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Credit Constraints and Creditless Recoveries: An Unsteady State Approach

Alexis Derviz*

Abstract

The paper investigates the behavior of credit demand and output arising from differences in productive capital sources in economies recovering from an adverse real shock. Beside physical capital, another form of capital – human capital – is available during the catch-up phase. Since a part of new physical capital must be debt-financed, whereas production is risky due to uncertain future total factor productivity, defaults happen with positive probability. The latter can be reduced by partially substituting physical capital for human, at a disutility cost. We ask whether a shift away from risky borrowed physical capital to human capital is able to generate a reduction in aggregate credit losses without too big a loss in output, thereby warranting a specific prudential policy. This question is addressed by means of a dynamic stochastic model with feedback decision rules, for which we develop a full-distribution numerical solution method. The long-term stationary limit distribution of the solution generalizes the steady state notion of deterministic models. Agents that start from relatively "poor" initial states are found to benefit from limits on unsecured borrowing at a very moderate cost in output terms, whereas for "rich" initial states, such limits prove to be largely redundant.

Abstrakt

Tato práce zkoumá chování poptávky po úvěrech a výstupu vyplývající z rozdílů ve zdrojích kapitálu pro ekonomiky zotavující se z nepříznivého reálného šoku. Kromě fyzického kapitálu je přítomen jeho další druh, a sice lidský kapitál, který je k dispozici v období dohánění. Poněvadž část nového fyzického kapitálu musí být financována na úvěr, zatímco výroba je zatížena rizikem v podobě nejisté budoucí celkové produktivity výrobních faktorů, selhání dluhu nastává s kladnou pravděpodobností. Tu lze snížit pomocí částečného nahrazení fyzického kapitálu lidským, avšak za cenu sníženého momentálního užitku. Ptáme se, zda odklon od riskantního fyzického kapitálu ve prospěch lidského kapitálu dokáže zajistit redukci agregátních ztrát z úvěrového selhání, aniž by způsobil příliš velký pokles výstupu, což by ospravedlnilo použití specifické obezřetnostní politiky. Tato otázka je zkoumána prostřednictvím dynamického stochastického modelu s rozhodovacími pravidly se zpětnou vazbou, pro který je vyvinuta metoda řešení zohledňující celkové pravděpodobnostní rozložení rizikových faktorů. Dlouhodobé stacionární limitní rozdělení tohoto řešení je zobecněním pojmu pevného bodu u deterministických modelů. Ukazuje se, že subjekty začínající v relativně „chudých" počátečních stavech profitují z uvalení limitů na nezajištěné úvěry při poměrně mírných nákladech z hlediska výstupu, zatímco v případě „bohatých" počátečních podmínek jsou tyto limity do značné míry nadbytečné.

JEL Codes: E22, G33, G38, C61, D92.

Keywords: Credit, dynamic stochastic equilibrium, human capital, prudential policy, recovery.

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Nontechnical Summary

This research aims at investigating bank credit behavior in the aftermath of a major adverse event, e.g., a recession, in a dynamic stochastic rational expectations model.

It is well known that the recovery from the latest crisis was accompanied by very modest growth, or even stagnation, of the usual credit aggregates (a "creditless recovery"), in contrast to most other known post-recessionary growth episodes. One should note that the years following the 2008 financial crisis have witnessed extraordinarily intense efforts by the authorities in the area of financial regulation. On the side of regulated institutions, this change has been accompanied by a somewhat jumpy attitude of major lenders towards both actual and presumed threats of a systemic event in their sector. This hypersensitivity has resulted in recurring reversals of credit flows, first of all cross-border, but sometimes also within borders, in reaction to a number of events or pseudo-events in the market that were, per se, either trivial or inconsequential for actual loan quality in banks' portfolios (such as the so-called "Fed tapering panic" and the "China slowdown panic," not to mention the numerous worldwide alarms issued in connection with the sovereign debt troubles in Europe). The result is an environment in which, due to both regulatory and private lender policy swings, sudden unpredictable lending constraints completely unrelated to borrowers' solvency or liquidity conditions are a fact of life.

In this environment of unpredictable credit squeezes, we look at a recovery after an adverse real shock. Agents have two options for working their way out of recession: they can either borrow a large amount immediately both to buy all the necessary physical capital and to maintain high consumption levels, or employ an additional capital category (human capital) to produce the desired output volumes, at a disutility, but with lower unsecured debt levels. The first variant means a low effort disutility from using human capital, but high credit risk and substantial losses given default. The second means a high effort disutility, but low aggregate credit losses. Our model includes basic attributes of risky lending such as causes and rules of default. The latter can be initiated by the lender as well as the borrower. The presence of human capital as a shunt around the post-recession investment gap is restricted to the intermediate period prior to the long-term steady state phase. At the inception of the latter, the human capital option becomes unavailable, but the average productivity level jumps upwards. This feature makes physical capital catch-up sufficiently attractive.

To facilitate policy analysis, we have constructed a model of lending compatible with the generally accepted framework of DSGE macroeconomics. Here, a somewhat controversial issue is the appropriate equilibrium concept. Our model solution is stated as a recursive Markov equilibrium, i.e., a mapping from the set of initial states to the set of probability distributions of next-period states, given optimal decisions. In particular, the limit behavior of the fundamentals is given by an ergodic distribution of the state vector (i.e., not by a single point in the state space).

Since dynamic optimization models of the aforementioned type are solved backwards starting from the long-term state, "modeling infinity" becomes the principle analytical challenge. In this regard, the philosophy of the present study requires one to look at long-term equilibrium as not just a point, but a whole carefully specified dynamic universe. This means rejecting the popular seduction of "linearization in the neighborhood of a non-stochastic steady state." Indeed, our findings would be either entirely outside the scope of popular non-stochastic steady state pseudo-analysis or, at the very least, badly misrepresented by it, given that the fixed point in the future to which the system is assumed to converge would be inconsistent with debt explosion in the intermediate periods.

Does this methodology have any other benefits (beside the minor consideration of replacing a formally faulty analysis with a correct one)? Some are visible from the policy perspective. If one narrowed the focus to a single long-term combination of fundamentals, long-term analysis of policy alternatives would be ruled out: the workings of any policy tool considered would be transient by construction, as the tool would only address transient phenomena. On the contrary, as a by-product of a full-distribution analysis, one is allowed to consider risk-return trade-offs not just for private individuals, but also for policymakers.

Under what circumstances is a prudential policy that penalizes unsecured debt during the recovery phase able to improve aggregate welfare by reducing present and future losses given default? Our model gives an answer that depends on the starting position of the economy. Roughly speaking, we find that initially "rich" economies, i.e., those with high current productivity and low unsecured debt levels, choose to behave prudently on their own and do not need an explicit policy to make them rely on human capital more than debt-financed physical capital. On the other hand, initially indebted and low-earnings economies might benefit from a policy that limits future unsecured debt. The reason is an immediate benefit from defaulting on the current debt stock, as the latter has a high probability of snowballing into even less sustainable indebtedness in subsequent periods.

Liberated from the artificial focus on a single point in the distant future, this description of state vector evolution returns the analyst's attention to the critical importance of the starting point of the analysis. This may be a setback for theorists accustomed to pronouncing their judgments in terms of the inevitable future state of affairs with just token concern for the initial conditions of the economy. Although easier than deriving state-conditioned prescriptions, such an approach is all the more wrong. Conversely, policy advice based on our approach has a chance to better differentiate across the variable national and cyclical realities for which such advice is being sought.

1. Introduction

The macroeconomic implications of credit cycles have moved to the forefront of research in financial economics after the experience of the world economy with the latest financial crisis and the subsequent Great Recession. This experience was in fact twofold. At the apex of the crisis, the economy got to feel the consequences of too light regulation of risky debt. Several years and several waves of regulatory crackdown on careless lending later, we find ourselves – at least in the developed world – in a new economic reality where growth in not only excessively risky credit, but virtually all credit, has come to a halt and, combined with the burden of legacy debt, is now effectively thwarting the much-needed return to pre-crisis growth standards.

A natural question is what effect randomly arriving credit constraints – originating either in purely exogenous financial shocks or in official prudential policies – are likely to have on aggregate credit risk, credit losses, and output in economies with varying dependence on debt-financed productive investment.

The real economic implications of investment financed by risky imperfectly secured debt are only gradually arriving at the center of standard macroeconomic modeling (see Clerc et al., 2015, for an analysis within the conventional DSGE framework). The present paper proposes a new perspective on the issue by setting up a dynamic model of a production economy with stochastic total factor productivity (TFP) and a positive probability of credit restrictions, for which full distribution solutions are calculated. The analysis starts at "infinity" by defining the long-term equilibrium (LTE) dynamics of consumption, investment, and debt under stationary random shocks to productivity and credit availability. The model possesses an LTE with positive probabilities of both endogenous and exogenous defaults. The next step is to investigate the period preceding the LTE phase. In particular, we look at a point of departure marked by a major adverse supply shock, from which agents need to recover in terms of capital accumulation and credit quality. In addition, to give a meaningful role to prudential regulation, we want the economy to be able to choose between quick but risky and slower but financially safer paths of recovery. This is done by giving the agents space for different intra-period uses of alternative resources on the financing and capital formation side, keeping the long-term dynamics fixed. Then we ask to what extent prudential regulation, which tries to push the economy in the latter direction, avoids unintended consequences on the borrower side, such as regulatory overkill, which above all damages financially fragile agents, or unstable borrowing behavior. In addition, we would like to relate the prudential policy stance to the aggregate credit losses following its implementation.

Given our preoccupation with the macroeconomic role of credit quality, we investigate the non-diversifiable probabilistic side of mass default and bankruptcy. In line with the prevailing convention in the theoretical literature, the central point of our analysis is a dynamic optimization problem of a typical investor-consumer in the presence of risky debt. This element appears in all existing models in this area, be they of the partial or general equilibrium variety. However, to move forward in addressing the aforementioned research questions, we need an appropriate numerical solution technique for this class of dynamic stochastic models. Indeed, when randomness (in our case, of borrower performance and credit availability) is present in every period, arguments based on analyzing a non-stochastic steady state neighborhood are of little help, given that the actual limit behavior of the state variables in the models in question follows an ergodic distribution. Its derivation and calculation constitute a considerable technical challenge. A major part of the background research underlying the present paper rotated around addressing this challenge. Eventually, a relatively simple method tailored to the problem class in question was developed and quantitative inferences derived.

Accordingly, the main original methodological contribution of the presented study is a numerical procedure solving the representative investor-cum-consumer agent's optimization problem. This procedure recognizes and exploits the convergence of the corresponding state variable vector to a limit random vector at infinity. The solution method for the long-term equilibrium dynamics is developed so that it can be conducted at an arbitrary degree of precision. After that, we proceed backwards in time by considering a period with the supply side properties modified in comparison to the subsequent long-term stationary dynamics. The model characteristics, such as the synthesis of exogenous and exogenous defaults, human capital utilization as a path around excessive credit risk, and optimal reactions to an unexpected credit supply squeeze, can all be seen as features of the testing ground on which we demonstrate the potential of the method. The present application remains (except for the labor market aspect, which plays a subordinate role in the current analysis) within the partial equilibrium category. However, subsequent extensions to full-fledged general equilibrium models will be a natural next step.

Default in the model happens with positive probability and can be initiated both by the lender and by the borrower. In principle, default can be avoided altogether if new loans of the necessary size are allowed to replace the old problematic ones. Conversely, if sudden refinancing stops are possible, there is space for default. We incorporate such sudden stops, arriving at random, in the basic risk structure of the model. A stop will cause default only if the debt is not fully collateralized by the totality of the borrower's assets. We will also associate prudential intervention with a ban on new unsecured debt in the current period. Besides that, the debt workout procedure may offer borrowers with certain characteristics, such as a substantial unsecured legacy debt, an attractive option of voluntary default. This means that we allow for both exogenous and endogenous defaults. The two named features, i.e., the possibility of debt rollover opportunity withdrawal and subsistence payments guaranteed at personal bankruptcy, will together work as a realistic backstop against unsustainable debt accumulation, extending the more abstract no-Ponzi-game restriction of many macroeconomic models.

This paper entertains the hypothesis that the effect of credit-targeting regulatory policies must depend on the relative importance of human, compared to physical, capital on the asset side of non-financial private agents. The reason is that physical capital is rarely financed with zero recourse to credit, whereas accumulation of human capital does not have such a direct relation to debt financing. For example, in economies experiencing a reduction in traditional ("daylight") bank credit supply and the weight of debt financing shifting towards shadow banks, physical capital markets may become tighter if shadow banks have a strong preference for borrowers with low opacity (e.g., big corporates to the detriment of SMEs and natural persons). As a result, one should expect more intensive use of human capital input. Eventually, the economy saves and invests accumulated earnings up to the optimal physical capital level in the long run. However, it takes longer, and with an ex ante higher utility sacrifice, to move along this safe path than in a world in which one could borrow all the necessary funds instantly, even though at high risk.

Among the main results of the present exercise are the following.

The presence of human capital can lead to multiple locally optimal borrowing and investment plans for a certain range of initial conditions (such as initial wealth or current after-interest income). A policy that discourages unsecured lending is not needed when the agent starts as rich, but can improve the welfare of initially poor ones.

Multiplicity of local optima naturally leads to instability in every period around the switching points in the initial condition space at which the globally optimal solution jumps to a new location. The effect appears only in economies with human capital (and consequently is not a part of the long-term equilibrium dynamics). Importantly, this discontinuity of globally optimal behavior is not an attribute of some extreme state of nature with a negligible probability, but happens over a range of fundamental values close to their means and modes, pointing at economic relevance of the phenomenon.

