ESTIMATING EXPECTED LOSS GIVEN DEFAULT

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This article discusses the estimation of a key credit risk parameter – loss given default (LGD) – and calculates it for selected companies traded on the Prague Stock Exchange. The importance of estimating LGD stems from the fact that a lender's expected loss is the product of the probability of default, the credit exposure at the time of default and the LGD. The Mertonian structural approach is used for LGD estimation. This technique enables us to derive LGD for publicly traded companies from a knowledge of their debt and share prices. It is reasonable to assume that the resulting LGD calculated for selected companies traded on the Prague Stock Exchange represents a lower estimate of this parameter for the entire corporate sector.

1. INTRODUCTION

The increase in global risk on world financial markets has underlined the importance of correctly estimating future credit losses. This recent experience shows how underestimation not only of the probability of default (PD), but also of the loss given default (LGD), can threaten the stability of financial markets. Over the past 20 years most effort has been put into estimating PD, but attention is now turning increasingly to estimation of the realised losses a lender will suffer if a counterparty defaults. These efforts were stepped up with the arrival of the New Basel Capital Accord (Basel II), which identifies LGD along with PD as a key risk parameter and, under the IRB[46] approach, allows banks to use internal methods to estimate it (see BCBS, 2005). Interest in LGD and in methods of calculating it has therefore been rising strongly in recent years.

As LGD is derived from many factors, such as the degree of subordination of debt, collateral and macroeconomic conditions, future LGDs are quite difficult to estimate. This article sets out to explain the LGD concept and to apply an LGD estimation method based on market data for the Czech economy.[47] The available data allow such techniques to be applied only to the corporate sector, not the household sector. Within the corporate sector, the methodology could only be applied to selected companies listed on the Prague Stock Exchange (PSE). For this segment the average LGD was constructed, which can be regarded as a lower estimate of the LGD for the corporate sector as a whole.

The next section presents the LGD concept and explains in detail the method used to estimate LGD based on market data. Section 3 is devoted to aggregate results for companies listed on the PSE. Section 4 compares the characteristics of the firms analysed with the indicators for the aggregate corporate sector and analyses the relationship between the estimated indicator and the probability of default. The concluding section summarises the results.

2. THE LGD CONCEPT

LGD is usually defined as the percentage loss rate suffered by a lender on a credit exposure if the obligor defaults. In other words, even if the counterparty defaults (fails to repay the amount owed), the lender will usually succeed in recovering some percentage of the current amount owed in the process of workout or sale of the obligor's assets. This percentage is termed the recovery rate (RR), i.e. the following relation holds: $RR = 1 - LGD$.[48] LGD can be estimated on the basis of historical data on realised losses.[49] Another modelling technique focuses on

46 The IRB (Internal Rating Based) approach allows banks in some cases to use internal PD and LGD estimates for calculating capital adequacy (see BCBS, 2005).
47 An alternative approach is to estimate LGD based on historical losses.
48 LGD also comprises other costs related to default of the obligor. A more accurate formula for the relationship between LGD and RR is therefore LGD = 1 – RR + Costs of Default. In the text that follows, however, we will regard RR and LGD as complements, as costs are relevant to only some types of LGD and tend to be negligible by comparison with RR (see Schuermann, 2004).
49 Although banks have started gathering such information in recent years, it is not publicly available.

the information contained in market prices of risky instruments and attempts to use this information for ex-ante estimation of future LGDs. Owing to the unavailability of relevant data covering realised losses, this article concentrates on the second method, deriving LGD from market data (implied market LGD).

