is 20%, the exposure is $100 mil and α_t is 0.805. Replacing all these values in the previous formula, we obtain the time t debt value equal to $92.4 mil.

4.2.3 Advantages and limitations

One of the advantages of this method is that the derivation of the default probability is more related to the firm's characteristics than to initial ratings. This firm-specific

approach is more sensitive to changes in the quality of obligors than the credit ratings approach. Market information is also incorporated in the model due to the fact that firms' equity is actively used in the computation of the credit risk. The choice of the time horizon is more flexible than in CreditMetrics as long as the variables used are market-driven and thus display more time variability.

Among the criticisms, the too simplistic capital structure of the firm has sometimes been put forward. Also, the relationship between the Distance to Default and EDF is based on US data, and its derivation is not thoroughly explained. Therefore, the model implementation in an environment outside the US depends upon the possibility to adapt such a relationship to the particular environment in question. Another drawback is related to the assumption that the portfolio is highly diversified. If this condition is not satisfied in real circumstances, then the tabulated values supplied by KMV can misrepresent the need of economic capital.

4.3 CreditRisk+

4.3.1 Definitions and assumptions

CreditRisk+ is a default mode model. In other words, it is assumed that each obligor can be in one of the two states, default or non-default, at the end of the risk period. CreditRisk+ estimates the portfolio's loss variable and assesses the need of economic capital within the Value at Risk framework.

From an internal, i.e. lending bank, point of view several assumptions are made.

First, CreditRisk+ can be implemented using either the assumption that the default rates of individual obligors are fixed or that they are variable. In the latter case, the default rates are modeled as continuous random variables whose volatilities incorporate uncertainty about the future state of the obligors (uncertainty that is linked to given systemic risk factors). In CreditRisk+, the default probability of an obligor is neither connected to its capital structure nor estimated based on a cross-sectional

historical data set. The model makes no assumption about the timing and causes of the default events.

Second, exposure to a large number of obligors is assumed to be such that each of them has a small probability of default. Conditional upon the realization of the systemic factors, the probabilities of default are mutually independent among single assets.

Third, the random variable giving the number of defaults for the portfolio (or sub-portfolios) over a given time period is supposed to follow a Poisson distribution:

$$P(\text{number of defaults} = k) = \frac{m^k e^{-m}}{k!}, \qquad\qquad k = 0, 1, 2, \ldots$$

where m stands for the expected number of defaults at the portfolio (sub-portfolio) level over the chosen time period.

Fourth, the model captures the concentration effect by allowing for differences between sectors. A diversified portfolio means that each exposure is assigned to a different sector while a concentrated portfolio means that all exposures are part of a single sector. A sector is a collection of homogeneous obligors under the influence of a single underlying factor. CreditRisk+ does not specifically mention how to assign obligors to different sectors. It just hints that sectors could be conceived as countries or industries to which obligors belong. The influence of systemic factors on the default rates is captured through default rate volatilities instead of default correlations between obligors (as was done in the previously discussed models).

4.3.2 Risk calculations at the asset and portfolio level

The basic inputs of CreditRisk+ are the individual assets' exposures and default probabilities (P_i). The model employs the common exposure of each obligor (v_i'), which is the ratio between the bank exposure to that obligor and a selected unit of exposure ($100,000 for example). The risk calculation at the asset level consists in the estimation of the expected loss (ε_i), which is the product of the common exposure and the probability of default of that asset.

The main idea is to ascribe the set of obligors to different bands such that each band contains obligors with the same rounded common exposure (v_j). It is assumed that a finite number of bands (m) is thus obtained.

The average number of defaults (m_j) in band j is approximated by the ratio between total loss (ε_j) and the common exposure characteristic to that band (v_j). The expected loss in each band is the sum of expected losses of all obligors belonging to that band.

Let L denote the loss value divided by the unit of exposure. Then, for each band j, a probability generating function (pgf) of the loss distribution characteristic to that band (L_j) can be constructed as

$$G_j(z) = \sum_{n=0}^{\infty} \Pr\{L_j = n\}z^n = \sum_{k=0}^{\infty} P(k \text{ defaults})z^{kv_j} = \sum_{k=0}^{\infty} \frac{m_j^k e^{-m_j}}{k!} z^{kv_j} = e^{-m_j + m_j z^{v_j}} \quad .$$

Based on the assumption that band exposures are independent, the probability generating function for the entire portfolio is simply the product of the individual pgfs:

$$G(z) = \prod_{j=1}^{B} G_j(z) = \prod_{j=1}^{B} e^{-m_j + m_j z^{v_j}} = e^{-\sum_{j=1}^{B} m_j + \sum_{j=1}^{B} m_j z^{v_j}}$$

The portfolio's probability generating function thus determined is used to derive the portfolio loss distribution. The central property of the probability generating function G is used, which relates the nth derivative of G evaluated at the origin to the probability that the portfolio loss equals n times the unit of exposure. Specifically:

$$P_n = P(\text{loss} = n \times \text{unit of exposure}) = \frac{1}{n!} \frac{d^n G(z)}{dz^n}\bigg|_{z=0} \qquad n = 0,1,....$$

To derive these probabilities, CreditRisk+ constructs a recurrence relationship

$$P_n = \sum_{\substack{k \\ s.t. v_j \le n}} \frac{v_j \times m_j}{n} P_{n-v_j} \tag{8}$$

that starts with the probability of no loss:

$$P_0 = P(\text{no loss}) = e^{-m} = e^{-\sum_{j=1}^{B} m_j} \quad .$$

Appendix A2 contains an example of calculations based on the above recursive formula.

