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An Economy in Transition and DSGE:
What the Czech National Bank’s New Projection Model Needs

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Abstract

Since the introduction of the inflation targeting regime in 1998 the Czech National Bank has made considerable progress in developing formal tools for supporting its Forecasting and Policy Analysis System. This paper documents the advances in the ongoing research aimed at developing a DSGE small open economy model designed to capture some of the most important features of the Czech economy—both the business-cycle regularities and the recent developments associated with the economy’s transition and its convergence towards the industrialized European countries. The model in its current form is able to capture trends in relative prices, allow for medium-convergence in expenditure shares, and deal with the undercapitalization and investment inflow issues. Besides the model exhibits real and nominal rigidities that are in line with the recent New Open Economy Macroeconomics literature built fully on first principles. The innovative features of our model include the international currency pricing scheme permitting flexible calibration of import and export price elasticities along with the disconnect of the nominal exchange rate, the policy reaction function with a parameterized forecast horizon, and a generalized capital accumulation equation with imperfect intertemporal substitution of investment.

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Nontechnical Summary

The introduction of the inflation targeting regime by the Czech National Bank in 1998 increased the demand for more advanced analytical tools and a consistent economic framework to be established within the institution. Since then the Bank has made considerable progress in developing its formal tools and models and organizing them in the Forecasting and Policy Analysis System (FPAS), reaching the stage of development where a forward-looking New Keynesian business-cycle model, the Quarterly Projection Model (QPM), plays the central role. However, the agenda addressed typically by the staff within the current FPAS during a prediction production cycle has gradually grown beyond the structure of the QPM, and a number of issues cannot be processed endogenously, such as the fiscal side or aggregate demand components. This was our primary motivation to develop a new, sufficiently detailed, core model built around the New Open Economy Macroeconomics (NOEM) paradigm, in line with the recent modeling strategy of other leading central banks.

The design and development of the new analytical framework has been based upon the following modeling strategy:

- a model technically tractable for model operators and understandable for all insiders within the Bank;
- a minimalist structure to guarantee the desired features—observed empirical regularities, country-specific convergence issues, and theoretical coherence;\(^1\)
- strict separation of steady-state versus transient parameters and properties of the model;
- full stock-flow consistency;
- convergence of the Czech economy to the ultimate steady state defined by the industrialized European countries;
- otherwise the standard NOEM building blocks.\(^2\)

The most important stylized facts and otherwise desired features include:

- both price and wage inflation inertia, generating costly disinflation, but monetary superneutrality in the steady state;
- realistic dynamic properties of real variables, in particular consumption and investment;
- sizable trends in relative prices motivated by differences in sectoral productivities;
- initial undercapitalization, and an investment boom induced thereby;
- considerable changes in the import intensity of aggregate demand, explained predominantly by a growing volume of re-exports;
- an explicit fiscal sector with the main expenditure and receipt categories.

The model has undergone a number of modifications or departures from the initial intentions on the road to its present design, e.g. from a single-good model via a tradable-nontradable model ultimately to a \( Y = C + G + J + X - N \) model, or from a small open economy via a two-country structure to a continuum of small economies. Moreover, the model has a number of aspects that

\(^1\)These first two requirements make our modeling strategy markedly distinct from the common practice of academic research.

\(^2\)Such as Calvo’s sticky price and wage setting, habit in consumption, time-to-build in capital formation, and exchange rate disconnect.
entail technical difficulties and require special attention and treatment, such as the problem of large deviations from the ultimate steady state and hence increased emphasis on the accuracy and efficiency of the numerical solution.

In the process of its development and benchmarking with similar developments at other central banks and other institutions, care has been taken to periodically review the model structure and discuss its policy usefulness. This paper is the first step in the process of general documentation of the model development project.
1. Introduction

The Czech National Bank adopted inflation targeting (IT) in January 1998. To support the implementation of monetary policy the Bank has developed a forecasting and policy analysis system (FPAS) whose earlier version is described in Coats et al. [2003]. The analytical framework has been established around a small open economy quarterly projection model (QPM). The model was primarily meant to provide a quantified story-telling device; its structure therefore contains a simple monetary transmission mechanism based on forward-looking agents and a proactive central bank reacting to deviations of the inflation forecast from the target. The QPM captures highly aggregated and stylized flow relationships in the Czech economy that are necessary to explain business cycle fluctuations in output, inflation, the exchange rate and the interest rate as viewed from the New Keynesian perspective. To this end, the model mechanisms primarily deal with deviations of the main macroeconomic variables from their long-run trends (i.e. a ‘gap’ logic), and gaps and trends are treated, to a large extent, separately. While the gap dynamics of the model can be easily linked to some underlying—rather standard—macroeconomic theories, the determination of long-run values lacks deeper supply-side or stock-flow consistency.

There were a number of good reasons for the Bank’s staff to start with a relatively simple, aggregated, and calibrated model to support the implementation of IT. The approximately four years of working experience with the FPAS centered around the QPM have revealed many advantages but also some limitations of the core model. The well organized forecasting process—i.e. smooth integration of short-term and expert inputs with the medium-term model mechanisms, communication between modelers and non-modelers, and communication between the staff and the Board—has led to overall acceptance of and reliance on the existing FPAS. On the other hand, the staff has successively expanded its expertise in areas that are not endogenously contained in the QPM. These areas have become an integral part of the non-model side of the FPAS, and have been extensively used as extra inputs in the model simulations; they often have a deep structural nature and can hardly be addressed without a microeconomically well-founded model. Examples include:

- model-consistent treatment of stocks and flows and their implications for the inflation outlook, particularly in the field of public finance,
- the key national accounts variables and demand components,
- identification and evaluation of the structural determinants of real appreciation,
- issues relating to the envisaged ERM2 entry.

The success of the existing FPAS played an important role in the approval of a research agenda by the Board more than two years ago, directed at developing a new, dynamic stochastic general

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3 The historical long-run trends for output, the real exchange rate, and the interest rate are determined simultaneously by a multivariate filter, and then projected to the future on the basis of expert evaluation.
4 For instance, prior to the QPM the Bank had not relied on any simultaneous-equations model with active monetary policy and forward-looking channels, and therefore there was no expertise in the real-time use of complex formal tools. Second, the modeling staff was initially not experienced enough to develop a fully optimizing DSGE model that could have provided more detail and macroeconomic rigor. Similarly, the non-modeling staff was not prepared to communicate on the basis of a highly formalized and sophisticated paradigm. Third, the process of inflation forecast integration (i.e. integration of the core model forecast with other short-term and expert information into a single staff projection, organizing meetings of the forecasting team with the Bank Board, etc.) was considerably smoother with a simpler model.
equilibrium framework. In line with growing demand for more detailed and micro-founded policy analyses at the Bank, the new model’s development has predominantly relied upon the recent advances in the New Open Economy Macroeconomics (NOEM) literature founded by Obstfeld and Rogoff [1995]. The fully optimizing DSGE framework borrowed from the real business cycle theory and enriched with New Keynesian nominal rigidities has reached a stage of development in which it exhibits realistic quantitative properties and predictions—even when contrasted with the performance of, for example, VAR models, as illustrated in Smets and Wouters [2003]—and may thus serve as a basis for monetary policy analyses. In addition, this class of models facilitates explicit statements about agents, markets, technologies and tastes, and provides a mapping to the economy’s deep parameters to address issues embedded in the microeconomic structure. On the implementation side, there are currently several policymaking institutions who have put into some level of operation DSGE models that derive from the NOEM paradigm, such as the IMF’s GEM, see Bayoumi [2004], the Bank of England’s BEQM, see Harrison et al. [2005], the Bank of Canada’s Totem, see Murchison et al. [2003], the Bank of Finland’s Aino, see Kuismanen et al. [2003], and the Bank of Norway’s Nemo, see Brubakk et al. [2005].