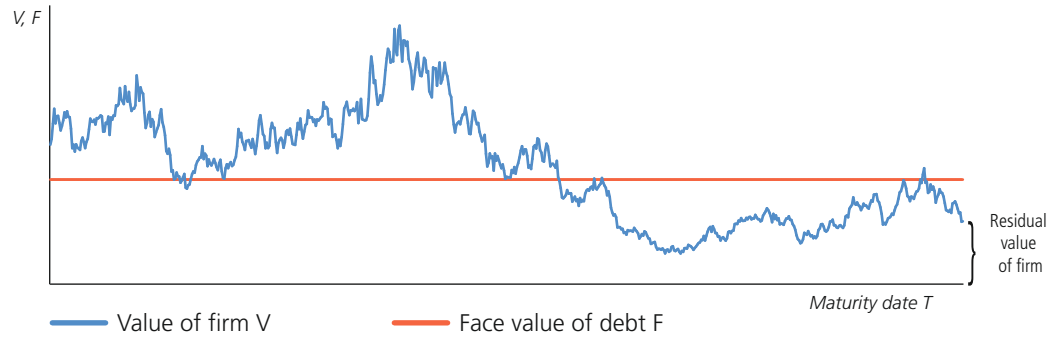
This approach is based on Merton's (1974) structural model, in which the theory of option pricing is used to determine corporate debt. Merton drew directly on the work of Black and Scholes (1973), who were the first to mention the possibility of valuing a firm's liabilities and equity as an option on the firm's assets. In the Mertonian approach, the firm's balance sheet consists of the market values of the individual items. The left-hand side represents the market value of assets *V*. On the liability side are the market value of equity *E* and the market value of debt *D*.[50] In this concept, the market value of the firm's assets *V* equals the sum of the market price of equity *E* and the market price of debt *D* with face value *F* maturing at time *T*.[51] In Merton's original approach, the firm defaults if, on the debt maturity date, the value of the firm is less than the face value of the debt.[52] In such case, it does not pay the equity holders to repay the debt. They do not exercise the option and hence they turn the firm over to the debtholders. The value of equity is therefore identical to the value of a European call option on the firm's value with an exercise price corresponding to the face value of the debt *F* (see Merton, 1974).[53]

The probability of default can be expressed as the probability that the value of the firm on the debt maturity date will be less than the amount the equity holders have to repay, i.e.

$$PD = \Pr(V_T \leq F) \qquad (1)$$

where V_T denotes the value of the firm on the maturity date. This situation is described by the following chart, which shows the evolution of the value of a firm that defaults on the maturity date ($V_T < F$). If this event occurs, the debtholders are only paid the value of the firm at the maturity date (V_T) instead of the original amount of their claim *F*. The ratio of the expected value of the firm at the maturity date to the face value of the debt can thus be regarded as the recovery rate (RR).[54]

Chart 1 – Possible evolution of value of firm over time



50 This approach can introduce ambiguities from the perspective of the traditional accounting approach. For example, the term "equity" in the Mertonian approach refers not to the book value of equity, but to the market value of equity, i.e. market capitalisation (see Merton, 1974, Jones et al., 1984, Hillegeist et al., 2004, etc.).

51 Another simplifying assumption is that there are no taxes. For this reason, the tax shield is not considered and the market value of the firm's assets *V* can be regarded as identical to the firm's total value.

52 In reality default can occur at any time before the maturity of the debt. According the Czech legislation in force, bankruptcy happens if a firm is (i) unable to pay, i.e. its monetary debts are more than 30 days past due and it is no longer able to repay them, (ii) overindebted, i.e. the sum of all the obligor's liabilities exceeds the sum of its assets. Moreover, the actual definition of default in the Mertonian approach corresponds more closely to bankruptcy than to the traditional Basel II definition of default (i.e. payment more than 90 days past due).

53 The value of the debt can be viewed as the value of a European put option with an exercise price of *F*. Unlike American options, European options cannot be exercised before the maturity date *T*.

54 More accurately this is the conditional mean of the value of the firm at the maturity date *T*, given the default event, i.e. $E(V_T \mid V_T < F)$.