CreditRisk+ provides some generalizations of the basic model such as: extensions over a multi-year period, loss distributions with variable default rates and, finally, a loosening of the one band – one factor relationship.

4.3.3 Advantages and limitations

An advantage of CreditRisk+ is that it requires a limited amount of data as inputs (basically only individual exposures and default probabilities), and the computation of the loan loss is rather easy to perform. A limitation of the model is that a lot of ambiguity surrounds the specification of the default rates for individual obligors, which are actually basic inputs of the method. In CreditRisk+, obligors are not assigned to rating classes, and their characteristics do not determine these default rates. It is implicitly assumed that banks know these probabilities and their volatilities, but a concrete method to derive them is not offered. Another limitation is that the model does not assume market risks.

4.4 CreditPortfolioView

4.4.1. Definitions and assumptions

In CreditPortfolioView both the default and transition probabilities are explicitly linked to macro-variables such as interest rates, the growth rate, unemployment rate, etc. It is assumed that in a recession, the default and downgrade transition probabilities are higher than the corresponding historical averages. In periods of boom, the opposite relation holds true. The model derives the distributions of the default (migration) probabilities conditional on the state of the economy and the distribution of portfolio loss at the risk horizon. The estimation of economic capital is performed in the Value at Risk framework.

4.4.2 Default (migration) probabilities conditional on the state of the economy

The probability of default (or migration to another credit quality) of an individual obligor in country (industry) i at time t is defined by a logit function:

$$P_{i,t} = \frac{1}{1+e^{-Y_{i,t}}}$$ (9)

The independent variable $Y_{i,t}$ is a country- (industry-) specific index that depends on a given set of macroeconomic factors (X):

$$Y_{i,t} = a_{i,0} + \sum_{j=1}^{m} a_{i,j} X_{i,j,t} + u_{i,t}$$ (10)

Each variable $X_{i,j,t}$ is assumed to follow an AR process of a specific order:

$$X_{i,j,t} = b_{i,j,0} + b_{i,j,1} X_{i,j,t-1} + b_{i,j,2} X_{i,j,t-2} + ... + v_{i,j,t}$$ (11)

The system (9)–(11) is calibrated. Then, a Monte Carlo method is applied to determine the distribution of the default probabilities conditional on the state of the economy. This is done in the following steps:

1. First, using a Cholesky decomposition of the variance-covariance matrix of the error terms in (9) and (10) and random draws of standard normal variables, a simulated value of the default probability in (8) is obtained.

2. This simulated value is compared with the unconditional value estimated as a historical average over more than 20 years and for many industries. It is assumed that the ratio between the simulated value and the historical average is greater than one in recessions and less than one in booms.

3. If this ratio is greater than one (recession), the unconditional transition matrix (its estimation is based on standard credit rating historical data) is adjusted to give increased probabilities of migration into downgrades and default and decreased probabilities of migration into upgrades. If the ratio is less than one (boom), the adjustment is done in the opposite direction. The elements of this new matrix give the transition probabilities conditional upon the state of the economy (one among them is the default probability).

Steps 1, 2 and 3 are repeated many times. As a result, the distribution of the default probability conditional upon the state of the economy is estimated. The mean value of this distribution represents the default probability of the obligor in question and is later used to estimate the portfolio loss.

Portfolio loss distribution

CreditPortfolioView derives the portfolio loss distribution assuming that the economy will find itself in a finite number of states over the risk horizon. Each potential loss position that the portfolio might encounter is then accompanied by an unconditional probability, which represents the sum of the corresponding conditional probabilities over all states of the economy. To make things clearer, let us assume a bank portfolio containing two assets with exposures of $100 and $200. The bank risk manager anticipates that the economy will go through a recession with a probability of 1/4 and through a boom with a probability of 3/4. Let us assume that, following the procedure exposed in 4.4.2.1, the default probabilities of the two obligors in each of the possible states (recession, boom) have been estimated as in Table 3.

Table 3
Default probabilities of the two obligors conditional on the state of the economy

State of the economy	Default probabilities (%)	
	Obligor 1	Obligor 2
Boom	2.1	1.8
Recession	3.2	9.4

The first obligor's default probabilities display less variation in rapport with the business cycle. Thus, it is likely that he represents an investment grade customer of the bank. The second obligor is more sensitive to economic fluctuations, thus he may be considered as a speculative grade customer.

At the end of the risk horizon any of the following four events can happen: none of the obligors default (event A), obligor 1 defaults but obligor 2 does not default (event B), obligor 2 defaults but obligor 1 does not default (event C) or both obligors default (event D). Assuming that the recovery rate is zero for both obligors, the potential loss values at the portfolio level are: 0 (event A), $100 (event B), $200 (event C), $300 (event D).

The portfolio distributions in the two states can then be represented as in the Table 4.

