Price Convergence: What Can the Balassa–Samuelson Model Tell Us?

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Abstract

The paper provides a theoretical reference point for discussions on adjustments in price levels and relative prices. The authors present a “nested” model integrating the Balassa–Samuelson model of the real equilibrium exchange rate with a model of accumulation of capital and with the demand side of the economy. Consequently, they show how the model can be generalised to a case of numerous commodities with different degrees of tradability. The predictions of the model are generally consistent with empirical findings for Central and Eastern European countries. The authors show how the theoretical model can be used for internally consistent simulations of the future convergence process in a transition economy.

JEL Codes: E31, E52, E58, F15, P22.
Keywords: Balassa–Samuelson model, inflation, relative prices.

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Nontechnical summary

The authors derive a model suited for studying in a consistent way the process of long-run convergence in productivity, output and prices. They extend the traditional Balassa–Samuelson framework by distinguishing between two different kinds of capital. Tradable capital is assumed to move freely among countries and be usable as collateral for borrowing, which means that it is financed from abroad in equilibrium. Nontradable capital, on the other hand, cannot be imported from abroad and has to be gradually accumulated via savings in the domestic economy. This accumulation of nontradable capital is the driving force of long-run economic growth, as well as of convergence in price levels.

An additional source of long-run convergence may be a falling risk premium on tradable capital investments, which increases the capital inflow into the economy. It is shown that a lower risk premium leads to higher output in both sectors of the economy, but also to a higher price of nontradables and thus a higher relative price level. These links between the risk premium, economic growth and price convergence are important when discussing the key equilibrium variables entering the macroeconomic forecasting process.

The two described mechanisms of convergence, i.e. accumulation of knowledge, infrastructure capital etc., together with falling risk premia and foreign capital inflows, are likely to be the main sources of economic growth in the EU accession countries, including the Czech Republic. To illustrate the potential use of the model in practice, the authors thus present two numerical simulations, one focusing on the role of nontradable capital, and the other on the impact of a falling risk premium. The convergence pace of nontradable capital towards the steady state represented by the EU is assumed to be in line with empirical cross-country studies on long-run economic growth. The supply-side of the model is calibrated using the empirically observed relationship between GDP and price levels from the International Comparison Project 1999. The starting position of the economy is chosen to correspond broadly to the current Czech case in terms of GDP per employee in purchasing power parity, which is about 50 percent of the EU average.

Both the simulations show that in the “baseline scenario” the converging country should achieve a growth differential vis-à-vis the EU of about 2 percent a year initially, with a gradually declining tendency. The implied equilibrium real exchange rate appreciation is slightly lower than that, starting from about 1.5–2.0 percent a year and declining to below 1 percent. It is important to keep in mind, though, that country-specific circumstances not captured by the present model may cause the reality to deviate substantially from this benchmark convergence scenario.

In the last part of the paper, the authors also show how the model can be generalised to a case of numerous consumer goods with different degrees of tradability. The implications for relative price dispersions are studied under various assumptions about consumer-goods production technology, the competitive nature of the markets, etc. It is shown that economic convergence to the EU should bring the price structures of converging economies more into line with the system of relative prices in the most advanced EU countries. This prediction of the model is generally consistent with the empirical cross-country findings.
1. Introduction and Review of the Literature

The purpose of this paper is twofold. Its general purpose is to contribute to the theory of real equilibrium exchange rates, thereby providing a theoretical reference point in discussions on adjustments in the price level and relative prices. Its specific purpose is to provide a theoretical foundation to the discussions on adjustment of relative prices in Central and Eastern European (CEE) countries to the European Union (EU).¹

The Balassa–Samuelson (B–S) model (see Balassa, 1964; Samuelson, 1964) is one of the cornerstones of the traditional theory of the real equilibrium exchange rate. The key empirical observation underlying the model is that countries with higher productivity in tradables compared with nontradables tend to have higher price levels.² The B–S hypothesis states that productivity gains in the tradable sector allow real wages to increase commensurately and, since wages are assumed to be linked between the tradable and the nontradable sector, wages and prices will also increase in the nontradable sector. This will lead to an increase in the overall price level in the economy, which will in turn result in an appreciation of the real exchange rate. The B–S effect and the basic version of the B–S model are well described in the theoretical literature (see, for instance, Mussa, 1984; Frenkel and Mussa, 1986; Asea and Corden, 1994; and Samuelson, 1994), including by Czech authors (see, for instance, Kreidl, 1997 and Čihák and Holub, 2001b). Therefore, we only briefly summarise it here.

Until recently, though, the literature did not satisfactorily incorporate both capital (in its various forms) and the demand side at once.³ Obstfeld and Rogoff (1996, pp. 214–216) sketched two examples of possible generalisations of the B–S model, but without formalisation. Asea and Corden (1994) included tradable capital in the model, and Asea and Mendoza (1994) presented the model in a general-equilibrium framework; however, neither of the models included nontradable capital goods and the demand side. Demand side factors have been incorporated in the dependent economy literature, following the pioneering work of Salter (1959), Swan (1960), and other authors. However, as regards capital, most of the dependent economy literature has arbitrarily made some of the following assumptions: (1) capital is tradable; (2) capital is nontradable; (3) the nontraded sector is capital intensive; or (4) the traded sector is capital intensive. For an example of (1) and (3), see Bruno (1982); for an example of (2) and (4), see Fischer and Frenkel (1974). These assumptions have been criticised as arbitrary or unrealistic (see Svensson, 1982; Fischer and Frenkel, 1974). Brock and Turnovsky (1994) and Turnovsky (1997) were the first to provide a comprehensive derivation of a dependent-economy model with both traded and nontraded capital. However, their model was not fully integrated with the B–S model.

¹ The present paper was originally written as companion paper to Čihák and Holub (2003), which analysed the adjustment of relative prices in the Czech Republic to the European Union using International Comparison Project data. Despite this specific purpose, the present paper is a self-contained piece that can be read on its own.

² The basic idea was known already to David Ricardo. Some authors, such as Obstfeld and Rogoff (1996), refer to this theory as the Harrod–Balassa–Samuelson theory, reflecting the fact that thirty years before Balassa and Samuelson, Harrod (1933) used this observation to explain the international pattern of deviations from purchasing power parity.

³ A possible reason for the lack of such analysis is the fact that the focus of the work on real equilibrium exchange rates has shifted from the B–S model to various newer fundamental and behavioural models of the equilibrium exchange rate (see Williamson, 1994, Clark and MacDonald, 1998; for a review, see Frait and Komárek, 1999).
Also, due to its high level of generality, it did not allow for closed-form solutions of the key variables. The model presented in the early parts of this paper can be viewed as a modified version of the model by Brock and Turnovsky (1994) and Turnovsky (1997). Compared with their general approach, ours is tailored to an analysis of monetary policy issues and allows for closed-form solutions for the key variables, which can be easily incorporated into other models and into simulations. We use the advantages of the closed-form solutions to show how the B–S model can be integrated with the new growth theory (which means incorporating both tradable and nontradable capital) and with a theory of the demand side in a comprehensive and internally consistent way. We illustrate the usefulness of this approach by showing how it can be used in simulations of price level convergence of the CEE countries to the EU.

The existing literature is also surprisingly thin on explicitly generalising the B–S model to a case with more goods, which would make the B–S model closer to reality. Even though the literature related to the International Comparison Program (ICP) allows for the existence of more than two goods (see Kravis and others, 1982; and Heston and Lipsey, 1999), it is focused on index numbers and other measurement issues relating to the ICP rather than on developing the B–S model. Several authors, including Obstfeld and Rogoff (1996, p. 214), suggested that such a generalisation of the B–S model is possible, but did not present it (and, to our knowledge, neither has anybody else). In the later parts of this paper, therefore, we show how more than two goods can be explicitly incorporated into the B–S model, allowing us to study relative price convergence in a more realistic theoretical setting.

The paper is organised as follows. After this introduction, the second section summarises the traditional B–S framework in the context of a single factor production function, which is typically presented in the literature. This section is included in the paper mainly to provide a departure point with which our analysis, contained in Sections 3-5, can be compared. The third section shows how the model changes with a three-factor production function, which helps us make the analysis consistent with the basic open-economy growth theory and broaden the discussion from the simplest factors to a richer set of explanatory variables. The fourth section presents a model of price adjustments that includes the demand side. In particular, the section provides a deeper look into how the speed of convergence and the allocation of labour across economic sectors are determined by consumers’ behaviour in interaction with the production and accumulation of nontradable (“human”) capital. The fifth section shows how the B–S model can be extended to the case where there are many commodity groups with different degrees of tradability. The sixth section concludes.