If we make some assumptions about the evolution of the value of the firm V over time,[55] we can obtain a closed-form formula for the expected value of the firm at the maturity date and hence for RR as well. If we further modify the original Mertonian model for dividend payouts, we obtain the final expression for estimation of the expected LGD (ELGD) as a function of the face value of the debt F, the value of the firm V, the maturity date T, the costs of default $(1-\varphi)$,[56] the dividend rate δ, the expected growth in the firm's assets μ_V and the volatility of those assets σ_V, i.e.

$$LGD = 1 - \varphi \cdot RR(F, V, T, \delta, \mu_V, \sigma_V) \tag{2}$$

A detailed description of the model and its derivation can be found in Seidler and Jakubík (2009). ELGD increases *(ceteris paribus)* with growing face value of the debt F and with rising volatility of the firm's value σ_V, which increases the probability that the firm's value on the maturity date will be less than the value of the debt. Longer debt maturity also causes greater uncertainty about the firm's future value. ELGD is therefore rising in T. Growth in the dividend rate δ also increases ELGD, because dividend payouts reduce the firm's value and thus make it more prone to default from the credit risk perspective. Conversely, ELGD falls with rising market value of the firm V and rising expected growth in its assets μ_V.

The empirical use of structural models, and thus also the calculation of ELGD from equation (2), is limited by the fact that variables such as the firm's value V and its volatility are not directly observable and must be estimated. The technique for obtaining these estimates was first proposed by Jones et al. (1984) and uses knowledge of the market value of equity E and its volatility σ_E, which are easy to determine. We thus obtain the estimated variables V and σ_V by simultaneously solving two equations based on the Mertonian approach which match the known values of E and σ_E with the estimated values of V and σ_V. Due to the non-linearity of those equations it is necessary to solve the system numerically (see Hillegeist et al., 2004).

We obtain the market value of equity E as the share price at the end of the fiscal year multiplied by outstanding number of stocks. Four methods were chosen for calculating volatility σ_E. The average of the two highest values then enters the system of equations for the calculation of V and σ_V.[57] For the sake of simplicity, we used the book value of liabilities at the end of the relevant year as the face value of the debt. The maturity of the debt was set arbitrarily at five years for all firms as the average maturity of short-term and long-term debt.[58] To extract δ we used the ratio of the absolute amount of dividend payouts in the given year to the value of the firm. The costs of default were set at 10% on the basis of the study by Andrade and Kaplan (1998).

Historical values of δ, volatility σ_E, the current market value of equity E and the book value of liabilities F can be obtained for each year. Using these values we can numerically calculate the firm's current market value V and its volatility σ_V. By substituting these variables into equation (2) we then obtain the ELGD at the five-year horizon.

55 We assume that the value of the firm follows a geometric Brownian motion over time (see Hull, 2002).

56 Owing to the costs of default, the final recovery rate achieved by the debtholders will be lower, i.e. $\varphi \cdot V_T$ expresses the residual value of the firm's assets at time T net of the costs of default. We can thus express ELGD as $ELGD = 1 - \varphi \cdot RR$.

57 This approach is chosen to capture the different values of σ_E over time and to better differentiate the current volatility from its historical values. For this reason we calculated (i) the basic volatility over 250 trading days, (ii) the five-year volatility using the exponentially weighted moving average, where more recent observations carry higher weights, (iii) the long-term volatility using GARCH(1,1) and (iv) the long-term five-year volatility.

58 For most firms the average debt maturity is shorter in reality. A longer time period was chosen for conservative reasons in order to ensure that the LGD estimates obtained were slightly overestimated rather than slightly underestimated.

3. RESULTS

The aforementioned approach was applied to non-financial corporations listed on the Prague Stock Exchange in 2000–2008. The least liquid companies were excluded from the sample. We also excluded firms that entered the PSE after 2007 on the grounds of an insufficiently long time series. Accounting data for the companies analysed were obtained from the Magnus (2008) database, and for some firms these were supplemented with information from the relevant annual reports. Share prices, share numbers and dividend payouts were obtained from the PSE website and internal CNB data.