Table 4

Portfolio loss distributions conditional on the state of the economy

Event	Recession (Prob.=1/4)		Boom (Prob.=3/4)	
	Portfolio loss ($)	Probability(%)	Portfolio loss($)	Probability(%)
A	0	21.9	0	71.1
B	100	0.72	100	2.3
C	200	2.27	200	1.5
D	300	0.07	300	0.05

The probabilities displayed in the third and fifth columns of Table 4 were derived assuming that the two obligors were independent once their dependence on common economic factors was controlled for. The probability of achieving no loss (event A) in the recession state, for example, represents the product of the probabilities that none of the obligors would default in recession, multiplied by the probability that a recession would occur ((96.8*90.6)/4 or 21.9%). The unconditional probability that no loss occurs is simply the sum of the probabilities of no loss across the two states, recession and boom: 21.9% + 71.1% = 93%. In a similar manner, all other unconditional probabilities can be derived. The unconditional portfolio loss distribution is then given in Figure 6.

Figure 6

The loss distribution according to CreditPortfolioView
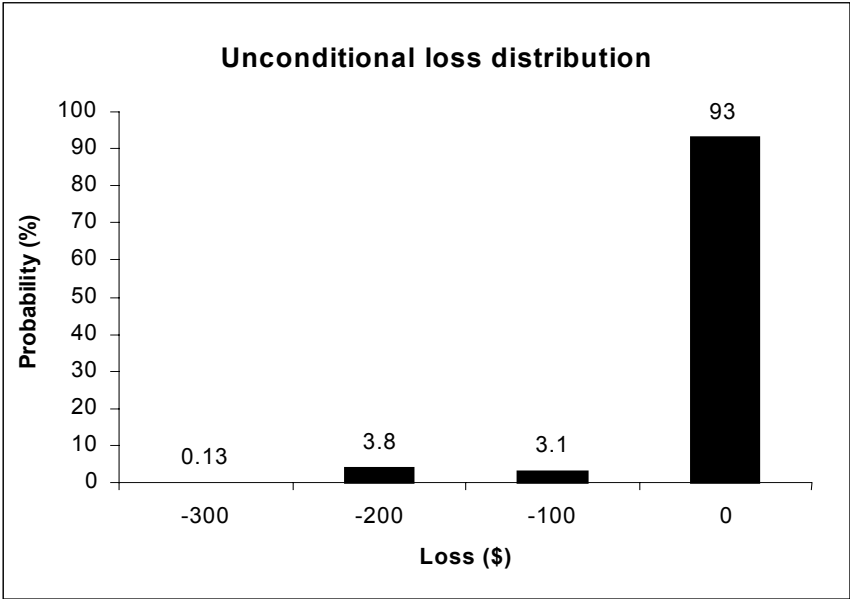
Once the portfolio asset composition is enlarged, the loss distribution would approach in shape the loss distribution presented in Sub-section 3.2. An estimation of the economic capital can thus be done in the context of the "Value at Risk" method

.

4.4.3 Advantages and limitations

The advantage of this method is that it is the only one that explicitly takes into account the dependence of the default events on the state of the economy. Nonetheless, this dependence between defaults and macroeconomic factors seems to be a rather strong assumption. Microeconomic factors play a role as well in generating default and credit quality migrations, and they are not mentioned in the model. Also data requirements might be demanding as long calibrations within countries and sectors are required. The more industries introduced in the model, the more the information about default events becomes sparse and thus the model's application is constrained.

5 Comparison of individual methods

5.1 Formal differences in individual methods

At first glance, the models appear to employ dissimilar assumptions, distributions, functional forms and methods of aggregating loss at the portfolio level. However, as Gordy, 2000, and Koyluoglu and Hickman, 1998, emphasized, theoretical constructs can be mapped from one model to another leaving only the choice of distributions as the main source of discrepancies across the models.

Let the (unconditional) probability of default of a given obligor be denoted by p. Let the vector-valued random variable Y comprise the background factors X and ε governing the uncertain default event (cf. the introductory part of Section 4). The conditional default probability given that Y takes the value of y, will be denoted by $p(y)$. If F is the cumulative distribution function of Y, then, obviously, $p = \int p(y) F(dy)$. From the formal point of view, the models discussed in the present paper differ mainly in their specification of functional dependencies $p(y)$ and the properties of F.

More precisely, Koyluoglu and Hickman, 1998, argued that the conditional default distributions and aggregation methods were the same, at least in the asymptotic sense, and only the treatment of the joint-default behavior caused different estimated results across the models. The parameters that describe the joint-default behavior are thus essential in producing differences over the range of the models' estimations. If the parameterization of the joint default behavior implies inconsistent means and standard deviations of the default rate distribution, then the models may differ significantly in their estimations. The joint-default parameterization is captured in a specific way by each model: Merton-based models use asset return correlations and coefficients that enter the corresponding functional forms, CreditRisk+ uses default rate volatilities and sector weights and the CreditPortfolioView use dependencies on macroeconomic factors and their coefficients. Koyluoglu and Hickman, 1998, also emphasized the main two sources that could be responsible for this parameter inconsistency: estimation error and model mis-specification. However, even if similar parameter values are obtained, the modeling approach can still make a difference, depending on the particular portfolio composition. Thus, for very high or very low quality portfolios, divergence in the distributions' tails for different models was detected. However, for more homogeneous portfolios, these differences were insignificant, and thus the models' outputs were similar.

Another important aspect that differentiates the models is data requirements and accessibility. Table 5 displays in a synthetic form the data requirements of the four models discussed in this paper.