Unsecured debt, which we can roughly associate with extra leverage or bank money additionally created each period in excess of the available reserves, plays two roles. It allows debt rollover and survival for borrowers with insufficient cash positions (a too low realized TFP or too high legacy debt service) and hence reduces loss given default (LGD) in the current period. At the same time, it creates a non-negligible default frequency and aggregate LGD next period. The latter would be impossible if the regulators issued a one-period ban on, or a similar restrictive measure against, unsecured debt. We find that numerical solutions of the model under a reasonable calibration generate unsecured debt and future LGD lying very close together, whereas the current-period LGD pre-empted by the unsecured debt extension is typically much lower. That is, it seems that, in our model, "money" printed to meet credit demand and thereby buy relief from debt overhang now, is bound to be largely used up on credit losses in the immediate future.

1.1 Related Literature

The present research sidesteps the prevailing body of macro literature dealing with financial frictions, with its too heavy dependence on linear around-the-steady-state analysis within the usual DSGE framework (this tradition started with the paper at the inception of the whole DSGE-with-financial-contracts literature, i.e., Bernanke et al., 1999, and has continued ever since; see, for example, Cristiano et al., 2008, or Clerc et al., 2015). There have been attempts to overcome the original steady state limitations which introduce the concept of a risk-adjusted steady state (e.g., Coeurdacier, Rey, and Winant, 2011, and Gertler, Kiyotaki, and Queralto, 2012). These extensions, although not solving the underlying conceptual problem completely, have the merit of recognizing its importance. Nevertheless, to get meaningful results about the interplay between financial credit imperfections and real activity, one needs, while still focusing on intermediate periods, a more convincing description of the long-term dynamics than what is usually offered by the DSGE class.

The general set-up of our model in terms of borrowing, investment, production, and consumption by a rational infinitely-lived household under random TFP shocks has been common ground in macro models with financial contracts since, essentially, Bernanke et al. (1999). The general setting of the yeoman economy (i.e., households directly controlling firms) may be seen as similar to that of Jermann and Quadrini (2012). The role of human (intangible) capital in both defaults and post-crisis recovery in our model has some general similarities with the intangible capital variable in Garcia-Macia (2015), although we do not emphasize the financing aspect of intangible capital as much as that paper.

Examples from the financial economics literature of working with full distribution solutions of stochastic dynamic models are less frequent than the mentioned "locally linearized" DSGE variety. Sometimes it helps to state the problem in continuous time at the cost of greatly restricting the class of risk distributions considered (He and Krishnamurthy, 2012; Brunnermeier and Sannikov, 2014). In discrete time, a global solution approach with some elements resembling our own, including calculation of the long-run equilibrium distribution of the state variable vector, can be found in Mendoza (2010). His numerical procedure employs an advanced and complex multidimensional finite element scheme.

Asynchronous credit and business cycles are both known from past recessions and captured – to some degree – by academic literature. It is widely acknowledged, at least for the United States, that the period (of about two years) immediately following the Great Recession most likely falls into the category of creditless recoveries (Abiad et al., 2011). According to recent data, Europe has been even more affected in this respect than North America, and the history of emerging markets contains a sufficient number of creditless recovery cases as well (Sugawara and Zalduendo, 2013). On the other hand, there is currently no canonical macroeconomic theory of the quantitative relation between the business and the credit cycle. If the two are not identical, then an economy that is not making use of some available bank credit should clearly rely on an alternative production model. As one possibility, it may draw on alternative sources of financing, for instance within shadow banking. Unfortunately, relatively little has been done so far to model the differences between shadow and daylight commercial banking in a broader dynamic economic context.

The history of economic thought knows several theories of capital misallocation as a result of overinvestment in bubbly assets at the origin of business cycles (Garrison, 2004). Conversely, theories of debt quality feeding into real economic activity are not so numerous. Mostly, such papers concentrate on explaining real activity and asset price data by an appropriately stylized credit friction. For that literature, credit is a tool needed to generate a directly unobservable financial shock, not a study object per se. In many cases, default as such is not even considered (see, for example, Jermann and Quadrini, 2012, and Shi, 2015).

Credit extended by different types of institutions has different implications for the balance sheets of both financial and non-financial agents. As emphasized recently by, for example, Benes, Kumhof, and Laxton (2014), daylight banks, with their legal power to create deposits as a part of lower-order monetary aggregates (buttressed by privileged deposit insurance and lender of last resort access in exchange for government regulation), would be much better represented by models that take account of their purchasing power creation ability ("medium-of-exchange models") than by standard "loanable funds" models, which reduce banks to mere intermediaries between savers and investors. (The economics on the monetary side of the accounting distinction in question was explored long ago in the "missing money" literature; see, for example, Goldfield, 1976, and Duca, 1992.) On the contrary, the loanable funds approach seems to be much better suited to describing the operation of the shadow banking sector. The modest role of shadow banking in generating predictable purchasing power is related to the limited ability of the markets to price shadow bank liabilities at least as accurately as they manage to price the usual claims on conventional commercial banks.

The latest financial crisis stimulated a lot of academic and policy research on prudential policies. The initial focus of attention was, understandably, daylight banks, since covering their crisis-related losses was associated with dramatic expansions of the balance sheets of several leading central banks (the already mentioned contribution by Benes, Kumhof, and Laxton, 2014, allows for an elementary explanation of this phenomenon), leading to a new, historically unique situation on the monetary policy front across the world. Naturally, the bulk of the considered policies addressed excessive leverage. This could not have failed to chase regulated activities away from the daylight into the shadow banking sector and lead to a corresponding slump not just in bank debt-conditioned leverage, but also in lending per se. The effect was hardly unanticipated: theoretical literature on prudential policies often predicts that high bank capital requirements will at some point become costly to economic activity and welfare (see, for example, Van den Heuvel, 2008, Derviz, 2012, and Clerc et al., 2015).

As regards debt and default, we depart from standard financial accelerator models by, first, giving the lender market power and, second, allowing for default on the unsecured portion of the debt with explicit losses for the lender. In doing so, we extend the previously developed "curtailed" semi-dynamic models with unconditional debt and defaults in equilibrium, as developed in Derviz (2012, 2014). Curtailing here means two things. First, there is a limited number of explicitly analyzed periods (usually just two or three), with evolution in subsequent periods summarized by exogenous continuation values of agents' decision problems. Second, the said continuation values are derived from a simpler model of infinite-horizon dynamics not involving the transient phenomena of interest.

The rest of the paper is organized as follows. Section 2 defines the model. Section 3 derives the optimal decision rules and the equilibrium. Section 4 explains the numerical solution of the model, provides calculation results, and discusses policy implications. Section 5 proposes avenues of future research and concludes. Proofs of technical statements are collected in the Appendix.

2. Model

There is a continuum of identical small agents (or households) that work, borrow, invest, produce, and consume, i.e., we consider a yeoman economy with backyard production. Therefore, in the sequel, the optimization problem and its solution will be derived for one representative household. This household has one member who works and another who runs the backyard firm.

The agent has one – even though infinitely divisible – unit of labor that is sold to other firms inelastically. The labor market is perfectly competitive so that each firm takes the wage as given.

Besides the yeoman household, there is also a representative bank which plays a passive role in the model. It grants loans, the volume of which is not a priori limited by any exogenous amount, although it can sometimes be constrained by the collateral the firm is able to offer. There is also a passive producer of physical capital who sells it to – or buys from – the firm at unit price without ex ante limits.

Time is discrete and the horizon is infinite.

Investment

There are two possibilities to invest: in a safe asset (cash) with gross rate of return R^M after one period, and in physical capital used in production. The firm can also borrow from the bank for one period at gross rate $R^B > R^M$. The amount of cash at the end of period t will be denoted by M_t, the debt taken in period t by B_t, and the physical investment undertaken in period t by I_t. Short positions in cash are not allowed ($M_t \geq 0$ for all t; in fact, an even stronger restriction will be imposed).

Cash holdings

Unless constraints on the minimum levels of cash buffers are imposed, agents would never choose to hold a positive M balance. Debt repayment is always preferable for the reason of the assumed relation between R^M and R^B, whereas investment in K, even adjusted for risk, dominates that in M provided the optimal choice of the marginal product of capital is made. So, we assume a mandatory prudential cash buffer in the form

$$M_t \geq \varphi K_{t+1} \tag{1}$$

for every t, with φ a small positive constant. This constraint will always be tight in the problems considered here.

Human capital

During intermediate periods, i.e., ones not belonging to the long-term equilibrium end-stage of the model, there is a second category of capital besides the physical one. We call it human capital (uppercase H with the corresponding time subscript in the notation) and think of it as individual ability the household can engage for the benefit of its own production if it is willing to accept a disutility. The latter will be assumed to be an additive term in the period utility function, equal to zero at the origin and with convex dependence on $H > 0$.

Human capital generated in one period is available for production in the subsequent period in that it is combined with the physical one, K, by means of a CES capital index Q,

$$Q = \gamma^{\frac{1}{1-\theta}} \left[\gamma^{\frac{1}{\theta}} K^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} (\varepsilon H)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

that enters the production function. Here, ε is the weight of human capital in the capital index that is allowed to be both above and below unity. Note that the usual CES expression is corrected by factor $\gamma^{1/(1-\theta)}$ so that Q reduces to the usual physical capital K as soon as $H = 0$.

The following features characterize human capital in our model, some of them distinguishing it from the conventional labor input:

- H is non-transferrable between households (i.e., can be only used in the household's own firm)
- It does not have a wage or other price depending on which it would be supplied in a bigger or smaller quantity; differently from labor it is not transacted in a market that has to be cleared
- It does not have an exogenous supply limit (any positive value can be chosen)
- It cannot be pledged as collateral
- It depreciates completely after one period, so that every new production cycle requires a new decision about its provision.[1]

Production

In period t, physical capital K_t formed at the end of period $t–1$ is used in production, after which it depreciates at rate δ. After new investment is made, the new capital stock

$$K_{t+1} = (1-\delta)K_t + I_t \tag{2}$$

is carried over to produce in period $t+1$. The same one-period delay holds for the utilization of human capital. After Q has been formed in period t by combining K_t and H_t, the usual Cobb-Douglas production function, magnified by a random total factor productivity factor A_t, is used to produce output:

$$Y_t = A_t Q_t^{\alpha} L_t^{1-\alpha} = A_t Q_t^{\alpha}. \tag{3}$$

(The second equality follows from normalization of the labor input to unity.) All firms are assumed to be ex ante identical at the end of period $t–1$ with respect to the future random realization of A_t. After the input choices are made, firms are endowed with different TFP values depending on the distribution of A_t.

After the value of A_t has been realized, production takes place and wages are paid. Due to the assumption made earlier about competitiveness of the labor market, the wage must be equal to the labor share $(1-\alpha) A_t Q_t^{\alpha}$.

Distributions of TFP in individual periods are assumed to be independent. They are also identical within the two stages considered: the intermediate, when human capital is available, and the final (for which long-term equilibrium, LTE, will be derived and calculated). However, we assume a lower mean of the TFP distribution in the intermediate periods, up to the last date on which H can be generated. In the subsequent period, output will still be generated with H, K, and L inputs by a production function with low average A, but only K can be picked for future use. The next production cycle, i.e., the first one at the LTE stage, already enjoys a high average TFP. The jump in average productivity approximately compensates for the no longer available additional production input. One should think of this arrangement as an opportunity for an economy hit by an adverse supply shock in the past to choose the optimal recovery path by balancing quick new physical capital investment with borrowed funds against more gradual capital accumulation with

[1] The assumption of a 100% depreciation rate is not central to the analysis, but allows us to highlight the advantageous and disadvantageous effects of "knowledge-based" production more easily. Particularly, in this way one rules out choices of high H levels at the beginning with only marginal additions to it afterwards, or the volatility of investment in H copying that of productivity.

less debt. At the end of this recovery phase one gets the same high-productivity economy, so the main dilemma is between converging to it at the cost of high effort or high credit risk.

Borrowing

As mentioned before, loans are for one period. Unless default occurs (see below), the agent repays the legacy debt from the previous period, B_{t-1}, and chooses the size of the current period loan, B_t. In each period, there are two possible credit regimes: unconstrained, or regular, and constrained. Their arrival is random and independent of the TFP distribution. In the regular regime, happening with probability $\xi \in (0,1)$, any value of B can be chosen regardless of the current balance sheet state or earnings (i.e., the TFP realization) of the firm. For simplicity, we assume that there is no default option in the regular regime.

In the constrained regime, the probability of which is $1-\xi$, B may not exceed the sum of the envisaged values of physical capital and cash:

$$B_t \leq K_{t+1} + M_t. \tag{4}$$

That is, the principal of the new loan must be fully collateralized.[2] This introduces two frictions into the decisions of the agent. The first one is an outright reduction of opportunities: in the constrained regime, if earnings are insufficient to repay the old debt, maintain the mandatory cash buffer, and consume a non-negative amount, the household is forced to default. The rules of default will be defined shortly. The second friction is a new choice that follows from the default option: it may happen that default is preferable to debt service even though earnings formally suffice to maintain a positive consumption level in the current period. The exact conditions follow from the modalities of the default procedure, to be specified next.

Default

A defaulting firm loses its assets, K and M, to the lender bank, and its liabilities, B, are written off. It is assumed that even a firm in default is obliged to pay outstanding wages, which means that the labor share is subtracted from the notional earnings and added to the bank loss. At the same time, we assume that the working member of the household earns wages in some other firm than its own backyard one, which is why he considers the labor income as given and does not internalize the effect of the household's own capital choices on the wage.

