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Effects of Fiscal Policy in the DSGE-VAR Framework: The Case of the Czech Republic

Jan Babecký, Michal Franta, and Jakub Ryšánek*

Abstract

In this paper we explore the potential of the DSGE-VAR modelling approach for examining the effects of fiscal policy. The combination of the VAR and DSGE frameworks leads theoretically to more accurate estimates of impulse responses and consequently of fiscal multipliers. Moreover, the framework allows for discussion about the differences of the effects of fiscal shocks in DSGE and VAR models and to some extent discussion about misspecification in fiscal DSGE models. The DSGE-VAR model is estimated on Czech data covering the period from 1996 to 2011 at quarterly frequency. The government consumption multiplier attains a value close to 0.4 at the horizon of four years. The public investment multiplier is about 0.4 higher, which confirms findings in the literature. On the other hand, the DSGE model alone implies a similar government consumption multiplier but a much lower public investment multiplier, suggesting misspecification of the fiscal DSGE model.

Abstrakt

V tomto článku zkoumáme potenciál modelovacího přístupu DSGE-VAR k hodnocení účinků fiskální politiky. Spojení metod VAR a DSGE vede teoreticky k přesnějším odhadům impulzních odezev a tedy i fiskálních multiplikátorů. Navíc tento modelový rámec umožňuje diskusi o rozdílech dopadů fiskálních šoků v modelech typu DSGE a VAR a zároveň do jisté míry diskusi o nepřesné specifikaci fiskálních modelů DSGE. Model DSGE-VAR odhadujeme na českých datech za období 1996 až 2011 v kvartální frekvenci. Multiplikátor vládní spotřeby dosahuje hodnoty blízko 0,4 v horizontu čtyř let. Multiplikátor vládních investic je cca o 0,4 vyšší, což potvrzuje závěry odborné literatury. Na druhou stranu, model DSGE samotný implikuje podobně velký multiplikátor vládní spotřeby, ale mnohem menší multiplikátor vládních investic, což naznačuje nepřesnou specifikaci fiskálního modelu DSGE.

JEL Codes: C11, E62, F41, H30.

Keywords: DSGE-VAR model, fiscal multipliers, fiscal shocks, identification, model misspecification.

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Nontechnical Summary

The ultimate purpose of this paper is to provide estimates of fiscal multipliers. Fiscal multipliers represent a measure of how domestic activity is affected by a fiscal instrument. We discuss fiscal measures in the form of a change in government consumption, a change in government investment and a change in the amount of other social benefits.

The literature discussing the values of fiscal multipliers is overwhelming, so why produce more estimates? The reason lies in the fact that we employ a novel methodology which is theoretically superior to the known approaches. We employ the DSGE-VAR modelling framework, which allows us to test cross-variable restrictions included in a DSGE model. More precisely, one could stick to the multiplier estimates produced by a calibrated or estimated DSGE model, but we take a step further and test whether various restrictions implied by the specification of the DSGE model are confirmed by data. This is what the VAR part brings into our modelling framework. Restrictions not confirmed by data through the VAR model are not taken into account, hence the multiplier estimates should be more precise.

In addition, we present impulse responses – the evolution of model variables after an unexpected fiscal shock. The impulse responses shed some light on why the multipliers estimated within the DSGE model and the DSGE-VAR framework are different. In consequence, the paper enriches the discussion on the differences between multipliers in structural and statistical models. Finally, the differences can be interpreted with respect to cross-variable restrictions implied by the DSGE model, and misspecification can thus be examined as well.

The model is estimated on Czech quarterly data covering the period 1996 to 2011. The results suggest that the government consumption multiplier attains values of around 0.4 after four years. Government investment affects the economy in a more pronounced way – the government investment multiplier four years after the spending occurs is 0.76. Regarding the differences between the DSGE and DSGE-VAR approaches, it turns out that the main difference concerns public investment, which affects domestic activity more within the DSGE-VAR modelling framework. The impulse responses suggest that a stronger effect of private consumption and a weaker effect of government consumption in the DSGE-VAR model explain the difference, thus suggesting misspecification of the fiscal DSGE model related probably to the assumed specification of the budget constraint.

1. Introduction

Assessment of the effects of fiscal measures on the economy is a topical issue. Such effects are quantified in terms of fiscal multipliers, which characterise the reaction of output to changes in selected fiscal instruments on the revenue and expenditure sides. The number of studies which estimate the values of fiscal multipliers for different countries and time periods and using various methods is rapidly increasing. The variety in the estimated size of fiscal multipliers has even been subjected to meta-regression analysis – a systematic quantitative literature review – with the objective of identifying regularity in the large amount of multipliers produced. Based on a review of 104 primary studies, Gechert (2015) finds that the size of fiscal multipliers depends, inter alia, on the method selected (time series approaches or vector autoregressions – VARs – result in higher multiplier values compared to structural macroeconomic models), on the fiscal instrument (the highest multipliers are typically associated with government investment) and on the share of constrained households who consume all their disposable income (a higher share of such households strengthens the effects of fiscal policy).

Three frameworks have been particularly popular in the literature investigating the effects of fiscal policy: (i) structural vector autoregression (SVAR) models (e.g. Blanchard and Perotti, 2002); (ii) the narrative approach (Romer and Romer, 2010); (iii) dynamic stochastic general equilibrium (DSGE) models (e.g. Galí et al., 2007). These frameworks differ in the way fiscal shocks – that is, unexpected fiscal policy changes – are treated. In SVAR models, fiscal shocks are identified within an unrestricted system; the narrative approach identifies fiscal shocks directly, by selecting those fiscal policy changes which are deemed to be exogenous; DSGE models identify fiscal shocks taking into account micro-level agents' behaviour.

The SVAR approach is often found to lack robustness due to relatively short and noisy fiscal data. Hence, Bayesian techniques become attractive, as they allow additional information to be incorporated into the estimation procedure through the imposition of priors on the model parameters. The choice of priors, though, is of critical importance. The common approaches to selecting VAR priors based on time series or statistical criteria have been criticised for a lack of economic interpretation. For example, the popular Minnesota prior assumes that the series simply follows a process close to a random walk. Such type of prior ignores potential interdependencies between the endogenous variables (Del Negro and Schorfheide, 2004).

A natural way of making priors economically consistent is to use a macroeconomic model to formulate them. Although this idea was already present in Ingram and Whiteman (1994), who considered the real business cycle (RBC) model, it took another decade to make this approach operational. In their influential study, Del Negro and Schorfheide (2004) introduce the methodology of the DSGE-VAR approach. They use a standard small-scale New-Keynesian DSGE model to generate a prior for a vector autoregression to examine the effects of monetary policy. Since then, the methodology has been employed several times, mainly for forecasting.[1]

[1] Applications of the DSGE-VAR approach focus mainly on forecasting and often suggest superior performance with respect to standard benchmarks. Lees et al. (2011) apply the DSGE-VAR methodology to estimate a five-variable system for New Zealand, Gupta and Steinbach (2013) develop a DSGE-VAR model of South Africa, and Langcake and Robinson (2013) develop a multi-sector DSGE-VAR model for Australia comprising ten endogenous variables.

Another issue that could be at least partially resolved by employing the DSGE-VAR framework is that of shock identification. The identification of fiscal shocks is inherently problematic and there is no consensus on an appropriate identification scheme. The micro-founded DSGE model provides the necessary theory to help link estimated residuals to structural shocks and thus to identify fiscal shocks within the VAR part of the model. Importantly, as discussed in detail in Del Negro et al. (2007), such bridge between unrestricted VAR and VAR implied by the DSGE model allows misspecification of the structural model to be discussed.[2]

The contribution of this paper is threefold. First, the main purpose is to provide estimates of fiscal multipliers employing a methodology that is theoretically superior to the approaches used so far. To the best of our knowledge, the paper represents the first attempt to apply the DSGE-VAR approach to the analysis of fiscal multipliers. So far, the focus has been solely on monetary DSGE-VARs. Second, the DSGE-VAR framework is used to examine the misspecification of a standard fiscal DSGE model implied by tight parametric restrictions and thus can enrich the discussion on the set-up of fiscal DSGE models in general. Finally, the comparison of fiscal multipliers implied by the DSGE model and the DSGE-VAR model can shed some light on the discussion about differences in multipliers in the DSGE and VAR modelling frameworks.

We estimate the models on Czech quarterly data covering the period 1996–2011. The end of the sample is chosen so as to avoid the zero lower bound (ZLB) period and the period of the exchange rate commitment. The fiscal multipliers presumable changed significantly after 2011. Moreover, the ZLB and the commitment are not reflected in the underlying DSGE model. The short time series available strengthen the importance of Bayesian estimation and appropriate formulation of priors. The fiscal DSGE model employed is a medium-scale model, all parts of which are currently standard in structural modelling of fiscal policy. The size of the model along with the short time series would be a problem for forecasting. However, our focus is on policy analysis and fiscal multipliers. Therefore, detailed modelling of various channels is an advantage even with the low number of observations used for the estimation.

In our analysis, we distinguish several fiscal measures on the expenditure side (government consumption, government investment, other social benefits) and on the revenue side (consumption tax, wage tax). The results show that impulse responses based on the DSGE and DSGE-VAR models exhibit a number of differences. This suggests the presence of misspecification in the DSGE part considered alone.

The multiplier estimates suggest that the government consumption multiplier attains values of around 0.4 after four years. Government investment affects the economy in a more pronounced way – the multiplier four years after the spending occurs is 0.76. The other social benefits multiplier is low at close to 0.2 at the horizon of four years. Regarding the differences between DSGE and DSGE-VAR, it turns out that the main difference concerns government investment, which affects domestic activity more within the DSGE-VAR modelling framework. The impulse responses suggest that a stronger effect of private consumption and a weaker effect of government consumption in the DSGE-VAR explain the difference, suggesting misspecification of the fiscal DSGE model probably related to the assumed specification of the budget constraint.

[2] The misspecification analysis introduced in Del Negro et al. (2007) was recently applied, for example, to misspecification related to expectations in Cole and Milani (2014).

The rest of the paper is organised as follows. After this introduction, Section 2 describes the underlying DSGE model and Section 3 presents the VAR counterpart. Section 4 discusses the identification of shocks. Issues related to the definition of fiscal multipliers are discussed in Section 5. Section 6 summarises the data used in the DSGE and VAR parts and the estimation procedures. Section 7 presents the DSGE-VAR results, showing the impulse responses and the multipliers obtained. The last section concludes. Finally, the marginal likelihood computation is presented in Appendix A, the list of model variables with their definitions is provided in Appendix B and convergence diagnostics of the estimation procedure are discussed in Appendix C.