To motivate our final choice of the theoretical structure of the model, we turn to a set of stylized facts of the Czech economy over the last decade that we find critical or important from the point of view of monetary policy implementation; the model then needs to contain devices to replicate, or to allow for, these facts. We simplify the exposition, making a distinction between the long-run trends and the business cycle fluctuations present in the Czech data.

**Balanced growth**—Because some of the cyclical properties of the model may be firmly tied to long-run economic developments, in particular when the economy is undergoing transition as in the Czech case (e.g. real interest rate issues), realistic forecasting properties require the model economy to enable analysis along a balanced growth path rather than a stationary steady state.

**Trends in relative prices**—Along the transition path the Czech economy has been exhibiting significant and permanent shifts in relative prices. Most importantly, given high productivity improvements mainly in the domestic export goods sector, the corresponding trend in relative prices has shown up in ongoing long-term real appreciation as measured by various definitions of the real exchange rate. We illustrate these facts in Figure 2 by the movement in the relative prices of the main expenditure deflators. As the transition in relative prices may in general have considerable effects on the inflation profile and the disinflation strategy, we should allow for it in the endogenous model mechanisms.

**Medium-run developments in expenditure shares**—When compared to developed economies, the expenditure shares of GDP in a transition economy tend to reveal a relatively high investment profile and a correspondingly lower consumption path; we provide a comparison with Germany and France in Figures 3 and 4. These may be explained by the capital accumulation process during the catch-up phase on the one hand, and by consumer credit rationing in the international financial market and consumer habit build-up on the other hand. Despite the fact that consumption and investment shares in some other European countries differ from those displayed in the two largest economies in Europe, it is likely that the Czech economy will ultimately converge in their direction.

**Initial undercapitalization of the economy**—One of the main transition issues is the process of replacement of the obsolete capital stock inherited from the command economy with new, more productive capacities. This can in fact be addressed, from an economic point of view, as a problem of initial undercapitalization, even though the old, less productive stock is rather large. The
investment projects carried out in the Czech economy during the last decade and the expected future trends in investment activity indicate a considerable scale of such undercapitalization.

**Imperfect substitution between old and new capital**—This issue follows on directly from the previous paragraph. The standard macroeconomic models usually assume perfect intertemporal substitution in capital accumulation—so that existing capacities make the same contribution to the new capital stock as newly undertaken investment projects. While this assumption may be valid in countries where economic development has been based on free market forces for a long period of time, it may fail to be appropriate for economies in which existing productive capacity still originates, to some extent, from the old command system. This is one of the reasons why we introduce a generalized capital accumulation equation, borrowed from Lucas and Prescott [1971] and Kim [2003].

**Labor market participation**—The other input factor has displayed a dramatic decline, accompanied by faster real wage growth than that observed in real consumption, as shown in Figure 5. This is hard to reconcile with a simple additive-separable utility function framework as in this model: the long-run elasticity of intertemporal substitution would have to be unrealistically large even for a relatively elastic labor supply curve. We therefore attribute these shifts to changes in the underlying tastes of households, and claim that it can be interpreted as households’ tendency to limit their labor market participation. We believe that this description might be valid for the Czech transition period.

**Growing import intensity**—Evident from Figure 6 is an ongoing increase in the import intensity of domestic production. This rapid build-up of imports is attributable to (a) the continuing investment boom, with investment goods being highly import intensive, and (b) a large increase in new production capacities oriented predominantly toward re-exporting industries.

While the stylized facts relating to the longer-term trends reflect a rapid transition process in the Czech economy, and often require extra devices to be incorporated into the model structure, the business cycle features can be addressed with the more standard tools found in the NOEM literature, relying upon various types of real and nominal frictions.

**Nominal rigidities and costly disinflation**—We use the Calvo [1983] mechanism augmented with backward indexation, as in Christiano et al. [2005], to generate endogenous inflation inertia, in both prices and nominal wages, consistent with an optimizing framework. The backward indexation is ingenious in that it introduces stickiness while maintaining steady-state monetary superneutrality and an interpretation close to that of the Phillips curves found in smaller and more ad-hoc models. The backward indexation also makes disinflation costly in terms of losses in real economic activity. Furthermore, stylized evidence from the Czech disinflation episode suggests that nominal wages are stickier than prices: when inflation falls the real wage tends to rise, as documented in Figure 8, which can be justified by mechanisms as in, for example, Spencer [1998], where relative wage-price stickiness is measured on the post-war U.S. data.

**Exchange rate disconnect**—The problem of nominal exchange rate disconnect is described both empirically and theoretically, for example, in Baxter and Stockman [1989], Flood and Rose [1995], Obstfeld and Rogoff [2000], Devereux and Engel [2002], and Obstfeld [2002]. We achieve realistic disconnect of excessive nominal exchange rate fluctuations from the real economy and prices by making specific assumptions about export and import price setting and international currency pricing, which resembles the usually adopted local currency pricing scheme but is more convenient for our multi-country framework for its analytical tractability.
The CNB’s New Projection Model

Real rigidities—The empirically observed hump-shaped and delayed reactions of real consumption and investment to both temporary and permanent shocks are generated by two, nowadays standard, mechanisms: external habit in consumption as originally introduced by Abel [1990], and the time-to-build constraint on capital accumulation proposed by Kydland and Prescott [1982]. The importance of habit formation in monetary-policy models is also emphasized, inter alia, by Fuhrer [2000].

The remainder of the paper is organized as follows. Section 2 describes the overall structure of the model economy, and emphasizes some of the features introduced for convenience and tractability. Sections 3 and 4 pose the problems solved by households and firms, and are supplemented by Section 5 to specify the behavior of the government and stochastic processes. Section 6 defines the general equilibrium, pointing to the implications of aggregation in monopolistic markets. Section 7 explains the benchmark calibration of the model and examines its basic properties. Section 8 concludes with some remarks on implementation issues. The paper has a series of appendices, namely, a diagram of goods flows in Appendix A, a summary of the first-order conditions in Appendix B, a table with the basic notational rules in Appendix C, an overview of the calibrated parameters in Appendix D, graphs to document the stylized facts of the Czech economy in Appendix E, and, finally, the shock response simulations in Appendix F.