In the period when default happens, the only income the household earns is the wage of its working member. Not all of it is necessarily consumed; the agent is free to save a portion, adding it to the new physical capital stock. This is to be expected particularly when default is caused by a high level of legacy debt, whereas the current TFP is sufficiently high to provide an attractive level of wage income. Under all other circumstances causing default, the agent consumes the whole labor share and borrows up to the collateral constraint in the current period: $B_t = K_{t+1} + M_t$. In other words, the agent finances asset acquisition for the next period entirely by debt and pledges these assets in full as the new collateral.

[2] Since the debt is due next period, one could have also added interest on M and the return on capital, simultaneously augmenting the due amount by the loan interest. However, the resulting changes in the definition of the credit constraint would play a negligible quantitative role while adding a lot of complexity to the algebra. So, for the sake of simplicity, we ignore the quantitatively insignificant discrepancy between the realized return on assets and due interest when assessing the collateral value.

As can be seen, the model accommodates both exogenous (when negative after-interest income is incompatible with non-negative consumption) and endogenous (when declaring default is preferable due to an attractive outside option) defaults. This is a step forward compared to many other models of production economies with financial frictions, in which either equilibrium allows no default at all or the latter is imposed by an exogenous rule not sufficiently anchored in individual choices.

Timeline of events and decisions within a period

At the beginning of period t, all agents are ex ante identical with respect to future TFP realization in that period. The representative agent inherits cash, physical capital, and the debt to be repaid. Then the agent learns whether the credit regime in that period will be regular or constrained. The backyard firm manager-member of the household learns the value A_t, hires labor, and produces. Physical capital depreciates after the conclusion of the production cycle. The household then decides whether to default or not. Next, the household chooses new levels of cash, investment in human (at the catch-up stage of the economy) and physical capital, and bank credit, subject to the standing constraints on cash (always) and loan size (in the constrained regime only). The household consumes and period t ends.

Optimization problem

The only actively optimizing agent in this version of the model is the yeoman household. The initial conditions in period t are A_t, H_t, K_t, M_{t-1}, and B_{t-1}, whereas the decision variables in that period are H_{t+1}, K_{t+1}, M_t, and B_t. The resource constraint in period t is

$$M_t + I_t - B_t + C_t = A_t Q_t^{\alpha} + R^M M_{t-1} - R^B B_{t-1} \tag{5}$$

if the firm repays the debt and

$$M_t + K_{t+1} - B_t + C_t = w_t \tag{6}$$

if it defaults. Here, w_t stands for the wage, which is taken as given by the working member of the household and is equal to $(1-\alpha)\ A_t Q_t^{\alpha}$ (recall that the labor force size is normalized to unity). Clearly, since default may only happen in the constrained regime, in which B_t is not allowed to exceed $K_{t+1} + M_t$, one is able to maintain non-negative consumption under all circumstances by moving the level of B up sufficiently close to the collateral level whenever the constrained regime applies.

Subject to the resource constraints and the relevant constraints on cash holdings and borrowing, at time zero the household maximizes the expected lifetime utility

$$E_0\left[\sum_{s>0} \beta^{s-1}\big(u(C_{s-1}) - v(H_s)\big)\right] \tag{7}$$

with respect to consumption and investment plans $(H_s, K_s, M_{s-1}, B_{s-1}, C_s)_{s>0}$, given the initial conditions $(A_0, H_0, K_0, M_{-1}, B_{-1})$. Expectations are taken over future TFP and credit regime realizations. Parameter β is the usual time-preference factor. Utility u of consumption has the usual growth, concavity, and behavior-at-infinity properties (in the calculations, a power utility will be used). Disutility v of human capital use is strictly growing and strictly convex on the positive half-axis, and zero at the origin (in the calculations, we take it to be a power function with exponent greater than one). In view of the form of the resource constraints, there is a one-to-one correspondence between choices of B and C, so that it is sufficient to characterize the optimal borrowing.

3. Solution of the Model

3.1 First-order Conditions and Credit Constraints

Before proceeding to the solution characterization, it is useful to introduce auxiliary notation. So, let $N_t = K_{t+1} + M_t - B_t$ denote the net assets of the agent held at the end of period t. This variable is allowed to be of any sign in the regular credit regime and must be non-negative in the restricted regime. Next, denote the period t Lagrange multipliers of the cash and the credit constraint by Λ_t^M and Λ_t^B, respectively. Clearly, the latter multiplier is always zero in the regular credit regime.

It is not difficult to figure out that, given the values (H_t, K_t, M_{t-1}, B_{t-1}) inherited from the previous period, the agent in period t faced with the credit constraint regime defaults for low TFP values A_t and repays the debt for high ones. The formal statement uses the notion of effective return on physical capital,

$$R^K = (1+\varphi)R^B - \varphi R^M = R^B + \varphi\,(R^B - R^M), \tag{8}$$

which is equal to the borrowing rate plus a mark-up equal to the mandatory cash-to-physical capital ratio times the spread between the borrowing and the savings rate. R^K will appear in many statements characterizing equilibrium behavior in the sequel. The exact result on the default threshold is summarized in the following.

Lemma 1 *If, in period t, the credit-constrained regime realizes, there exists a TFP value A^D such that firms with A_t realizations below A^D choose to default on debt B_{t-1} and those with A_t realizations above A^D choose to repay. If, in period t–1, the cash constraint (1) was tight, the default threshold value is given by*

$$A^D = \frac{(R^K - 1 + \delta)K_t - R^B N_{t-1}}{\alpha Q(H_t, K_t)^\alpha}. \tag{9}$$

If A^D in the above expression is non-positive, all firms repay the debt.

Proof: see the Appendix.

When, in the constrained regime, the realized TFP value is such that constraint (4) is tight but the firm does not default, the choice of B (or, equivalently, C) is trivial: one borrows exactly the targeted asset level $M + K$ as given by the right-hand side of (4). This happens for TFP values below the threshold value, which we denote by A^T. When $A^T < A^D$, all surviving firms optimally borrow strictly below the limit; in the opposite case, there are some surviving credit-constrained firms. Naturally, at every time t, both thresholds are functions of the initial conditions (A_t, H_t, K_t, M_{t-1}, B_{t-1}). We will use the simplified double-index notation A_{t+1}^D, A_{t+1}^T whenever this does not cause ambiguity.

According to our assumption regarding the subsistence income at default, the latter is taken as given, i.e., the dependence of wage income on own investment decisions is not internalized by the household member who runs the firm. Accordingly, when this firm manager calculates expectations conditional on the credit-constrained regime in force, only period utilities of consumption for TFP realizations above the default threshold A_{t+1}^{D} are in his eyes dependent on the choice variables. For firms that enter period t as very rich (i.e., they have high positive N_{t-1} values and/or high A_t realizations), it may happen that, even for zero TFP, default is suboptimal, in which case we set A_{t+1}^{D} equal to zero.

Let us denote by ψ the density of the TFP distribution (recall that it is the same for all periods in each of the two stages of the model). The preceding discussion was meant to explain that the portion of the expectation of any random variable z of the next-period TFP A_{t+1} conditional on time t information that may depend on the decision variables is given by

$$E_t^R[z] = \xi \int_0^\infty z(X)\psi(X)dX + (1-\xi)\int_{A_t^D}^\infty z(X)\psi(X)dX. \tag{10}$$

Superscript R in this expression should remind one of the repayment decision necessary to generate quantities dependent on own choices. The first term in (10) is the expectation conditional on the regular credit regime in place (the probability of getting this regime is ξ). The second term corresponds to the constrained credit regime (which occurs with probability $1-\xi$), when the conditional expectation is only taken over high TFP realizations not causing default.

It is useful to establish once and for all that the solutions we are interested in imply tightness of the M constraint at all times. The following lemma is proved in the Appendix.

Lemma 2 *Assume* $R^B > R^M$. *For consumption and investment plans that solve optimization problem* (5)-(7), *the constraint on cash is always tight. Accordingly*, $M_t = \varphi K_{t+1}$ *for all t.*

Another supporting statement links the first-order optimality condition for physical capital in the agent's optimization problem with the conventional marginal product of capital formulae. The exact statement is summarized as

Lemma 3 *If cash and credit volumes are chosen optimally, then the first-order condition of optimality for physical capital* K_{t+1} *chosen at time t is*

$$E_t^R\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}\left(A_{t+1}\frac{\partial Q(H_{t+1},K_{t+1})^\alpha}{\partial K_{t+1}} - R^K + 1 - \delta\right)\right] = 0. \tag{11}$$

In periods when human capital is present, its first-order optimality condition is

$$E_t^R\left[\beta u'(C_{t+1})A_{t+1}\frac{\partial Q(H_{t+1},K_{t+1})^\alpha}{\partial H_{t+1}}\right] - v'(H_{t+1}) = 0. \tag{12}$$

Proof: see the Appendix.

The above lemma states that, in expectations corrected for the price kernel valuation of future TFP realizations ("the risk-neutral density"), the marginal product of capital must be equal to the net effective return on capital, R^K–1, plus the depreciation rate.[3]

Let us define function G as follows:

$$G(A,H,K,N) = AQ(H,K)^{\alpha} - (R^K - 1 + \delta)K + R^B N.$$

At the LTE stage of the model, it has a simpler counterpart

$$G^{LT}(A,K,N) = AK^{\alpha} - (R^K - 1 + \delta)K + R^B N.$$

The agent's consumption in period t, provided there is no default, is equal to earnings less net investment:

$$C_t = A_t\, Q(H_t,K_t)^{\alpha} + (1-\delta)K_t - R^B B_{t-1} + R^M M_{t-1} - M_t + B_t - K_{t+1}$$

(note that the labor income, subtracted from the earnings of the firm-owning member of the household, is added again as the income of the working member, i.e., it is cancelled out in the consolidated household budget). Using Lemma 2 and the definition of net assets, the above expression can be rewritten as

$$C_t = A_t\, Q(H_t,K_t)^{\alpha} - (R^K - 1 + \delta)K_t + R^B N_{t-1} - N_t = G(A_t,H_t,K_t,N_{t-1}) - N_t. \quad (13)$$

Thus, the G term stands for the after-interest earnings of a firm that does not default. In a defaulting firm, the G term is replaced by the labor share.

3.2 Equilibrium Conditions

The rest of this section derives a parsimonious representation of the equations characterizing equilibrium, to be used in the calculations.

The value function of the agent's problem is equal to the objective function (7) calculated at the optimal consumption and investment plan. In general, as follows from (13), it is a function of initial values A, H, K, N (i.e., it is sufficient to consider one variable for net assets instead of M and B separately). Equation (13) allows us to achieve a further reduction of the number of variables. Clearly, in non-constrained states of nature, that is, when either credit is unconstrained or $A_t > A_t^T$ in the constrained regime, the value function can be considered a function of the single variable $g_t = G(A_t,H_t,K_t,N_{t-1})$. The first derivative of this function will be denoted by j (or by h at the LTE stage of the model). It turns out that the first-order conditions of optimality for the agent's problem can be reduced to a system of three (or two at the LTE stage) integro-difference equations involving j or h and next-period feedback rules $\boldsymbol{H}$, $\boldsymbol{K}$ for human and physical capital as functions of g_t.

[3] Although the result is quite in line with the numerous similar ones given by textbook models of optimal investment, one should note the presence of default contingencies in the operator used to calculate the expected marginal product of capital. The influence of default will turn out to be quantitatively important in the model solution.

Define by q_K (q_H) the partial derivative of the capital index Q w.r.t. K (H). It can be easily checked that these derivatives can be written as

$$q_K(H,K) = \left(\frac{Q(H,K)}{K}\right)^{\frac{1}{\theta}},\ q_H(H,K) = \varepsilon\left(\frac{Q(H,K)}{\varepsilon H}\right)^{\frac{1}{\theta}}, \tag{14}$$

whereby q_K collapses to unity in periods in which human capital H is identically zero, i.e., at the long-term equilibrium stage.

Proposition 1 *Conditioned on the subspace of the even space in which the firm is not credit-constrained and does not default, the value function* $V(A_t,H_t,K_t,N_{t-1})$ *of the agent's problem in period t is a function J of the single variable* $g_t = G(A_t,H_t,K_t,N_{t-1})$. *This function is smooth and its first derivative j is equal to the marginal utility of optimal consumption*:

$$J'(g_t) = j(g_t) = u'(g_t - \boldsymbol{N}_t). \tag{15}$$

Here, $\boldsymbol{N}_t$ *denotes the optimal (unconstrained) choice of net assets for period t.*

The first-order conditions of optimality for unconstrained investment plans $(\boldsymbol{H}_s,\boldsymbol{K}_s,\boldsymbol{N}_{s-1})_{s>0}$ *can be stated as*

$$j(g_t) = \beta R^B E_t^R[u'(G(A_{t+1},\boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1},\boldsymbol{N}_t) - \boldsymbol{N}_{t+1})] \tag{16}$$

(*the Euler equation*),

$$E_t^R[u'(G(A_{t+1},\boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1},\boldsymbol{N}_t) - \boldsymbol{N}_{t+1})(A_{t+1}\alpha Q(\boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1})^{\alpha-1}q_K(\boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1}) - R^K + 1 - \delta)] = 0 \tag{17}$$

(*the marginal product of physical capital equation*), *and*

$$E_t^R[u'(G(A_{t+1},\boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1},\boldsymbol{N}_t) - \boldsymbol{N}_{t+1})A_{t+1}\alpha Q(\boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1})^{\alpha-1}q_H(\boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1})] = v'(\boldsymbol{H}_{t+1}) \tag{18}$$

(*the marginal product of human capital equation*).

Proof: see the Appendix.