2. Model – DSGE Part

In this section we provide a summary of the DSGE model from which we intend to form priors for the DSGE-VAR estimation. A more detailed description of the DSGE model can be found in Ambriško et al. (2015) and Ambriško et al. (2012).

The model blocks of the structural (DSGE) model are built along the lines of the CNB's g3 model as put forward by Andrle et al. (2009). The model is based on a small open economy set-up in which the foreign environment is assumed to be strictly exogenous in the econometric sense. The central bank operates in an inflation-targeting regime and the interest rate is set according to the forward-looking Taylor rule. The implementation of the production structure of the economy mimics the main links within the system of national accounts in that capital and labour are the sources of domestic intermediate production, which – taken together with imports – forms the input for the final use sectors: consumption, investment and exports. The model is closed by an uncovered interest parity condition augmented by an adjustment term sensitive to the net foreign asset position of the economy (known as the debt-elastic risk premium).

The modelling approaches to all sectors in the economy share an identical strategy – the sector inputs are aggregated using a constant elasticity of substitution production function, and the pricing in all sectors builds on the sticky prices premise as introduced by Calvo (1983):

$$\int_0^1 X_t\left(z^X\right)dz^X = \left[\left(\omega_X\right)^{\frac{1}{\eta_X}}\left(N_t^X\right)^{\frac{\eta_X-1}{\eta_X}} + \left(1-\omega_X\right)^{\frac{1}{\eta_X}}\left(Y_t^X\right)^{\frac{\eta_X-1}{\eta_X}}\right]^{\frac{\eta_X}{\eta_X-1}}, \tag{2.1}$$

$$\log\frac{\Pi_t^X}{\Pi_{t-1}^X} = \beta\log\frac{\Pi_{t+1}^X}{\Pi_t^X} + \frac{\left(1-\xi_X\right)\left(1-\beta\xi_X\right)}{\xi_X}\log\left(RMC_t^X\Theta^X\right) + \varepsilon_t^X, \tag{2.2}$$

where X stands for a sector label (i.e. intermediate production, private consumption, investment and exports), z^X is a sector-specific firm identifier[3], ω_X is the share of imported inputs, N_t^X,

[3] The usual strategy is to index all sector-specific agents on a unit mass continuum so that the sector aggregates equal per capita values.

Y_t^X represent the amounts of imported/domestic production inputs, $\eta_X > 0$ is the elasticity of substitution parameter between domestic and foreign inputs, Π_t^X denotes price growth in sector X, which is a function of the real marginal costs RMC_t^X, ε_t^X is a cost-push shock and β, ξ_X and Θ^X are fixed parameters.

The core model behaviour is influenced by the treatment of the households sector. Following Galí et al. (2007) we assume two types of households – optimisers and "rule-of-thumb" consumers. The households with optimising behaviour tend to generate savings based on current economic conditions and their expectations. The other type of households is treated as non-Ricardian and always consumes its entire disposable income. We also adopt the recipe of Coenen et al. (2012) in that households' utility is partially affected by consumption of government sector goods:[4]

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t),$$

$$C_t = \left[(\alpha_C)^{\frac{1}{v_C}} \left(C_t^P\right)^{\frac{v_C - 1}{v_C}} + (1 - \alpha_C)^{\frac{1}{v_C}} \left(G_t\right)^{\frac{v_C - 1}{v_C}} \right]^{\frac{v_C}{v_C - 1}}, \tag{2.3}$$

where the consumption of households, C_t, is a CES aggregate of the output from private consumption goods producers, C_t^P, and government goods, G_t, and v_c is the elasticity of substitution. Equation (2.3) imposes the condition that private and government consumption are substitutes.

The division of households into two types results in a pair of budget constraints. The "rule-of-thumb" households consume the entire after-tax income from the labour they supply (augmented by unemployment and other benefits). On top of this specification, the optimising households split their income into consumption and investment. The investment expenditures are further divided into capital investment and purchases of domestic bonds. On the income side, the optimising households gather yields on previous investment on top of their after-tax labour income. Specifically, the budget constraints are as follows:

optimisers:
$$\left(1+\tau_t^c\right)P_t^C C_t^{po} + P_t^I I_t^{po} + B_t^o = \left(1-\tau_t^w+\tau_t^{UB}\right)\int_0^1 W_t(i) L_t^o(i)\,di + P_t^C\left(OB_t^o - T_t^o\right)$$
$$+ \left\{\left(1-\tau_t^K\right)P_t^K + \tau_t^K \delta^p P_t^I\right\} K_{t-1}^{po} + R_{t-1} B_{t-1}^o + \left(1-\tau_t^D\right) D_t^o, \tag{2.4}$$

"rule-of-thumb":
$$\left(1+\tau_t^c\right)P_t^C C_t^{pr} = \left(1-\tau_t^w+\tau_t^{UB}\right)\int_0^1 W_t(i) L_t^r(i)\,di + P_t^C\left(OB_t^r - T_t^r\right),$$

where τ_t^c, τ_t^w, τ_t^K, τ_t^D represent the tax rates (consumption, wage, capital and dividend respectively) set by the government and τ_t^{UB} is the unemployment benefit rate; C_t^{po} and C_t^{pr}

[4] The utility function itself furthermore features habit formation in consumption.

denote private consumption of optimising and "rule-of-thumb" households respectively; I_t^{po} is optimisers' investment in private capital K_t^{po} and the depreciation rate of capital is denoted by δ^p; W_t is the wage rate and L_t is the amount of labour supplied; domestic bond purchases are labelled as B_t^o and R_t is the nominal interest rate; the term $\left(OB_t - T_t\right)$ represents all benefits (other than unemployment benefits) free of lump-sum taxes; D_t^o are dividends from monopolistic firms; and $P_t^{\forall}$ stands for sector-specific prices.

The labour market is embedded into the model using the concept of Galí (2011), according to which unemployment is a result of workers' market power and unemployment fluctuations arise because of the existence of nominal rigidities. We model the development of wage dynamics using a Calvo-type Phillips curve (similar to equation 2.2) in which we assume a simple formula in place of the marginal costs term:

$$RMC_t = \left(u_t\right)^{-\varphi}, \tag{2.5}$$

where u_t is the unemployment rate and φ is the sensitivity parameter.

The fiscal authority has under its control a set of fiscal instruments (i.e. various tax rates and individual expenditure components). Following Leeper et al. (2010) the fiscal block is modelled by simple backward-looking rules. Each of the above instruments at time *t*, denoted again by X_t, is a function of the output gap, Y_t, and the gap of indebtedness, B_t. Cross-correlations among the instruments are also allowed for. Specifically:

$$gap\left(X_t\right) = gap\left(Y_t\right)^{\phi_{yx}} gap\left(B_t\right)^{\phi_{bx}} \left(\left(u_t^1\right)^{\phi_{1x}} \left(u_t^2\right)^{\phi_{2x}} \cdots \left(u_t^n\right)^{\phi_{nx}}\right), \tag{2.6}$$

where the sequence of shocks $u_t^{[\cdot]}$ can potentially influence multiple instruments at one time. This way we model the gaps of government consumption, government investment, social benefits expenditures and tax rates.

The government furthermore operates subject to its budget constraint, which states that the current debt stock (as of time t) is given by the debt stock in the previous period augmented by the associated interest payments and the actual primary surplus (–) /deficit (+), or:

$$B_t = R_{t-1} B_{t-1} + PD_t,$$

$$PD_t = P_t^G G_t + P_t^I I_t^g + \tau_t^{UB} W_t L_t - \tag{2.7}$$

$$\tau_t^C P_t^C C_t^p + \left(\tau_t^W + \tau_t^S\right) W_t L_t + \tau_t^K \left(P_t^K - \delta^p P_t^I\right) K_{t-1}^p + \tau_t^D D_t + P_t^C T_t,$$

where the primary deficit, PD_t, is taken as the difference between government expenditures and government revenues (thus this measure can be negative); G_t and I_t^g stand for government consumption and investment respectively.

3. Model – VAR Part

Following Del Negro and Schorfheide (2004) we consider a VAR(p) model for an $n \times 1$ vector of observed variables y_t:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + u_t , \tag{3.1}$$

where $u_t \sim N(0, \Sigma_u)$ is an $n \times 1$ vector of error terms. Defining the $T \times n$ matrix $Y \equiv [y_1', \ldots, y_T']'$ and the $T \times (1+np)$ matrix $X \equiv [x_1, \ldots, x_T]'$, where $x_t = [y_{t-1}', \ldots, y_{t-p}']$ and $\Phi \equiv [\Phi_0, \ldots, \Phi_p]'$, the system (3.1) can be rewritten into matrix form:

$$Y = X\Phi + U , \tag{3.2}$$

where U is a $T \times n$ matrix with u_t' in rows. Usually, matrix x_t is defined with ones in the first column to play the role of intercepts. However, since the DSGE model's steady-state growth rates are demeaned and the data are linearly detrended, the VAR specification without intercepts is used.

Modelling fiscal policy requires a DSGE model with many variables (especially in comparison to DSGE models focused on monetary policy). The number of variables in the VAR part of the model equals the number of observables in the DSGE part ($n = 26$). A lower number of variables in the VAR part would imply misspecification of the VAR model. So, dealing with fiscal policy implies a medium-scale VAR, hence the reasons for using the Bayesian approach become even more relevant in comparison to monetary DSGE-VARs.

Another issue relates to the question of when it is correct to approximate a DSGE model by a VAR structure. As noted by Giacomini (2013), the observable part of a log-linearised DSGE model can be represented in VARMA format, and furthermore, the VARMA itself can be written as VAR(∞) provided that the VARMA structure is invertible. In our case, VAR(∞) exists and thus a VAR(*p*) approximation can be used.

The problem with invertibility of the MA part relates to fiscal foresight. It is suggested by the literature (e.g. Leeper et al., 2012) that a process reflecting fiscal foresight leads to non-invertible MA processes. If the DSGE model cannot be expressed as a VAR(∞) structure, straightforward use of the DSGE-VAR methodology is not possible.