Table 5

Data requirements of individual models

Model	Data Requirements
CreditMetrics	- Assets' characteristics (rates of return, maturities, recovery rates) - Equity returns - Rating systems and transition matrices - Forward zero curves - Country and industry indices and individual assets' exposures to different sectors of the economy
KMV	- Assets' characteristics (rates of return, recovery rates) - Capital structure of firms - Relation between DD a EDF - Risk-free interest rate - Country and industry indices and individual assets' exposures to different sectors of the economy
CreditRisk+	- Probabilities of default of individual obligors - Bank exposures to individual obligors
CreditPortfolioView	- Macroeconomic variables (interest rates, growth rate, unemployment rate, etc.) - Calibration coefficients specific for different countries and sectors - Asset characteristics

Based on this criterion, CreditMetrics is probably the most demanding model. It requires long cross-sectional data sets that, especially for a country in transition, are not available. It also relies heavily on ratings provided by rating agencies, country- and industry-specific equity indices and Stock Exchange data. All these elements may represent additional costs for banks wanting to implement this model. The KMV model also relies on historical data in estimating default probabilities but, nevertheless, its emphasis is on firm characteristics. For its implementation, banks need information about their clients' capital structure and debt. This type of data might be more accessible, especially for corporate exposures and large clients. The use of CreditPortfolioView may be constrained by the unavailability of sectoral data, especially concerning default events. CreditRisk+ seems the less demanding with regard to data inputs. It requires only the level of exposure and default probabilities for individual exposures, and these elements can be estimated separately by each bank that decides to use this model.

For an overall review of the models, Table 6 presents in a succinct form the main intermediary steps and output estimations of the models.

Table 6

Intermediary steps and output estimations of the models

Model	Intermediary steps	Output
CreditMetrics	- Asset value distributions - Risk calculations at the asset level	Analytical estimates of risk at the portfolio level
	- Asset return correlations - Threshold values marking changes in future ratings	The portfolio value distribution based on a Monte Carlo simulation
	- Joint migration probabilities	An estimation of the economic capital done in the "Value at Risk" framework
KMV	- Expected Default Frequency - Asset valuations - Asset return correlations	Analytical solutions of the portfolio loss and economic capital
CreditRisk+	- Preliminary risk calculations at the asset level	Portfolio loss distribution
	- Band partition of the portfolio - Recurrence relationship for the probability of loss at the portfolio level	An estimation of the economic capital done in the "Value at Risk" framework
CreditPortfolio View	- Default (migration) probabilities dependent on the state of the economy	Portfolio loss distribution
	- Loss distributions dependent on the state of the economy - Loss distributions independent of the state of the economy (convolution technique)	An estimation of the economic capital done in the "Value at Risk" framework

Table 7 translates the four basic elements that enter the NBCA risk calculations and emphasizes the way in which they are employed by the credit risk models. Even if their definitions and estimation methods can be different in different circumstances, the four risk characteristics given below represent basic inputs for all models.

Table 7

NBCA elements and their equivalents employed by credit risk models

Model	NBCA Elements			
	PD (Probability of default)	EAD (Exposure at Default)	LGD (Loss Given Default)	G (Granularity)
CreditMetrics	Estimation based on internal rating models or supplied by rating agencies	The principal in case of bonds Specific estimation for other types of assets	1-Recovery	Portfolio diversification – based on asset return correlations
KMV	Risk neutral probability	Specific estimation	LGD	Portfolio diversification – based on asset return correlations
CreditRisk+	Not specified but basic input in the model	Not specified but basic input in the model	1-Recovery	Sectoral analysis – based on the standard deviation of the default probability
CreditPortfolio View	Estimation based on a Monte Carlo simulation	Specific estimation	1-Recovery	Portfolio diversification

5.2. Advantages and limitations of individual methods

5.2.1 Advantages

The regulatory view publicly acknowledges that some characteristics of credit risk modeling present net benefits from a regulatory perspective. These models incorporate obligor-related risk weights and portfolio concentration risks, issues that have just recently been addressed within the regulatory norms. Another advantage of these models is that economic capital can be further used in the computation of risk-adjusted performance measures such as Rorac (Return on risk-adjusted capital) or Rarorac (Risk-adjusted return on risk-adjusted capital) which contribute to more efficient capital allocation across different business lines and within the financial sector as a whole. Regulators also praise the informative impact of data collection and model implementation.

The models display relative advantages among themselves. Thus, from the banks' standpoint, some models may gain comparative advantages depending on the type

of data they require, the compatibility between models and bank portfolio compositions, specific assumptions they make, and the ease of implementation of the estimation techniques. As many of these aspects have been already mentioned or will be mentioned latter, they are not further discussed in this sub-chapter.

5.2.2 Limitations

Several limitations can be put forward with regard to the current state of modeling techniques. For example, models that are not linked to macroeconomic factors can under- or over-estimate the transition probabilities in the short run by relying exclusively on long-run historical averages that do not reflect business fluctuations. On the other hand, conditional models may under- or over-estimate the transition probabilities at the outset of a downturn (upturn) in the credit cycle.

Mutual correlations between probabilities of default, loss given default and exposure are assumed to be zero even if real life examples may show that this is not necessarily the case. For a loan commitment, for example, the drawn part may extinguish the entire line of credit as the obligor's rating deteriorates towards default. Therefore, the level of exposure and probability of default seem to be positively correlated.

The methods that model correlations across obligors based on a set of background factors (CreditMetrics, KMV) are highly sensitive to the evaluation and measurement of correlations among these underlying risks. Departures in the industrial structure of a country from the historical pattern may generate biased estimates of the concentration of exposure of individual portfolios. A similar result could be generated if, for example, relationships among particular indices are duplicated from one country to another due to the unavailability of data in the latter case. On the other hand, flaws in the selection of macroeconomic variables (in CreditPortfolioView) and the implementation of the method may invalidate the results of the econometric tests.