The aggregate results for the 15 selected companies are given in the following table.[59]

Table 1 – ELGDs

LGD (%)	2000	2001	2002	2003	2004	2005	2006	2007	2008*
Maximum	78.4	76.6	58.5	57.9	45.0	28.9	29.5	41.2	73.6
Minimum	24.0	15.8	21.7	16.5	15.6	13.0	13.2	13.2	14.5
75th percentile	67.5	53.2	43.3	34.0	31.3	20.0	22.3	25.8	53.7
25th percentile	32.7	28.4	28.5	19.7	20.6	15.9	15.8	16.4	22.7
Median	42.9	48.3	36.2	26.2	21.6	18.5	18.7	19.2	33.9
Mean	**48.8**	**44.8**	**37.0**	**29.6**	**25.4**	**18.8**	**19.5**	**22.2**	**39.9**
Std. Dev.	**19.4**	**19.1**	**12.1**	**13.2**	**9.2**	**4.4**	**5.0**	**9.3**	**19.6**

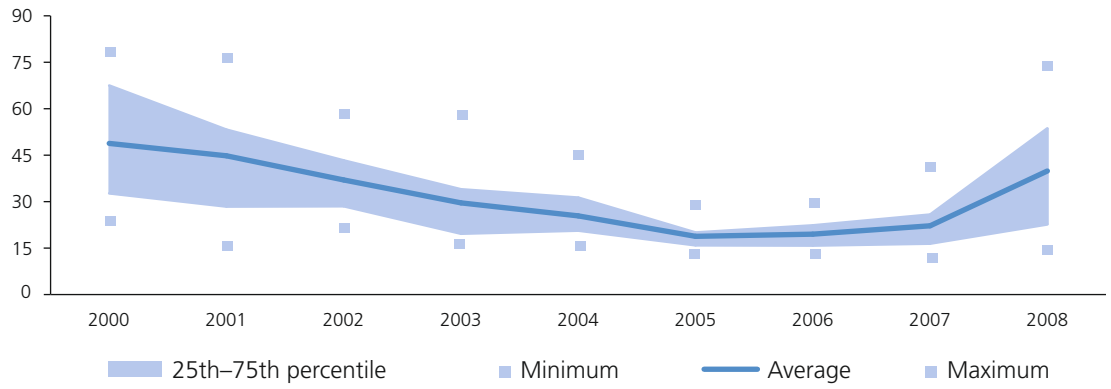
Source: Authors' calculations
* The estimates for 2008 draw on equity market information which, however, is based on 2007 accounting data

The highest ELGD, exceeding 78%, occurred in 2000 in a company that posted a large loss. This negative result led to a sharp decline in the firm's share price and an almost 24% drop in the value of its assets. Some companies recorded a rise in ELGD despite posting profits. This was usually due to negative stock market developments and growth in the volatility of the firm's assets. Some firms recorded a sharp fall in value together with a high dividend payout and thus also a sizeable increase in ELGD.

The calculations for 2008 are still based on the previous year's accounting data and are therefore more an illustrative example of how stock market developments influenced the LGD estimates in 2008. Some companies showed moderate LGD growth differing little from the previous years' values, whereas others recorded sharp growth several times higher than the historical values of this indicator. The latter were mostly companies that had been listed on the PSE for a short time only. However, where the value of their liabilities fell significantly compared to 2007, the given ELGD estimates may be considerably overestimated.

59 The 15 companies under analysis account for around 7% of the corporate sector's total assets.

Chart 2 – Evolution of average ELGD (%)



Source: Authors' calculations

Chart 2 displays the evolution of the average LGD and its volatility over time. It is apparent that the average LGD of the sample of firms analysed was falling until 2006, as was its volatility as illustrated by the 25th–75th percentiles. Although the average LGD rose slightly in 2007, the growth was fairly insignificant, so it seems that the stock market in 2007 still did not contain information on the subsequent financial turbulence. Given the stock market falls in 2008, we can expect sharp growth in the average LGD in that year. This is confirmed by twofold growth in the preliminary values of this indicator.