So, the DSGE part of the model provides priors for the VAR in (3.2) such that if we define functions:

$$\Phi^*(\theta) \equiv \Gamma_{xx}^{*-1}(\theta)\Gamma_{yx}^*(\theta) \tag{3.3}$$

$$\Sigma_u^*(\theta) \equiv \Gamma_{yy}^*(\theta) - \Gamma_{yx}^*(\theta)\Gamma_{xx}^{*-1}(\theta)\Gamma_{xy}^*(\theta)$$

then the prior distribution of the VAR parameters Φ and Σ_u is of the normal inverted Wishart form:

$$\Sigma_u \mid \theta \sim IW\left(\lambda T \Sigma_u^*(\theta), \lambda T - k, n\right) \tag{3.4}$$

$$\Phi \mid \Sigma_u, \theta \sim N\left(\Phi^*(\theta), \Sigma_u \otimes \left(\lambda T \Gamma_{xx}^*(\theta)\right)^{-1}\right).$$

$\Gamma_{xx}^*(\theta)$, $\Gamma_{yx}^*(\theta)$ and $\Gamma_{yy}^*(\theta)$ are the population moments implied by the DSGE model: for example $\Gamma_{xx}^*(\theta) = E_\theta\left[x_t x_t'\right]$. They can be computed analytically from the state space representation of the DSGE model. Parameter λ is the weight of the prior relative to the data sample. So, it drives the prior variance of the mean and both parameters of the inverse Wishart distribution. Vector θ collects deep parameters from the DSGE model and $k \equiv 1 + np$. If $\lambda T \geq k + n$ and $\Gamma_{xx}^*(\theta)$ is invertible, then the prior density is proper and nondegenerate and the marginal likelihood can be computed.

The posterior distribution of the VAR parameters then belongs to the same family of distributions. More precisely, it follows the following distributions:

$$\Sigma_u \mid Y, \theta \sim IW\left((\lambda+1)T\tilde{\Sigma}_u(\theta), (\lambda+1)T - k, n\right) \tag{3.5}$$

$$\Phi \mid Y, \Sigma_u, \theta \sim N\left(\tilde{\Phi}(\theta), \Sigma_u \otimes \left(\lambda T \Gamma_{xx}^*(\theta) + X'X\right)^{-1}\right),$$

where $\tilde{\Phi}(\theta)$ and $\tilde{\Sigma}_u(\theta)$ are the maximum likelihood estimates based on the data sample and the sample produced by the DSGE model with the parameter vector θ :

$$\tilde{\Phi}(\theta) = \left(\lambda T \Gamma_{xx}^*(\theta) + X'X\right)^{-1}\left(\lambda T \Gamma_{xy}^*(\theta) + X'Y\right) \tag{3.6}$$

$$\tilde{\Sigma}_u(\theta) = \frac{1}{(\lambda+1)T}\left[\left(\lambda T \Gamma_{yy}^*(\theta) + Y'X\right) - \left(\lambda T \Gamma_{yx}^*(\theta) + Y'X\right)\left(\lambda T \Gamma_{xx}^*(\theta) + X'X\right)^{-1}\left(\lambda T \Gamma_{xy}^*(\theta) + X'Y\right)\right]$$

Parameter λ is obtained by maximising the marginal likelihood of the DSGE-VAR model. The computation of the marginal likelihood is described in Appendix A.

4. Identification of Shocks

Identifying shocks basically means finding a linear relationship between the uncorrelated structural shocks ε_t and the model residuals u_t. In VAR models, the relationship can be expressed as follows:

$$u_t = \Sigma_{tr}\Omega\varepsilon_t, \tag{4.1}$$

where Σ_{tr} is an $n \times n$ lower triangular matrix obtained by Cholesky decomposition of the estimated variance-covariance matrix Σ_u and Ω is an arbitrary $n \times n$ orthonormal matrix. The matrix Σ_{tr} reflects an assumption regarding the contemporaneous causation between variables. The choice of Ω cannot be based on the observed data, as the likelihood function is not affected by the choice of the matrix. Ω is usually based on some theoretical considerations.

When looking for a theoretically grounded Ω, the theory included in the micro-founded DSGE part of the model can be employed. So, Del Negro and Schorfheide (2004) suggest using the orthonormal matrix $\Omega^{DSGE}(\theta)$ from the DSGE model, which can be obtained from the unique LQ factorisation of the matrix of impacts of shocks $A_0(\theta)$:

$$A_0(\theta) \equiv \frac{\partial y_t}{\partial \varepsilon_t^{DSGE}} = \Sigma_{tr}^{DSGE}(\theta)\Omega^{DSGE}(\theta). \tag{4.2}$$

The relationship between the structural shocks from the DSGE model ε_t^{DSGE} and the estimated one-step-ahead forecast errors u_t is then expressed by the following formula:[5]

$$u_t = \Sigma_{tr}\Omega^{DSGE}(\theta)\varepsilon_t^{DSGE}. \tag{4.3}$$

Since the DSGE model includes 30 structural shocks ε_t^{DSGE} and 26 observed variables, the lower triangular matrix Σ_{tr} is extended by a block of zeros in order for the dimensions of multiplied matrices to be aligned. In this way, we impose zero restrictions on four DSGE structural shocks which we find irrelevant a priori. However, imposing zero effects on some shocks affects the results, as the unexplained variation in the observed variables is linked to a lower number of shocks. The strength of the influence is related to how much of the variation is explained by the

[5] The approach to identification in Del Negro and Schorfheide (2004) is subject to criticism. First, it is not guaranteed that the lower triangular matrices from the VAR and DSGE model are the same. Del Negro and Schorfheide (2004) note that "in our experience, the difference is for practical purposes negligible". The issue is partially addressed by Liu and Theodoridis (2012), who take the triangular matrix "closest" to the original matrix implied by the VAR error covariance estimate. However, their approach is not computationally feasible for medium-scale models. The problems with identification within the DSGE-VAR framework are also discussed in Sims (2008).

neglected shocks in the data-generating process. Quantification of this effect is beyond the scope of this paper.

There are several theoretical reasons why impulse response functions (IRFs) and consequently fiscal multipliers could be estimated more accurately using the DSGE-VAR framework. First of all, the DSGE-VAR framework relaxes the cross-variable restrictions from the DSGE model and lets the data speak through the unrestricted VAR. In this way, the potential misspecification implied by the limitations of the DSGE framework can be dealt with and the DSGE-VAR model parameters should be estimated more accurately. For example, the DSGE model indirectly imposes a trade-off between private investment and private consumption (through the budget constraint of a representative household). Both private investment and consumption affect the resulting fiscal multipliers in the same direction. So, the trade-off implies that an increase in the multiplier due to a more profound impact of the fiscal measure on consumption is dampened by the opposite impact of private investment. Such a restriction is not present a priori in the VAR model and thus the data can suggest relaxing this restriction with a corresponding impact on the multiplier value.[6]

Of course, if the underlying DSGE model describes the data-generating process perfectly, the VAR part does not add anything to the prior belief and the parameters of the VAR corresponding to the DSGE model do not change.

5. Multipliers

In general, fiscal multipliers are defined as the ratio of a change in an economic activity variable to a change in a fiscal policy instrument. Since fiscal tools affect the economy with a lag, cumulative multipliers at specific horizons are usually considered. The cumulative multiplier is defined as the cumulative change in economic activity over the cumulative change in a fiscal policy instrument at a given horizon.

As the economic activity variable we employ the sum of private consumption (*PC*), government consumption (*GC*), private investment (*PI*) and government investment (*GI*). Such a measure coincides with GDP for a closed economy.[7] The list of fiscal measures considered in our analysis includes government consumption, government investment and other social benefits (*OSB*).[8] The change in the instrument is represented by an unexpected one-period unit shock.

[6] Using prior predictive analysis, Leeper et al. (2015) demonstrate how the structure of the model used to estimate fiscal multipliers implies a priori a range of multiplier values. The DSGE-VAR approach relaxes the imposed ranges, drawing on observed data.

[7] There are two reasons why we focus on all components of GDP except net exports. First, we focus on the effect of a fiscal instrument on domestic economy. Second, there are trends in the components of real GDP. The problem is that the trends in exports and imports differ from the trends in the other components. Multipliers including net exports would thus be time-varying.

[8] Other fiscal measures, such as unemployment benefits, consumption tax revenues and wage tax revenues, could potentially be examined, as they are included in the set of endogenous variables of the model. However, the computation of the corresponding fiscal multipliers is problematic. For example, consumption tax revenues can be computed in the model as a product of consumption tax and private consumption. However, the ratio of consumption tax revenue to the components of domestic demand is not constant in the data and thus equation (5.1) cannot be used without making some strong assumption about the relevant steady states.

The variables in the model are expressed in demeaned quarter-on-quarter changes (not annualised) with zero as their steady state. The impulse response functions are defined in terms of the difference from the steady state, i.e. in percentage points. To obtain the multipliers, the ratio of cumulative differences needs to be multiplied by the average ratio of the activity measure to the corresponding fiscal measure. More precisely, first we compute the fiscal multiplier for the components of the economic activity measure at horizon h. For example, for government consumption as a component of the activity variable and other social benefits as a fiscal measure, the fiscal multiplier is:

$$F_{GC}(h)=\frac{\Delta GC}{\Delta OSB}=\frac{CIRF_{GC}(h)}{CIRF_{OSB}(h)}*\frac{SS_{GC}}{SS_{OSB}}, \qquad (5.1)$$

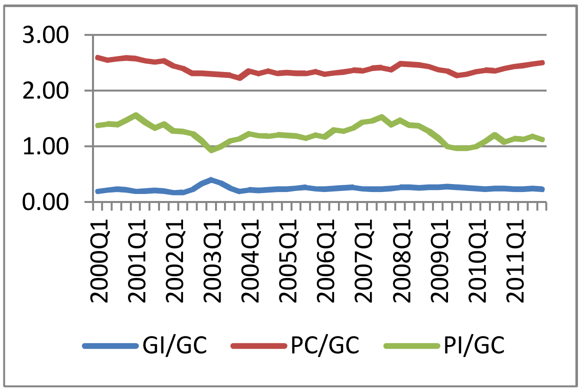
where $CIRF_i(h)$ is the cumulative impulse response at horizon h and SS_i is the steady-state value of the corresponding variable i. The ratio of the two steady-state values is approximated by the observed mean ratio in the data. The implicit assumption is that this ratio does not change over time, as demonstrated in Figure 1 (panels A–C). The total fiscal multiplier at horizon h is then the weighted average of the corresponding economic activity components:

$$F(hh)=w_{GC}F_{GC}(h)+w_{GI}F_{GI}(h)+w_{PC}F_{PC}(h)+w_{PI}F_{PI}(h). \qquad (5.2)$$

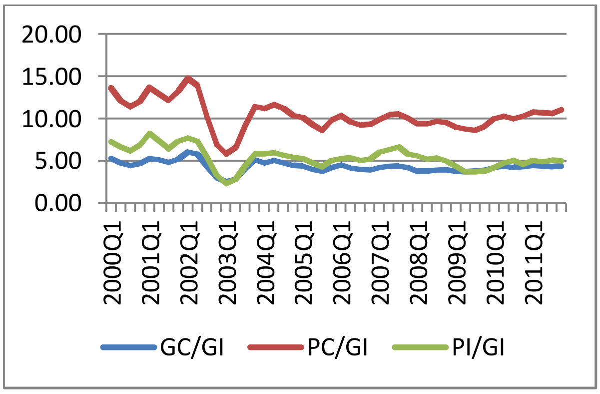
Again, the weights are approximated by the observed ratios, which do not change much over time, as demonstrated in Figure 1 (panel D).