2. Structure of the Model and Features of Convenience

The World economy consists of a continuum of small countries indexed on the interval [0; 1]; we name one of them Home and the others Rest-of-World. All countries are identical; they produce their own variety of final goods that are then either consumed/invested domestically, or sold abroad as intermediate inputs to foreign production. Even though each economy has its own currency, there is an international currency (IC) used in the international financial and goods markets. Home final goods are produced from two inputs, the import intermediate goods purchased from importers, and the Home intermediate goods produced by intermediate producers. The Home intermediate producers employ two domestic input factors, capital and labor. Home households consume domestically produced consumption goods, supply labor, and accumulate physical capital (or productive capacity, in general). In addition they may hold Home currency bonds, and cash money. IC bond trading is delegated to foreign exchange dealers. Nominal wages as well as export, import, and intermediate goods prices are sticky. To this end we introduce tastes for variety and monopolistic competition in the respective markets. Nevertheless, since price stickiness is the only purpose of these extra assumptions, we use the same elasticity of substitution for all these four varieties, denoted by ε throughout the paper. A sketch of the basic structure of the model’s flows of goods and value added can be found in Appendix A.

The multi-country framework provides extra flexibility in calibrating the international expenditure-switching effects of the exchange rate, to an extent that cannot be easily achieved within a more usual two-country model. In particular, we can maintain different price elasticities of export and import demand functions at the national level; this turns out to be one of the critical degrees of freedom for an open-economy monetary policy model to generate realistic business-cycle profiles. For easier analytical tractability of this multi-country World we consequently also introduce international currency pricing as a substitute for the traditional local currency pricing scheme: exporters are assumed to set their prices sticky not in local currency (recall that there is a whole continuum of local currencies in World), but in the common IC. In the orig-
inal LCP hypothesis exporters absorb in their profit margins a certain portion of the bilateral exchange rate volatility between the exporting and importing market. In our World, the bilateral exchange rate is approached in two stages: exporters’ profit margins are only affected by domestic exchange rate movements against the IC, while foreign importers, who are also sticky, absorb the fluctuations in their own exchange rate against the IC.

Furthermore, we maintain strict separation of the parameters in line with their desired effect on the determination of the model’s steady state. Several parameters emerge only to describe purely business-cycle phenomena, and this fact influences the way we define some of the preferences and technologies. Specifically, parameters that relate to mechanisms introduced for a realistic business-cycle profile, such as habit ($\chi$) or imperfect substitution between old and new capital ($\eta$), are arranged so not to have any impact on the steady-state properties of the economy.

Finally, we try to obey the following rules in our notation: The lowercase letters denote (i) quantities demanded and (ii) individual prices of differentiated goods. On the other hand, the uppercase letters (together with several Greek uppercase ones too) denote (i) quantities supplied, (ii) sector- or economy-wide price indexes, and (iii) competitive prices. Calligraphic letters are reserved for functions, and lowercase Greek letters for parameters.

3. Households

The economy is populated by a continuum of households indexed on the interval $[0, 1]$. Each household is a consumer of a variety of consumption goods, a monopoly supplier of a differentiated labor service, and a competitive supplier of capital services; investment in the capital stock needs time to build, as in Kydland and Prescott [1982]. Households can domestically trade in Home-currency non-contingent bonds; all Home trading in International-currency bonds is delegated to forex dealers who act in households’ interests, see Devereux and Engel [2002]. Households then receive a net payment from dealers, negative or positive.

Furthermore, when setting their wage rates households face random duration of contracts, as originally introduced by Calvo [1983] and modified herein in the direction proposed by Christiano et al. [2005]. In each period, a constant fraction $1 - \xi_w$ of households receive a signal to re-optimize their wage contracts; the remaining proportion $\xi_w$ of households must maintain the previous period’s wage rate updated at the previous period’s market-wide rate of nominal wage inflation. In addition, the signal non-receivers always fully satisfy demand for their labor services. The signal receivers are chosen randomly, with each household having an equal probability $1 - \xi_w$ of being drawn. Under these circumstances, the risk of not receiving the signal is idiosyncratic: households would end up heterogenous not only in their current wage rates and labor effort, but also in consumption and wealth. As a consequence, the analytical tractability of the model would dramatically decline. However, given the fact that individual but no aggregate uncertainty exists in the labor market there is scope for insurance. We therefore suppose that there is a competitive market with insurance companies that offer one-period insurance against the wage-setting risk. At date $t$ each household may purchase any positive or negative amount $v_t$ of insurance at the price $P_r^v/i_{1t}$, where this is the riskless nominal gross rate. Consecutively, if the household fails to receive the re-optimization signal at date $t + 1$, the company makes or
collects the payment \( v_t \). The \( s \)-th household’s budget constraint at date \( t \) reads

\[
i_{t-1}b_{t-1}(s) + P^K_t u_t(s) K_{t-1}(s) + \Phi_t(s) + \Psi_t(s) + P^C_t f_t(s) = \\
b_t(s) + P^C_t c_t(s) + \sum_{k=0}^{T} z_{t-k}(s)/(T + 1),
\]

where \( b_t \) are holdings of nominal Home currency bonds; \( u_t K_{t-1} \) is the volume of capital services sold to intermediate producers at date \( t \), compounded by productive capacity available at \( t \), denoted by \( K_{t-1} \), and the utilization rate, \( u_t \); \( \Phi_t \) is the net labor income defined below; \( \Psi_t = \Psi_t^f + \Psi_t^Y + \Psi_t^X \) are firms’ (positive or negative) net cash-flows paid lump-sum to households; \( P^C_t f_t \) are government lump-sum transfers; \( c_t \) is real consumption; and \( z_t \) denotes an investment plan initiated at time \( t \) and resulting in an increase in productive capacity available after \( T + 1 \) periods.\(^5\)

Net labor income includes compensation for labor effort plus the net payment made with the insurance companies, i.e.

\[
\Phi_t(s) = \left(1 - \tau^w_t\right) w_t(s) L_t(s) - P_t^V(s) v_t(s) + \begin{cases} 0, & \text{if signal received,} \\ v_t(s), & \text{if signal not received,} \end{cases}
\]

where \( w_t(s) \) is the nominal wage rate, \( L_t(s) \) is the labor effort exercised, and \( \tau^w_t \) is the labor income tax rate.

Our insurance market is in fact analogous to the life insurance used by Yaari [1965]. An alternative assumption found in the sticky-price literature is the existence of state contingent bonds that are able to provide the same insurance against the wage-setting risk, see e.g. Woodford [1996] or Erceg et al. [2000].