Although it provides information on all three decision processes of the agent, the equation system (16)-(18) would be incomplete without a proper characterization of the credit tightness threshold A^T. This is because the operator E_t^R introduced in (10) may include integration over non-default but credit-constrained states of nature in situations in which $A_{t+1}^D < A_{t+1}^T$. To summarize the properties of A^T (in Lemma 4 below), we first need to introduce the auxiliary notion of borderline values of the choice variables.

Let, in period t, the legacy excess debt be zero: $N_{t-1} = 0$. Also, let the current TFP be exactly at the threshold level: $A_t = A^T$ (we omit the time subscript t for now). This means that the agent optimally chooses $N_t = 0$. Denote the corresponding optimal choices of H and K by H^{T0} and K^{T0}, respectively. Clearly, with A_t and N_t fixed, H^{T0} and K^{T0} must be functions of (H_t,K_t). We are interested in the pair of initial conditions (H_t,K_t) serving as the unique fixed point of this optimal choice mapping, i.e., such that $H^{T0} = H_t$, $K^{T0} = K_t$.

Assuming this pair of capital choices exists[4] let us call g^0 the value of after-interest earnings (under zero unsecured legacy debt) that is generated by them at the tightness threshold:

$$g^0 = G(A^T, H^{T0}, K^{T0}, 0).$$

The credit tightness threshold for an arbitrary vector of initial conditions can be characterized by the following:

Lemma 4 *Assume there exists a unique vector of initial conditions of the form* (A^{T0},H^{T0},K^{T0},0) *for period t such that the optimal choices of* H_{t+1}, K_{t+1}, *and* N_t *for period* t+1 *coincide with* (H^{T0},K^{T0},0) *and the tightness threshold is again equal to* A^{T0}. *Then, for arbitrary initial conditions* (A_t,H_t,K_t,N_{t-1}), *the credit tightness threshold* A^T *is given by*

$$A^T = Max\left\{0, \frac{g^0 - R^B N_{t-1} + (R^K - 1 + \delta)K_t}{Q(H_t, K_t)^\alpha}\right\} \tag{19}$$

and has the following properties:

- *fo*$R^B N_{t-1} < (R^K - 1 + \delta)K_t - \frac{\alpha}{1-\alpha} g^0$*r r, it holds that* $0 < A^T < A^D$, *meaning that all surviving firms in the next period borrow below their collateral limit regardless of the credit regime tightness* (*i.e., they choose* $N_t > 0$ *whenever* $A_t > A^D$)
- *for* $(R^K - 1 + \delta)K_t - \frac{\alpha}{1-\alpha} g^0 \leq R^B N_{t-1} < (R^K - 1 + \delta)K_t$, *it holds that* $A^T \geq A^D > 0$
- *for* $(R^K - 1 + \delta)K_t \leq R^B N_{t-1} < (R^K - 1 + \delta)K_t + g^0$, *no firm defaults (i.e.,* $A^D \leq 0$*), whereas* $A^T > 0$
- *for* $(R^K - 1 + \delta)K_t + g^0 \leq R^B N_{t-1}$, *no firms are either credit-constrained or default in the credit-constrained regime*; *all firms choose* $N_t > 0$ *regardless of the* A_t *value*.

Here, A^D *is the default threshold defined in* (9).

Proof: see the Appendix.

For the case of unconstrained credit, one can obtain a one-to-one correspondence between income, consumption, and excess borrowing. To state it, one needs to use the inverse of the marginal utility function, which we denote by y. For strictly increasing and strictly concave utility functions, y is strictly decreasing. Therefore, (15) can be transformed to obtain both consumption and net assets as a function of current after-interest income g:

$$C = y(j(g)),\ N = g - y(j(g)). \tag{20}$$

The equation system (16)-(18) can be further streamlined depending on the position of the time period t in one of the model stages. It is useful to move backwards in time and consider equilibrium conditions in the second model stage first.

[4] Informally, the existence here is a simple consequence of boundedness: there are natural upper and lower bounds for values of H, K that solve (16)-(18) given N = 0. Accordingly, an iterative procedure based on (16)-(18) must converge, since it is confined to a compact set, providing at least one fixed point. Uniqueness then follows from monotonicity in Q and degree-zero homogeneity in (H,K) of the right-hand sides of (17) and (18).

3.3 Long-term Equilibrium

To begin with, assume that both periods t and t+1 belong to the LTE stage. This means that, first, there is no human capital choice to be made in either period and, second, A_t and A_{t+1} have the same distribution. The argument presented in Section 3.2 shows that the value function under slack credit constraints collapses to a one-variable function, the first derivative of which is denoted by h. By using an analogue of (20), one can rewrite (16) and (17) as

$$h(g_t) = \beta R^B E_t^R \left[h\left(G^{LT}\left(A_{t+1}, \boldsymbol{K}_{t+1}, g_t - y\big(h(g_t)\big)\right)\right)\right] \qquad (21)$$

and

$$E_t^R \left[h\left(G^{LT}\left(A_{t+1}, \boldsymbol{K}_{t+1}, g_t - y\big(h(g_t)\big)\right)\right)\left(\alpha A_{t+1}\boldsymbol{K}_{t+1}{}^{\alpha-1} - R^K + 1 - \delta\right)\right] = 0. \qquad (22)$$

One of the useful consequences of the equation system (21), (22) is that, absent credit restrictions, the optimal physical capital for period t+1 is a function of g_t or, alternatively, N_t, alone. Also note that, as long as function h is known, the condition on $\boldsymbol{K}$ as stated in (22) is an algebraic equation with a well-defined single solution, since the right-hand side of (22) is strictly decreasing in $\boldsymbol{K}$ on the interval containing potential zeroes of that equation.

As regards physical capital under the tight credit constraint, it turns out to be a constant K^0 depending only on the parameters of the model. The value of K^0 is pinned down by the special case of (17) in which $\boldsymbol{H}_{t+1}$ and $\boldsymbol{N}_t$ are set equal to zero (or, equivalently, by (22) in which $N_t = g_t - y(h(g_t))$ is set equal to zero):

$$E_t^R\left[h\big(G^{LT}(A_{t+1}, K^0, 0)\big)\big(\alpha A_{t+1}(K^0)^{\alpha-1} - R^K + 1 - \delta\big)\right] = 0. \qquad (23)$$

It remains to observe that the default and the credit tightness thresholds are now described by simplified versions of (9) and (19):

$$A_t^D = Max\left\{0, \frac{(R^K - 1 + \delta)K_t - R^B N_{t-1}}{\alpha K_t{}^\alpha}\right\},\ A_t^T = Max\left\{0, \frac{g^{LT0} + (R^K - 1 + \delta)K_t - R^B N_{t-1}}{K_t{}^\alpha}\right\}, \qquad (24)$$

with g^{LT0}, a special case of g^0, defined as $g^{LT0} = G^{LT}(A^{TL0}, K^0, 0)$. Here, $(A^{TL0}, K^0, 0)$, defined implicitly by (23), (24) with $K_t = K^0$, $N_{t-1} = 0$, are the *borderline* (see the discussion prior to Lemma 4 in Section 3.2) initial conditions on the LTE stage that result in the optimal $(K^0, 0)$ choice and the tightness threshold A^{TL0} next period. The appropriate simplified version of Lemma 4 is straightforward.

Summarizing, we can define the long-term equilibrium of the model as a representative net saving and investment plan $(K_s, N_{s-1})_{s>0}$ that maximizes the objective function

$$E_0\left[\sum_{s>0}\beta^{s-1}\left(u\big(\tilde{G}^{LT}(A_s, K_s, N_{s-1}) - N_s\big)\right)\right],$$

given the initial conditions (A_0, K_0, N_{-1}) valid in the first period of the LTE stage. Symbol $\tilde{G}^{LT}$ stands for the earlier defined function G^{LT} in a non-defaulting firm and for the labor share, that is, $(1-\alpha)AK^\alpha$, in a defaulting firm.

The equilibrium can be represented in a parsimonious way by the pair of integro-difference equations (21), (22) subject to constraints (23), (24), plus the usual transversality condition on N at infinity. In practical terms, function h, the first derivative of the objective function for the LTE stage conditioned on no credit constraint, provides an exhaustive characterization of this equilibrium. The same marginal utility can also be used to fully characterize the value function, a result with which we conclude this subsection.

Denote the value function in a period s belonging to the LTE stage by VLT. It is a function of three variables (initial conditions), A_s, K_s, and N_{s-1}. From the defined within-period timeline of the model it follows that there are no optimizing decisions to be made between the period start and the announcement/revelation of the credit regime. Therefore, VLT at the start of the period is a weighted sum of two value functions, VLT^u (unrestricted) and VLT^s ("squeezed"), valid under regular and credit-constrained regimes:

$$VLT(A_s, K_s, N_{s-1}) = \xi\, VLT^u(A_s, K_s, N_{s-1}) + (1-\xi)VLT^s(A_s, K_s, N_{s-1}).$$

As mentioned earlier, VLT^u can be reduced to a function, to be denoted JLT, of a single variable $g_s = G^{LT}(A_s, K_s, N_{s-1})$. The same reduction of dimensionality obtains for TFP values higher than

$$A_s^S = Max\{A_s^D, A_s^T\}.$$

The following proposition, proved in the Appendix, provides a calculable formula for VLT^u and VLT^s, in terms of $h = JLT'$ and A_s^S.

Proposition 2 *Depending on the credit regime realizing at the start of period s, the value function equals either*

$$VLT^u(A_s, K_s, N_{s-1}) = JLT(G^{LT}(A_s, K_s, N_{s-1})) = V^0 + \int_{G^{LT}(A^{TL0},K^0,0)}^{G^{LT}(A_s,K_s,N_{s-1})} h(x)dx$$

under no credit constraints or

$$VLT^s(A_s, K_s, N_{s-1}) = \mathbf{1}_{[0,A_s^S)}(A_s)\left(u\left(\tilde{G}^{LT}(A_s, K_s, N_{s-1})\right) + \beta V^0\right)$$
$$+\mathbf{1}_{[A_s^S,+\infty)}(A_s)JLT\left(G^{LT}(A_s, K_s, N_{s-1})\right)$$

in the credit-constrained regime. Here, boldface unities stand for indicator functions of the interval given in the subscript and V^0 *is the expected value function at* (A^{TL0},K^0,0) (*the borderline credit tightness threshold* A^{TL0} *exceeds the corresponding* A^D *value, so that* $A^S = A^{TL0}$). *For* V^0, *the expression*

$$V^0 = \frac{1}{1-\beta}\int_0^\infty \left\{\mathbf{1}_{[0,A^{TL0})}(X)u\left(\tilde{G}^{LT}(X,K^0,0)\right) + \mathbf{1}_{[A^{TL0},\infty)}(X)u\left(G^{LT}(A^{LT0},K^0,0)\right)\right\}\psi(X)dX$$
$$+\frac{1}{1-\beta}\int_0^\infty \int_{G^{LT}(A^{TL0},K^0,0)}^{\tilde{G}^{LT}(X,K^0,0)} h(g)dg\,\psi(X)dX \tag{25}$$

is valid.

3.4 The Last Intermediate Period

The focus of analysis in this paper is on the period immediately prior to the first LTE one. Human capital is formed for the last time in this period to become an input of the production process next period (the first at the LTE stage). In the absence of the credit constraint (or when this constraint is slack), this output is a part of the lump income variable that determines subsequent decisions on future physical capital and unsecured debt (see the discussion at the beginning of Section 3.1). The first-order optimality conditions (16)-(18) reduce to the following (note the appearance of the marginal indirect utility h, applicable at the LTE stage):

$$u'(G(A_t, H_t, K_t, N_{t-1}) - \boldsymbol{N}_t) = \beta R^B E_t^R\big[h\big(G(A_{t+1}, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)\big)\big], \tag{26}$$

$$E_t^R\big[h\big(G(A_{t+1}, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)\big)\big(\alpha A_{t+1}\boldsymbol{K}_{t+1}{}^{\alpha-1} - R^K + 1 - \delta\big)\big] = 0. \tag{27}$$

$$E_t^R\big[h\big(G(A_{t+1}, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)\big)A_{t+1}\alpha Q(\boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1})^{\alpha-1} q_H(\boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1})\big]$$
$$= v'(\boldsymbol{H}_{t+1}). \tag{28}$$

The three variables featured in boldface are the ones to be chosen optimally. The default and the credit tightness thresholds for the first LTE period (indexed as t+1 in (26)-(28)) needed to apply the expectation operator E^R are calculated according to (24), but with t replaced by t+1.

When h is known from the LT equilibrium, the optimal choices of $\boldsymbol{H}$, $\boldsymbol{K}$, and $\boldsymbol{N}$ under the non-default decision can be recovered from (26)-(28). An important difference compared to the LTE equations is the possibility of multiple solutions in the transition period. These local optima can be ordered in agent welfare terms to select the global optimum, an exercise to be described in the next section. We conclude this section with the characterization of the agent's value function at the start of the last intermediate period, to be used for ordering local maxima of the optimization problem. The value turns out to be fully characterized by the previously defined function h that describes the LTE. For a more compact statement of the results, we introduce the following step function on the positive half-axis: St(ξ,A;X) equals ξ for $0 \le X < A$ and unity for $A \le X$.