Figure 1: Ratios of the Components of the Activity Measure and the Fiscal Measure (Panels A–C) and Weights of the Components of the Activity Measure (Panel D)

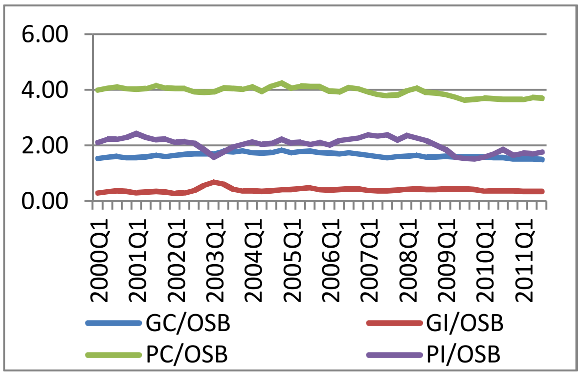
Panel A: Denominator – government consumption



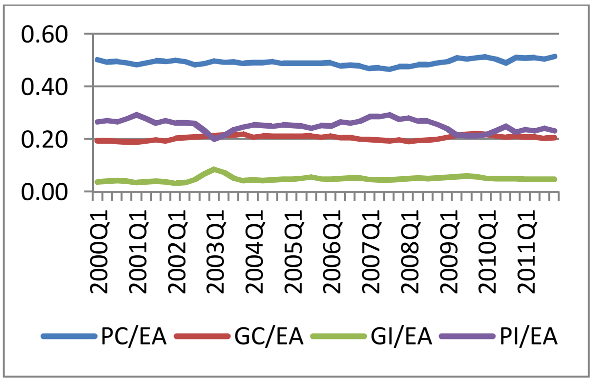
Panel C: Denominator – government investment



Panel C: Denominator – other social benefits



Panel D: Denominator – economic activity



Note: PC – private consumption, GC – government consumption, PI – private investment, GI – government investment, OSB – other social benefits, EA – economic activity.

6. Data and Estimation

The data used are at quarterly frequency, covering the period from 1996 to 2011. In 1998, the monetary policy regime changed and inflation targeting was introduced. For the DSGE model, the regime change is not an issue in terms of the Lucas Critique. For the VAR part, the regime change could imply changes in the coefficients. The estimation sample for the VAR part is, however, left to start at 1996 due to the low number of observations.

The underlying DSGE model comprises 26 observed variables. These variables and the structural shocks are listed in Appendix B. The features of the Czech fiscal data and details on adjusting for data quality, such as the use of Kalman filtration and the application of manual adjustment for one-off government transactions, are discussed in Ambriško et al. (2012).

The DSGE part of the model is estimated using Bayesian techniques. We keep approximately half of the model parameters calibrated and for the rest of the parameters we apply the following strategy. For the prior distributions we assume normal distribution for the majority of the model parameters. Due to the zero lower bound on the standard deviations of the shocks, we apply inverse gamma prior distributions. The posterior modes of the estimated parameters are derived numerically and we subsequently use the Metropolis-Hastings (MH) algorithm to learn about the posterior distributions. In order to ensure sufficient convergence, we run the MH procedure twice, each run consisting of 200,000 draws. The two simulations yield similar acceptance rates (over 50%).

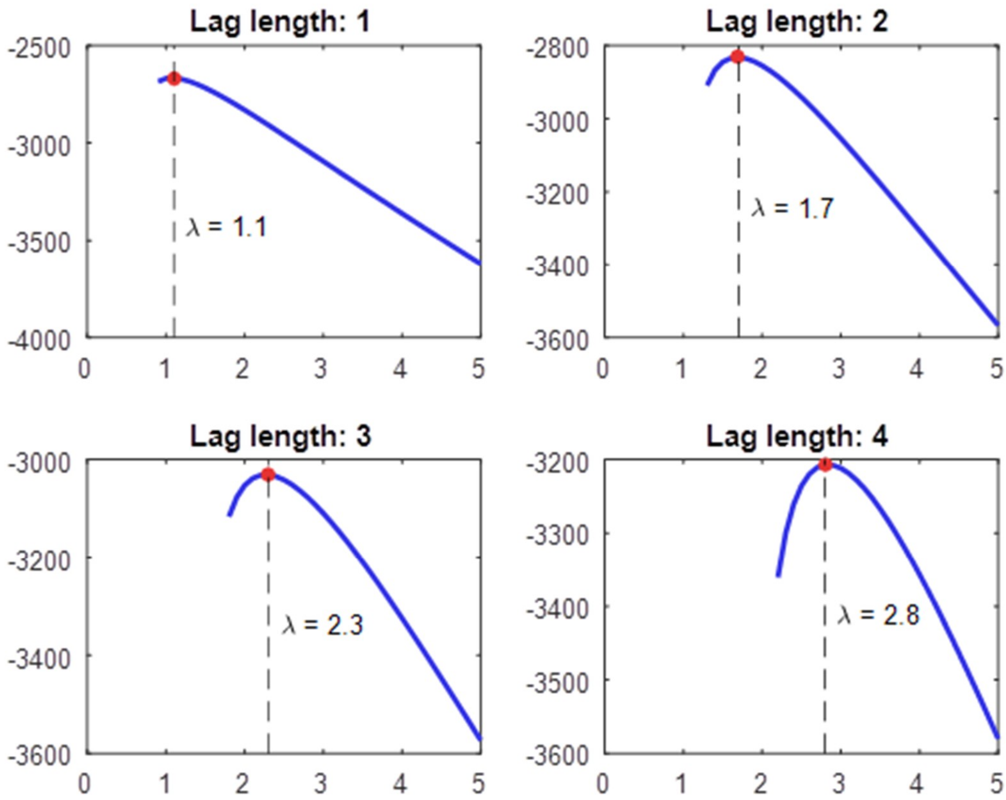
The VAR part of the estimation is based on a standard Gibbs sampler employed for the case of the normal inverted Wishart prior. The conditional posterior distributions are known in closed form and 5,000 draws are taken for posterior inference. The convergence diagnostics of the sampler for the DSGE-VAR model are discussed in Appendix C.

7. Results

Parameter λ drives the tightness of the prior in the VAR part of the model around the parameter vector implied by the DSGE part of the model. Two extreme cases emerge. For $\lambda = 0$ the DSGE-VAR model reduces to the unconstrained VAR model, and for $\lambda \to \infty$ the DSGE-VAR model approaches the DSGE model, with all the restrictions imposed by the structure of the model. The value is chosen based on the marginal likelihood measure. Figure 2 reports the evolutions of log marginal likelihood over values for λ for different numbers of lags in the VAR part. Even though the magnitude of the marginal likelihood suggests using one lag, we stick to four lags ($p = 4$). The reason is that the DSGE model structure suggests four lags in some equations, and imposing zero where the DSGE suggests non-zero goes against the logic of the DSGE-VAR framework. On the other hand, if the ultimate aim is model fit, one lag is a viable alternative. So, as a robustness check, we also estimate the DSGE-VAR(1) model.

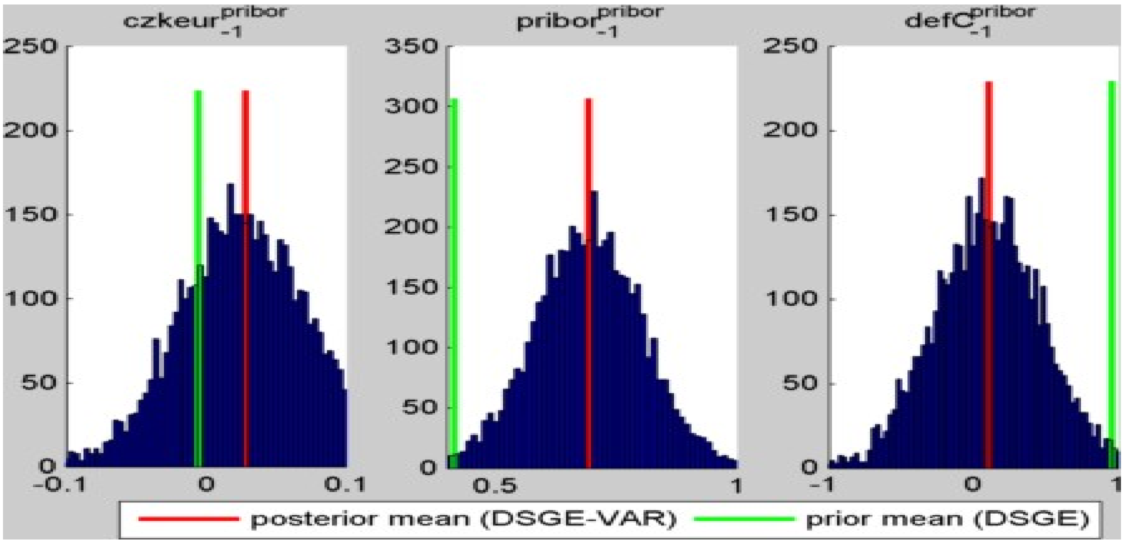
The maximum marginal likelihood for the model with four lags is obtained for $\lambda = 2.8$, which means that the DSGE restrictions improve on the unrestricted VAR estimates. Lambda is higher than one, implying that the data puts larger weight on the DSGE than on the VAR. However, the immediate fall in the log marginal likelihood after lambda reaches its maximum suggests some degree of misspecification of the DSGE model. More details about the sources of misspecification are discussed below when the impulse responses from the DSGE and DSGE-VAR models are compared.