Provided that households are risk averse and face actuarially fair prices of insurance (in that \( P^V_t \) equals the probability \( \xi_w \), this requirement being justified by perfect competition in the insurance market where profits are zero), they will always decide to completely pool the wage-setting risk. As a consequence, randomness of re-optimization signals will not affect the shadow value of their wealth.\(^6\) Moreover, if preferences and initial wealth are identical across households, then all of them will choose identical consumption and asset holding plans, whereas heterogeneity will only prevail in the wage rates earned and labor effort exercised. This assumption allows us to solve the households’ optimization problem separately for representative consumption and asset holdings on the one hand, and for distribution of the individual wage rates and labor supply on the other hand.

### 3.1 Representative Consumption and Asset Holdings

Throughout this subsection we drop indexation relating to a particular household, as we deal with representative decisions made identically by all households. The household seeks to maximize a lifetime objective

\[
E_t \sum_{0}^{\infty} \beta^k \left[ \mathcal{U} (c_{t+k} - \chi h_{t+k}) + \psi(1 - L_{t+k}) \right]
\]

\(^5\) Each project initiated at time \( t \) requires real investment actions equally distributed over \( T + 1 \) periods: \( t, t + 1, \ldots, t + T \).

\(^6\) Herein the shadow value of wealth is represented by the Lagrange multipliers associated with nominal bonds and physical capital.
where the utility derives from consumption adjusted for external habit and leisure. We assume the following instantaneous utility functions

$$U \equiv (1 - \chi) \log(c_t - \chi h_t), \quad V \equiv a^t_L (1 - L_t),$$

where $$\chi \in (0, 1)$$ reflects the importance of habit in consumption, and $$a^t_L$$ is an economy-wide stochastic process that controls households’ willingness to participate in the labor market. The linearity of the utility from leisure—or, in other words, constant marginal disutility of labor giving rise to an infinitely elastic labor supply curve—arises here because of the indivisibility of labor imposed on the labor market as in Hansen [1985].

The process of capital accumulation follows a law of motion with $$T + 1$$ periods to build, where we allow for a general—though constant—elasticity of substitution between the capital already in place and the newly installed one, as proposed by Lucas and Prescott [1971] and generalized by Kim [2003],

$$K_t = \mathcal{K} (K_{t-1}, a^t_L z_{t-T}),$$

where $$\mathcal{K}$$ is a CES function of the previously installed capital, $$K_{t-1}$$, the finished investment plan, $$z_{t-T}$$, and an economy-wide shock to the productivity of capital accumulation, $$a^t_L$$. We find three motivations for the more general accumulation equation: (i) $$K_t$$ may now have a more general interpretation of overall productive capacity rather than physical capital alone; in fact, we will be using the two terms physical capital and productive capacity interchangeably in this paper; (ii) it provides a device similar to the more usual intertemporal capital adjustment costs; and (iii) in the context of a transition economy the equation may well describe a process of replacement of the old capacity (inherited from the past) with the new, more productive one.

The capital accumulation function has the form

$$\mathcal{K} \equiv \left[ v \left( \alpha K_{t-1} \right)^{\frac{\eta-1}{\eta}} + (1 - v) \left( \frac{a^t_L z_{t-T}}{1 - v} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $$\alpha$$ is the steady-state gross rate of real growth of the economy, and $$\eta > 0$$ is the elasticity of substitution. The functional form (3.6) is in fact an extension of Kim [2003] for a balanced-growth economy. Total demand for investment goods by each household at time $$t$$ is then given by

$$j_t = \frac{1}{T+1} \sum_{k=0}^T z_{t-k}.$$

When setting $$v = (1 - \delta) / \alpha$$, the seemingly complicated structure of (3.6) permits us to achieve two convenient features of the model: first, the steady-state investment projects-to-capital ratio is independent of the elasticity of substitution, $$\eta$$, and equals $$z_t / K_t = (\alpha - 1 + \delta) \alpha^{T-1}$$, second, for $$\eta \to \infty$$ we obtain the standard identity $$K^*_t = (1 - \delta) K^*_{t-1} + z^*_{t-T}$$.

The first-order conditions necessary to maximize (3.3) s.t. (3.1) and (3.5) taking net labor income as given are listed in Appendix B.1, equations (B.28) to (B.31).

Finally, we assume that external habit evolves according to the last economy-wide per-capita consumption, subject to i.i.d. habit shocks,

$$h_t = c_{t-1} \exp e^H_t.$$
3.2 Staggered Wage Setting

To describe the aggregate wage dynamics we need to derive the wage rate set by those households who receive the re-optimization signal; the non-receivers simply follow the imposed rule,\[ w_t(s) = w_t(s) \cdot W_{t-1}/W_{t-2}, \]
where the market-wide wage, \( W_t \), is introduced as the standard minimum-expenditure index of employment intermediaries in Appendix B.3, equation (B.35).

First note that because no other idiosyncratic uncertainty exists for households in the model, all time \( t \) signal receivers will choose the same rate regardless of their own history. For this reason we drop individual indexation again and denote by \( w_{t+k|t} \) the wage rate valid at time \( t+k \), which was last optimized at time \( t \) and has been updated since then, and by \( L_{t+k|t} \) the corresponding labor effort supplied at time \( t+k \) by a household who last received the signal \( k \) periods ago.\(^8\)

The wage setter optimizes (3.3) over all future states of nature in which she consecutively fails to receive the re-optimization signal and must keep updating the wage rate decided today. This effectively means to maximize\[ E_t \sum_{k=0}^{\infty} (\xi_t \beta)^k \left[ y(1 - L_{t+k|t}) + \lambda_t(1 - \tau_t^w)w_{t+k|t}L_{t+k|t} \right], \quad (3.7) \]

taking \( \lambda_t \) as given, subject to the demand function for her own labor service (derived later in the section, see Appendix B.3, equation B.34),
\[ L_{t+k|t} \leq \ell_{t+k|t} = \left( w_{t+k|t}/W_{t+k} \right)^{\epsilon} \ell_{t+k}, \quad (3.8) \]

and the wage-indexation scheme, converted here to relative wage terms,
\[ \frac{w_{t+k|t}}{W_{t+k}} = \frac{w_{t|t}}{W_{t}} \cdot \frac{W_{t+k-1}/W_{t-1}}{W_{t+k}/W_{t}} = \frac{w_{t|t}}{W_{t}} \cdot \Omega_t / \Omega_{t+k}, \quad (3.9) \]

where \( \Omega_t = W_t/W_{t-1} \) denotes the market-wide gross rate of wage inflation. Finally, recall that our other assumption is \( L_{t+k|t} = \ell_{t+k|t} \) for \( \forall k > 0 \).

The first-order conditions are listed in Appendix B.2. In Section 6 we derive the law of motion for the market-wide wage rate and its inflation, and quantify the effect of the wage and labor effort dispersion that prevails in the economy out of steady state due to stickiness.