Proposition 3 *Consider the last period t in which a positive level of human capital is selected to be used in production. Let the agent enter this period with initial conditions* (A_t,H_t,K_t,N_{t-1}). *Then, using the notation of Proposition 1 in Section 3.2 for the optimal choices, the value function in this period in the regular regime can be written as*

$$u(G(A_t, H_t, K_t, N_{t-1}) - \boldsymbol{N}_t) + \beta E_t[V^*(A_{t+1}, \boldsymbol{H}_{t+1}\boldsymbol{K}_{t+1}, \boldsymbol{N}_t)],$$

where the expectation is over the two credit regimes and the realizations of TFP A_{t+1} in the next period and V^* is the value function for the first period, t+1, *in which new human capital formation is absent (although the output is produced with the help of the previously formed one for the last time). The expected continuation value $EV^* = E_t[V^*(A_{t+1}, \boldsymbol{H}_{t+1}\boldsymbol{K}_{t+1}, \boldsymbol{N}_t)]$ can be calculated as*

$$EV^* = \int_0^\infty \Big\{St(\xi, A_{t+1}^S; X)u\Big(\tilde{G}(A_{t+1}^S, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)\Big) + \mathbf{1}_{[0,A_{t+1}^S)}(X)(1$$
$$- \xi)u\Big(\tilde{G}(X, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)\Big)\Big\}\psi(X)dX$$

$$+\beta V^0 + \int_0^\infty \int_{\tilde{G}(A_{t+1}^S, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)}^{G(X, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)} h(g) dg\, \psi(X) dX \quad (29)$$

with $A_{t+1}^S = Max\{A_{t+1}^D, A_{t+1}^T\}$ *and the threshold values defined by* (9) *and* (19) *for the corresponding optimal choice values* ($\boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t$). *The continuation value* V^0 *is given by* (25).

Proof: see the Appendix.

4. Computation and Numerical Results

4.1 Calibration

The three groups of parameters to be calibrated in this model are: agent's preferences, interest rates and production possibilities, and risk distributions.

The agent's period utility of consumption will be taken as a power (constant relative risk aversion) function with inverse elasticity parameter $\mu = 0.3$:

$$u(C) = \frac{C^{1-\mu}}{1-\mu}.$$

The disutility of human capital provision will also be a power function

$$v(H) = \upsilon H^{1+\chi}$$

with $\upsilon = 0.1$ and $\chi = 0.05$. The time preference parameter in the utility is $\beta = 0.97$. The minimum required cash-to-physical capital ratio is $\varphi = 0.1$.

The interest rates are $R^M - 1 = 0.025$ on cash and $R^B - 1 = 0.03$ on loans. The capital share is $\alpha = 1/3$. The parameters of the CES capital index are $\theta = 2$ and $\gamma = 0.4$. The physical capital depreciation rate is $\delta = 0.02$.

There are two risk sources in the model: arrival of the credit-constrained regime and TFP realization. The first uncertainty is assumed binary, with the probability of the regular (unconstrained) credit regime within a period being $\xi = 0.9$. The TFP realizations are assumed to have a gamma density with parameters $\omega = 1.5$ and $\sigma = A^m/\omega$. The mean TFP value A^m is equal to unity at the intermediate stage (when human capital is involved) and to 1.2 at the LTE stage. The choice of ω implies that TFP is almost always strictly positive, with mode 0.5 and vanishing at both zero and infinity. The gamma distribution was selected to allow for fat tails in the default frequency (a lognormal choice would have generated unrealistically low probabilities of default).

4.2 Long-term Equilibrium Behavior

In LTE, the choice variables of the model in a given period are functions of the realizations of exogenous risks, i.e., the latest TFP value and the current credit regime, and the initial value of a two-dimensional state vector. (Components of the state vector can be associated, for instance, with current income and the labor share.) From the computational point of view, these transition functions can be obtained from function h discussed in section 3.3. Namely, in the absence of default and tight credit constraints, the physical capital choice under known h is given as a solution to (22), whereas consumption and unsecured debt follow from the analogues of (20): $c = y(h(g))$, $n = g - y(h(g))$. Finally, the representative agent welfare is uniquely defined by h and the initial conditions in view of Proposition 2. In short, LTE is fully characterized by function h both theoretically and numerically. A numerical procedure for solving (21) is all one needs to make quantitative inferences about equilibrium behavior.

Differently from linearized pseudo-solutions of DSGE models around an inconsequential "non-stochastic steady state," impulse response exercises make no sense when one considers well-defined feedback solutions of dynamic stochastic optimal control problems. Therefore, our approach to presenting the numerical results of the model analysis will concentrate on showing the dependence of key endogenous fundamentals on the initial state of the economy. If one sets aside the possibility of default in the current period, this initial state is sufficiently characterized by the after-interest income (see Proposition 1). In other cases, it is important to know the dependence of optimal decisions on the size of unsecured legacy debt, holding the initial physical capital and TFP values fixed. Finally, after having looked at decisions of the representative agent, we are going to examine characteristics of the whole household-firm population, such as aggregate unsecured debt and aggregate loss given default.

The first task of the numerical part of this analysis is to obtain a reasonably precise algorithm for calculating the marginal utility of consumption function h as defined in Section 3.3 and characterized by equations (21), (22). Note that, in theory, h is defined on the whole real line. However, for calculation purposes one needs to use a suitable approximation for both highly negative (near-certainty of default) and highly positive (near-certainty of survival) values of the argument. We observe that at the negative infinity limit, equation (21) implies a power asymptotic for h with the exponent defined by the parameters of the model. At the positive infinity limit, a solution to (21) can be satisfactorily approximated by function $S_0(g-S)^{-\mu}$ with constants S_0 and S also being functions of the model parameters only. On the finite interval between these extremes, we then use a finite element procedure specially tailored for the class of optimization problems considered.

The result of the currently employed approximation procedure is illustrated in Figure 1. (The dots overlaying the schedule indicate the values of the right-hand side of the Euler equation (21) on a grid with step-length 0.2.)[5]

To exemplify the way this solution transmits into inferences about other fundamentals of this economy at the LTE stage, we show the dependence of the unsecured debt quantity preferred by the agent for the subsequent period in the absence of credit constraints, denoted by n, on current-period after-interest income g. In other words, we graph the second of the expressions given in (20) as a function of current after-interest income g under no credit constraints, albeit with the marginal utility function h relevant for the LTE case replacing the generic indirect marginal utility function j introduced in Section 3.2. See Figure 2 for the calculation outcome. Note the quick dissolution of unsecured debt for values of g increasing from about 5.25 to about 5.5. A similar comparative static exercise for the last intermediate period (with human capital) will result in a singularity, i.e., a sudden jump from a negative to a positive N value, in the interval of income

[5] The calculations were conducted in Mathematica®.

values for which the LTE economy showed regular, although accelerated, growth. See the next subsection for a discussion.

The remaining results pertaining to LTE will be shown in conjunction with the analogous statements referring to the last intermediate period.

4.3 Human Capital and Preference for Secured Debt

The quantitative investigation in this paper will be limited to the last period in which human capital is still being generated, with the issues arising under multiple human capital-utilizing periods being relegated to future research. The main question we ask is how the optimizing consumer-cum-producer deals with the trade-off between the technological opportunity to diversify into production with low financial risk (physical capital that needs to be financed by borrowing can be partially substituted by human capital that needs no financial investment) and the corresponding effort costs that weigh on the speed of wealth accumulation. After that, assuming that the social planner has her own unspecified costs associated with credit losses, we look at the possible aggregate consequences of regulation aimed at penalizing unsecured debt (and thereby indirectly encouraging more extensive use of human capital) in this environment. We also investigate the role of the debt position with which the economy enters the period in the resulting new borrowing decisions and aggregate losses from default.

Technically, the task boils down to solving (26)-(28) for ($\boldsymbol{H}_{t+1}$,$\boldsymbol{K}_{t+1}$,$\boldsymbol{N}_t$) under varying initial after-interest income $g_t = G(A_t,H_t,K_t,N_{t-1})$. Conveniently, (27) and (28) do not depend on g_t. Therefore, the solution can be obtained by solving (27), (28) with respect to ($\boldsymbol{H}_{t+1}$,$\boldsymbol{K}_{t+1}$) for any selected range of $\boldsymbol{N}_t$ values, substituting the solutions in (26) and then inverting this equation wherever the inverse is well-defined to derive the dependence of ($\boldsymbol{H}_{t+1}$,$\boldsymbol{K}_{t+1}$,$\boldsymbol{N}_t$) on g_t.

It turns out that, for the selected functional forms and parameters, there are multiple (more exactly, triple) solutions to the system (26)-(28) on a particular interval of g_t values. Since one of them (the one that gives rise to the $\boldsymbol{N}_t$ value lying between the other two) corresponds to a local minimum of the agent's utility function, we are left with just two relevant ones, standing for two local optima of the agent's utility. One is associated with positive unsecured debt values (i.e., negative $\boldsymbol{N}_t$ values), relatively low human capital, and relatively high physical capital, and the other with incompletely utilized borrowing capacity (positive $\boldsymbol{N}_t$ values) and relatively high human and relatively low physical capital levels. In short, they represent exactly what one has intuitively associated with the dilemma between high-risk quick physical investment and low-risk gradualist physical investment decisions. The ambiguity disappears for both sufficiently low (only the high-risk solution is available) and sufficiently high (only the low-risk solution remains) initial income levels.

To find out which of the available local optima will be preferred privately, we calculate the indirect utility of the agent by using Proposition 3 of the previous section. Figure 3 shows the result on the whole initial income interval for which local optima multiplicity obtains. For reference purposes, the graph is complemented by the utility calculated under the exogenous ban on unsecured borrowing treated as a tight constraint (for example, as if the social planner emulated the credit-constrained regime of the model by imposing certainty on it). We see that, on the interval of initial income values considered, the agent will not, actually, even require any outside regulation to select the low-risk solution. The preferred debt level will not just be fully collateralized but, moreover, lie strictly below the possible externally imposed limit. This happens in spite of the fact that the low-risk alternative involves both a lower output and a lower utility level in the current period. The key benefit of the low-risk behavior is associated with its high continuation value. The higher future utility is due to substantially reduced losses under default of any agent who avoids entering the subsequent period with a high unsecured debt level. The latter happens to lead to a rapid increase (on average) of the default threshold, which entails rapidly growing default probabilities in future periods. Also, credit tightness thresholds are much higher under the high-risk alternative (Figure 4), meaning a further reduction of the expected utility as the agent is being pushed into involuntary default with a high probability in future periods.

For initial income levels above the multiplicity interval, the picture is even more clear-cut: the low-risk solution is the single optimum. On the other hand, for low initial incomes (below the interval of multiplicity) the low-risk option disappears, since the only global optimum is the high risk borrowing and investment plan (positive unsecured debt). One can see what happens in Figure 5, in which only the value function levels for the globally optimal solution are shown, together with the already mentioned reference value function levels under the credit constraint enforced with equality.

The first thing to observe is that, differently from high initial income cases, poorer agents would be better off even under such a blunt regulatory policy as the hundred percent ban on unsecured debt might seem. Whereas for relatively rich agents, the regulatory constraint of this kind will be slack, the relatively poor ones will benefit from the impossibility to choose an immediate "escape" to high unsecured debt and high future default probabilities: although regulation only saves them from the debt-financed consumption trap in the current period, the effect of keeping future average default thresholds low is enough to improve the future expected utility.

The second observation concerns the value function jump at the level of initial income corresponding to the switch between the high-risk and the low-risk behavior. Even for a minor undershooting of this critical income level, the agent's optimal behavior changes abruptly in all relevant respects. This is further discussed in the next subsection.

4.4 Regulatory Credit Limits and Unstable Borrowing Behavior

Naturally, the singularity in the private welfare dependence on initial income is but a derivative of the singularities appearing in optimal decisions. We illustrate this by calculating the globally optimal unsecured debt level as a function of current income. Figure 6 shows the result together with the corresponding dependence in the no-human-capital (LTE) case (the latter is a function analogous to the one already featured in Figure 2, but on a narrower range selected to concentrate attention on the jump neighborhood). Similar dependencies for physical capital and other fundamentals, showing the difference compared to the LTE stage, can be obtained and would exhibit the same qualitative traits.

What can be immediately observed is that the human capital option makes the biggest difference in the vicinity of the income value at which the jump occurs. In more extreme cases (i.e., of both much richer and much poorer agents) the quantitative importance of human capital for borrowing behavior is much smaller. Clearly, when the starting debt levels are either too low or too high, the implications of unsecured debt stock for optimal behavior are much stronger than partial substitution of production inputs. On the contrary, for intermediate-income agents, the presence of human capital introduces an unexpected source of instability. And, as the discussion in the previous subsection indicates, this instability cannot be removed by simply penalizing unsecured debt overhang (this would, at best, only reduce it). Smoother dependence of unsecured debt on income, similar to the no-human-capital case, can possibly be achieved by a fine-tuned tax regime that would address investment "feats" out of line with the debt position of the firm. This issue, however, is outside the scope of the present paper.

4.5 Present and Future Losses from Default and Bank Balance Sheet Expansion

We would now like to look at the aggregate consequences of unsecured debt, as well as the restrictions thereon, from the bank balance sheet perspective. A positive quantity of unsecured debt (i.e., a negative N value) in an individual firm-bank relationship is a simple (at least as long as capital requirements are not violated) accounting matter of the bank expanding its balance sheet in accordance with unsecured credit demand. At the same time, a part of the unsecured debt will be defaulted on, leading to aggregate credit losses for the banking sector – a potentially systemic event. On the other hand, if the regulator attempts to curb unsecured debt provision at short notice, there will still be other defaults, namely, in that part of the private sector that needs to roll over the existing debt to maintain activity. So, there is a trade-off between accepting some defaults now and later. We will now try to get an idea of the relative magnitudes of expected credit losses at different horizons depending on whether and when brakes on unsecured debt are introduced.