Figure 2: Graphs of the log Marginal Likelihood for the DSGE-VAR(p) Model with Different Numbers of Lags in the VAR Part



As an example, Figure 3 illustrates how the combination of the DSGE model and the VAR model within the DSGE-VAR framework affects the estimated VAR parameters. It reports the posterior distributions of the coefficient at the first lag of selected variables in the equation with the interest rate (Pribor) on the left-hand side. For example, the DSGE part suggests a lower smoothing parameter in the interest rate rule ($pribor_{-1}^{pribor}$). On the other hand, the VAR part suggests a non-zero reaction of the interest rate to the observed exchange rate, which is ruled out by the structure of the interest rate rule in the DSGE model ($czkeur_{-1}^{pribor}$). Finally, the reaction of the interest rate to inflation according to the VAR part of the model is much lower than the response imposed/estimated by the DSGE model.

Figure 3: Selected Posterior Distributions with Prior and Posterior Means



Note: Posterior distributions of the coefficient at the first lag of the exchange rate (czkeur), the interest rate (pribor) and the consumption deflator (defC) in the interest rate equation for $\lambda = 2.8$.

7.1 Impulse Responses

We present the results for both the impact matrix $A_0(\theta)$ taken from the DSGE model completely and the combination of the VAR lower triangular matrix and the rotation from the DSGE model: $\Sigma_{tr}\Omega^{DSGE}(\theta)$. The reason is that such a distinction allows us to compare the IRFs and fiscal multipliers between the DSGE and VAR approaches. The sizes of the fiscal shock are equal to one in all cases, so they represent unexpected one percentage point changes in the quarter-on-quarter (QoQ) growth of the fiscal variable lasting for one quarter.

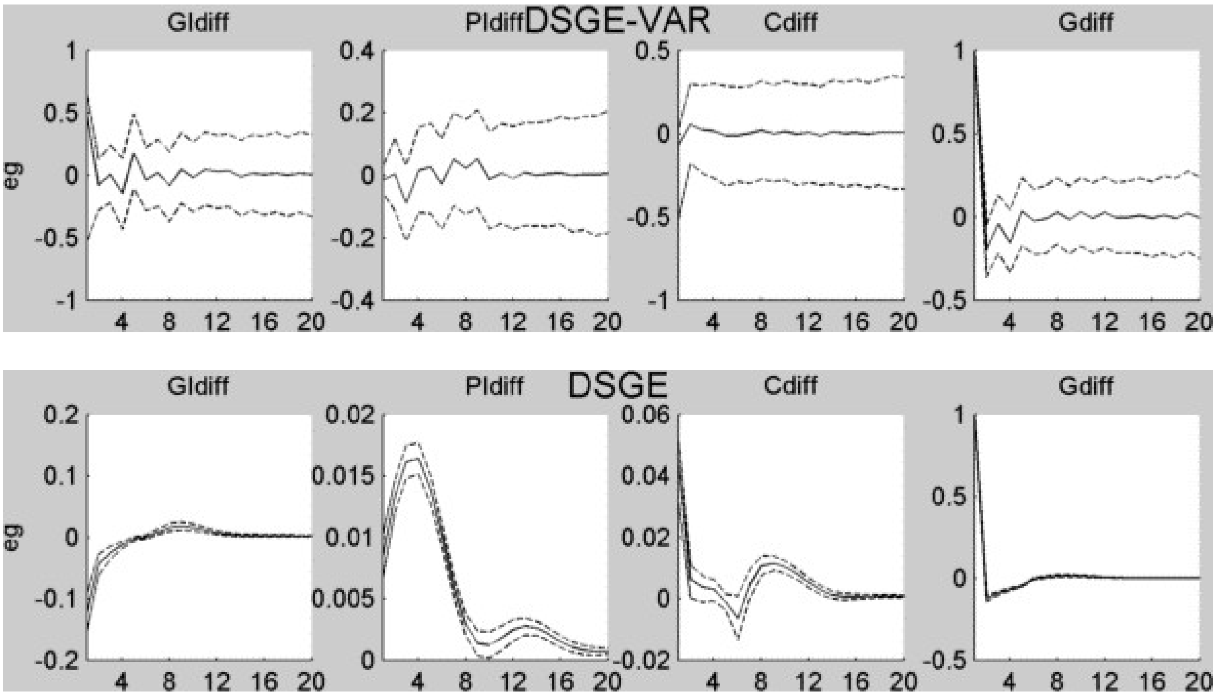
Regarding the shock to government consumption (*eg*), the IRFs based solely on the impact matrix from the DSGE model are different from those implied by the combination of the VAR and DSGE impact matrices (Figure 4).[9] The immediate positive reaction of private consumption (*Cdiff*) to the increase in government consumption is in line with the Keynesian view that is inherently present in the DSGE model (the magnitude of the response depends inter alia on the share of rule-of-thumb households; this share is estimated and set to 16% according to the posterior mean). On the other hand, the median reaction adjusted by the VAR is negative on impact, in line with the neoclassical approach, in which after a rise in consumption the consumer expects a future increase in taxes and adjusts consumption accordingly. Relaxing the assumptions of the DSGE model implies a move from Keynesian behaviour toward neoclassical reactions.

The reaction of government investment (*GIdiff*) is also remarkably different. The DSGE model suggests that government investment reacts in the opposite direction to the government consumption shock. The DSGE-VAR model suggests the opposite, although the reaction is not

[9] The IRFs are reported as the median response with the centred 68% of the posterior distributions of the IRFs.

statistically significant. Modelling the expenditure side of the government budget in the DSGE implies restrictions that are not convincingly confirmed by the data.

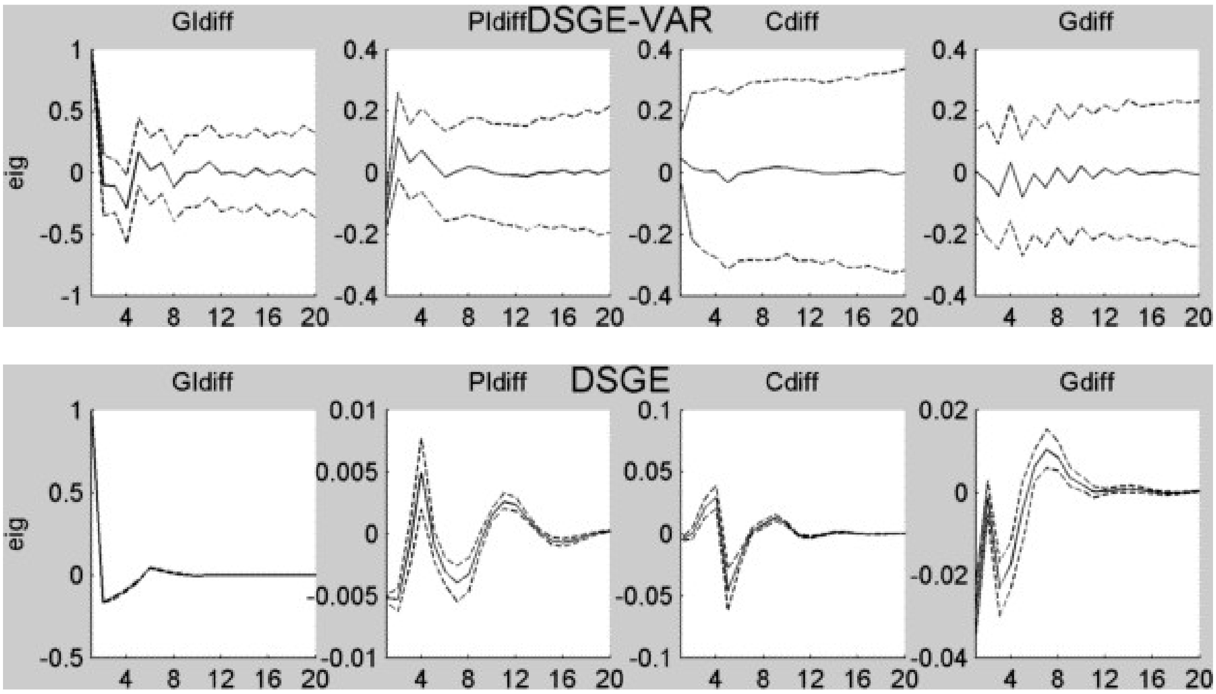
Figure 4: Effects of the Government Consumption Shock in the DSGE-VAR and DSGE Models



Note: Government consumption spending shock (***eg***) – an unexpected one-quarter 1 pp increase in government consumption (***Gdiff***). Responses: government investment (***GIdiff***), private investment (***PIdiff***), private consumption (***Cdiff***) and government consumption (***Gdiff***).

Similar IRFs between the DSGE and DSGE-VAR approaches can be observed for the shock to government investment (Figure 5). A negative response on impact for private investment (*PIdiff*) is reported in both approaches, so the restriction yielding a crowding-out effect of government investment is confirmed through the VAR. The response of private consumption (*Cdiff*) is zero in the DSGE model and positive in the DSGE-VAR model. However, the reaction of consumption in the DSGE-VAR model is not statistically significant. Government consumption (*Gdiff*) does not react on impact in the DSGE VAR model, but decreases immediately in the DSGE model.