All the methods assume no market risks in the sense that yield spreads and exposures are derived from deterministic trends. Market risk and credit risk are analyzed separately.

6 The potential impact on banks and on credit allocation in the Czech Republic

6.1 Current state and perspectives

Banks in the Czech Republic allowed enterprises to operate on a relatively soft budget constraint during the transition period. This led to the accumulation of non-performing loans on banks' balance sheets and to increased spending of budgetary funds on bank bailouts. The recent progress in the bank privatization process has already led to strengthened competitive pressures and thus to an increased need for reassessing banks' lending practices and strategies. For all these reasons, the implementation and active use of the internal credit risk models is likely to have a positive impact within the Czech banking system.

A potential outcome that might be expected is a decrease in the volume of non-performing loans once an active management of credit risk is introduced. Supplementing the traditional subjective analyses, currently focused on borrowers' reputations and bank-client relationships, with more objective methods may lead to a better assessment of the risks and early detection of problem-causing borrowers. Banks could also strengthen their bargaining power relative to problem customers once the prediction of the models assesses that a higher recovery rate should be expected or more collateral should be asked for. This is an important aspect in the

Czech context as long as a common explanation for the high loss rate is that, for example, collateral seizure and asset transfers after a bankruptcy have been rather poorly enforceable until very recently.

A second potential positive impact is the signal that the adoption of internal credit risk models would send to the national regulators and international financial institutions. The Basle Committee, for example, recommends the use of these models even if the estimation of economic capital is still considered inadequate for regulatory purposes. Banks could also more convincingly prove to the national regulators that they are concerned with prudential regulation once they signal that a regular assessment of risks takes place with regard to their operations. At the same time, the effort to adapt to new regulatory pressures could be considerably easier once a bank regularly and accurately assesses the economic capital it must hold. By applying methods that tend to become a standard practice at the international level, Czech banks could also become more competitive and more integrated in the world economy. Another externality that could arise is that rating agencies would become more and more involved in the Czech market and thus would provide a self-enforcing mechanism of data collection.

A third advantage consists in the informative impact not only from the point of view of one's own portfolio but also with regard to the general trends in the economic environment in which the bank operates. Active use of the credit risk models requires, among other things, monitoring of forward rates, equity price movements, migrations between credit classes and interaction among the different business sectors in the economy. Increased awareness about these underlying factors may lead to more efficiency in lending, the discovery of new credit opportunities or better prediction of the obligors' performance.

Nonetheless, active implementation of the internal credit risk models by Czech banks might prove to be a lengthy process due to the lack of data or know-how or simply because strong bank-client relationships may prohibit a more technical assessment of lending practices. Even for the large, international banks, reliable data collection is one of the most difficult aspects in the process of implementation of internal credit risk models. In general, long time series are necessary to construct their historically

based transition matrices and the country- or industry-specific indices. Moreover, defaults are rather infrequent, and information about them is not easily accessible due to a bias in the publicly available information away from negative trends. Czech banks might also face difficulties with regard to effective implementation of the models, at least at an early stage. Usually the models' theoretical constructs are quite sophisticated, and the estimation techniques are built upon advanced programming tools. Therefore, their implementation must be supported by the banks' willingness to invest in know-how, staff training and the acquisition of new software.

An aspect that might potentially endanger credit allocation in the Czech Republic, at least in the short run, is a reduction in bank lending caused by the use of internal credit risk models. This is more likely to happen if the application of the models is done mechanically and disregards specific conditions in the Czech economy. Considering the perceptions of higher risk usually associated with a country in transition, a certain tendency towards pessimistic views and lower credit ratings may appear. If this bias is reflected in the outputs of the credit risk models, then banks might have the incentive to reduce lending below the proper level. This fact, in turn, might negatively affect the functioning of the business sector, particularly in periods of recession. To find the demarcation line between sound lending practices and an unreasonable reduction in credits is hard but, supposedly, this is what the Czech banks will have to do. Probably the use of the KMV or CreditRisk+ models would target this demarcation line with the most precision in Czech conditions. CreditMetrics heavily relies on credit rating systems, which, in turn, would impose large capital requirements on Czech banks. In contrast, KMV is more obligor-specific while CreditRisk+ needs only individual probabilities of default that are not necessarily linked to ratings.

6.2 Recommendations for the Czech Republic

With regard to individual methods, the choice of the most adequate method rests largely on the availability of data and the specificity of the bank's portfolio. Thus, if a large share of the bank's assets represents corporate credits, then the best models are CreditMetrics and KMV. The assumptions of CreditRisk+ are better suited to

portfolios containing a large number of obligors, and thus, its use may be the most appropriate for big retail exposures. In general, CreditPortfolioView is better suited to portfolios containing speculative grade obligors that are more sensitive to the turns of the credit cycle. Of course, idiosyncratic reasons play a role, and banks may chose the method they want to implement based on other considerations, such as costs, availability, expertise, and so on.

Secondly, Czech banks could develop their own credit ratings systems and, eventually, methods to measure credit risk prior to the implementation of the standard credit risk models. In principle, this solution could have some advantages. First, these alternative methods would be simpler, would use available data, better capture the peculiarities of the Czech loan market and, more than likely, would be less costly. Second, the knowledge and expertise acquired with these simpler methods may have a spillover effect during the next stage when the industry-sponsored models are implemented. Third, they could form the basis for the internal approach proposed by the Basle Committee, and therefore, serve two purposes at the same time. Several methodologies could serve this goal: the logit and probit analysis, the discriminant analysis or the linear probability model.