The corresponding exercise consists in calculating three aggregate quantities (that is, taking expectations over producers' TFP realizations): economy-wide unsecured debt N in the next period assuming a laissez-fair credit policy (no constraints on N) at present, aggregate loss given default (LGD) next period under the same laissez-fair credit policy, and, finally, aggregate LGD in the current period if the policy prohibits negative N in it. (Observe that, in the present model, by construction, a ban on negative N now guarantees no or negligible LGD next period; LGD can, absent new restrictions, only reemerge in later periods.) The results are illustrated by the two graphs in Figure 7, in which, this time, the independent variable is the size of legacy unsecured debt (i.e., the value of N inherited from the previous period), denoted by n_{-1} in the LTE case and N_{-1} in the human capital case, ranging between -3 and +1 units. The dependent variables are shown with the original signs (i.e., the loss, if incurred, is negative, and N is negative if there is unsecured debt).

We see that LGD due to restrictive policies at present is clearly dominated by the avoided LGD next period. Unsurprisingly, this feature is most salient for low legacy debt levels, given that LGD only becomes quantitatively relevant when a sufficient amount of past unsecured debt is involved.

Further, we note that the presence of human capital makes a difference in at least two regards. First, LGD in the current period always has the natural (minus) sign even for low or altogether missing legacy unsecured debt. On the contrary, the physical capital-only economy without a debt overhang sometimes even generates positive average earnings on the seized collateral of defaulting firms. This circumstance has to do with the altogether low – in relation to both output and debt – physical capital quantities chosen by agents with access to human capital, and the latter, as we know, cannot be pledged as collateral. Second, when human capital is present, future LGD remains slightly below the aggregate extended unsecured credit for negative legacy N values. In physical capital-only economies, aggregate losses given default always exceed the unsecured credit extended. The difference is due to a heavier dependence of physical capital-only economies on debt needed to finance production. Nevertheless, with regard to the “extra credit creation vs. LGD” criterion, the presence of human capital is less quantitatively important for the equilibrium outcome than from the viewpoint of borrowing behavior stability discussed in Section 4.4.

Finally, we observe that the magnitudes of the optimally privately demanded unsecured credit (the size of the additional credit creation) and the losses on it are comparable. Roughly speaking, all the balance sheet expansion incurred to finance extra credit will be devoured by loan losses. Of course, the quantitative side of this particular result ought to be corrected for the inevitable crudeness of the present theoretical exercise, above all the simplistic nature of the agents’ balance sheets. Staying within the limits of the current model, one can argue that the “pure balance expansion accommodating extra credit”, although, indeed, turning out to be of the same order as the subsequent LGD, has an immediate benefit of avoiding LGD at present, at least for economies already facing a debt overhang.

5. Conclusion

This paper dealt with the implications of occasional restrictions on unsecured borrowing for future credit, economic activity, and losses on bad loans. This was done in a dynamic stochastic model of a production economy with risky debt, to which we applied a solution concept that improves on the popular linear approximations in the DSGE-with-financial-frictions class. By focusing on a small number of intermediate periods on the one hand and on full distribution solutions instead of linearized steady state neighborhoods on the other, we were able to shed light on the trade-off between stabilizing the observable credit volumes and unintentionally destabilizing latent dimensions of private sector decision making.

Importantly, the method used in the present paper allows one to analyze situations inaccessible through traditional linearized DSGE-model-with-financial-frictions solutions. The fact that our model, at the cost of technically challenging numerical procedures, is solved properly in the form of feedback optimal plans opens the door to applications that trade off expected private utility on infinitely expanding future risk spaces against contemporaneous utility gains/losses. This is a feature generically absent from the “non-stochastic steady state” method, including its various modifications. With a bit of simplification, we outlined here an approach to macroeconomic modeling with financial frictions in which not just private agents but also policymakers can weigh risk against return. In particular, we were able to assess the consequences of macroprudential policies that curb insufficiently collateralized lending, for the risk/return properties of future private credit and output. We not only preserve the dynamic stochastic optimization for individual agents, which is necessary in the mainstream micro-founded analysis, but also allow aggregate uncertainty to show up in the computed solution, which should facilitate policy inference in matters involving systemic risk.

What can be imagined under a regulatory ban on unsecured lending examined in our model? One can think of dramatically scaled-up supervision of unsecured lending in economies with tail risk-concerned policymakers that, through the corresponding capital and administrative costs for banks, induces a resource shift from conventional bank credit-dependent to (quasi)credit-independent projects. For example, investment can be withdrawn from small entrepreneurs, who tend to be opaque, and expanded in the sector of big firms who get financing from shadow banks.

Disintermediation, the rise of shadow banking, and other departures from the usual "European" bank financing paradigm were both expected and registered during several business cycles in the past. Still, the extent of the currently observed weakness of bank lending growth in Europe in the aftermath of the Great Recession is exceptional both in length and in magnitude. Explanations of this phenomenon vary from calling it a permanent secular shift of financing demand toward opaque forms poorly observable by policymakers (including the use of jurisdictions with light supervision), to blaming the credit supply side (i.e., banks) weakened by the twin global financial and European sovereign debt crises and tight regulation. One can conceive of various situations in which credit stimuli on the one hand and regulatory policies on the other, if designed in isolation from each other, collide.

These and other applications are accessible by our approach of working out a full distribution numerical solution of a sort that is, due to the analytical challenges involved, often avoided in applications that rely on macroeconomic models with financial frictions. This can be seen as the principal methodological contribution of this paper.

As a possible avenue of future research, an extension of the proposed model may be useful for addressing the interplay of the medium-of-exchange contribution of daylight banks, the loanable funds contribution of shadow banks, and the monetary-cum-macroprudential authority that is better informed about the daylight than about the shadow banking sector risks. One may be able to assess how much a given policy mix, such as a monetary easing combined with a macrofinancial tightening, notwithstanding the joint objective of affecting the daylight credit costs and fighting financial distress, supports migration of economic activity towards shadow bank financing. (The counterpart on the finance supply side would be yield search and/or regulatory arbitrage.) Additionally, one will be able to express predictions as to what pattern of production factors (defined by the typical share of physical vs. human capital) will make the economy more susceptible to regulatory overkill.

The presented approach can help expand the existing picture of traditional monetary and fiscal stimulation in an environment of enhanced credit risk awareness in both the private and the official sectors. The anemic response of conventional bank credit to a monetary policy stimulus is often met with incomprehension by those central bankers who do not have a clear idea about the influence of macrofinancial measures on the real economy. Still, recent academic literature offers enough examples of monetary and macroprudential policies, when they go in opposite directions, being able to partially balance each other's effect on headline macro fundamentals (this effect can be discerned from dynamic macro models with a financial sector such as Iacoviello, 2005, Monacelli, 2009, and Stein, 2012; it is spelled out more explicitly in Derviz, 2012). Specifically with regard to credit, it is probably not very surprising that easy monetary policy and financial repression, by making it more attractive to channel savers' funds through shadow banking, discourages rather than encourages traditional bank balance sheet expansion, instead supporting deleveraging in the daylight banking sector. On the surface, too easy money goes hand in hand with creditless recovery.

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Appendix

Proof of Lemma 1

The general statement follows from the observation that, in our model, the consequences of default are concentrated within one period. This holds because default is understood as an option belonging to the integral set of conditions characterizing credit-constrained states of nature, valid for a given period only. Consequently, once the credit-constrained period is over, default has no further implications for choices in subsequent periods.

A^D is the TFP value at which the agent is indifferent between defaulting and repaying the debt. Now, observe that at this state of indifference, cash holdings cannot be above the lower bound, as defined by (1). Indeed, if they were, the agent could have improved future utility by borrowing a little less – hence generating a lower debt service – at the same time reducing the saved cash by the same amount (recall that cash earns less than what borrowing costs), while keeping current consumption constant. This would contradict the assumption of indifference between repayment and default.

Given that continuation values coincide for the default and the repayment choices, the only condition defining A^D is the equality of current consumption levels under default and repayment. Further, at A^D, the credit constraint is either tight or slack. In the former case, (4) is satisfied with equality and the consumption level is equal to earnings, to be compared under default and repayment. If the credit constraint is slack at $A_t = A^D$, then the same $N_t > 0$, H_{t+1}, and K_{t+1} are chosen regardless of either the default decision or past history, meaning the same continuation value. But then, since net savings N_t are the same, indifference between default and repayment is equivalent to equality of current-period earnings with and without default. In both cases, the latter are increasing in A_t, but the slope of the no-default earnings line is higher. The lines either do not intersect at all (this is the case when no default occurs) or have a single intersection point for a non-trivial threshold value.

Now we turn to the calculation of the threshold value given a tight cash constraint in the preceding period. A^D must satisfy

$$A^D Q(H_t,K_t)^\alpha + (1-\delta)K_t - R^B B_{t-1} + R^M M_{t-1} - M_t + B_t - K_{t+1} = (1-\alpha)\, A^D Q(H_t,K_t)^\alpha .$$

If, as assumed, (1) held in period t–1, then

$$-\, R^B B_{t-1} + R^M M_{t-1} = R^B N_{t-1} - (R^B + \varphi(R^B - R^M)\,)K_t = R^B N_{t-1} - R^K K_t .$$

This allows us to write

$$A^D Q(H_t,K_t)^\alpha + (1-\delta)K_t - R^B B_{t-1} + R^M M_{t-1} = A^D Q(H_t,K_t)^\alpha + R^B N_{t-1} - (R^K - 1 + \delta)K_t \,.$$

Equating the latter expression with the labor share $(1-\alpha)A^D Q(H_t,K_t)^\alpha$ and solving for A^D, one gets (9) ●

To prepare a common ground for the proofs of other statements, let us denote by $\tilde{\Lambda}_t^B$ the stochastic process equal to the Lagrange multiplier Λ_t^B of the B constraint at time t in the constrained regime and zero otherwise. The objective function of the agent at time 0 in the Lagrange multiplier form is

$$L^{HF} = E_0\left[\sum_{t\geq 0}\beta^t\left\{u(C_t) - v(H_{t+1}) + \Lambda_t^M(M_t - \varphi K_{t+1}) + \widetilde{\Lambda}_t^B(M_t + K_{t+1} - B_t)\right\}\right]. \qquad \text{(A1)}$$

(Superscript *HF* stands for "household-firm.") It is to be maximized with respect to the consumption and investment plans $(H_s, K_s, M_{s-1}, B_{s-1}, C_s)_{s>0}$ and the non-negative Lagrange multiplier sequences $(\Lambda_s^M)_{s>0}$ and $(\Lambda_s^B)_{s>0}$, subject to resource constraints (5), (6).

Recalling the discussion of Section 2.2 about the (in)dependence of expectations over default TFP states on decision variables, and introducing the auxiliary notation

$$MRS_t^{t+1} = \frac{\beta u\,'(C_{t+1})}{u\,'(C_t)},\ MPH_{t+1} = \frac{\partial Q^\alpha}{\partial H_{t+1}},\ MPK_{t+1} = \frac{\partial Q^\alpha}{\partial K_{t+1}},$$
$$\lambda_t^M = \frac{\Lambda_t^M}{u\,'(C_t)},\ \tilde{\lambda}_t^B = \frac{\widetilde{\Lambda}_t^B}{u\,'(C_t)},$$

we can write the expressions for the partial derivatives of L^{HF} as follows:

$$\frac{1}{u\,'(C_t)}\frac{\partial L^{HF}}{\partial M_t} = R^M E_t^R[MRS_t^{t+1}] - 1 + \lambda_t^M + \tilde{\lambda}_t^B, \qquad \text{(A2)}$$
$$\frac{1}{u\,'(C_t)}\frac{\partial L^{HF}}{\partial B_t} = -R^B E_t^R[MRS_t^{t+1}] + 1 - \tilde{\lambda}_t^B, \qquad \text{(A3)}$$
$$\frac{1}{u\,'(C_t)}\frac{\partial L^{HF}}{\partial K_{t+1}} = E_t^R[MRS_t^{t+1}(A_{t+1}MPK_{t+1} + 1 - \delta)] - 1 - \varphi\lambda_t^M + \tilde{\lambda}_t^B, \qquad \text{(A4)}$$
$$\frac{\partial L^{HF}}{\partial H_{t+1}} = E_t^R[\beta u\,'(C_{t+1})A_{t+1}MPH_{t+1}] - v\,'(H_{t+1}). \qquad \text{(A5)}$$

Since the objective function L^{HF} has the usual strict concavity properties with respect to the decision variables, in common with a multitude of analogous ones used in neoclassical and New Keynesian stochastic consumption and investment models, the first-order conditions that require the four above partial derivative sequences to be equal to zero are necessary for optimality. (Another necessary condition is the usual transversality, but even when it is satisfied, there can be multiple local optima, as discussed in Section 4.)