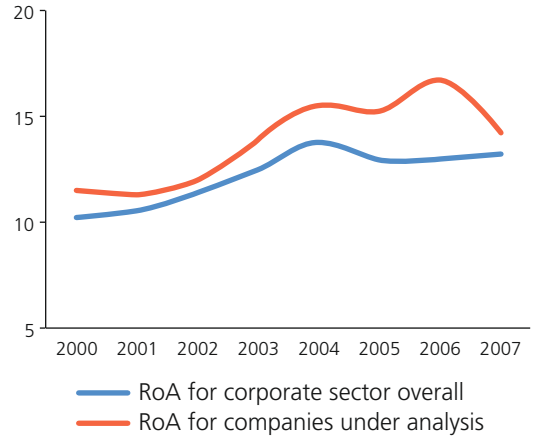
4. APPLICATION OF THE RESULTS

The above estimated LGDs for the set of firms under review can be used only as a rough estimate of the LGD for the corporate sector as a whole. It is reasonable to assume that firms traded on the PSE are less risky, hence their LGD is the lower estimate of the average LGD for the corporate sector overall. For the LGD calculation method used, this assumption corresponds to lower leverage and greater potential for growth in the firm's value. If we use return on assets (RoA) as a proxy for growth in the firm's value, a comparison of the time series suggests higher growth for the companies under analysis than for the corporate sector as a whole (see Chart 3).[60] The evolution of RoA also reveals that the profitability of the firms under analysis rises more relative to the corporate sector average at a time of economic growth. Conversely, during an economic slowdown the differences are less significant. Likewise, firms that use the stock market to raise funds are less leveraged relative to the corporate sector average (see Chart 4).[61]

60 The profit time series correlation between the aggregate corporate sector and the firms under analysis is 90%.

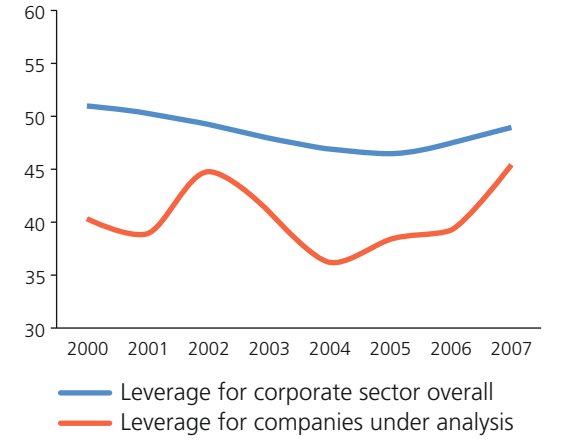
61 Leverage was defined as the ratio of total payables and total liabilities and return on assets as the ratio between the sum of depreciation, costs and pre-tax profits and total assets.

Chart 3 – RoA for companies under analysis vs. aggregate corporate sector (%)



Source: CZSO, Magnus

Chart 4 – Leverage for companies under analysis vs. aggregate corporate sector (%)



Source: CZSO, Magnus

The LGD estimate we obtain gives us some idea about the true value of this indicator for the corporate sector and, together with the aggregate default rate, enables us to better estimate banks' future potential losses on corporate loans.[62] These estimates can also be used to refine the banking sector stress tests used for financial stability analysis. The World Bank gives a recovery rate of claims on insolvent firms of 21% for the Czech Republic, which equates to an LGD of 79%.[63] Owing to a different calculation methodology, however, this figure cannot be compared directly with our implied market estimate. Correct LGD assessment in our case requires all future information to be contained in market data. The indicator thus does not reflect future growth in risk due to developments not expected by market participants. In this regard, our concept is relatively procyclical.[64] It is clear from the following chart that the LGD estimate is quite strongly correlated with the corporate sector default rate (see Chart 5).[65] The expected losses of debtholders correspond to the product of these two parameters, and the credit exposures will differ from the actual losses by the square of the prediction errors of these two parameters. This can lead to significant underestimation of the total loan loss. Our conclusions are in line with the empirical evidence of the correlation between PD and LGD in the literature (e.g. Altman et al., 2005).