2. The Balassa–Samuelson Model with a One-Factor Production Function

In this section, we summarise the standard version of the B–S model, using a single-factor aggregate production function. This version of the model is included mainly to provide a
comparison point for the analysis contained in Sections 3–5. Let us assume, for simplicity, that the production functions of tradable (T) and nontradable (N) goods have the following form:

\[ Y_T = A_T L_T \]  
\[ Y_N = A_N L_N \]  
\[ Y_T^* = A_T^* L_T^* \]  
\[ Y_N^* = A_N^* L_N^* , \]  

where \( Y \) is production, \( A \) is a constant describing technology, and \( L \) is employment. Foreign economies (modelled as a single economy for simplicity and denoted by an asterisk) employ the same kind of technology as the domestic economy (variables without an asterisk), but may differ from it in the value of the technological parameter, \( A \). Let us assume that the law of one price holds for tradable commodities and let us set (without loss of generality) the world price of tradable commodities equal to one. Finally, let us assume perfect labour force mobility among sectors within an individual economy, but zero mobility of labour among the economies.

From equation (1), it follows that the wage in the domestic economy \((w)\), expressed in terms of tradable commodities, must satisfy the following condition:

\[ w = A_T. \]  

Due to perfect mobility of labour within each economy, the same wages must also prevail in the sector of nontradable commodities. The price of nontradable commodities (again in terms of the world price of tradables) must thus satisfy:

\[ p_N = \frac{w}{A_N} = \frac{A_T}{A_N}. \]  

If we define the price index as a weighted geometric average of prices of tradable and nontradable commodities,\(^5\) we obtain

\[ P \equiv (p_T)^{\gamma} (p_N)^{1-\gamma} = 1^{\gamma} \left( \frac{A_T}{A_N} \right)^{1-\gamma} = \left( \frac{A_T}{A_N} \right)^{1-\gamma} , \]  

where \( \gamma \) is the share of tradable goods in private consumption. If we assume that this share is the same at home as abroad, the relative price level vis-à-vis the outside world is:

\[ \frac{P}{P^*} = \left( \frac{A_T/A_N}{A_T^*/A_N^*} \right)^{1-\gamma} . \]  

\(^5\) The geometric average is the optimal price index, provided that we assume a C–D utility function with unitary elasticity of substitution between tradable and nontradable commodities; the traditional arithmetic average can be thought of as a log-linear approximation of this optimal price index (see, for instance, Obstfeld and Rogoff, 1996).
In an empirical analysis, we should thus ideally use a regression of the relative price levels on the relative productivities in the tradable and nontradable economic sectors at home and abroad in order to test for the theory’s predictions. The expected elasticity would then be \((1-\gamma)\). In reality, however, the technological parameters are not directly observable and are often approximated by GDP (see, for instance, Čihák and Holub, 2003). If the model’s assumptions hold, such an approximation may indeed be reasonable. This can be seen if we express the “nominal GDP” per employee (i.e., GDP in the units of tradable goods, or in a currency in practical use, divided by total employment \(L=L_T+L_N\)) as

\[
\text{GDP}_{\text{nom}} = A_T \frac{L_T}{L_T + L_N} + P_N A_N \frac{L_N}{L_T + L_N} = A_T ,
\]

where we have substituted for \(p_N\) from equation (4). Equation (7) means that nominal GDP expressed in terms of international prices of tradable commodities exactly reflects productivity in the tradable sector. Equation (6) can then be transformed into

\[
\frac{P}{P^*} = \left( \frac{\text{GDP}_{\text{nom}}}{\text{GDP}^*} \right)^{1-\gamma} \left( \frac{A_N}{A_N} \right)^{1-\gamma},
\]

which means that the relative price level directly depends on the ratio of nominal GDPs at home and abroad, the elasticity being \((1-\gamma)\), which is the share of nontradables in GDP.

A natural approach thus would be to regress the relative price levels on the ratio of nominal GDPs. In Čihák and Holub (2003), for example, we carried out such a regression and the estimated elasticity of the relative price level with respect to nominal GDP was 0.47. This would imply that the share of nontradables in GDP is about one half, which is in the range of realistic values. One can thus conclude that the B–S model is supported by the basic empirical cross-country estimates in both qualitative and quantitative terms.

3. The Balassa–Samuelson Model with Tradable and Nontradable Capital

The one-factor production function, with labour as the single factor of production, is a convenient assumption for the first exposition of the B–S model. However, to gain a better understanding of the relationship between economic growth and price adjustment, it is useful to assume a more elaborate production function, similar to the new literature on economic growth. In particular, the growth literature indicates the key role of accumulation of physical and human capital in economic growth and convergence (see, for instance, Solow, 1956; Barro, Sala-i-Martin, 1995; Čihák and Holub, 2000). In this section, therefore, we address the question of whether and how much the price adjustment process changes if we take the various forms of capital into account.

If we wanted to proceed in several steps, the next one could be to take into account internationally mobile (“tradable”) capital, and only in the last step to introduce non-mobile (“nontradable”) capital. Nonetheless, the intermediate step (presented, for instance, in Obstfeld and Rogoff, 1996 and Asea and Corden, 1994) brings no additional insights compared to the other two. Moreover,
the implications of the assumed perfect capital mobility for economic growth are difficult to reconcile with the empirical evidence (see, for instance, Barro and Sala-i-Martin, 1995; or Čihák and Holub, 2000). We therefore proceed directly to a model that includes both forms of capital.

Let us now consider an economy with a stock of capital consisting of two parts, namely “tradable capital” and “nontradable capital,” with the production function specified in Cobb–Douglas (C–D) form,

\[ Y_T = A_T L_T^{1-\gamma} K_T^\alpha H_T^\gamma \]
\[ Y_N = A_N L_N^{1-\varphi} K_N^\beta H_N^\varphi, \]

where \( K \) denotes tradable capital and \( H \) nontradable capital. We will assume that \( \alpha > \beta \) and \( \gamma > \varphi \), i.e. that the production of tradable goods is more intensive in terms of both tradable and nontradable capital, while the production of nontradable goods is more labour intensive.\(^6\)

The difference between \( K \) and \( H \) is that tradable capital can be moved freely among countries, while nontradable capital cannot. The free mobility of capital also means that \( K \) can serve as collateral for foreign borrowing, while \( H \) cannot. The notation for tradable and nontradable capital was chosen to be consistent with the growth literature, where \( K \) traditionally stands for physical capital, while \( H \) stands for human capital.\(^7\) However, since the distinction between \( K \) and \( H \) consists in their mobility across country borders rather than their natural characteristics, we refer to them as tradable and nontradable capital goods, following the terminology used in the dependent-economy literature.

We assume that nontradable capital cannot flow between economies, so that its total amount in the domestic economy (i.e., \( H_T + H_N \)) in a given period is fixed at the level \( \bar{H} \). The convergence of nontradable (“human”) capital in per-capita terms to the level of advanced countries, \( h^* \), will be only gradual. We assume that nontradable capital can flow freely between the two sectors, which means that in equilibrium, its marginal return must be the same in both sectors.\(^8\)

We will discuss the accumulation of nontradable capital in more detail later on. At this point, let us start with the static point of view. The above assumptions imply that

\(^6\) Some authors have challenged this assumption by pointing out that nontradables include some capital-intensive public utilities such as electricity generation and transport (see Neary and Purvis, 1982). However, the empirical literature tends to support the assumption of higher labour intensity in nontradables. Moreover, for most of our key conclusions, it is sufficient to assume that \( \eta/(1-\alpha) > \varphi/(1-\beta) \), i.e. that the ratio of nontradable capital intensity with respect to all nontradable factors (labour plus nontradable capital) is greater in the tradable sector than in the nontradable one.

\(^7\) Another reason for this notation is that we want to distinguish the abbreviations for capital from those for goods.