Figure 5: Effects of the Government Investment Shock in the DSGE-VAR and DSGE Models

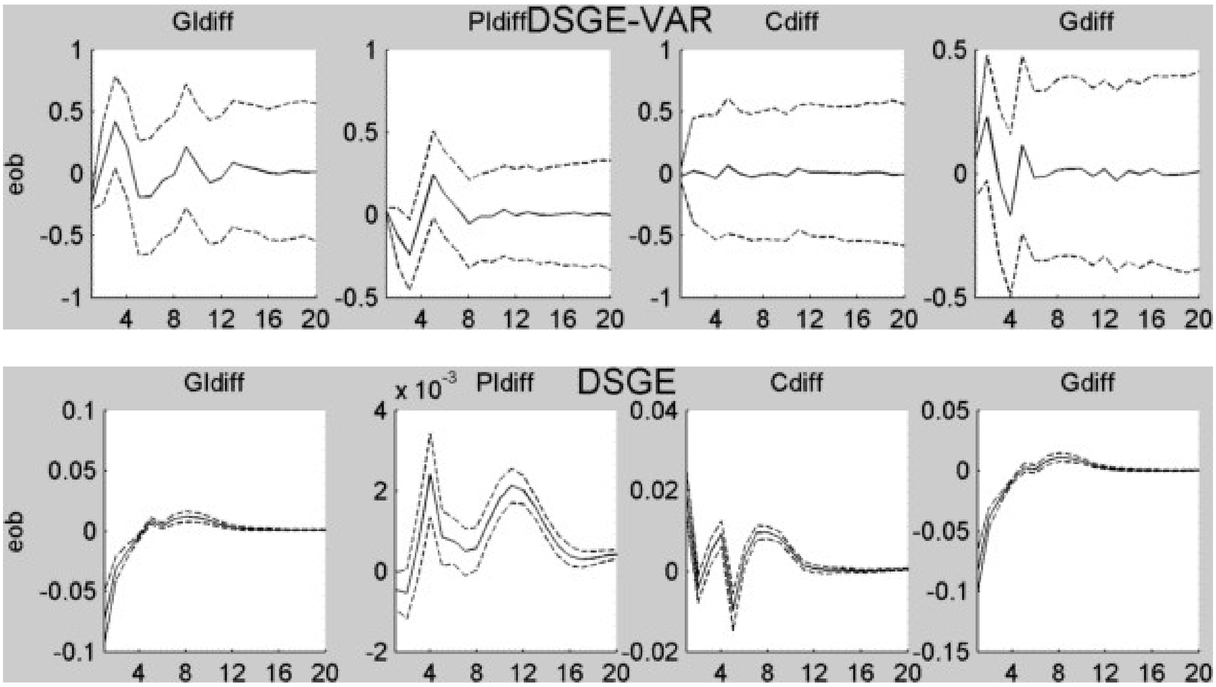


Note: Government investment shock (***eig***) – an unexpected one-quarter 1 pp increase in government investment (***GIdiff***). Responses: government investment (***GIdiff***), private investment (***PIdiff***), private consumption (***Cdiff***) and government consumption (***Gdiff***).

A positive other social benefits shock (*eob*) has a negative impact on government investment (*GIdiff*) accompanied by a decline in private investment (*PIdiff*) in both approaches – see Figure 6. The response of government consumption (*Gdiff*) is negative in the DSGE model and insignificant in the DSGE-VAR model.

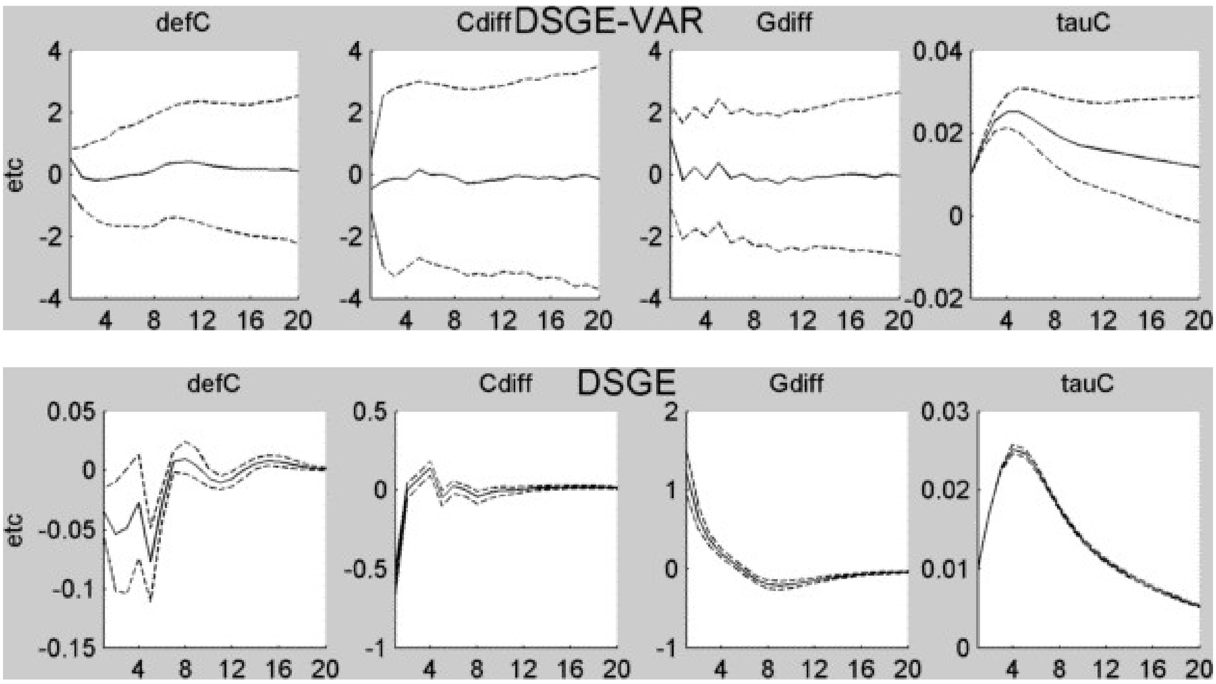
Finally, Figure 7 presents the IRFs of the consumption tax shock (*etc*), defined as an unexpected increase in consumption tax by one percentage point. The IRFs from the DSGE model are in accordance with the theory: an immediate fall in private consumption (*Cdiff*) associated with a decrease in the consumption deflator (*defC*) is observed. The DSGE-VAR framework provides no statistically significant results. A similar picture is obtained when the labour tax shock is examined.

Figure 6: Effects of the Other Social Benefits Shock in the DSGE-VAR and DSGE Models



Note: Other social benefits shock (***eob***) – an unexpected one-quarter 1 pp increase in other social benefits (***sbdiff***). Responses: government investment (***GIdiff***), private investment (***PIdiff***), private consumption (***Cdiff***) and government consumption (***Gdiff***).

Figure 7: Effects of the Consumption Tax Shock in the DSGE-VAR and DSGE Models

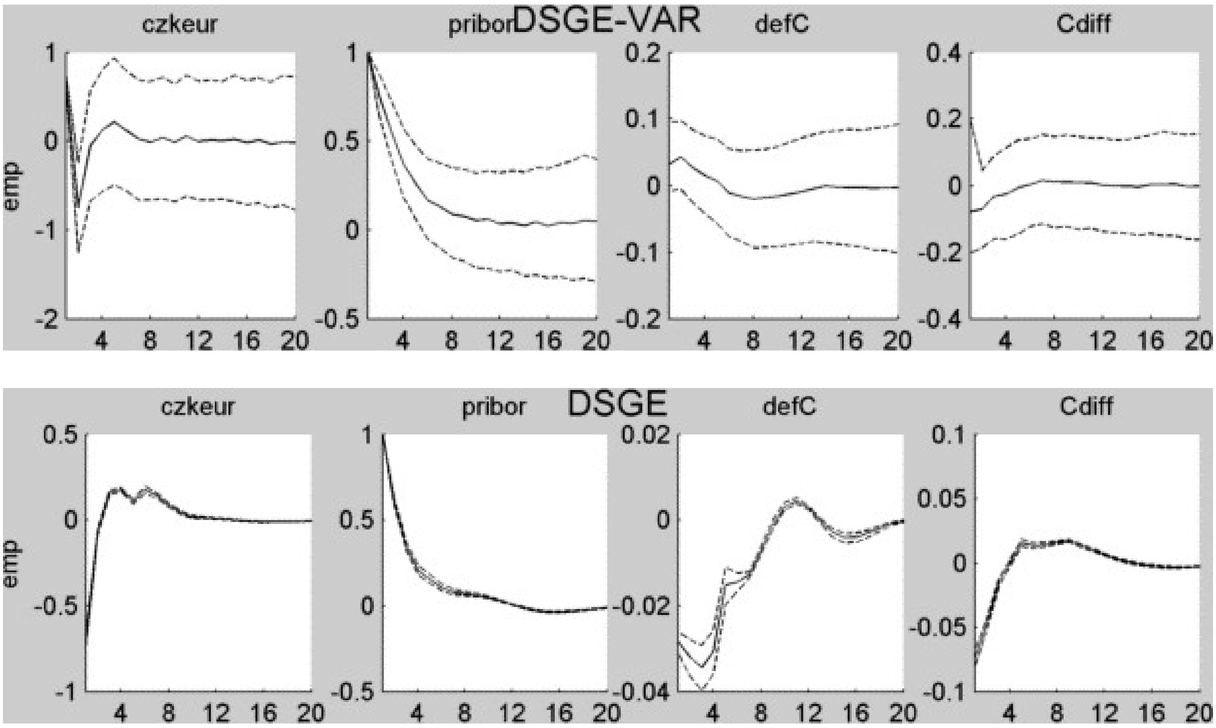


Note: Consumption tax shock (***etc***) – an unexpected one-quarter 1 pp increase in the consumption tax rate (***tauC***). Responses: consumption deflator (***defC***), private consumption (***Cdiff***), government consumption (***Gdiff***) and the consumption tax rate (***tauC***).

Rossi and Zubairy (2011) demonstrated on US data how important it is to consider both monetary and fiscal policy shocks for an appropriate estimation of the effect of either of them. Our model includes both fiscal and monetary policy variables and thus from this point of view is suitable for both monetary and fiscal shock identification. In addition, checking the effects of monetary policy shocks (which are examined in much greater detail than fiscal shocks in the literature) could provide some justification for the estimated effects of fiscal shocks.

Figure 8 presents the IRFs of a monetary policy shock of size 1 (*emp*), i.e. an unexpected one-quarter one percentage point increase in the interest rate. The responses of the DSGE model (i.e. with the shock impacts taken completely from the DSGE model) follow the standard patterns – an immediate appreciation of the exchange rate (*czkeur*) and a fall in the consumption deflator (*defC*) and consumption (*Cdiff*). Relaxing the restrictions imposed by the theory leads to changes in the responses of the exchange rate and the consumption deflator. The so-called price puzzle – a temporary increase in prices following an unexpected increase in the interest rate – can be observed from the DSGE-VAR model (although the response is not statistically significant). The patterns in the data do not justify the restrictions imposed by the DSGE model. However, the different IRFs do not necessarily demonstrate misspecification in the DSGE model. A possible explanation is that we do not control for some variables that are important for the evolution of inflation and the VAR model assigns the variation in prices to monetary policy shocks. In the DSGE model the problem is fixed by an imposed restriction, while in the VAR framework we need to add the variable we do not control for.