Finally, to avoid data collecting by each bank and thus replicating similar databases many times, central but more complex databases could be created. It would allow each bank to extract the exact information it needs and also to contribute to the database construction.

7 Conclusion

The New Basle guidelines intend to make the estimation of regulatory capital requirements in large, international banks more flexible. Even if the industry-sponsored models are perceived as a potential alternative in the long run, their use for regulatory purposes still has to pass the test of time. The validation of these models from a regulatory perspective still entails problems that are difficult to overcome such as backtesting, stress testing and a sensitivity assessment of the models with regard to the assumptions they employ.

Nevertheless, in developed countries, the credit risk models have gradually acquired recognition within the banking sector. Increased use of these models by banks sends two-way feedback between banks and model developers, and thus continuously add complexity in the models' theoretical framework. The necessity of adopting standards in use at the international level makes the adoption and active implementation of these models an imminent requirement also in the Czech Republic.

This paper offered a survey of the practices in use both from the regulatory and banking sector perspectives. The emphasis put on Czech banks has shed some light on the problems they might face once the models start to be actively implemented in this country. Closely related to the content of this paper, two potential improvements could be made. First, a closer parallel between the regulatory and standard industry

approaches could provide evidence on the similarities and differences between these two approaches. The comparative analysis in this paper was conducted mainly on the basis of the four internal models, but little was done to project them into the regulatory perspective. A second essential improvement should consider the Czech sector more attentively. As Czech banks start to implement internal credit risk models, an analysis of their adapting strategies should be supported by more detailed evidence and, whenever possible, by bank-level data.

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Appendix:

Examples of calculations in individual models

A1 CreditMetrics

The following example[5] shows how CreditMetrics can be used in practice. For simplicity, let us assume that a bank portfolio consists of only two bonds. The characteristics of the bonds are given in Table A1. Tables A2, A3 and A4 contain the forward zero curves, a transition matrix and the recovery rates, respectively. All these elements figure as inputs in the CreditMetrics method and will be used in this exposition to determine the two bonds' risk characteristics.

Table A1

Bank portfolio asset composition

Characteristics	Bond 1	Bond 2
Initial Rating	BBB	A
Face value ($)	100	100
Coupon (%)	6	5
Maturity (years)	5	3
Seniority	2	3

Table A2

Forward zero-curves (%)

Grade Year	AAA	AA	A	BBB	BB	B	CCC
2	3.6	3.65	3.72	4.1	5.55	6.05	15.05
3	4.17	4.22	4.32	4.67	6.02	7.02	15.02
4	4.73	4.78	4.93	5.25	6.78	8.03	14.03
5	5.12	5.17	5.32	5.63	7.27	8.52	13.52

Table A3

The transition matrix (%)

Rating	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.2
CCC	0.22	0	0.22	1.3	2.38	11.24	64.86	19.79

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[5] The majority of data used in this example were taken from the CreditMetrics-Technical Document.

Table A4

Recovery rates

Seniority Class	Recovery Rates (%)
1	53.8
2	51.13
3	38.52
4	32.74
5	17.09

Let us consider the first bond. Assume that it migrates from the initial BBB to the AAA grade over a one-year horizon. Then its value (see formula (1), forward zero curves and bond characteristics) at the end of the first year is:

$$V_1^{AAA} = 6 + \frac{6}{(1+0.036)} + \frac{6}{(1+0.0417)^2} + \frac{6}{(1+0.0473)^3} + \frac{106}{(1+0.0512)^4} = 109.37$$

If the bond migrates to the AA grade, its value becomes:

$$V_1^{AA} = 6 + \frac{6}{(1+0.0365)} + \frac{6}{(1+0.0422)^2} + \frac{6}{(1+0.0478)^3} + \frac{106}{(1+0.0517)^4} = 109.19$$

This procedure continues until the bond is valued in all states. Only in the case of default, the bond value equals the product between its face value and the corresponding recovery rate:

$$V_1^{Default} = 100 * 0.5113 = 51.13$$

The bond value distribution thus obtained is given in Table A5:

Table A5

The value distribution of Bond 1

Year-end rating	Value ($)	Migration Probabilities (%)
AAA	109.37	0.02
AA	109.19	0.33
A	108.66	5.95
BBB	107.55	86.93
BB	102.02	5.30
B	98.10	1.17
CCC	83.63	0.12
Default	51.13	0.18

Applying a similar methodology, the distribution of the second bond is obtained. This distribution is presented in Table A6.

Table A6

The value distribution of Bond 2

Year-end rating	Value	Migration Probabilities (%)
AAA	106.59	0.09
AA	106.49	2.27
A	106.3	91.05
BBB	105.64	5.52
BB	103.15	0.74
B	101.39	0.26
CCC	88.71	0.01
Default	38.52	0.06

Risk calculation at the asset level

The two bonds' means, variances and standard deviations can be easily computed. For example, in the case of bond 1, the mean is:
$$\mu[V_1] = 109.37 \times 0.0002 + 109.19 \times 0.0033 + ... + 51.13 \times 0.0018 = 106.99$$
The variance and standard deviation are equal to:
$$\sigma^2[V_1] = (109.37 - 106.99)^2 \times 0.0002 + ... + (51.13 - 106.99)^2 \times 0.0018 = 8.95$$
$$\sigma[V_1] = \sqrt{VAR[V_1]} = 2.99$$

A similar analysis can be done with regard to bond 2. In this case, the mean, variance and standard deviation are:
$$\mu[V_2] = 106.16 \quad , \quad \sigma^2[V_2] = 2.93 \quad , \quad \sigma[V_2] = 1.71$$
It is obvious that the second bond exposes a lower return but also less expected deviations from this average return. Thus, even if it promises a lower return, it is less risky.