\(^8\) In other words, we make the same assumptions about nontradable capital as about labour in the previous version of the Balassa–Samuelson model, i.e. that it is mobile within a country, but immobile internationally. In order to keep the model as simple as possible at this stage, we do not consider the case of sector-specific nontradable (“human”) capital, even though this is a fruitful stream of research – see, for instance, Dickens and Katz (1987), Fallick (1993), Gibbons and Katz (1989), Helwege (1992), Katz and Summers (1989), Neal (1995), or Strauss (1998).
\[ \eta A_T L_T^{1-\sigma-\eta} K_T^\eta H_T^{\eta-1} = \varphi p_N A_N L_N^{1-\beta-\varphi} K_N^\beta \left( \bar{H} - H_T \right)^{\varphi-1}. \]  
(11)

We assume that tradable capital is perfectly mobile among countries, possibly up to some risk premium \( \sigma \). This assumption means that the marginal product of tradable capital must be equal to the world interest rate \( r^* \) plus the domestic risk premium \( \sigma \) in both economic sectors:

\[ \alpha A_T L_T^{1-\sigma-\eta} K_T^\eta H_T^n = r^* + \sigma = \beta p_N A_N L_N^{1-\beta-\varphi} K_N^\beta \left( \bar{H} - H_T \right)^{\varphi}. \]  
(12)

We will also maintain the B–S assumption concerning no labour mobility among countries, but perfect labour mobility between economic sectors within a country. This implies that the marginal product of labour must be equal to the same wage level in both sectors, i.e. that

\[ (1-\alpha-\eta)A_T L_T^{1-\sigma-\eta} K_T^\eta H_T^\eta = w = (1-\beta-\varphi)p_N A_N L_N^{1-\beta-\varphi} K_N^\beta \left( \bar{H} - H_T \right)^{\varphi}. \]  
(13)

Dividing equations (13) and (11) and rearranging, we get the equilibrium allocation of nontradable capital to the tradable sector as a function of total nontradable capital accumulated in the country and the equilibrium allocation of labour between the two economic sectors

\[ H_T = \bar{H}Z; \quad Z \equiv \frac{\eta(1-\beta-\varphi)\gamma_E}{\varphi(1-\alpha-\eta)(1-\gamma_E) + \eta(1-\beta-\varphi)\gamma_E}, \]  
(14)

where \( \gamma_E \equiv L_T / (L_T + L_N) \) is the tradable sector’s share in total employment. Combined with (12), this equation yields the equilibrium levels of tradable capital,

\[ K_T = \left( \frac{\alpha A_T L_T^{1-\sigma-\eta} \left( \bar{H}Z \right)^{\varphi}}{r^* + \sigma} \right)^{\frac{1}{1-\sigma}}, \]  
(15)

\[ K_N = \left( \frac{\beta p_N A_N \left( L_N \right)^{1-\beta-\varphi} \left( \bar{H}(1-Z) \right)^{\varphi}}{r^* + \sigma} \right)^{\frac{1}{1-\beta}}. \]  
(16)

Note that a higher price of nontradable commodities attracts a higher volume of capital into the nontradable sector. This is an important adjustment mechanism in this model (and also in the simple two-factor production function version). Poor technology in the tradable sector means that this sector attracts little tradable capital despite the perfect capital mobility among countries. Poor technology and little tradable capital means low labour productivity, and thus low wages in the tradable sector. These low wages, in turn, imply low prices in the nontradable sector. The low prices, however, mean that nontradable production attracts little tradable capital, too, even if its technological level is the same as abroad and tradable capital is perfectly mobile. Only as the country converges to the advanced world in terms of technology in the tradable sector can it converge in terms of labour productivity in the nontradable sector as well, even assuming that there are no technology differences in this sector among countries. In addition to this mechanism, the stock of tradable capital in both economic sectors depends positively on the accumulated stock of nontradable capital, which influences the productivity of tradable capital.
Finally, after combining (11), (14), (15), and (16), and some rearranging, we get an expression for prices in the nontradable sector as a function of the per capita nontradable capital stock \( h = H/(L_T + L_N) \), the domestic interest rate, the sectoral allocation of labour, and the technological coefficients.\(^9\)

\[
P_N = \frac{A_T^{1-\beta}}{A_N^{1-\gamma}} \left( \frac{\eta}{1-\phi} \right)^{\phi(1-\gamma)/(1-\phi)} \left( r^* + \sigma \right)^{\frac{\beta(1-\alpha) - \phi(1-\beta)}{1-\alpha}} \frac{X}{h^{1-\alpha}}. \tag{17}
\]

This directly yields the ratio of price levels at home and abroad.

\[
\frac{P}{P^*} = \left( \frac{A_T}{A_T^*} \right)^{(1-\gamma)(1-\beta)/(1-\alpha)} \left( \frac{A_N^*}{A_N} \right)^{(1-\gamma)} \frac{X}{X^*} \left( \frac{r^* + \sigma}{r^*} \right)^{\frac{(1-\gamma)(\beta(1-\alpha) - \phi(1-\beta))}{1-\alpha}} \frac{h^*}{h^{1-\alpha}}. \tag{18}
\]

These two equations resemble equations (4) and (6) of the one-factor production function model above. For example, the elasticity of nontradable prices (and thus also of the overall price level) with respect to the technological coefficient of nontradables is here the same as before (i.e. unitary). The technological coefficient in the tradable sector has the same qualitative impact, even though with a modified elasticity (i.e. greater than unitary in equation (17) under our assumptions). However, in this case, the technological coefficients are not very important, as they are not needed in explaining the process of GDP and price convergence. (It might be possible to assume that these coefficients are equal across all countries.) The mechanism that explains convergence is the accumulation of per-capita nontradable capital \( \bar{h} \), and perhaps also a gradual reduction in the risk premium. Note also the variable \( X \) in equations (17) and (18), which depends not only on the parameters of the model, but also on the share of employment in the tradable sector (i.e. \( \gamma_k \)).

Having derived the model, we can focus on the impact of the following three changes in the model: (i) an increase in the world interest rate and the risk premium; (ii) an increase in nontradable capital; and (iii) an increase in the share of the nontradable sector in total employment. These three issues are discussed below.

Firstly, an increase in the world interest rate and/or the risk premium, which together determine the equilibrium domestic interest rate, reduces the relative price of nontradable goods – and the overall price level – under the realistic assumption that \( \alpha > \beta \). This is due to the fact that a higher domestic interest rate reduces the tradable capital stock in both sectors, but its impact on labour productivity is more pronounced in the tradable sector, which is more tradable-capital intensive. The marginal product of labour thus goes down more in the tradable sector than in the nontradable one, implying reduced unit labour costs in the production of nontradables, given the fact that the wage rate is determined in the tradable sector. If there is no risk premium, however, a change in the world interest rate has no effect on the relative price level, as it affects all countries

\(^9\) In which we defined

\[
X = \frac{\alpha^{1-\alpha}}{\beta^{1-\gamma}} \frac{1-\alpha - \eta}{1 - \beta - \phi} \left( \frac{Z}{\gamma_k} \right)^{1-\beta} \left( \frac{1 - Z}{1 - \gamma_k} \right)^{\phi} = \frac{\alpha^{1-\alpha}}{\beta^{1-\gamma}} \frac{\eta}{\phi} \left( \frac{1 - \alpha - \eta}{1 - \beta - \phi} \right)^{1-\beta} \left[ \gamma_k + \frac{\phi}{\eta} \frac{1-\alpha - \eta}{1 - \beta - \phi} (1 - \gamma_k) \right]^{1-\alpha}. 
\]
symmetrically (provided that \(1-\gamma\) is the same at home and abroad). This is not true for changes in the risk premium, which can be country-specific. A higher/lower risk premium leads to a lower/higher price of nontradables and thus also to a lower/higher relative price level. This is important to keep in mind, as a gradual decline in the risk premium has been (and is likely to remain) an integral part of all the transition countries’ convergence processes, including that of the Czech Republic. This decline in the risk premium not only decreases the equilibrium real interest rates in these economies, but also increases the equilibrium speed of the price and GDP convergence process.

Secondly, there is a positive relationship between per-capita nontradable capital on the one hand, and the price of nontradables and the overall price level on the other. As shown in (17), the elasticity of nontradables prices with respect to the per-capita stock of nontradable capital is \(\eta(1-\beta) - \varphi(1-\alpha)/(1-\alpha)\). If we recall our assumptions that \(\alpha>\beta\) and \(\eta>\varphi\) (or at least that \(\eta/(1-\alpha)>\varphi/(1-\beta)\)), the elasticity is unambiguously positive, which means that a higher per-capita stock of nontradable capital increases the price of nontradables (and the price level), and vice versa. The exact magnitude of this effect depends on the values of parameters \(\alpha, \beta, \eta,\) and \(\varphi\).