3.3 Employment Intermediation

The individual labor services are aggregated by competitive employment intermediaries. The aggregation is described by a CES function defined over the labor variety. We may therefore write for the sector-wide labor aggregate\(^9\)
\[ \ell_t = \left[ \int_0^1 \ell_t(s)^{\epsilon - 1} ds \right]^{\frac{1}{\epsilon}}, \quad (3.10) \]

\(^8\) Hence, the choice variables of our interest are \( w_{t|t} \), the currently optimized wage rate, and \( L_{t|t} \), the optimizer’s labor supply. This fact is reflected in the way we express the indexation scheme in (3.9).

\(^9\) Note that \( \ell_t(s) \) denotes the sector-wide demand functions for the \( s \)-th labor service, and not demand by the \( s \)-th intermediary.
The aggregate labor service, $\ell_t$, is supplied to the intermediate producers. The cost minimization of each intermediary leads to the sector-wide demand for individual labor services described in Appendix B.3, equations (B.34) to (B.35).

### 3.4 Foreign Exchange Dealers

There is one forex dealer, who is delegated to buy or sell International currency bonds. She is assumed to behave competitively on both sides, and to be short-lived: she only exists for two consecutive periods. At date $t$ she collects payments from households and invests the amount in the international financial market; at date $t + 1$ she transfers all net cash-flows, positive or negative, back to households and dies. The dealer chooses the stock of International currency bonds, $\tilde{b}_t$, to maximize the expected net cash-flow evaluated at the households’ shadow value of wealth, $\beta^k \lambda_{t+k}$, facing a quadratic cost of portfolio adjustment,\(^10\)

$$a^S_t E_t \left( \beta \lambda_{t+1} \cdot S_{t+1} \tilde{b}_{t+1} \right) - \lambda_t \cdot S_t \tilde{b}_t (1 + \frac{1}{2} \zeta_a a_t \tilde{b}_t)$$

(3.11)

where $a^S_t$ captures both the expectational errors and the ex-ante risk premium (we will refer to this term as the premium henceforth); the former can potentially be serially correlated because the short-lived dealer is unable to learn. The net cash-flow maximizing first-order condition can be found in Appendix B.4, equation (B.36).

### 4. Production

There are two varieties of intermediate goods produced in the economy: Home intermediate goods and import intermediate goods. The former variety is produced by a continuum of Home intermediate firms indexed on the interval $[0, 1]$; the latter is produced by a continuum of Home residing importers indexed again on $[0, 1]$. The intermediate goods are used by the final goods producers to assemble four types of goods: consumption, government consumption, investment, and export.

#### 4.1 Intermediate Producers

There is a continuum of intermediate producers in Home indexed on the interval $[0, 1]$ who combine capital services, $k_t$, and labor services, $\ell_t$, rented or hired from households in competitive markets, to produce a variety of intermediate goods. These goods are then demanded by final goods producers. The intermediate producers are homogenous in the technology they use, including a labor-augmenting technological process, and in the factor prices they face. Assuming CRS-CES technology we may reduce the notational burden by expressing directly the sector-wide level of output as a function of the sector-wide quantities of the two input factors employed, assuming unit elasticity of substitution between them,\(^11\)

$$\int_0^1 Y_t(s) ds = Y_t = k_t^{1-\gamma} (a_t \ell_t)^\gamma,$$

(4.12)

\(^{10}\)The portfolio adjustment cost is one of the convenient ways of achieving a unique stationary distribution of households’ wealth and consumption plans in steady state in small open economy models, see Schmitt-Grohé and Uribe [2003].

\(^{11}\)Note that demand for productive capacity utilized today, $k_t$, is matched by supply, denoted by $K_{t-1}$, see the market clearing in Section 6.
with $a_t$ denoting the sector-specific technological process. Obviously, each producer’s cost minimization leads to the following sector-wide relative factor demand

$$ k_t/\ell_t = (1 - \gamma)/\gamma \cdot (1 + \tau_{i}^s)W_t/P^K_t, $$

where $P^K_t$ denotes the rental price of capital services, whereas $W_t$ is the wage rate paid for a unit of the aggregated labor service; $\tau_{i}^s$ is the rate of social and health insurance payments imposed on employers. Furthermore, the marginal, as well as average, costs are identical for all producers and independent of the level of their production; they can be expressed in nominal terms as

$$ Q^Y_t(s) = Q^Y_t = \frac{(1 + \tau_{i}^s)W_t\gamma_{k}P^K_t}{\gamma_{(1 - \gamma)}^{1-\gamma}}. $$

The instantaneous sector-wide net cash-flow is

$$ \Psi^Y_t = \int_{0}^{1} p^Y_t(s)Y_t(s)ds - P^K_tk_t - (1 + \tau_{i}^s)W_t\ell_t, \quad (4.13) $$

Each producer chooses her output, $Y_t(s)$, and price, $p^Y_t(s)$, to maximize the net cash-flow over an infinite horizon evaluated at households’ present shadow value of wealth, $\beta^k \lambda_t^{k}$, subject to two restrictions: (i) the demand function for her own output,

$$ Y_t(s) \leq y_t(s) = \frac{[p^Y_t(s)/P^K_t]^{-\xi}y_t}, \quad (4.14) $$

and (ii) the sticky-price constraint. Without the latter limitation, e.g. if the producer were allowed to optimize freely at each date, she would simply maintain the price equal to her nominal marginal cost, or in other words, the real marginal cost equal to 1, $Q^Y_t(s) = Q^Y_t = 1$, (recall that markups arising from monopoly power are offset by the subsidy), and produce output to satisfy the corresponding demand. This, however, only holds in steady state, and we instead assume that intermediate price setting is staggered in a similar way as the nominal wage. A fixed fraction $1 - \xi^Y$ of producers is randomly drawn each period and allowed to re-optimize. The remaining proportion of $\xi^Y$ can only adjust their previous period’s prices for the previous period’s sector-wide rate of inflation,

$$ \Pi^Y_{t-1} = P^K_{t-1}/P^K_{t-2}. \quad (4.15) $$

The sticky-price problem is described in a separate subsection along with the other sectors.

4.2 Importers

There is a continuum of importers indexed on the interval $[0, 1]$ who purchase export goods abroad and costlessly produce a variety of differentiated import intermediate goods using a CES aggregator. Moreover, the imports from individual countries themselves are assumed to be CES bundles defined over the variety of that country’s national exporters. That is, each national exporter competes with other national exporters, but not with any foreign one, whereas the aggregate national export as a whole competes with other countries’ aggregate exports. Consequently, neither individual nor national types of exports are directly confronted with the intermediate goods produced domestically. The multi-stage import structure provides enough
flexibility to calibrate the expenditure switching in response to nominal exchange rate fluctuations. There are three distinct elasticities of substitution on the import side: first, $\varepsilon$ between any pair of goods produced by exporters from the same country, introduced because of the sticky-price assumption; second, $\theta$ between any two national composite imports, introduced to redirect demand internationally; and third, $\zeta$, see Subsection 4.3, between import goods and domestically produced intermediates.