The next step is to derive all the possible values of the portfolio containing the two bonds. Let us assume, for example, that the originally BBB- and A-rated bonds both upgraded to AAA over the one-year period. Then the portfolio value is 215.96, which simply represents the sum of the two bonds' values in the AAA state. Since eight

migrations are possible for each bond, the portfolio value can take any of the potential 64 values given in Table A7.

Table A7
Year-end values for the two-bond portfolio ($)

| | Bond 2 | | | | | | | |
Bond 1	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	215.96	215.86	215.67	215.01	215.52	210.76	198.08	147.89
AA	215.78	215.68	215.49	214.83	212.34	210.58	197.90	147.71
A	215.25	215.15	214.96	214.30	211.81	210.05	197.37	147.18
BBB	214.14	214.04	213.85	213.19	210.70	208.94	196.26	146.07
BB	208.61	208.51	208.33	207.66	205.17	203.41	190.73	140.54
B	204.69	204.59	204.40	203.74	201.25	199.49	186.81	136.62
CCC	190.23	190.13	189.94	189.28	186.79	185.03	172.35	122.15
Default	157.72	157.62	157.43	156.77	154.28	152.52	139.84	89.65

Asset return correlations

In our particular case, let us assume that the firms that issued the two bonds are exposed to the Czech food (CZF) and chemicals (CZC) industries and to the German agricultural machinery (GAM) industry, respectively. Table A8 displays the level of exposure of each firm.

Table A8
Country and industry exposure of the two firms (%)

Index Firm	Czech food (CZF)	Czech chemicals (CZC)	German agricultural machinery (GAM)
Firm 1	70	10	0
Firm 2	0	0	80

In practical terms, this means that 70% of the first firm's volatility of equity returns is explained by the Czech food index and 10% by the Czech chemicals index. The rest of the 20% is due to firm-specific factors. Similarly, 80% in the second firm's volatility of equity returns is explained by the German agricultural machinery index and the rest is due to firm-specific factors.

Also available are tables containing the correlations of the country- and industry-specific indices. For example, in our case, they could look like the elements of Table A9.

Table A9
  Volatilities and correlations of the country- and industry-specific indices

Index	Correlations (%)		
	CZF	**CZC**	**GAM**
CZF	1	0.16	0.48
CZC	0.16	1	0.34
DEE	0.48	0.34	1

According to the specific country and industry exposure, the first firm's standardized asset return can be written as (see equation (2)):

$$R = w_1 x_{CZF} + w_2 x_{CZC} + w\varepsilon \qquad ,$$

where w_1= 0.7, w_2= 0.1 and w is determined in such a way as to make the volatility of the asset return R equal to one. Knowing that all indices are standard normally distributed, the weight of the firm-specific term for the first firm is:

$$w = \sqrt{1 - w_1^2 - w_1^2 - 2\text{corr}(x_{CZF}, x_{CZC})} = \sqrt{1 - (0.7)^2 - (0.1)^2 - 2 \times 0.16} = 0.42$$

The standardized return of the second firm can be written as:

$$R' = w_1' x_{GAM} + w'\varepsilon' \qquad ,$$

which gives a weight of the firm-specific factor equal to 0.6.

In conclusion, the standardized asset returns for the two firms can be written as:

$$R = 0.7 x_{CZF} + 0.1 x_{CZC} + 0.42\varepsilon \quad \text{and} \quad R' = 0.8 x_{GAM} + 0.6\varepsilon' \qquad .$$

The correlation of the asset returns is then equal to:

$$\begin{aligned}
\rho(R, R') &= \text{corr}(0.7 x_{CZF} + 0.1 x_{CZC} + 0.42\varepsilon \ , \ 0.8 x_{GAM} + 0.86\varepsilon') \\
&= 0.7 \times 0.8 \times \text{corr}(x_{CZF}, x_{GAM}) + 0.1 \times 0.8 \times \text{corr}(x_{CZC}, x_{GAM}) \\
&= 0.7 \times 0.8 \times 0.48 + 0.1 \times 0.8 \times 0.34 = 0.3,
\end{aligned}$$

i.e. 30%.

Joint migration probabilities

In the case of the two-bond portfolio, for example, we need the portfolio values (given in Table A7) and the joint migration probabilities to all sixty-four possible states. The way in which these joint migration probabilities are obtained was presented in the main text. In the context of our example, they could look like the elements of Table A10.