Thirdly, an increase in the share of the nontradable sector in total employment has a positive impact on the price of nontradable goods and the overall price level. The explanation can be derived from (14). If the country decides to allocate more labour to the nontradable sector (and thus increase the share of nontradable goods in its GDP), which is less capital intensive, its constraint stemming from the limited stock of nontradable capital becomes less “binding” than it used to be. The country responds by shifting part of its nontradable capital stock from the tradable sector to the nontradable sector, which eventually increases nontradable capital per employee in the production of nontradables but cannot be enough to prevent nontradable capital per employee from rising in the tradable sector, too. The latter factor dominates in equilibrium, being magnified by the responses of tradable capital investments, implying a bigger increase of labour productivity in the tradable than in the nontradable sector. This implies a higher price of nontradables, having the same effect as an increase in relative productivity in the tradable sector in the simpler version of the model in Section 2. The situation is illustrated in Figure 1, in which we used equation (19) to derive an upward-sloping supply curve of nontradable goods\(^{10}\) and combined it with a downward sloping demand curve (see Section 4). Figure 1 shows that an exogenous increase in the demand for nontradables raises the price of nontradables. This contrasts with the simple version of the B–S model in Section 2 (and also with the two-factor production function case), in which the price level of nontradables is determined solely by supply-side factors. This fact may be important for post-communist countries as an additional explanation for the observed real exchange rate appreciation, because economic transition is – at least in its earlier stages – connected with a growing importance of the services sector in the countries’ employment.

\(^{10}\) Equation (17) shows a positive relationship between the price of nontradables and employment in this economic sector. At the same time, it is not difficult to show there is a positive relationship between employment and production in nontradables, which is a very intuitive result. This implies an upward-sloping supply curve.
As the next step in our analysis, we can use the results obtained so far to derive an expression for nominal GDP, in analogy with equation (7) of Section 2,\textsuperscript{11}

\[ GDP_{nom} = \left( A_T \right)^{1-\alpha} \left( \frac{\alpha}{r^* + \sigma} \right)^{\alpha} \left( \frac{\eta}{h} \right)^{1-\alpha} R. \] \text{(19)}

We can see that in the present setting with tradable and nontradable capital, there is no longer such a simple one-to-one relationship between nominal GDP and the tradable productivity parameter as in the case of the one-factor production function. First, the relationship to the technology coefficients is still a direct one, but the exponent differs from one, reflecting the impact of technology on the tradable capital stocks invested in the economy. Second, an important role is played by nontradable capital, which influences labour productivity in a similar way as the technological coefficients. Note that the exponent of nontradable capital reflects the relative importance of nontradable and tradable capital in the tradable sector. Third, an important factor is the risk premium. It has a negative impact in both cases, as an increase in the risk premium reduces the capital stock and thus production per employee in both economic sectors for a given level of technology. Fourth, nominal GDP is also influenced by the allocation of labour between the two sectors. In spite of these complications, nominal GDP remains a fairly good proxy for productivity in the nontradable sector.\textsuperscript{12}

\textsuperscript{11} Where we define \( R = \left( \frac{Z}{y} \right)^{\frac{\beta}{\gamma_e + (1-\gamma_e)\frac{1-\alpha-\eta}{1-\beta-\varphi}}}. \)

\textsuperscript{12} In the empirical analysis of Čihák and Holub (2003), we used nominal GDP as a proxy for tradable productivity. We also applied overall labour productivity (i.e., output per employee) in the nontradable sector in our estimates, which in the current setting equals \( \frac{prodNT}{prodNT^*} = \left( \frac{A_T}{A_N} \right)^{1-\eta} \left( \frac{r^*}{r^* + \sigma} \right)^{\frac{\beta}{0}} \left( \frac{P}{P^*} \right)^{\frac{\beta}{\gamma_e + (1-\gamma_e)\frac{1-\alpha-\eta}{1-\beta-\varphi}}}. \) If we combine this with (20) and (21), we can get

\[ P = \left( \frac{GDP_{nom}}{GDP_{nom}^*} \right)^{\left( 1-\eta \right)^{\left( 0 \right)}} \left( \frac{prodNT}{prodNT^*} \right)^{\left( 0 \right)-\left( 1-\gamma_e \right)^{\left( 0 \right)}} \left( \frac{S}{S^*} \right)^{\left( 1-\gamma_e \right)^{\left( 0 \right)}}; \quad S = \left( \gamma_e + (1-\gamma_e)\frac{1-\alpha-\eta}{1-\beta-\varphi} \right). \] Note that this closely resembles equation (8), which was used as a starting point in Čihák and Holub (2003). The only exception is the last term \( S \), which reflects the sectoral allocation of labour. This factor was for simplicity ignored in Čihák and Holub (2003), as its relationship to the price level is nonlinear even after log-linearisation.
To close the model, equations (18) and (19) can be used to write down an expression for relative GDP in purchasing power parity PPP,

\[
\frac{GDP_{ppp}}{GDP^*_ppp} = \left(\frac{A_T}{A_T^*}\right)^{1-(\gamma)(1-\rho)} \left(\frac{A_N}{A_N^*}\right)^{(1-\gamma)} \left(\frac{\rho^*}{\rho^* + \sigma}\right)^{(1-\gamma)pR^*} \left(\frac{h}{h^*}\right)^{(1-\gamma)pR^*} \frac{R}{R^*} \left(\frac{X}{X^*}\right)^{1-\gamma}. \tag{20}
\]

Given our previous results, it is not surprising that GDP in PPP also depends on the nontradable capital stock, risk premium, and sectoral allocation of labour.

We can now use equations (18), (19), and (20) to illustrate the results of the model graphically. We provide two examples, one focusing on the role of nontradable capital, the other on the impact of the risk premium. To make the illustrations easier, we make two simplifying assumptions for both examples. First, we treat the sectoral distribution of labour as constant. Second, we assume the speed of convergence towards the steady state to be constant, too. We calibrate the examples consistently with the empirical estimates of the convergence speed from cross-country regressions (see, for instance, Barro, 1991). In particular, we assume that the country tends to close about 2.7 percent of the GDP gap relative to its steady state, implying a 2.7 percent speed of convergence also for nontradable capital.\(^{13}\)

To derive a “benchmark” convergence scenario, let us set \(\alpha=\eta=0.4, \beta=\phi=0.1\), and \((1-\gamma) = 0.60\). These values are broadly consistent with the cross-country empirical estimates in Čihák and Holub (2003).\(^{14}\) Figure 2 illustrates how the key macroeconomic variables develop as functions of the nontradable capital stock if all the other parameters of the model are the same at home and abroad and if there is no risk premium.

In the first example, shown in Figure 3, we focus on the role of nontradable capital. In particular, we consider a country that starts with a nontradable capital stock equal to 15 percent of the steady state value and no risk premium. These values imply nominal GDP per employee equal to 28 percent of the steady state, GDP in PPP equal to 50 percent of the steady state, and a price level equal to 57 percent of the steady state. Figure 3 shows that such a country – which is close to the Czech Republic in terms of GDP in PPP per employee – should achieve a growth rate of nontradable capital starting from above 5 percent a year and gradually declining to below 3 percent over 25 years. As a result, the country’s real growth differential vis-à-vis a steady-state economy (here represented by the EU) should achieve a growth rate of nontradable capital starting from above 5 percent a year and gradually declining to below 3 percent over 25 years. The implied equilibrium real exchange rate appreciation is slightly lower than that (starting from about 1.5 percent a year and declining to below 1 percent). The B–S effect is thus important in this “benchmark” convergence scenario, but its impact is smaller than the inflation differential allowed under the Maastricht criteria, and much

\(^{13}\) A full-fledged discussion would need to analyse the determination of labour allocation and convergence speed, which is related to the discussion of consumers’ behaviour and the production of nontradable capital. Such an analysis is presented in the next section.

\(^{14}\) The estimated slope coefficient was roughly 0.9 for the price level regression on GDP in PPP. The estimated elasticity of the price level with respect to nominal GDP was 0.47.
smaller than the nominal exchange rate appreciation allowed under the ERM II regime, if membership in it is minimised to two years.