Proof of Lemma 2

At the optimum, the right-hand side of (A3) must be equal to zero, implying

$$E_t^R[MRS_t^{t+1}] = \frac{1-\tilde{\lambda}_t^B}{R^B}. \qquad \text{(A6)}$$

This is one of the possible ways to state the Euler equation of the problem. Since the marginal rates of substitution here are strictly positive everywhere, we see that $\tilde{\lambda}_t^B$ must lie strictly between 0 and 1. Next, we substitute the right-hand side of (A6) into (A2) and reorganize to obtain

$$\frac{1}{u\,'(C_t)}\frac{\partial L^{HF}}{\partial M_t} = \lambda_t^M - \left(1 - \frac{R^M}{R^B}\right)\left(1 - \tilde{\lambda}_t^B\right).$$

The second term on the right-hand side of this expression is strictly negative. Now, if the *M* constraint were slack at the optimum, the corresponding Lagrange multiplier, and hence λ_t^M as well, would be zero. This would render a negative partial derivative on the left-hand side, implying that the optimal *M* value must lie below the current one – a contradiction. The argument holds for any candidate optimal M above the lower bound given by (1), i.e., the optimal value can only be the lower bound itself ●

Proof of Lemma 3

Optimality of M and B choices implies fulfilment of the first-order conditions (A6) and the analogous one for the right-hand side of (A2). Combining them, we get the following relation between the two Lagrange multipliers:

$$\lambda_t^M = \frac{R^B - R^M}{R^B}(1 - \lambda_t^B).$$

Accordingly, the last three terms on the right-hand side of (A4) can be transformed as follows:

$$-1 - \varphi\lambda_t^M + \tilde{\lambda}_t^B = -\left(1 + \varphi\frac{R^B - R^M}{R^B}\right)\left(1 - \tilde{\lambda}_t^B\right) = -(R^B + \varphi R^B - \varphi R^M)E_t^R[MRS_t^{t+1}].$$

The second equality follows from (A6). Recalling the definition of R^K, we get (11) ●

Proof of Proposition 1

Equation (15) is the usual equality between direct and indirect marginal utilities known from other dynamic stochastic models of optimal consumption, with the specific modifications dictated by our set-up. It follows from the Envelope Theorem. Equation (16) is a re-statement of (A6) in the unconstrained case (when the Lagrange multiplier vanishes) and the definition of MRS. Finally, (17) and (18) are restatements of (11) and (12) from Lemma 3 ●

Proof of Lemma 4

Recall that, by definition of the credit tightness threshold, for A_t above A^T, the possible credit constraint is slack and, by Proposition 1, the value function degenerates to a function of the single variable $g_t = G(A_t,H_t,K_t,N_{t-1})$. As A_t approaches A^T from above, g_t must approach g^0 since the latter is the unique level of after-interest earnings at which credit constraint slackness turns into tightness. (Put differently, for TFP levels above A^T, the exact composition of (A_t,H_t,K_t,N_{t-1}) does not matter; any ingredients resulting in the same g_t, including $(A0_t,H^{T0},K^{T0},0)$ for a properly chosen $A0_t > A^{T0}$, render the same optimal choices for the next period as the original (A_t,H_t,K_t,N_{t-1}).) Therefore, the basic characterization of A^T is given by the equality $G(A^T,H_t,K_t,N_{t-1}) = g^0$, which is equivalent to (19) when the maximum is strictly positive.

However, there are situations in which this equality cannot be obtained. This happens when the legacy debt is well below the allowed limit, i.e., N_{t-1} is big and positive, as specified in the 4th case of the lemma.

For somewhat lower, but still sufficiently big N_{t-1} (the 3rd case of the lemma), the expression in (19) is positive ($A^T > 0$), whereas the one in (9) for A^D is negative, i.e., there are no defaults, but some firms become credit-constrained if only secured debt is allowed. The 2nd case of the lemma is the one in which, depending on the TFP realization, the firm can either default, or survive but be credit-constrained, or remain unaffected by the constraints. Finally, for low enough values of N_{t-1} (the 1st case of the lemma), the default threshold is so high ($A^D > A^T$) that any credit-constrained firm also automatically belongs among the ones that default, so that the exact level of A^T itself is irrelevant for the outcome ●

Proof of Proposition 2

The first statement of the proposition regarding VLT^u is straightforward in view of the discussion at the beginning of Section 3.2. For the same reason, the value of VLT^s for A_s above A^S is also evident. As regards the case of $A_s < A^S$, this inequality implies that the agent's credit constraint is tight. In other words, N_s must be equal to zero, the agent consumes either the after-interest income if the debt is repaid or the labor share if it is defaulted on (note the tilde over the G function in the formula). Besides, the subsequent period is started with the optimal choice K^0 of physical capital (as is always the case when the unsecured debt is chosen to be zero), meaning that the continuation value is, indeed, V^0. It remains to establish that the tightness threshold A^{TL0} is bigger than the default threshold for $(K_{s+1},N_s) = (K^0,0)$. This follows immediately from applying (24) to this special case, as long as $g^{LT0} > 0$. The latter inequality is a consequence of the definitions of the thresholds A^D and A^T, since, for TFP values slightly above that one threshold which is bigger, the agent repays the debt and even selects a positive new N value. However, these decisions could not have been taken under non-positive after-interest income without violating the consumption non-negativity requirement.

Finally, we will derive (25). This can be done by taking $(A_s,K_s,N_{s-1}) = (A^{TL0},K^0,0)$, inserting these data in the already established formulae for VLT^u and VLT^s, and taking expectations. In this way, one gets V^0 on the left-hand side of the equality. Next, shorthand g^X for $G^{LT}(X,K^0,0)$ and observe that

$$JLT(g^X) = JLT(g^{LT0}) + \int_{g^{LT0}}^{g^X} h(g)dg. \qquad \text{(A7)}$$

But, by definition of g^{LT0} as the borderline after-interest income at which the agent chooses $N = 0$ in the next period, $JLT(g^{LT0}) = u(g^{LT0}) + \beta V^0$. Now, taking the expectation over X on the positive half-line and collecting terms, one gets (25) ●

Proof of Proposition 3

The statement of this proposition follows from the observation that, although the optimal human capital $\boldsymbol{H}_{t+1}$ is generically positive, there is no subsequent human capital decision in period t+1, as the firm produces output using human capital input $\boldsymbol{H}_{t+1}$ for the last time. That output is taken as given when decisions about debt repayment, consumption, borrowing, and investment are made. Therefore, one can proceed by analogy with Proposition 2. Specifically,

$$V^u(A_{t+1}, \boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1}, \boldsymbol{N}_t) = J(G(A_{t+1}, \boldsymbol{H}_{t+1},\boldsymbol{K}_{t+1}, \boldsymbol{N}_t)) \qquad \text{(A8)}$$

and

$$V^s(A_{t+1}, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t) = \mathbf{1}_{[0,A^S_{t+1})}(A_{t+1})\left(u\left(\tilde{G}(A_{t+1}, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)\right) + \beta V^0\right)$$
$$+\mathbf{1}_{[A^S_{t+1},+\infty)}(A_{t+1})J\left(G(A_{t+1}, \boldsymbol{H}_{t+1}, \boldsymbol{K}_{t+1}, \boldsymbol{N}_t)\right). \qquad \text{(A9)}$$

Observe, i.a., the term βV^0 in (A9): the continuation value appearing there refers to the period t+2 belonging to the LTE stage. Therefore, it is given by (25). Now, using the obvious analogue of (A7), taking expectations over (A8), (A9), and collecting terms, one obtains the expression for the expected continuation value EV^* given in (29) ●

Figure 1: The Marginal Utility of Consumption Function as a Solution of the Time-homogenous Euler Equation in Long-term Equilibrium

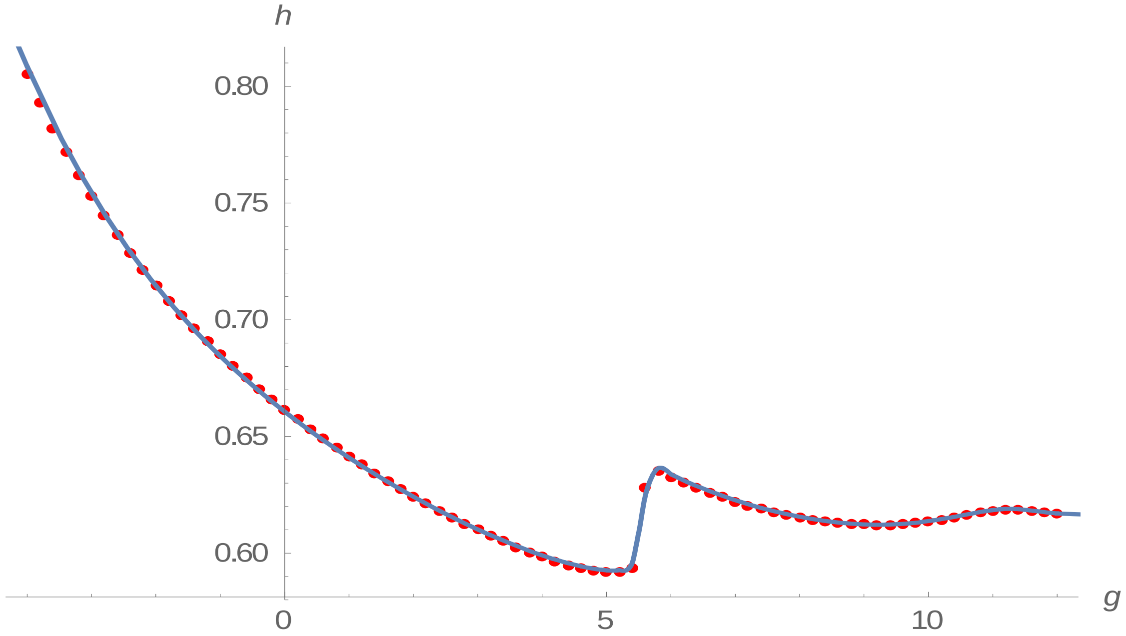


Figure 2: Unsecured Debt in LTE as a Function of After-interest Income in the Regular Credit Regime

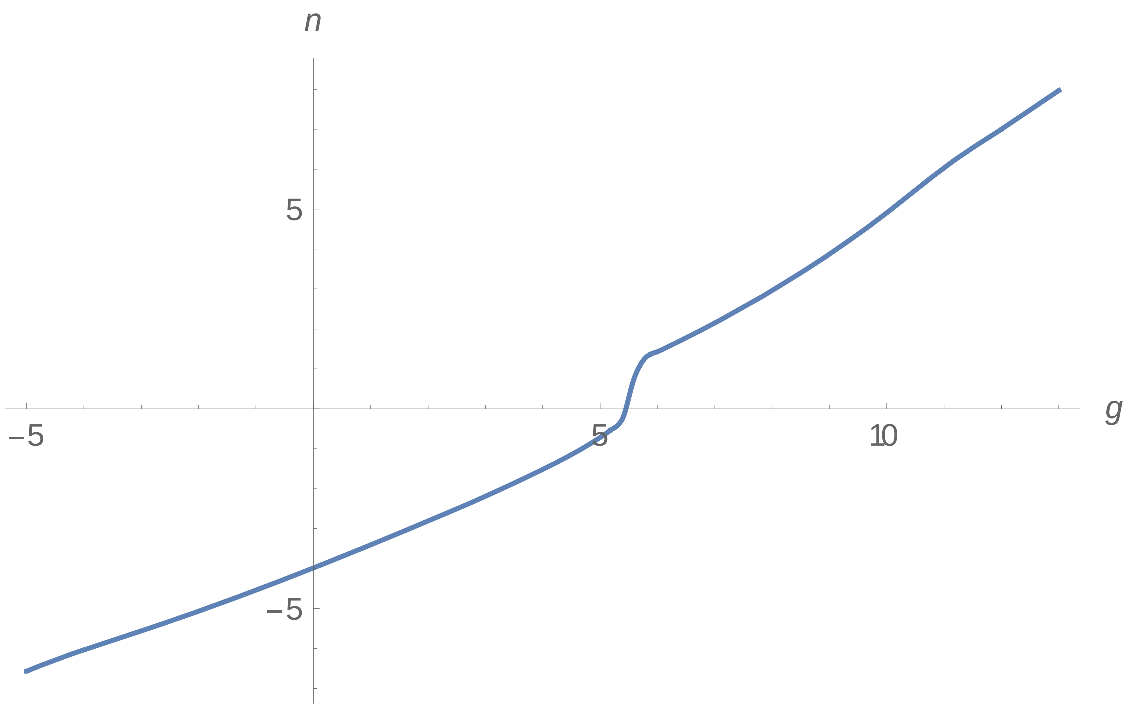


Figure 3: Private Welfare Levels for High-risk (blue line), Low-risk (green line), and Tightly Credit-constrained (orange line) Locally Optimal Borrowing and Investment Plans, as a Function of Initial Income

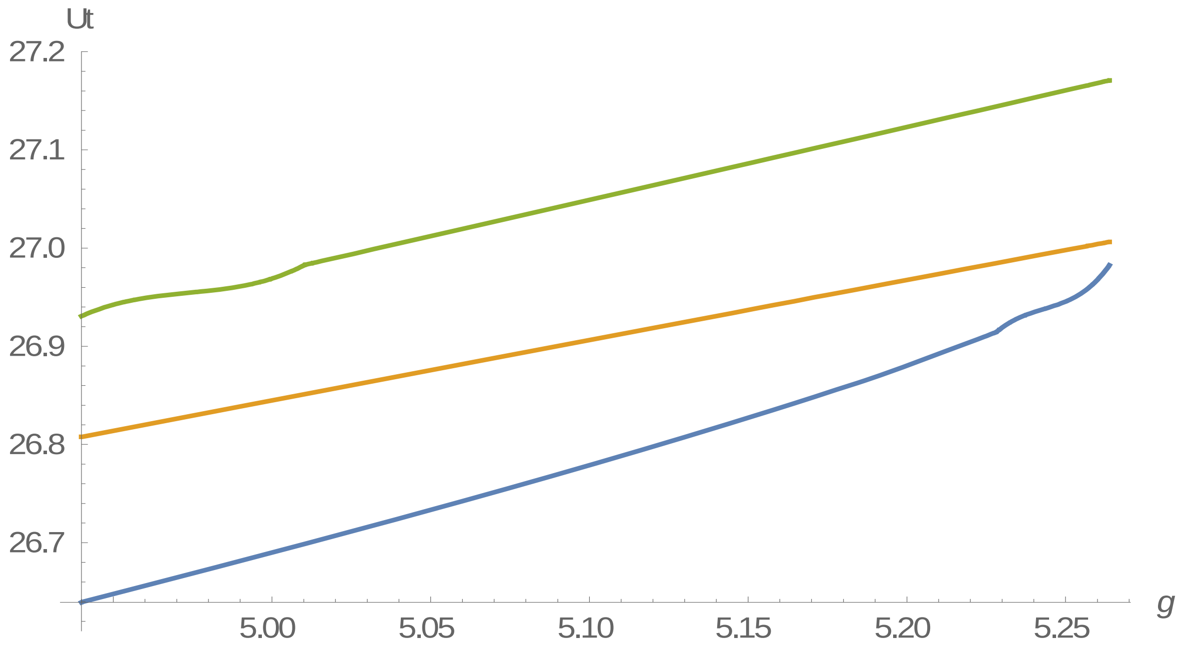


Figure 4: Credit Tightness TFP Threshold Levels for Low-risk (blue line) and High-risk (orange line) Locally Optimal Borrowing and Investment Plans, as a Function of Initial Income