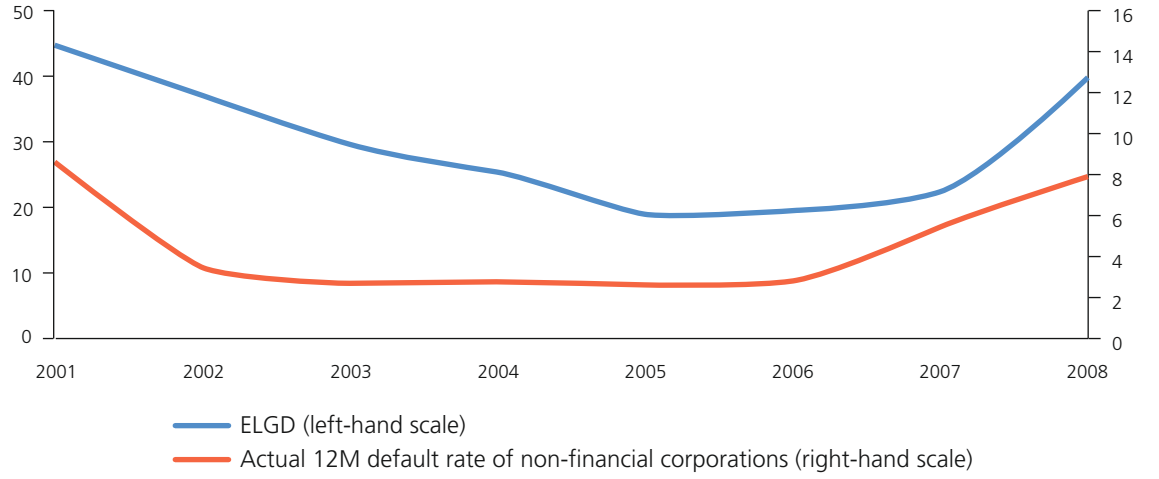
62 Banks' average losses can be estimated as the product of credit exposure, the average probability (rate) of default and the average LGD.

63 This figure is based on data for 6/2006–6/2008. According to these data, the recovery rate for insolvent Czech firms is one of the lowest in Europe. The figures for other countries are: Poland 30%, Slovakia 46%, Germany 52%, Italy 57%, Portugal 69%, the UK 84% and Belgium 86% (The World Bank, 2008).

64 However, share prices can also be influenced by events that are not directly linked with changes in expectations regarding future risks – for example a fight for a majority. Such jumps in prices, however, will be corrected in time and the short-term change in volatility will not be significant enough in the five-year volatilities used to further affect the future LGD estimates.

65 The 12M default rate of non-financial corporations expresses the actual default rate in the 12 months following the period under review. The 2007 figure expresses the default rate during 2008. The 2008 figure is thus only an estimate. LGD refers to the expected percentage loss given default at the five-year horizon.

Chart 5 – Evolution of average ELGD and corporate sector default rate (%)



Source: CCR and authors' calculations

5. CONCLUSIONS

In this article we presented a method for estimating loss given default for firms traded on the Prague Stock Exchange. LGD is dependent on numerous factors and so the methods used to estimate it are not straightforward and still rank among the open problems of contemporary credit risk management. This study describes the LGD concept and uses a modified Mertonian structural model to estimate the five-year LGD for selected companies listed on the PSE in the 2000–2008 period. The chosen method does not use historical LGDs, but attempts to extract LGDs from market data. For this reason, the methodology is applicable to publicly traded companies only. The use of market information allows us to incorporate current impulses from the financial markets into the LGD estimates. On the other hand, one should bear in mind that these estimates may be distorted by over-optimistic expectations about future economic conditions.

The calculations reveal that the average LGD of the sample of firms analysed varies over time from 20 to 50%. The approach described above is based on a number of simplifying assumptions. There are no taxes, the firm's overall debt structure is represented by a single zero-coupon bond, and default can occur only on maturity of the debt. The calculations also abstract from the various degrees of debt subordination in the capital structure of firms. Despite all these assumptions, though, our constructed indicator of the average LGD of the publicly traded non-financial corporations under analysis probably equates to a lower estimate of the corporate sector LGD.

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