**Figure 2: Convergence with Nontradable Capital**

![Figure 2](image)

*Note:* $\alpha = \eta = 0.4; \beta = \varphi = 0.1; 1-\gamma = 0.60.$

**Figure 3: Growth Rates of GDP and the Price Level in the “Benchmark Scenario”**

![Figure 3](image)

*Note:* $\alpha = \eta = 0.4; \beta = \varphi = 0.1; 1-\gamma = 0.60; \text{convergence speed} = 2.7\% \text{ a year.}$
In the second example, we focus on the impact of the risk premium. In particular, we consider a country that starts with a nontradable capital stock equal to 23 percent of the steady state, and we assume that the country faces a risk premium of 3 percentage points compared with the world interest rate of 6 percentage points (i.e., the domestic real interest rate is 1.5-times the world level). As in the benchmark case, these values mean that nominal GDP per employee is roughly 28 percent of the steady state, GDP in PPP 50 percent of the steady state (close to the Czech starting point) and the price level 57 percent of the steady state. After the initial period, the risk premium starts declining by 15 percent each year, converging gradually towards zero. The results are illustrated in Figure 4. It shows that there is very little convergence in the starting period. The economy’s output is far from the EU levels, but given the risk premium it is not very far from its own steady state. Once the risk premium starts to decline, however, the GDP growth both in nominal terms and in PPP, as well as the price level convergence, go up immediately, in line with equations (18), (19), and (20). Moreover, this primary effect is further magnified by the fact that the reduced equilibrium real interest rate creates an additional motivation for nontradable capital accumulation, which gradually starts to gain momentum. As a result, the real GDP growth differential increases to above 2 percentage points, and then only gradually declines towards 1 percentage point. The real exchange rate appreciation goes up to 2 percentage points initially, then gradually decreases to below 1 percentage point. The results are thus fairly similar to the above “benchmark” scenario of Figure 3, even though the driving force is different here, at least initially (later on, the accumulation of nontradable capital takes over as the main convergence factor in this second example, too). The contribution of this alternative simulation consists mainly in illustrating the links between the risk premium, GDP growth and real exchange rate appreciation, which are usually the key equilibrium variables entering the forecasting process at the central bank (see, for instance, Beneš et al., 2002).

**Figure 4: The Impact of a Declining Risk Premium**

![Figure 4: The Impact of a Declining Risk Premium](image)

**Note:** $\alpha=\eta=0.4$; $\beta=\sigma=0.1$; $1-\gamma=0.60$; convergence speed = 2.7% a year.
4. The Demand Side of the Economy and the Speed of Convergence

In Sections 2 and 3, we focused on the supply side of the economy. In particular, we assumed that the speed of convergence was constant (which is broadly consistent with the results of cross-country regressions by Barro, 1991 and others) and the allocation of labour across economic sectors was fixed for each economy. In this section, we provide a deeper look into how the above two factors are determined by consumers’ behaviour in interaction with the production and accumulation of nontradable capital. The discussion in this section is based on the Ramsey model in an open economy with nontradable (“human”) capital (see Cohen and Sachs, 1986, and Barro and Sala-i-Martin, 1995), extended for the distinction between tradable and nontradable goods (see, for instance, Obstfeld and Rogoff, 1996).

Let us adopt several assumptions to make the analysis easier to present. First, we assume that preferences between tradable and nontradable goods have a Cobb–Douglas (C–D) form, implying unitary elasticity of substitution between them (see, for instance, Obstfeld and Rogoff, 1996, p. 222). Second, we assume that while one unit of tradable capital is equivalent to one unit of a tradable good, in order to produce a unit of nontradable capital, one needs to give up one unit of the consumption basket $C$ (see equation (23) below). Third, we assume that the borrowing constraint is binding for a converging economy, implying that the sum of tradable capital and foreign borrowing is zero, and the net assets of the economy are thus equal to the nontradable capital only. This allows us to formulate the optimisation problem of a representative consumer in time $s$ as follows:

$$\max_{c_s} U_s = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^{t-s} u(c_t), \quad u(c_t) = \frac{(C_t)^{1-\theta} - 1}{1 - \theta}, \quad C_t = (c_{T,t})^{-\gamma} \left( c_{N,t} \right)^{\gamma}, \quad \theta > 0$$

s.t.  \( P_t \left( H_{t+1} - H_t \right) = w_t - \left( c_{T,t} + p_{N,t} c_{N,t} \right) + \tilde{r}_t P_t H_t \)

\( (21) \)

where $c_{T,t}$ is the consumption of tradable goods in time $t$, $c_{N,t}$ is consumption of nontradable goods in the same period, $\theta$ is the coefficient of relative risk aversion, $\rho$ is the subjective discount rate, and $\tilde{r}_t$ is the market return on renting nontradable capital in time $t$ (which is in equilibrium equal to its marginal product). $P_t$ is the optimal price index for the C–D utility function, defined in

\[15\] The assumption that the production of nontradable capital requires nontradable goods fits quite well with the assumption that nontradable capital cannot move across borders, since nontradables cannot be transferred abroad in the case of default, either. The assumption is also convenient, for two reasons. First, the price of nontradable capital develops in the same way as the price of the consumer basket, which means that we do not have to take into account relative price changes of nontradable capital in the Euler equation (24). Second, the assumption means that the employment distribution between the tradable and nontradable sectors is constant (see below). A more general assumption would be that nontradable capital is produced as a C–D combination of the tradable and nontradable good, with weights $\gamma_t$ and $(1-\gamma_t)$ respectively. Here we restrict our attention to the case of $\gamma_t = \gamma$. Note that in this respect we differ from the earlier dependent-economy literature, which assumed that nontradable capital is identical with the nontradable good (see Brock and Turnovsky, 1994; Turnovsky, 1997), i.e., another special case with $\gamma_t = 0$. 
Similarly to Section 3, $H_t$ and $w_t$ denote the level of nontradable capital and the wage level respectively.

The above specification of the utility function implies that the demand functions for tradable and nontradable goods are given by (see, for instance, Obstfeld and Rogoff, 1996)

$$c_{T,t} = \gamma P_t C_t; \quad c_{N,t} = \left(1 - \gamma\right) \frac{P_t}{P_{N,t}} C_t.$$  \hspace{1cm} (22)

Note that the C–D utility function leads to constant nominal shares of tradable and nontradable goods in consumer expenditures in equilibrium. This allows us to rewrite the flow budget constraint of equation (21) as

$$(H_{t+1} - H_t) = \frac{w_t}{P_t} - C_t + \bar{r}_t H_t,$$  \hspace{1cm} (23)

which is similar to the budget constraint of a representative agent in an economy without the distinction between tradable and nontradable goods.

When we maximise the utility function of equation (21) subject to (23), we get a standard optimality condition (“Euler equation”),

$$\frac{C_{t+1}}{C_t} = \left(\frac{1 + \bar{r}_t}{1 + \rho}\right)^{1/\gamma}.$$  \hspace{1cm} (24)

If we assume away depreciation of nontradable capital, the return on nontradable capital should be equal to its marginal product (denoted $f_{h,t}$) in equilibrium. Thanks to the assumption of perfect mobility of nontradable capital – and thus equality of its marginal product – between sectors, we can concentrate on the marginal product of nontradable capital in the tradable sector only.\(^{17}\) Using our previous results from equations (11), (14), and (15), this marginal product can be expressed as\(^{18}\)

\[^{16}\] Note that in the more general case of $\gamma_f \neq \gamma_h$, nontradable capital would be associated with a different price index, the weight of tradable prices being $\gamma_f$. Consequently, this price index would also appear in equation (25) and also in the Euler equation (24), having an implication for savings behaviour and the convergence speed. We avoid this complication here, but a generalisation would not be too difficult.

\[^{17}\] This assumption was adopted mainly for computational simplicity. Section 3 includes references to the stream of research that argues that a substantial portion of nontradable (“human”) capital is in fact sector-specific.

\[^{18}\] Note that this is the derivative of production per employee in the tradable sector, which is given by

$$\frac{Y_T}{L_T} = B_T \left(\frac{\bar{h} Z}{y}\right)^{\varepsilon}; \quad B_T = (A_T)^{(1-\alpha)} \frac{\alpha}{r^* + \sigma} \left(\frac{r^* + \sigma}{1-\alpha}\right)^{1/(1-\alpha)}; \quad \varepsilon = \frac{\eta}{1-\alpha},$$

with respect to tradable capital, multiplied by $(1-\alpha)$. The term $\alpha B_T \hat{h}^{\varepsilon}$ that we effectively deduct from the reduced-form production function equals the flow of rents from tradable capital.
\[ f_{h,t} = \eta B_{T,t} \left( \frac{\bar{h}_{T,t} Z_{T,t}^{1 \alpha}}{Y_{E,t}} \right)^{\frac{1}{1-\alpha}} = \varepsilon (1 - \alpha) B_{T,t} \left( \frac{\bar{h}_{T,t} Z_{T,t}^{1 \alpha}}{Y_{E,t}} \right)^{\frac{1}{1-\alpha}}; \quad B_{T,t} = \left( A_{T,t} \frac{1}{r_{t}^{\alpha}} \left( \frac{\alpha}{r_{t} + \sigma} \right) \right)^{\frac{1}{1-\alpha}}; \quad \varepsilon = \frac{\eta}{1 - \alpha}. \quad (25) \]

Equations (24) and (25), together with the standard transversality condition,\(^{19}\) describe the consumption and investment behaviour in the model.