Figure 8: Effects of the Monetary Policy Shock in the DSGE-VAR and DSGE Models



Note: Monetary policy shock (***emp***) – an unexpected one-quarter 1 pp increase in the 3M Pribor (***pribor***). Responses: CZK/EUR exchange rate (***czkeur***) – a decrease corresponds to appreciation; 3M Pribor (***pribor***), consumption deflator (***defC***) and private consumption (***Cdiff***).

The difference in the IRFs between the DSGE and DSGE-VAR models suggests possible misspecification of the DSGE model in the form of invalid restrictions implied by the DSGE model itself. The extent of this form of misspecification is summarised by the estimate of parameter λ. If there were no misspecification present in either modelling framework, the IRFs of the two modelling approaches would coincide.

7.2 Multipliers

Table 1 presents the cumulative fiscal multipliers at the horizons of 1, 4, 8 and 16 quarters for the DSGE-VAR and DSGE models. Regarding the DSGE-VAR multipliers listed in the upper part of the table, several observations can be made.

First, the government consumption multiplier approaches 0.4 within four years. The multiplier is in the range of zero and one (albeit at the lower bound) commonly reported in the literature: according to a recent meta-regression analysis of 104 studies on multiplier effects conducted by Gechert (2015), government consumption multipliers are close to one. There is, however, substantial heterogeneity in the multipliers; their magnitude depends, *inter alia*, on the estimation method.

Second, the government investment multiplier is about 0.4 units larger, approaching 0.8. This confirms the prevailing findings in the literature: according to Gechert (2015), government investment multipliers are, on average, about 0.5 units larger than government consumption ones.

Third, the other social benefits multiplier is considerably lower (approaching 0.2 at the horizon of 16 quarters). Its magnitude lies in the range of 0.1 and 0.3 reported in those few available studies which examine detailed expenditure categories (see Ambriško et al., 2012, 2015).

Comparing the multipliers resulting from the DSGE model (see Table 1, lower part) one can see that the government consumption and other social benefits multipliers are similar to those from the DSGE-VAR at the horizon of 16 quarters, while the government investment multiplier resulting from the DSGE (0.03) is much lower than the one from the DSGE-VAR (0.76). This difference can be traced to the impulse response functions, specifically the near-zero responses of private and government consumption in the DSGE shown in Figure 5. Such reactions, in turn, are due to the underlying model structure. The critical restriction in the fiscal DSGE model that implies a fall in government consumption following a government investment shock is related to the government budget constraint – see equation (2.7). The data suggest through the VAR part that this restriction is invalid and the specification of the budget constraint in the DSGE model should be reconsidered. The zero reaction of private consumption on impact relates to the fact that government consumption, not government investment, is included in the utility function of a representative household. Similarly, cross-variable restrictions implied by the budget constraint probably underlie the difference in the other social benefits multiplier in the DSGE-VAR and DSGE models at the horizon of 1 to 4 quarters. The immediate fall in government consumption after the shock to other social benefits is not observed in the DSGE-VAR framework (see Figure 6), resulting in positive (near-zero) multiplier values, while in the DSGE-VAR model the other social benefits multipliers are negative (near-zero) at impact.

It is important to emphasise that the confidence bands of the multiplier estimates are wide and the vast majority of the differences are not statistically significant in this sense.

Table 1: Cumulative Fiscal Multipliers

DSGE-VAR					
	horizon:	1	4	8	16
Government investment		0.15	1.02	0.50	0.76
Government consumption		0.15	0.32	0.34	0.38
Other social benefits		-0.02	-0.24	0.01	0.17
DSGE					
	horizon:	1	4	8	16
Government investment		-0.01	0.33	-0.14	0.03
Government consumption		0.26	0.32	0.35	0.42
Other social benefits		0.01	0.00	0.06	0.13

Note: The fiscal stimulus lasts for one period.

The differences in the multipliers between the DSGE and DSGE-VAR frameworks, in particular the higher values of the government consumption multiplier in the DSGE-VAR one, are broadly in line with the evidence from the literature (Gechert, 2015): DSGE models were found to result in somewhat lower multipliers compared to VAR estimates. The DSGE-VAR model therefore occupies the territory between the two modelling approaches.

Restricting the number of lags to one, as suggested by the marginal likelihood and discussed above, yields fiscal multipliers similar to the case of four lags, as demonstrated in Table 2, for the DSGE and somewhat lower multipliers in the DSGE-VAR. Given the uncertainty related to the responses provided by the DSGE-VAR, the differences are not that substantial or statistically significant.

Table 2: Cumulative Fiscal Multipliers

DSGE-VAR					
	horizon:	1	4	8	16
Government investment		-0.03	0.08	0.08	0.13
Government consumption		0.08	0.10	0.13	0.15
Other social benefits		-0.01	-0.25	-0.33	-0.26
DSGE					
	horizon:	1	4	8	16
Government investment		-0.01	0.05	0.05	0.07
Government consumption		0.26	0.32	0.40	0.42
Other social benefits		0.01	0.05	0.14	0.18

Note: The fiscal stimulus lasts for one period.

Our multiplier values obtained from the DSGE approach are broadly in the range reported by other studies for the Czech Republic employing similar modelling frameworks. The closest results are those reported by Ambriško et al. (2015), as this study has the same model but uses a slightly different definition of shocks (lasting for four periods or longer), while the present study uses one-period shocks, which results in fiscal multipliers at the lower end of the scale. On the other hand, our multipliers are similar (for government consumption) or somewhat larger (for government investment) compared to those reported by Klyuev and Snudden (2011). The authors employed the IMF's Global Integrated Monetary and Fiscal Model (GIMF) calibrated on the Czech data, yielding government consumption and investment multipliers of about 0.4, as compared to our estimate of 0.4 for government consumption and 0.8 for government investment.

Government consumption and investment multipliers obtained for the Czech Republic from the VAR model (Franta, 2012) exhibit a much wider range of values – see Table 3. The government spending multipliers implied by the AB model (Breitung, 2004) take values of 7.5–10.7, which is an order of magnitude higher than the lower bound values of 0.23–0.35 that follow from recursive identification, or those derived from DSGE models.

Table 3: Cumulative Fiscal Multipliers Based on the VAR Model

Government consumption + investment:				
horizon:	1	4	8	20
Recursive identification	0.23	0.32	0.35	0.35
Sign restriction	1.43	1.43	1.46	1.47
AB model (restrictions advocated by Cuaresma et al., 2011)	5.28	6.89	6.83	7.52
AB model (restrictions advocated by Valenta, 2011)	6.65	8.90	9.09	10.76

Note: The multipliers are computed as the ratio of the cumulative response of GDP and government spending for a given quarter.
Source: Franta (2012)

Table 3 also suggests that the VAR identification method itself is the key determinant of the reported multipliers, as different identification methods yield different ranges of estimates: 0.23–0.35 for recursive identification, 1.4–1.5 for sign restriction and 5.3–10.7 in the case of the AB model.

8. Conclusions

In this paper we built a medium-scale DSGE-VAR model and applied it to the analysis of fiscal policy. The model, comprising 26 variables, was estimated on Czech data covering the period 1996:Q1–2011:Q4. Estimations were performed using Bayesian techniques. The priors for the VAR part and the identification scheme were derived from the underlying DSGE model.

The DSGE-VAR framework was then used to examine the extent and sources of misspecification of the underlying fiscal DSGE model. The results suggest that misspecification of the standard fiscal DSGE model is present. The parameter λ that drives the weight of information from the DSGE model and the information from the data captured by the VAR takes the value of 2.8. It was estimated by maximisation of the joint density of the data and the DSGE model parameters. Values of λ higher than one imply that the data puts larger weight on the DSGE part than the VAR part. Nevertheless, relatively low values of λ show that information from the data helps improve the model estimates (if the underlying DSGE model were able to describe the data-generating process perfectly, the value of λ would go to infinity). Furthermore, a comparison of the impulse responses from the DSGE and DSGE-VAR models sheds light on the particular sources of misspecification of the DSGE model considered alone.

Finally, the paper presents a comparison of fiscal multipliers based on the DSGE and DSGE-VAR frameworks. The government consumption multiplier resulting from the DSGE-VAR model is 0.38 at the horizon of four years and the public investment multiplier is 0.76. These values are in the range commonly reported in the literature. On the other hand, the DSGE model considered alone implies a similar effect of a government consumption stimulus (0.42) but a much lower impact of public investment (close to zero). The impulse responses suggest misspecification of the fiscal DSGE model, which is in turn related to the model assumptions regarding the budget constraint and the utility function of households.

Our findings have implications for model building. First, the results show that the DSGE-VAR framework is a promising modelling tool: while popular VAR models lack robustness due to relatively short and noisy fiscal series (and, in addition, are very sensitive to the choice of identification scheme), the DSGE-VAR framework combines the advantages of structural models with good data fit.

Second, while it would still be too ambitious to generalise that the DSGE-VAR framework is a remedy for modelling, the application of the DSGE-VAR framework for a particular country- and model-specific purpose yields valuable information on how well the underlying DSGE model fits the data, thus helping in the assessment of whether any essential features are missing from the model.