We again express the sector-wide production function, which takes as inputs the export goods from all countries, 

$$\int_0^1 N_t(s) \, ds = N_t = a_t^N \left\{ \int_0^1 \left[ \int_0^1 x_t(z,s) \frac{e^{-1}}{e^{(\theta-1)/\theta}} \, ds \right] dz \right\}^{\theta/\theta-1}, \quad (4.16)$$

where $\theta > 1$ is the elasticity of substitution between any pair of countries, $x_t(z,s)$ denotes the quantity used sector-wide of the $s$-th type of good originating from the $z$-th country, and $a_t^N$ is an import-specific technological process. The marginal, and average, costs faced by all importers are again obviously identical. We may write for the sector-wide nominal marginal cost

$$Q_t^N(s) = Q_t^N = \frac{S_t}{a_t^N} \left\{ \int_0^1 \left[ \int_0^1 \tilde{p}_t^{X*}(z,s)^{1-\varepsilon} \, ds \right]^{1-\theta} dz \right\}^{\theta/\theta-1} = \frac{S_t}{a_t^N} \tilde{P}_t^{X*},$$

where $\tilde{p}_t^{X*}(z,s)$ are the export prices defined in conformity with $x_t(z,s)$ and quoted in International currency, and $\tilde{P}_t^{X*}$ is an aggregate price index introduced for convenience of description of the aggregate model behavior. The cost minimization performed by and aggregated over all importers obviously the demand functions at the national level,

$$x_t(z) = \left[ \frac{S_t}{a_t^N} \tilde{P}_t^{X*}(z)/Q_t^N \right]^{-\theta} N_t,$$

where we have integrated away the individual export goods from $x_t(z,s)$ and $\tilde{p}_t^{X*}(z,s)$. This first-order condition is important in that it establishes World-wide demand for Home exports, and will be employed in Subsection 4.3.

The choice made by an importer is fully analogous to that of an intermediate firm: by setting her output, $N_t(s)$, and the price, $p_t^N(s)$, she maximizes the net cash-flow over an infinite horizon subject to demand for her own production,

$$N_t(s) \leq n_t(s) = \left[ \frac{p_t^N(s)}{P_t^{X*}} \right]^{-\varepsilon} n_t,$$

and the sticky-price constraint. Finally, the sector-wide instantaneous net cash-flow is

$$\Psi_t^N = \int_0^1 p_t^N(s) N_t(s) \, ds - \int_0^1 \int_0^1 x_t(z,s) \tilde{p}_t^{X*}(z,s) \, dz \, ds. \quad (4.17)$$

\[12\] This is indeed the simplest possible structure that needs to be imposed on the foreign trade flows to achieve distinct elasticities of aggregate import and export functions along with price stickiness.
4.3 Final Goods Producers

There are four types of final goods producers: exporters, and consumption, government, and investment goods firms; to assemble their output they use the two intermediate goods with various intensity. For the reasons laid out in the introduction we assume the following: (i) the investment goods are fully imported, (ii) the government goods are only produced from Home intermediates, (iii) the export and consumption goods have identical import intensities and elasticities of substitution between Home intermediates and imports, (iv) exporters’ prices are sticky in International currency—we therefore need to introduce monopolistic competition in the export sector, while the other producers operate in competitive markets.

Accordingly, we write the sector-wide production functions as

\[
\int_0^1 X_t(s) \, ds = X_t = a_i^X \left[ \omega_x (n_t^X) \frac{\epsilon-1}{\tau} + (1 - \omega_x)(y_t^X) \frac{\epsilon-1}{\tau} \right]^{\frac{\tau}{\epsilon-1}},
\]

\[
C_t = a_i^C \left[ \omega_c (n_t^C) \frac{\epsilon-1}{\tau} + (1 - \omega_c)(y_t^C) \frac{\epsilon-1}{\tau} \right]^{\frac{\tau}{\epsilon-1}},
\]

\[
J_t = a_i^J n_t^J,
\]

\[
G_t = a_i^G y_t^G,
\]

where \(a_i^X, a_i^C, a_i^J, a_i^G\) are sector-specific technological processes, \(n_t^k\) and \(y_t^k\) are the sector-wide input factor indexes related to the respective intermediate goods, \(\omega_x, \omega_c \in (0, 1)\) controls the import intensity of exports and consumption respectively, and \(\zeta > 0\) is the elasticity of substitution in final goods production. The input factor indexes, \(n_t^X, n_t^C, n_t^J, n_t^G\) and \(y_t^X, y_t^C, y_t^J, y_t^G\), entering the four production functions are assumed to be CES functions defined over the variety of the two intermediate goods.

The cost minimization leads to the standard price-elastic demand curves for intermediate inputs in the export and consumption sectors, see equations (B.37) to (B.40) in Appendix B.5 for the sector-wide curves. Demand for import (Home) intermediate goods in the investment (government) sector is trivial.

The individual and export and consumption sector-wide nominal marginal costs before indirect taxation are

\[
Q^X_t(s) = Q^X_t = \frac{1}{a_i^X} \left[ \omega_x P_t^{n_t^X-\zeta} + (1 - \omega_x)P_t^{y_t^X-\zeta} \right]^{\frac{1}{1-\zeta}},
\]

\[
Q^C_t = \frac{1}{a_i^C} \left[ \omega_c P_t^{n_t^C-\zeta} + (1 - \omega_c)P_t^{y_t^C-\zeta} \right]^{\frac{1}{1-\zeta}},
\]

whereas for the production of investment and government goods they simply equal \(Q^I_t = P_t^N/a_i^I\) and \(Q^G_t = P_t^G/a_i^G\). In the three competitive sectors, prices are driven to the respective nominal marginal costs, and profits to zero, instantaneously; consumption sales are, nevertheless, subject to taxation at the rate \(\tau^C\). We thus have \(P_t^C = (1 + \tau^C)Q^C_t, P_t^I = Q^I_t, P_t^G = Q^G_t\). Exporters each face their own world-wide demand curve, and are therefore constrained by

\[
X_t(s) \leq x_t(s) = \left( \frac{\tilde{P}^X_t(s)}{P^X_t} \right)^{-\epsilon} x_t,
\]
and set their prices sticky in International currency. The export price dynamics are described later. For future reference we introduce the net cash-flow of the export sector,

\[ \Psi_t^y = \int_0^1 p_t^X(s) X_t(s) ds - P_t^y y_t^X - P_t^Y n_t^Y. \]  

(4.22)

Finally, we may append the sector-wide demands for the individual types of the two intermediate goods, which follow from the cost minimization again,

\[ n_t(s) = n_t^X(s) + n_t^C(s) + n_t^I(s) = \left[ p_t^N(s)/P_t^N \right]^{-\epsilon} n_t, \]

\[ y_t(s) = y_t^X(s) + y_t^C(s) + y_t^G(s) = \left[ p_t^Y(s)/P_t^Y \right]^{-\epsilon} y_t, \]

where \( n_t \) and \( y_t \) are the respective economy-wide input factor indexes defined as CES functions over the variety of individual quantities, and \( P_t^N \) and \( P_t^Y \) are the cost-minimizing price indexes defined in Appendix B.5, equations (B.41) to (B.42), which are identical for all producers. Because of linear homogeneity and the fact that all producers face identical intermediate prices and hence always decide to employ the varieties of the individual input factors in identical proportions, we may easily prove that \( n_t = n_t^X + n_t^C + n_t^I \) and \( y_t = y_t^X + y_t^C + y_t^G \).