To close the model, we need to find a general equilibrium in which demand equals supply in both the tradable and nontradable sector. In other words, we are looking for a situation in which

\[ c_{T,t} + I_{T,t} = (1 - \alpha)Y_{T,t}; \quad c_{N,t} + I_{N,t} = Y_{N,t}, \quad (26) \]

where \( I_{T,t} \) is the amount of tradable goods used for investment in nontradable capital in time \( t \) and \( I_{N,t} \) is the volume of nontradable goods invested in nontradable capital. Note that the first equation reflects the fact that a fraction \( \alpha \) of the tradable production goes as a reward to tradable capital, which is fully foreign-owned in equilibrium.

The assumption that nontradable capital is a C–D combination of tradable and nontradable goods helps to simplify the computations, as it means that it is optimal to use the inputs into the investment process so that:

\[ I_{T,t} = \frac{\gamma}{1 - \gamma} p_{N,t} I_{N,t}, \quad (27) \]

which is an analogy to equation (22).\(^{20}\) Combining equations (22), (26), and (27) with the expressions for tradable production and the nominal value of nontradable production, which we had to derive already when expressing nominal GDP in equation (19) of Section 3, we find that the equilibrium requires the following equality.

\[ \gamma_{E} = \frac{1 - \alpha - \eta}{(1 - \alpha - \eta) + (1 - \alpha)(1 - \beta - \phi)}. \quad (28) \]

This means that in the equilibrium of this simplified version of the model, the labour shares of the tradable and nontradable sectors are constant, determined by the technological parameters. As a result, the price level, nominal GDP, and real GDP are all functions of per-capita nontradable capital only (plus the risk premium; see Section 3), which makes the analysis of the convergence process much easier. Note that with the values of technological coefficients that we used in

\(^{19}\) The transversality condition says that the present value of the household’s assets must approach 0 as time approaches infinity. It can be expressed as \( \lim_{t \to \infty} \left[ H(t) \left( \frac{1}{1 + R(t)} \right)^{t} \right] = 0 \), where \( t \) is time, and \( R(t) \) is the geometric average of the interest rate between now and time \( t \).

\(^{20}\) Note that in the more general case, \( \gamma \) would be replaced with \( \gamma_{E} \) in this equation.
Figures 2 and 3 (i.e. $\alpha=\eta=0.4$; $\beta=\varphi=0.1$), the labour share of the tradable sector would be about 30 percent. This approximately corresponds to the median share of industry and agriculture in employment in the sample of countries used in Čihák and Holub (2003), increasing our confidence that these calibrations are not completely unrealistic.21

We can use our results to discuss the process of real and price convergence in the model. Mathematically, convergence in the standard closed-economy (i.e. Ramsey) growth model is described by the equation

$$\log(y(t)) = e^{-Bt} \log(y(0)) + (1-e^{-Bt}) \log(y^*)$$

where $B>0$. The logarithm of output per unit of effective labour over time is therefore a weighted average of the initial value and the steady-state value, with the weight of the initial value declining exponentially. The speed of convergence, $B$, depends on parameters determining technology and preferences. For the Cobb–Douglas function,

$$2B = \left[ \omega^2 + 4 \left( \frac{1-\tilde{\alpha}}{\theta} \right) (\rho + \delta + \theta g) \left( \frac{\rho + \delta + \theta g}{\tilde{\alpha}} - (n + g + \delta) \right) \right]^{1/2} - \omega,$$  \hspace{1cm} (29)

where $\omega$ is defined as $\omega = \rho - n - (1-\theta)g > 0$ and $\tilde{\alpha}$ denotes the capital-intensity coefficient for a single-good closed economy.22 In our open-economy version of the model, however, we have to replace $\tilde{\alpha}$ in equation (29) with $\gamma \gamma_{t}$, as defined in equation (25).

For the usual values of the other parameters,23 the speed of convergence is from 1.5 percent per year (for $\alpha/\eta=0$) to about 3 percent per year (for $\alpha/\eta=1$) and further to for instance 5 percent (for $\alpha/\eta=3$).24 If we assume that less than half of the total capital is tradable in tradable goods production (i.e., $\alpha/\eta<1$), then the predicted speed of convergence would be in the range of 1.5–3.0 percent, which is consistent with most empirical studies (see Barro and Sala-i-Martin, 1995). Note that our calibration used in Figure 2 would imply a convergence speed close to the upper end of this interval. We used such a convergence speed (in particular 2.7 percent a year) in our beta-convergence scenarios of Section 3.

21 In reality, however, employment in the tradable sector tends to be smaller for richer countries. For example, when we used the sample of countries from Čihák and Holub (2003), and regressed employment in agriculture and manufacturing on the log of GDP in PPP, we got a negative slope coefficient of −0.2 that was highly statistically significant. The above simplified model captures this tendency only partially – the indebted converging countries always have to allocate more labour to the tradable sector to be able to service their borrowing of tradable capital (see the term $1-\alpha$ in the denominator of equation (30)). Another way to introduce such a tendency would be to assume that $\gamma > \gamma$ in the production of tradable capital, which would mean that a converging – and thus investing – country would have to allocate more labour to tradable production.

22 Where $\delta$ denotes capital depreciation, $n$ population growth and $g$ exogenous technological progress. We have so far ignored these factors for simplicity, but their introduction would be straightforward.

23 The usual values of the parameters are $\alpha = 1/3$, $n = 1$ percent, $g = 2$ percent, and $\delta = 3$ percent. As regards the parameters of impatience, “reasonable” values include $\rho$ of about 0.02 and $\theta$ lower than 10 (see Barro and Sala-i-Martin, 1995).

24 For $\eta=0$, all capital in the economy would be tradable, so that the economy would effectively return to the case of an open economy with perfect capital mobility. Indeed, $\eta=0$ means $\varphi=0$, and the convergence coefficient, $B$, is infinitely high. The case of $\alpha=0$, on the contrary, would correspond to a closed economy, because there is no tradable capital. In this case, $\varphi=\eta$, and the convergence coefficient is the same as in the case of a closed economy.
5. The Balassa–Samuelson Model with More than Two Commodities

One of the problems of the standard B–S approach is that the distinction between tradable and nontradable commodities is largely artificial. In empirical cross-country comparisons, each commodity group generally behaves as a blend of tradable and nontradable elements rather than as purely tradable or purely nontradable (see Čihák and Holub, 2003).

To introduce such a variety of goods into the model, let us assume that there are \( x \) consumption goods – produced by combining tradable and nontradable goods – which, in turn, are subject to the production process described in the B–S model of Section 3. In contrast to the standard B–S model, the tradable and nontradable goods would not be consumed directly in this setup, but would rather serve as intermediate inputs into the production of consumption (retail) goods. For simplicity, let us initially assume that the shares of tradable and nontradable inputs are fixed; later on, we will introduce the possibility of substitution between these two inputs.

**Example 1: Fixed proportions of tradable and nontradable inputs**

Let us start with the assumption of a zero elasticity of substitution between tradable and nontradable goods, meaning that consumption good \( i \) is characterised by fixed proportions of tradable and nontradable factors. This can be formalised by assuming that the production of consumption good \( i \) takes place according to the following formula,

\[
y_{i,t} = \min \left( \frac{T_{i,t}}{w_i}, \frac{N_{i,t}}{1-w_i} \right); \quad 0 < w_i < 1.
\]

where \( y_{i,t} \) is output of good \( i \) (=1, 2, \ldots \( x \)) at time \( t \), \( T_{i,t} \) is the tradable input, \( N_{i,t} \) is the nontradable input, and \( w_i \) is the weight of the tradable input in consumption good \( i \). Equation (30) is a formalisation of the idea that each consumer good contains some nontradable elements (such as transportation costs, wholesale margins, and retail margins) and some tradable elements (the part of the good after adjustment for all the nontradable elements).