Table A10

Joint migration probabilities for the two bonds

Bond 1		Bond 2 (A)							
		AAA	AA	A	BBB	BB	B	CCC	Default
(BBB)		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0	0	0.2	0	0	0	0	0
AA	0.33	0	0.04	0.29	0	0	0	0	0
A	5.95	0.02	0.39	5.44	0.08	0.01	0	0	0
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0	0.02	4.47	0.64	0.11	0.04	0	0.01
B	1.17	0	0	0.92	0.18	0.04	0.02	0	0
CCC	0.12	0	0	0.09	0.02	0	0	0	0
Default	0.18	0	0	0.13	0.04	0.01	0	0	0

$$\mu_p = \mu[V_1] + \mu[V_2] = 106.99 + 106.16 = 213.15$$

Returning to the previous example, the mean of the two-bond portfolio becomes:

The portfolio variance and standard deviation are:

$$\sigma_p^2 = (215.96 - 213.15)^2 \times 0 + (215.86 - 213.15)^2 \times 0 + (215.67 - 213.15)^2 \times 0.2 + ...$$
$$+ (89.65 - 213.15)^2 \times 0 = 11.61,$$

$$\sigma_p = \sqrt{\sigma_p^2} = 3.4$$

Marginal risk contributions

For example, for the two-bond portfolio considered before, adding the second bond to the initial portfolio consisting of only the first bond increases the portfolio standard deviation by 0.41 (thus the difference between the two-bond portfolio's standard deviation of 3.4 and the first bond's standard deviation of 2.99). This value represents

the second bond's marginal risk contribution. Similarly, when the first bond is added to the second bond, it generates an increase in the portfolio's standard deviation of 1.69 (i.e. 3.4-1.71). This value represents the first bond's marginal risk contribution.

A2 CreditRisk+

The following example shows how CreditRisk+ works in practice. Let us assume that a bank portfolio contains assets from ten different obligors. Table A11 shows the bank exposure to each of them and their probabilities of default and expected loss.

Table A11

Basic inputs to CreditRisk+

Obligor i	Exposure ($)	Common Exposure (v_i')	Common Exposure Rounded to multiples of $100.000 (v_i)	Probability of Default (P_i)	Expected Loss $(\varepsilon_i = v_i' \times P_i)$
1	450 000	4.5	5	0.25	1.125
2	225 000	2.25	3	0.3	0.675
3	789 000	7.89	8	0.1	0.789
4	345 650	3.45	4	0.5	1.725
5	275 000	2.75	3	0.3	0.825
6	330 000	3.3	4	0.15	0.495
7	215 000	2.15	3	0.4	0.86
8	125 000	1.25	2	0.7	0.875
9	680 000	6.8	7	0.2	1.36
10	255 000	2.55	3	0.4	1.02

Six different bands have been thus obtained with rounded common exposures of 2, 3, 4, 5, 7 and 8, respectively. Table A12 illustrates the partition of the portfolio in bands and the characteristic elements for each band.

Table A12 – Band partition of the portfolio

Band j	Rounded Common Exposure in band j (v_j)	Number of obligors in band j	Expected loss in band j (ε_j)	Expected number of defaults in band j ($m_j=\varepsilon_j/v_j$)	Probability generating function for band j
1	2	1	0.875	0.4	$\exp(-0.4+0.4z^2)$
2	3	4	3.38	1.1	$\exp(-1.1+1.1z^3)$
3	4	2	2.22	0.5	$\exp(-0.5+0.5z^4)$
4	5	1	1.125	0.2	$\exp(-0.2+0.2z^5)$
5	7	1	1.36	0.2	$\exp(-0.2+0.2z^7)$
6	8	1	0.789	0.1	$\exp(-0.1+0.1z^8)$

The probability generating function for the entire portfolio is the product of individual probability generating functions given in the last column of Table A12. More precisely, in this particular example, it is given by:

$$G(z) = e^{-\sum_{j=1}^{B} m_j + \sum_{j=1}^{B} m_j z^{v_j}} = e^{-2.5+0.4z^2+1.1z^3+0.5z^4+0.2z^5+0.2z^7+0.1z^8}$$

Thus, the probability of no loss is equal to $P_0 = \exp(-2.5)$ or, approximately, 0.08. The probabilities with which the portfolio loss equals multiples of the unit of exposure are then computed according to the recurrence relationship given by (8). In what follows, several such probabilities are derived. Considering that the total amount of bank exposure in this example is \$3 689 650, the maximum loss that can occur is also bound by an upper limit due to this value. Therefore, to derive the full loss distribution, 36 probabilities have to be estimated. The first three probabilities and the last one are computed below.

$$P_1 = P(\text{Loss} = \$100000) = \sum_{\substack{j \\ s.t. v_j \leq 1}} \frac{v_j \times m_j}{1} P_{1-v_j} = 0$$

$$P_2 = P(\text{Loss} = \$200000) \sum_{\substack{j \\ s.t. v_j \leq 2}} \frac{v_j \times m_j}{2} P_{2-v_j} = \frac{v_1 \times m_1}{2} P_0 = \frac{2 \times 0.4}{2} 0.08 = 0.032$$

$$P_3 = P(\text{Loss} = \$300000) = \sum_{\substack{j \\ s.t. v_j \leq 3}} \frac{v_j \times m_j}{3} P_{3-v_j} = \frac{v_1 \times m_1}{3} P_1 + \frac{v_2 \times m_2}{3} P_0 = \frac{3 \times 1.1}{3} 0.08 = 0.09$$

....

$$P_{36} = P(\text{Loss} = \$3\,600\,000) = 0.000275$$

The entire loss distribution is represented in Figure A1. An estimation of the economic capital needed could be obtained based on the VaR methodology presented in Subsection 3.2.

Figure A1

The loss distribution according to CreditRisk+