### 4.4 Sticky Price Setting

In this subsection we derive the optimal prices set by signal receivers in the intermediate, import, and export goods sectors. The three problems are indeed close parallels, so that we only describe in full the behavior of Home intermediate producers. Some differences nevertheless arise, as exporters set their prices in International currency but face nominal marginal costs sticky in Home currency, while the reverse holds for importers. 13

As in the case of wage-setting households, all the time \( t \) optimizing intermediate producers choose the same price, and hence we drop the index \( s \). Next we denote by \( P_{t+k|t}^y \) the price valid at time \( t+k \) which was last optimized at time \( t \) and has been updated since then; correspondingly, we denote by \( Y_{t+k|t} \) the output supplied at time \( t+k \) by a producer who last received the signal \( k \) periods ago.

The producer optimizes her lifetime cash-flow function over all future states of nature in which she consecutively fails to receive the re-optimization signal and must keep updating the price decided today,

\[ E_t \sum_0^{\infty} \left( \xi f \right)^k t_{s} Y_{t+k|t} \left( P_{t+k|t} - Q_t^f \right) \]  

(4.23)

where the sector-wide nominal marginal cost, \( Q_t^f \), has been defined earlier, and the discount factor is based on the present shadow value of wealth, \( t_t = \lambda_t/(\beta^t \lambda_{t+k}) \).

The optimization is performed subject to the demand function for her own production (4.14), and the price-indexation scheme, converted again to relative price terms,

\[ \frac{P_{t+k|t}^y}{P_t^y} = \frac{P_{t+k}^y}{P_t^y} \cdot \frac{P_{t+k-1}^y/P_{t}^y}{P_{t+k}^y/P_t^y} = \frac{P_{t+k}^{y}}{P_t^y} \cdot \prod_t^{y} / \prod^{y}_{t+k}. \]  

(4.24)

13 However, bear in mind that both of them maximize Home-currency profits.
The first-order conditions for all sectors are summarized in Appendix B.6, equations (B.43) to (B.46); the laws of motion for the sector-wide price indexes and the gross rates of inflation can be directly derived from their definition; we state them in equations (B.49) to (B.51).

5. The Rest of the Model

5.1 Government

The government’s fiscal policy ensures intertemporal balance through a certain fiscal rule that will be imposed in the testing and validation stage later. The government’s income consists of tax revenues (direct and indirect taxes and social and health insurance), while its expenditure consists of goods purchased, \( P^G_t g_t \), and transfers to households, \( P^C_t F_t \). Furthermore we assume that the government does not intervene in the foreign exchange market, and the exchange rate floats freely. These assumptions describe the following nominal intertemporal budget constraint,

\[
B^G_t + P^G_t g_t + P^C_t F_t = i_t - 1 B^G_{t-1} + (\tau^W_t + \tau^S_t) W_t \ell_t + \tau^C_t P^C_t c_t, \tag{5.25}
\]

where \( B^G_t \) is the government’s net position. For the sake of the simulation experiments herein we adopt “neutral” fiscal policy, in that the nominal government consumption share of domestic value added, \( \Sigma^G_t = P^G_t g_t / \Sigma_t \), fluctuates around a fixed target value, \( \sigma_G \),

\[
\log \Sigma^G_t = \rho_G \log \Sigma^G_{t-1} + (1 - \rho_G) \log \sigma_G + e^G_t, \tag{5.26}
\]

where the three tax rates, \( \tau^W_t \), \( \tau^S_t \), and \( \tau^C_t \) are kept constant at \( \tau^W \), \( \tau^S \), \( \tau^C \), respectively, and the government’s net asset position is balanced through non-distortionary (potentially negative) transfers to households.

Monetary policy acts proactively to achieve a target specified in terms of consumer inflation. We adopt the following log-linear inflation-forecast rule,

\[
\log i_t = \rho_M \log i_{t-1} + (1 - \rho_M) \left( \log \bar{i}_t + \psi \log \bar{\Pi}_t \right) + e^M_t, \tag{5.27}
\]

in which policy reacts\(^{14}\) to the \( \rho \)-discounted sum of current and future deviations of the model-consistent consumer log-inflation forecast, \( E_t \log \Pi^*_t+\kappa \), from the respective log-targets, \( \log \Pi^*_t+k \),

\[
\log \bar{\Pi}_t = (1 - \kappa) E_t \sum_{k=0}^{\infty} \kappa^k (\log \Pi^*_t+k - \log \Pi^*_t+k). \]

The rule (5.27) is flexible in the sense that by changing the parameter \( \kappa \in [0, 1) \) we can smoothly adjust the average forecast horizon which is monitored by the policy-maker; the steady-state average horizon can be shown to be equal to \( \kappa/(1 - \kappa) \). Furthermore, the policymaker has the option of engaging in interest rate smoothing by setting \( \rho_M > 0 \).

The policy neutral rate, \( \bar{i}_t \), used as a reference level in the rule, is defined as the nominal rate consistent with the steady-state real rate of interest (fundamentally determined by \( \alpha/\beta \)) should expectations be anchored to the target,

\[
\log \bar{i}_t = \log(\alpha/\beta) + \log \Pi^*_t+1. \]

\(^{14}\) The residual term, \( e^M_t \), can be assigned various interpretations, e.g. disturbances in the money market, monetary policy surprises, etc.
The target itself can vary over time in general; nevertheless, it is ultimately assumed to converge to a pre-specified long-run rate of inflation denoted by $\pi$. This is to allow for disinflation episodes.

### 5.2 Stochastic Processes

Basically, there are three types of stochastic processes (called generally residuals herein) introduced in the model economy: the domestic labor-augmenting technological process, $a_t$, the other sector-specific technological processes, $a_{Lt}$, $a_{Zt}$, $a_{Xt}$, $a_{Nt}$, $a_{Jt}$, $a_{Gt}$, along with the time-varying premium, $a_S$, and the i.i.d. shocks, $e_{Ht}$, $e_{Gt}$, $e_{Mt}$. Each group of stochastic processes plays a distinct role in the model dynamics and interpretation.