Profit maximisation under this production function implies that the ratio of tradable and nontradable factors in optimum has to be \( w_i/(1-w_i) \). The marginal cost is then given by

\[
MC_i = \frac{\partial(T_i + p_N N_i)}{\partial T_i} = w_i + p_N (1 - w_i).
\]

Assuming for the time being that the consumption goods markets are perfectly competitive, the price of consumption good \( i \), \( p_i \), would be equal to the marginal costs, \( w_i + p_N (1 - w_i) \). This means that \( p_i \) is a weighted average of the price of tradable goods (normalised at 1) and the price of nontradable goods, the weights being the shares of tradable and nontradable inputs respectively.

\[25\] In Čihák and Holub (2003), we have attempted to proxy the “degree of nontradability” of a commodity group as a slope coefficient in a cross-country regression between the commodity group’s price and GDP. We have found that in the 30 commodity groups covered under private consumption in the 1999 International Comparison Project, the empirical degrees of nontradability ranged from 10 to 85 percent. In other words, there was no purely tradable or purely nontradable commodity group.
For a given price of nontradables, the price of a consumption good would be a linear function of the weight of the tradable element, $w$, increasing from $p_N$ to 1 for $p_N<1$ and decreasing from $p_N$ to 1 for $p_N>1$ (Figure 5a).

An increase in $p_N$ would increase the price of each individual consumption good, $p_i$, thereby raising the aggregate price level. The increase will be higher the less tradable is the good, since $\frac{\partial p_i}{\partial p_N} = 1 - w_i$. For $p_N \geq 1$, an increase in $p_N$ would increase the overall dispersion of the individual prices, while for $p_N<1$, it would decrease the price dispersion. To see this, we can consider two goods ($i$ and $j$) with different degrees of tradability. The impact of a change in $p_N$ on the difference of the prices of these goods corresponds to the difference in the weights of nontradable inputs in these two goods, namely,

$$\frac{\partial (p_i - p_j)}{\partial p_N} = \frac{\partial [w_i - w_j + p_N(w_j - w_i)]}{\partial p_N} = w_j - w_i.$$  \hspace{1cm} (32)

Let us take a good $i$ that is less tradable than a good $j$, that is, $w_i < w_j$. Equation (32) implies that an increase in $p_N$ always increases the price of good $i$ more than it increases the price of good $j$, because of its higher share of the nontradable element (i.e. its lower $w$). For $p_N \geq 1$, it holds from (31) that $p_i \geq p_j$, so the increase in $(p_i - p_j)$ as a result of the increasing $p_N$ means an increase in the differences between the two prices. For $p_N<1$, it holds from (31) that $p_i < p_j$, so the increase in $(p_i - p_j)$ as a result of the increasing $p_N$ means a decrease in the differences between the two prices. Figure 5b illustrates this conclusion, showing the relationship between the price level ($P$), defined as the weighted average of the individual prices, and the price dispersion ($\rho$), defined as the weighted deviation of the individual prices, with the weights corresponding to the shares of the individual goods in overall consumption. The relationship between $P$ and $\rho$ is negative for low price levels ($P<1$) and turns positive for high price levels ($P<1$). To plot this curve, we have generated 10,000 consumer goods with $w_i$s drawn from a uniform distribution over the interval $(0,1)$.

**Figure 5: Fixed Proportions of Tradable and Nontradable Factors**

(a) Price vs. tradability

(b) Price level vs. price dispersion

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26 For the time being, we can leave aside the exact definition of the price level. Since all individual prices rise, the aggregate price level, which is some form of weighted average of the individual prices, needs to increase as well.
Example 2: Substitution between tradable and nontradable factors

As a second example, let us assume that the tradable and nontradable factors are, to some extent, substitutable. We can assume that the production function has a constant elasticity of substitution (CES) form,

$$y_{i,t} = \left[ wT^\psi + (1 - w)N^\psi \right]^{1/\psi}, \quad (33)$$

where $1/(1-\psi)$ is the elasticity of substitution between the two factors.\(^{27}\) For $\psi \to \infty$, the CES function has a zero elasticity of substitution, and becomes the fixed-proportions technology discussed in Example 1. The other extreme case is $\psi \to 1$, when the elasticity of substitution is infinite and the production function becomes linear,

$$y_{i,t} = wT + (1 - w)N. \quad (34)$$

With infinite elasticity of substitution, the model would collapse back into the two-goods model of Sections 2–4. All goods for which $p_N > w_i/(1-w_i)$ would be produced from tradable factors only, while all goods for which $p_N < w_i/(1-w_i)$ would be produced from nontradable factors only. This outcome, meaning that all goods are either tradable or nontradable, would not be consistent with our empirical findings (see Čihák and Holub, 2003). The assumption of infinite elasticity of substitution does not seem to be realistic, since it is not always easy to substitute nontradable factors (such as transportation costs, wholesale margins, and retail margins) for tradable goods.

The interesting cases are therefore those with an elasticity of substitution that is higher than zero, but substantially less than infinity. Let us therefore assume that the production function of the consumption goods has unitary elasticity (i.e., $\psi \to 0$), which implies a C–D form,

$$y_{i,t} = (T_{i,t})^{w_i} (N_{i,t})^{-w_i} ; \quad 0 < w_i < 1, \quad (35)$$

where the variables have the same meaning as in the first example, except for $w_i$, which is the elasticity of $y_i$ with respect to the tradable input, rather than simply the share of the tradable input. The assumption of a C–D form is both convenient and consistent with the exposition in Sections 2–4, which were also based on the assumption of a C–D production function.

Unlike in the previous example, the two inputs are substitutable. As a result, the optimum share of tradable and nontradable factors for consumption good $i$ is not fixed. Rather, it is given by $(p_N w_i/(1-w_i))$. This means that as the price of nontradables increases, the relative share of nontradables in the production of good $i$ declines. It is possible to think of examples such as retailers shifting towards “no-frills” outlets in reaction to an increase in wages of qualified shop assistants. This assumption may be more realistic than the assumption of fixed proportions of tradables and nontradables in Example 1.

\(^{27}\) Barro and Sala-i-Martin (1995) discuss properties of the CES function, albeit in a different context.
Similarly to the previous example, we will for the time being assume perfect competition in the goods markets, meaning that the price of consumption good $i$ equals its marginal cost,

$$p_{i,t} = MC_{i,t} = D_i \left(p_{N,t}\right)^{-w_i}; \quad \text{where} \quad D_i = \left(w_i\right)^{w_i} \left(1 - w_i\right)^{w_i-1}.$$ (36)

Similarly to Example 1, the price of each consumption good increases with the price of the nontradable input, the increase being greater for goods with a lower degree of tradability. More exactly, the elasticity of $p_{i,t}$ with respect to $p_{N,t}$ is positive and equal to the degree of nontradability,

$$\frac{\partial p_{i,t}}{\partial p_{N,t}} \frac{p_{N,t}}{p_{i,t}} = D_i (1 - w_i) (p_{N,t})^{-w_i} \frac{p_{N,t}}{D_i (p_{N,t})^{-w_i}} = 1 - w_i.$$ (37)

which implies that a 1 percent increase in the price of nontradables will result in a $(1-w_i)$ percent increase the price of good $i$, where $(1-w_i)$ is the share of the nontradable element.

Unlike in the previous example, the relationship between the weight of the tradable element and the price of the goods is hump-shaped rather than linear (Figure 6a), with $p_i$ increasing from $p_N$ (for $w_i=0$) to $(1+p_N)$ and then declining to 1 (for $w_i=0$). The hump shape reflects the substitutability between the two factors: as the weight of one of the factors increases close to 1, the marginal product of the other factor becomes very high, making the marginal cost (and therefore the price) relatively lower. Similarly to Figure 5b in the previous example, Figure 6b also shows the relationship between the price level ($P$) and the price dispersion ($\rho$), both the definitions and the method of calculation being the same as in the previous example. Similarly to the fixed-parameter technology in Example 1, the relationship is negative for lower price levels and positive for higher price levels. However, thanks to the substitutability between the inputs, the negatively sloping portion is longer and the transition from a negative slope to a positive one is gradual.