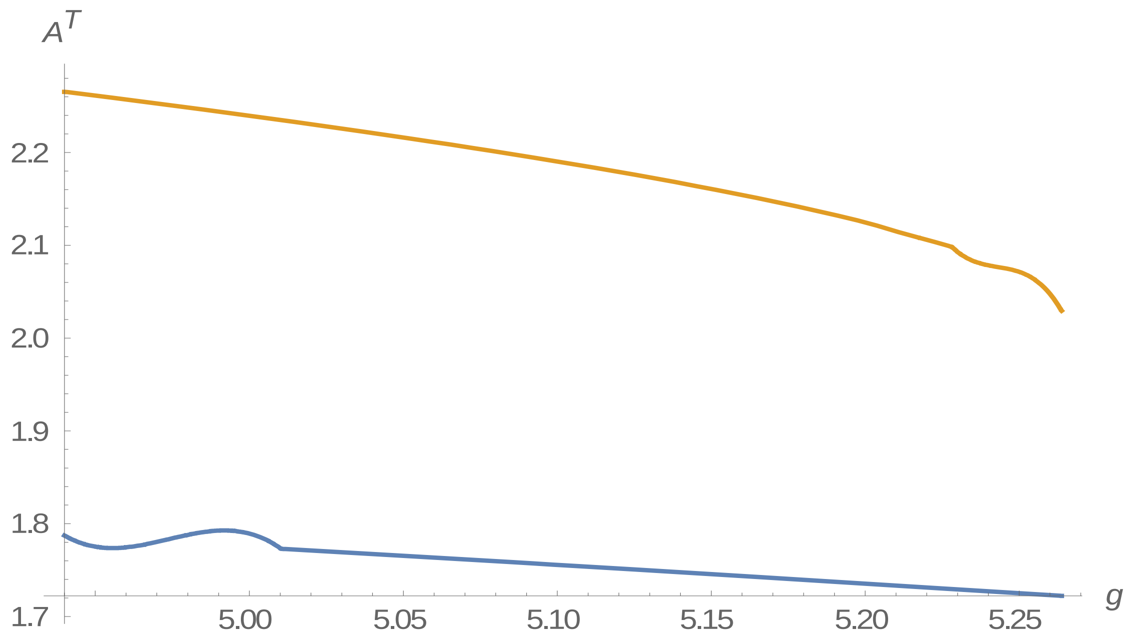


Figure 5: Private Welfare Levels for Globally Optimal Borrowing and Investment Plans (blue line) and Tightly Credit-constrained Plans (orange line), as a Function of Initial Income

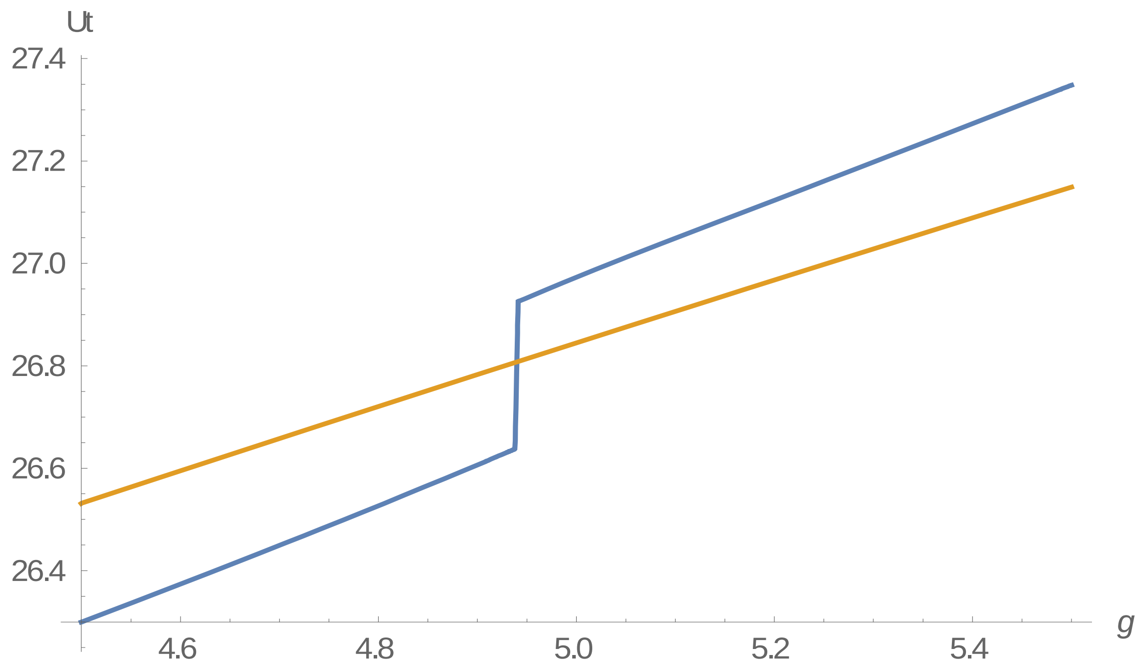


Figure 6: Unsecured Debt as a Function of After-interest Income in the Regular Credit Regime, the Last Human Capital Period (blue line), and LTE (orange line)

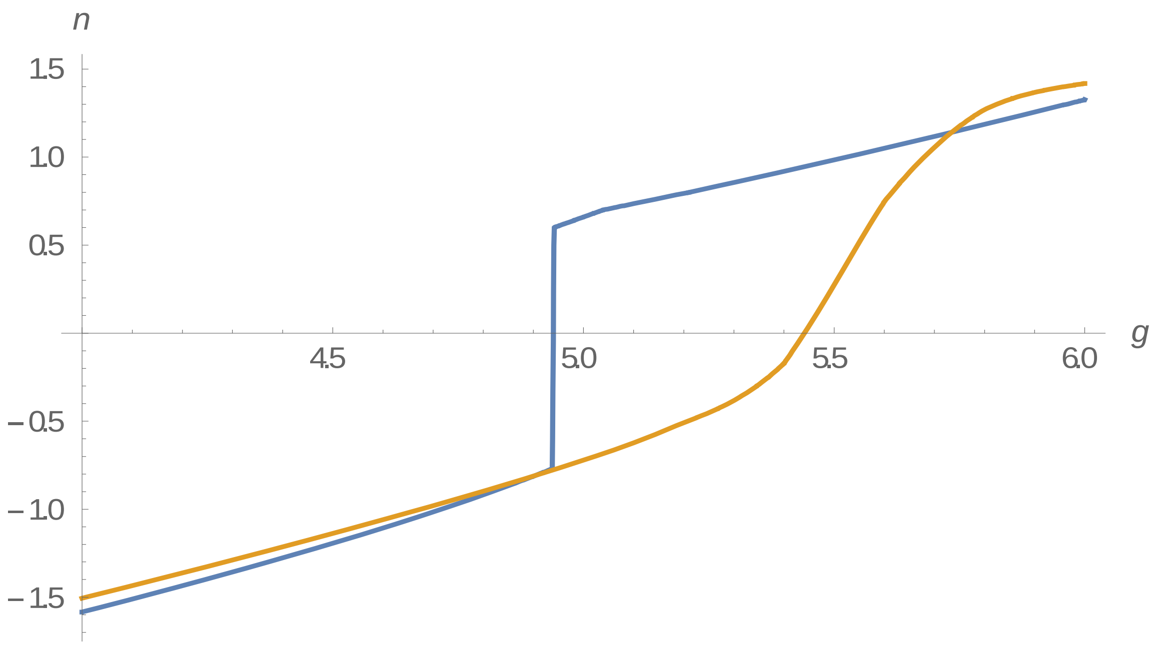
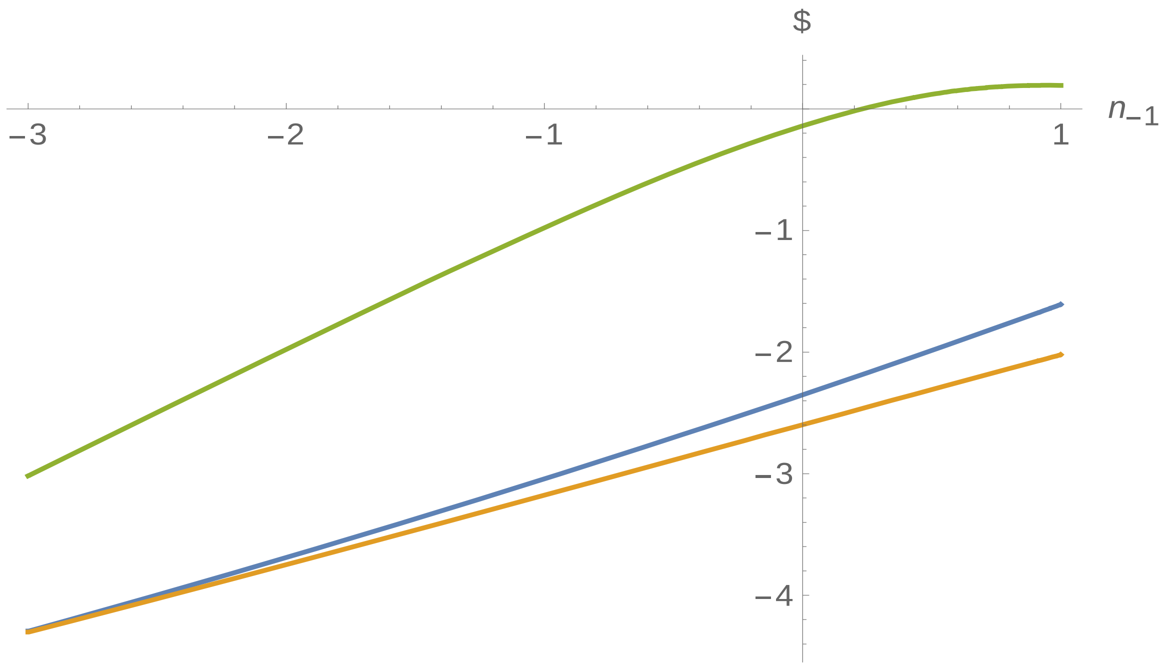
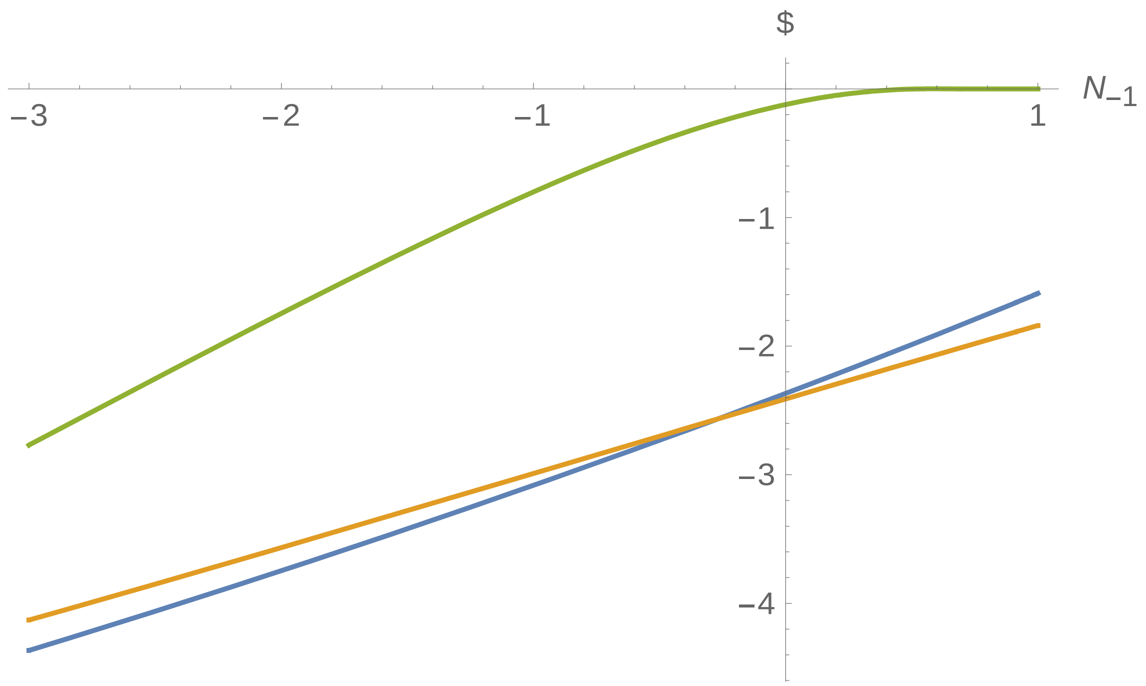


Figure 7: Aggregate Future Unsecured Debt Under No Current Credit Restrictions (blue), Future Aggregate LGD Under No Current Credit Restrictions (orange), and Current Aggregate LGD Under a Complete Contemporary Ban on Unsecured Debt (green), as a Function of Legacy Unsecured Debt

(a) LTE stage



(b) The last period with human capital utilization



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