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Appendix A: Marginal Data Density

Parameter λ determines how much the parameters from the DSGE part of the DSGE-VAR model affect the VAR parameters. It is obtained by maximising the marginal data density

$$p_\lambda(Y) = \int p_\lambda(Y \mid \theta) p(\theta) d\theta \tag{A.1}$$

over the grid for λ covering non-negative values. Gelfand and Dey (1994) noted that

$$\frac{1}{p_\lambda(Y)} = \int \frac{f(\theta)}{p_\lambda(Y \mid \theta) p(\theta)} p_\lambda(\theta \mid Y) d\theta = E\left[\frac{f(\theta)}{p_\lambda(Y \mid \theta) p(\theta)}\right] \tag{A.2}$$

because the posterior $p_\lambda(\theta \mid Y)$ can be expressed as

$$p_\lambda(\theta \mid Y) = \frac{p_\lambda(Y \mid \theta) p(\theta)}{p_\lambda(Y)}. \tag{A.3}$$

Formula (A.2) suggests that we need not evaluate posterior $p_\lambda(\theta \mid Y)$ to obtain the marginal data density. The inverse of the marginal data density in (A.1) is then numerically approximated using Geweke's (1999) modified harmonic mean estimator:

$$\frac{1}{M} \sum_{j=1}^{M} \frac{f(\theta_j)}{p_\lambda(Y \mid \theta_j) p(\theta_j)}, \tag{A.4}$$

where θ_j is a draw of the DSGE parameter vector from the Metropolis Hastings algorithm and function $f(\cdot)$ is defined as follows:

$$f(\theta_j) \equiv \frac{1}{p(2\pi)^{k/2}} \left|\widetilde{\Sigma}_\theta\right|^{-1/2} \exp\left[-\frac{1}{2}\left(\theta_j - \widetilde{\theta}\right) \widetilde{\Sigma}_\theta^{-1} \left(\theta_j - \widetilde{\theta}\right)'\right] \times I(\theta_j), \tag{A.5}$$

where $\widetilde{\theta}$ and $\widetilde{\Sigma}_\theta$ are the posterior mean and covariance respectively, *k* denotes the number of parameters and the indicator function $I(\theta_j)$ equals one if

$$\left[\left(\theta_j - \widetilde{\theta}\right) \widetilde{\Sigma}_\theta^{-1} \left(\theta_j - \widetilde{\theta}\right)'\right] \le \chi_{1-p}^2(k). \tag{A.6}$$

$\chi_{1-p}^2(k)$ is the inverse-chi-squared cumulative distribution function with *k* degrees of freedom and probability *p*. The likelihood function at a draw $p_\lambda(Y \mid \theta_j)$ is actually a marginal data density from the VAR part of the DSGE-VAR model:

$$p_\lambda(Y \mid \theta_j) = p_\lambda(Y \mid \Phi, \Sigma) p_\lambda(\Phi, \Sigma \mid \theta_j) / p_\lambda(\Phi, \Sigma \mid Y) =$$

$$\frac{\left|\lambda T \Gamma_{xx}^*(\theta_j) + X'X\right|^{\frac{-n}{2}} \left|(\lambda+1) T \widetilde{\Sigma}_u(\theta_j)\right|^{\frac{-(\lambda+1)T-k}{2}}}{\left|\lambda T \Gamma_{xx}^*(\theta_j)\right|^{\frac{-n}{2}} \left|\lambda T \Sigma_u^*(\theta_j)\right|^{\frac{-\lambda T-k}{2}}} \frac{(2\pi)^{\frac{-nT}{2}} 2^{\frac{n((\lambda+1)T-k)}{2}} \prod_{i=1}^{n} \Gamma[((\lambda+1)T-k+1-i)/2]}{2^{\frac{n(\lambda T-k)}{2}} \prod_{i=1}^{n} \Gamma[(\lambda T-k+1-i)/2]}. \quad \text{(A.7)}$$

The marginal likelihood is approximated using 200,000 draws from the MH algorithm.

Appendix B: List of DSGE Variables and Structural Shocks

	Variables	Description	Units	ss[1]		Shocks	Description
1	'data_czkeur'	Exch. rage CZK/EUR	QoQ [%]	0	1	'eg'	Government consumption spending shock
2	'data_pribor'	3M Pribor	%	2.48	2	'eig'	Government investment shock
3	'data_Ldiff'	Employment	QoQ [%]	0	3	'eub'	Unemployment benefits shock
4	'data_xdiff'	Exports	QoQ [%]	0	4	'eob'	Other social benefits shock
5	'data_ndiff'	Imports	QoQ [%]	0	5	'ea'	Technological shock
6	'data_GIdiff'	Gov. investment	QoQ [%]	0	6	'etc'	Consumption tax shock
7	'data_PIdiff'	Private investment	QoQ [%]	0	7	'etk'	Capital tax shock
8	'data_defC'	Consumption deflator	QoQ [%]	0	8	'etw'	Labour tax shock
9	'data_defI'	Investment deflator	QoQ [%]	0	9	'ets'	Social security tax shock
10	'data_defG'	Government deflator	QoQ [%]	0	10	'etd'	Dividend tax shock
11	'data_defX'	Export deflator	QoQ [%]	0	11	'et'	Lump-sum tax shock
12	'data_defN'	Import deflator	QoQ [%]	0	12	'eprem'	Risk premium shock
13	'data_Wdiff'	Nominal wage	QoQ [%]	0	13	'emp'	Monetary policy shock
14	'data_Istar'	3M EURIBOR	%	2.48	14	'euip'	Uncovered interest parity shock
15	'data_Nstar'	Foreign demand	QoQ [%]	0	15	'edemwedge'	Export/foreign demand wedge shock
16	'data_Pstar'	Foreign PPI prices	QoQ [%]	0	16	'enstar'	Foreign demand growth shock
17	'data_Cdiff'	Private consumption	QoQ [%]	0	17	'erstar'	Foreign monetary policy shock
18	'data_Gdiff'	Gov. consumption	QoQ [%]	0	18	'epstar'	Foreign price growth shock
19	data_undiff'	Unemployment rate	QoQ [%]	0	19	'ecostpushW'	Wage stickiness (Calvo) shock
20	'data_bdiff4'	Indebtedness	QoQ [%]	0	20	'ecostpushPI'	Investment price stickiness (Calvo) shock
21	'data_tauC'	Consumption tax rate	0.2=20%	0.2	21	'ecostpushPY'	Domestic intermediate price stickiness (Calvo) shock
22	'data_tauK'	Capital tax rate	0.19=19%	0.19	22	'ecostpushPC'	Consumer price stickiness (Calvo) shock
23	'data_tauL'	Labour tax rate	0.1=10%	0.1	23	'ecostpushPG'	Government deflator stickiness (Calvo) shock
24	'data_ubdiff'	Unempl. benefits	QoQ [%]	0	24	'ecostpushPN'	Import price stickiness (Calvo) shock
25	'data_sbdiff'	Social benefits	QoQ [%]	0	25	'ecostpushPX'	Export price stickiness (Calvo) shock
26	'data_pbdiff'	Primary deficit	QoQ [%]	0	26	'emeipo'	Marginal efficiency of private investment shock
					27	'emeig'	Marginal efficiency of government investment shock
					28	'ehabito'	Habit shock (optimising households)
					29	'ehabitr'	Habit shock (rule-of-thumb households)
					30	'eimport'	Real import shock

Note: [1] steady-state values (the data mapping is carried out on the basis of demeaned series, therefore the steady-state growth rates are all zero).

Appendix C: Convergence Diagnostics

This appendix contains statistics suggesting convergence of the sampler used to estimate the DSGE-VAR model. The statistics employed are: autocorrelation of the chain at a lag equal to 10, the inefficiency factor and Raftery and Lewis (1992) statistics showing the number of draws needed to get a stationary distribution for the Gibbs sampler. Figure C1 shows the statistics for all the autoregressive parameters of the VAR model (2,704 parameters), while Figure C2 displays the statistics for the draws of elements of the variance-covariance matrix of the VAR model in the first row of the matrix (26 parameters). All the statistics demonstrate convergence of the sampler.

Figure C1: All Autoregressive Parameters of the VAR

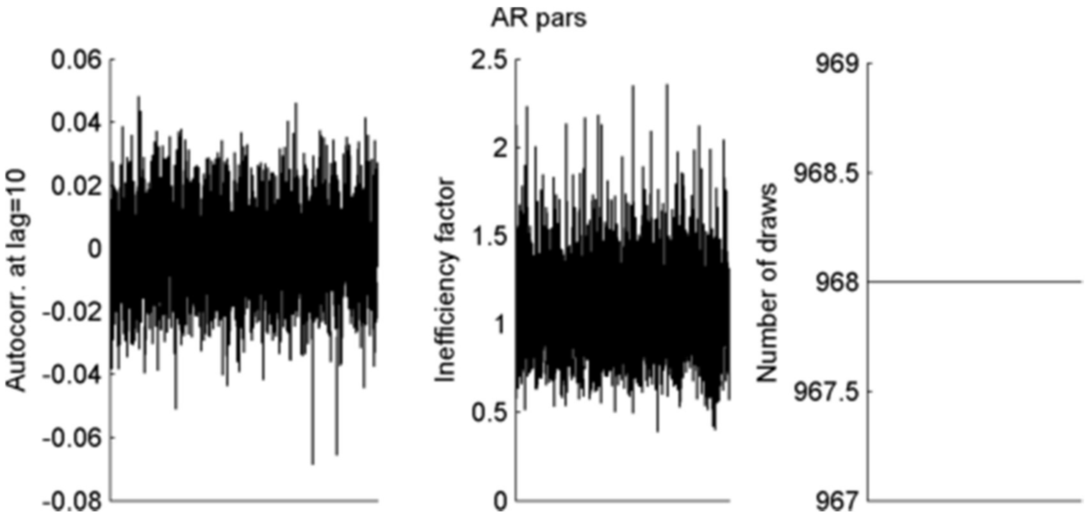
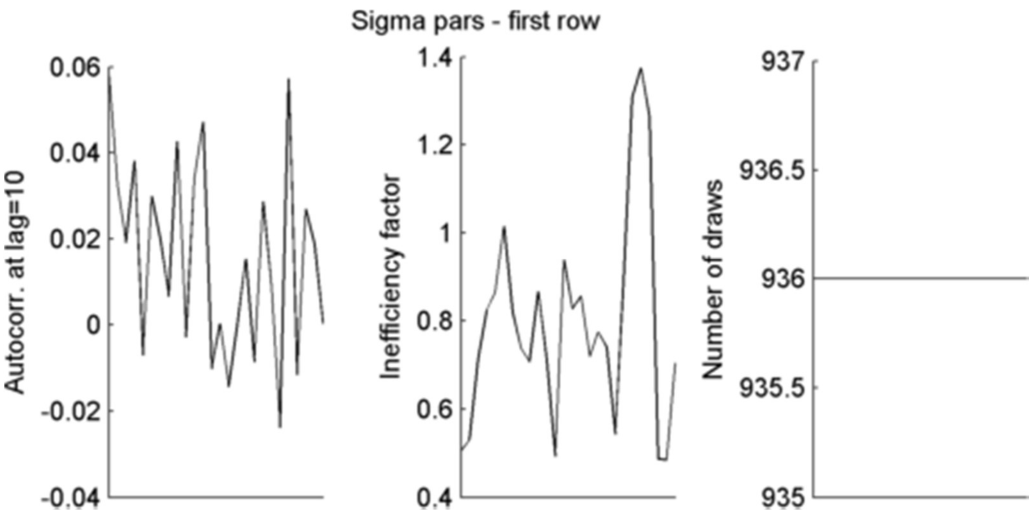


Figure C2: First Raw Parameters of the Variance-covariance Matrix of the VAR



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