Figure 6: Substitution between Tradable and Nontradable Factors

(a) Price vs. tradability

(b) Price level vs. price dispersion
Example 3: Imperfect competition with substitutable factors

Both the price level and the relative price structure in each economy depend on the competitive structure in the consumption goods markets. So far, we have assumed that producers are price takers, i.e., that perfect competition exists in all consumption goods markets. Alternatively, we could assume that producers in each market face a downward sloping demand function and are able to set their price above marginal costs, depending on the price elasticity of demand. Carrying on the assumption of the C–D production function from Example 2, we obtain

\[ p_{i,j} = \frac{MC_{i,j}}{1 - \frac{1}{\xi_i}} \left( \frac{P_{N,j}}{P_{i,j}} \right)^{-w_i} ; \quad \xi_i > 1 , \]  

(38)

where the variables and parameters have the same meaning as in the previous examples, and \( \xi_i \) is the perceived elasticity of demand for good \( i \), which can be decomposed as,

\[ \xi_i = - \frac{dc_i}{dp_i} \frac{p_i}{c_i} - \sum_{j \neq i} \left( \frac{\partial c_i}{\partial p_j} \frac{p_j}{c_i} \right) \left( \frac{dp_j}{dp_i} \frac{p_i}{p_j} \right) , \]  

(39)

where \( c_i \) denotes consumers’ demand for good \( i \) (\( c_i = y_i \) in equilibrium). The first term on the right-hand side is the own price elasticity of demand of good \( i \), and the expression in the sum is a product of the cross-price elasticity of demand with respect to good \( j \) and the price conjectural elasticity with respect to the price of good \( j \). Equation (38) means that there is a negative relationship between the elasticity of demand and the mark-up that a monopolistic firm is able to charge over the marginal cost.

The way in which the “price vs. tradability” curve changes in this case compared with Figure 6a depends on whether there is a systematic relationship between the tradability parameter, \( w_i \), and the mark-up over marginal cost. The answer to this question is not straightforward, since these two variables are determined by two different sets of factors. Namely, the weight \( w_i \) reflects the production technology of the consumption good, while the mark-up is determined by demand factors such as the competitive structure of the particular market and the availability of substitutes (reflected in the price elasticity of demand, as per (39), and thereby in the mark-up, as per (38)).

It would be possible to argue that more tradable goods would tend to have lower mark-ups. Empirical evidence exists that could be used in support of this assumption.\(^{28}\) If there indeed were such a general relationship, the presence of monopolistic competition would tend to increase the relative price of less tradable commodities, as illustrated in Figure 7a. As regards the relationship between price dispersion (\( \rho \)) and the price level (\( P \)), the introduction of imperfect competition would not change the basic conclusion that for lower price levels, the relationship tends to be negative, while for higher price levels, it tends to be positive, even though both the price level and

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the price dispersion would tend to be higher under imperfect competition than under perfect competition (Figure 7b).\footnote{The definitions and the methods of calculation in Figure 7 are the same as in Figures 5 and 6.}

Even though the assumption of a negative relationship between tradability and price mark-ups can be backed by some empirical evidence, there are also important deviations from this assumption. Utilities such as electricity are examples of industries with arguably a high share of tradable inputs, but highly monopolistic markets for outputs. As an opposite example, a haircut has a high share of the nontradable input; however, since there are typically many barbers in the market, the price elasticity of demand may be relatively high, despite the absence of foreign demand.

**Figure 7: Imperfect Competition**

(a) Price vs. tradability

(b) Price level vs. price dispersion

As an alternative hypothesis, therefore, one could assume that there is no relationship between $w_i$ and the mark-up. However, even in such a case, the conclusion from the perfect competition model about the relationship between the price level and price dispersion would not be affected. Both the aggregate price level and the dispersion of the individual prices would be different from the case of perfect competition (the former will be higher, while the latter will depend on the distribution of own-price, cross-price, and price-conjectural elasticities), but their relationship will still be driven by a hump-back shaped curve of $p_i$ and $w_i$, similar to Figure 6(a).

Imperfect competition would have an impact on the relationship between price levels and price dispersion if there was a systematic relationship between the price of nontradables in a country, and the price elasticities. This is indeed possible and consistent with the empirical evidence. For instance, we could assume that the elasticity of demand for the same good differs in individual countries and that companies are – due to their monopolistic power – able to set different prices in
different countries.$^{30}$ As a result, the price of the same good would be higher in the countries with
less elastic demand. Demand functions are generally likely to be less elastic in more advanced
countries (i.e., countries with higher prices of nontradables), as higher income levels enable
consumers to put more emphasis on non-price factors such as the brand or the perceived quality of
the good. This effect would tend to decrease the measured price dispersion for countries with low
price levels (and high price elasticity of demand), while increasing it for countries with high price
levels (and low elasticity of demand). However, there are also factors attenuating this effect, in
particular the fact that goods exported from less developed countries to developed countries tend
to face a more elastic demand function, because of their perception as being “inferior.”$^{31}$

Finally, cross-country price differences may also be influenced by government policies such as
competition policy and taxation, which may influence the mark-ups over marginal costs. As a
result, a part of the price convergence process may be related to these effects, rather than to the B–
S effect associated with the changing relative price of tradable and nontradable goods.

**Consistency with empirical findings**

In a series of empirical papers on relative prices and price levels (see Holub and Čihák, 2000;
Čihák and Holub, 2001a,b, 2003), we have found that for a pooled sample of EU countries and
CEE transition countries, there is a significant negative relationship between the degree of
deviations of relative prices in a given country vis-à-vis the system of relative prices in the
reference country and the price level. This result appears to be relatively stable across time.$^{32}$ We
have also found that the relationship is still negative, but less so, if we look only at EU countries,
and that the relationship is very weak when applied to a world-wide sample of countries.

These empirical findings are consistent with the theoretical calculations presented in this section.
The theoretical calculations indicated that for lower price levels, the relationship between the
price level and price dispersion should be negative. This is consistent with the negative correlation
found for CEE countries, where the price levels were relatively low compared with the EU
countries. The theoretical model also suggested that the relationship will become less negative or
positive with increasing price level. This is also consistent with the finding of the less negative
correlation in the case of the EU countries and the weak relationship found for the world-wide
sample (which included countries with low and high price levels).

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$^{30}$ In a review of the literature, it appears that the car industry is particularly affected by price discrimination. For
example, Verboven (1996) finds that European car markets are segmented, with producers gaining higher mark-
ups in their home country than in export destination countries.

$^{31}$ In Čihák and Holub (2001a,b; 2003), we found empirical evidence supporting this view for the case of EU and
CEE countries.

$^{32}$ The degree of deviations of relative prices was measured as a weighted standard deviation of comparable price
levels of individual goods in the given country relative to the average comparable price level,

$$\rho = \frac{1}{\mu} \sqrt{\sum_i w_i (P_i - \mu)^2},$$

where $w_i$ are the weights of the individual commodities in the consumption basket
and $\mu$ is the average price level of consumption. This is consistent with the definition of the price dispersion in the
theoretical calculations in Figures 5b, 6b, and 7b. The only difference is that the empirical comparable price levels
are measured as ratios to the same prices in a reference country, whilst here we measure all prices in terms of a
single theoretical numeraire, i.e. the price of a tradable input.
An interesting aspect of the above model is that it assumes that the price adjustment takes place along an equilibrium path (either a perfectly competitive equilibrium or a monopolistic equilibrium). Despite this assumption, the model is able to derive results that are consistent with the empirical evidence. Nevertheless, more theoretical work would be needed to investigate the possibility that the adjustment takes place from an out-of-equilibrium situation. Also, more work is needed to fully incorporate other factors outside of the B–S framework that have an important role in price convergence, such as the structure of foreign trade, the competitive structure in the economy, and government policies.

6. Conclusion

In this paper, we provided a theoretical reference point in the discussions on adjustments in the price level and relative prices. We presented a “nested” model integrating the Balassa–Samuelson model of real equilibrium exchange rates with a model of accumulation of capital consistent with the new growth literature, and with the demand side of the economy. We also showed how the model can be generalised to a case of more than two commodities.

The presented extensions to the B–S model provide several useful insights into the likely price adjustment process in the Czech economy. For instance, the simulations based on the model with nontradable capital, presented in this paper, allow us to assess in a consistent framework the links between the risk premium, GDP growth, and real exchange rate appreciation, which are typically the key equilibrium variables in the forecasting process at the central bank.

The extension of the model to the case of more than two commodities also provided intriguing insights into the nature of the price adjustment process. The calculations suggest that for countries with relatively low price levels, there should be a negative relationship between the price dispersion and price levels; the relationship should become less negative for countries with a higher price level, and eventually turn positive with increasing price levels. This prediction is consistent with empirical findings based on data for CEE and EU countries.
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