The labor-augmenting technological process is to stimulate the economy’s growth through the production function (4.12). We assume for convenience that it follows a log-linear time-trend stationary process,

$$A(q)(a_t - \bar{a}_t) = e_t^A,$$

where $A(L)$ is an absolutely summable generating function of the lag operator $q$, whereas $\bar{a}_t = \alpha_t$ is the deterministic component of the technology trend shared across World, with $\alpha$ being the steady-state real growth of the economies, and $e_t^A$ is an i.i.d. productivity shock. Note that Home economy’s initial underdevelopment is captured by an initial deviation $a_t - \bar{a}_t$. The deterministic component, $\bar{a}_t$, is also our numeraire for stationarizing the real sector of the economy.

Next, the other technological processes are log-linear stationary around deterministic trends; however, in contrast to $a_t$, the deterministic part of each of them converges to a stable asymptote and is considered the driving force of the economic transition. As claimed in the introduction, such deterministic trends are critical in our model to address some of the convergence and transition issues, in particular those related to relative prices, including various measures of the real exchange rate.\(^{15}\) Introduced formally, e.g.

$$A^L(q)(a_{Lt} - \bar{a}_{Lt}) = e_{Lt}^A,$$

where

$$\lim_{t \to \infty} a_{Lt} = a^L,$$

and $a^L$ is treated as a model parameter, and $e_{Lt}^A$ is again an i.i.d. shock. Defined this way, the processes guarantee that relative prices and the investment-to-capital and capital-to-output ratios are stable in the ultimate steady state where the economy replicates the structure of its developed neighbors.

Last, the i.i.d. shocks put into the preference and technology definitions are meant (i) as devices to examine the impulse response functions of the model, (ii) to add a sufficient number of degrees of freedom for matching the population stochastic properties, e.g. the cross-spectra and correlation functions, and (iii) to permit historical simulations with the model. More specifically, the historical simulations are performed as a solution to an exactly identified system where the number of observed data points to be replicated is matched by the number of unknown residuals.

\(^{15}\) The potential for fast export-specific technological progress generating effects similar to the more traditional Balassa-Samuelson models with a tradable-nontradable structure has been recognized, for example, by Gregorio and Wolf [1994].
6. Equilibrium and Aggregation

In this section we define the general equilibrium for Home, point to some implications of aggregation in a model with monopolistic competition and sticky prices, and finally develop the notion of national accounting within the given structure of the model.

6.1 The General Equilibrium

Taking the variables determined in Rest of World, $\tilde{t}_t$, $\tilde{P}^X_t$, $\tilde{Q}^N_t$, $N_t^*$, as given, and having observed realizations of the stochastic processes and random distribution of optimization signals, we define the Home economy’s equilibrium as follows:

1. Households maximize (3.3) s.t. (3.1) and (3.5) taking as given their individual wage and labor supply.

2. Households maximize (3.7) s.t. (3.8) and (3.9) taking as given the shadow value of wealth.

3. Employment intermediaries minimize the cost of aggregating (3.10).

4. Home intermediate goods firms minimize the cost of producing (4.12).
   (a) Signal receivers maximize the lifetime cash-flow (4.23) s.t. (4.14) and (4.24).
   (b) Signal non-receivers set the price according to (4.15) and fully satisfy demand.

5. Import intermediate goods firms minimize the cost of producing (4.16).
   (a) Signal receivers maximize the import-sector equivalent of (4.23) s.t. the sector equivalents of (4.14) and (4.24).
   (b) Signal non-receivers set the price according to the import sector equivalent of (4.15) and fully satisfy demand.

6. Final goods firms minimize the cost of producing (4.18), (4.19), (4.20), (4.21).

7. In the export sector:
   (a) Signal receivers maximize the export-sector equivalent of (4.23) s.t. the sector equivalents of (4.14) and (4.24).
   (b) Signal non-receivers set the price according to the export sector equivalent of (4.15) and fully satisfy demand.

8. Firms’ net cash-flows, (4.13), (4.17), (4.22), are distributed equally to households.

9. Exports at the national level are constrained by Rest-of-World’s demand,
   \[ x_t = \left( \frac{\tilde{P}^X_t}{\tilde{Q}^N_t} \right) \theta N_t^*, \]

10. The government collects taxes, sets its consumption according to (5.26) and the interest rate according to (5.27), and distributes the transfers to keep $B_t^G$ zero.
11. Government transfers are distributed equally to households, \( F_t = f_t(s) \), so that \( P_t^C F_t = \int_0^1 P_t^C f_t(s) \, ds \).

12. Markets clear:

(a) Labor market:

\[
L_t(s) = \ell_t(s), \quad \forall s \in [0, 1].
\]

(b) Import intermediate goods market:

\[
N_t(s) = n_t^X(s) + n_t^C(s) + n_t^I(s) = n_t(s), \quad \forall s \in [0, 1].
\]

(c) Home intermediate goods market

\[
Y_t(s) = y_t^X(s) + y_t^C(s) + y_t^G(s) = y_t(s), \quad \forall s \in [0, 1].
\]

(d) Export goods market

\[
X_t(x) = x_t(s).
\]

(e) Consumption goods market

\[
C_t = \int_0^1 c_t(s) \, ds.
\]

(f) Investment goods market

\[
J_t = \int_0^1 j_t(s) \, ds.
\]

(g) Government goods market

\[
G_t = g_t.
\]

(h) Productive capacity market

\[
k_t = \int_0^1 K_{t-1}(s) \, ds.
\]

(i) Home currency bonds market

\[
\int_0^1 b_t(s) \, ds = 0.
\]
The market-clearing conditions (1212a), (1212b) and (1212c) for the individual types of labor services and goods do not obviously provide a description of the connection between their sector-wide counterparts. When wages and prices are sticky and hence symmetric equilibria fail to hold in these markets, then the wage and price dispersion among households and producers have real effects and cause inefficiency in resource allocation. This of course matters for the aggregation and simulations of the model. For example, in the Home intermediate goods sector in general,

\[ Y_t = \int_0^1 Y_t(s) \, ds \neq \left[ \int_0^1 y_t(s) \frac{\tau}{1-\tau} \, ds \right]^{\frac{\tau}{\tau-1}} = y_t. \]

unless symmetric equilibrium prevails therein, for which \( y_t(s) = y_t, \, \forall s \in [0, 1] \).

We may, however, express the volume of supplied goods as an integral of the quantities demanded and proceed as follows,

\[ Y_t = \int_0^1 y_t(s) \, ds = y_t \int_0^1 [p^y_t(s)/p_t^y]^{-\varepsilon} \, ds = y_t D^y_t, \]

where \( D^y_t \) measures the price dispersion. Due to Jensen’s inequality the price dispersion term is clearly bounded from below by 1, with the equality holding in symmetric equilibrium only, see Schmitt-Grohe and Uribe [2005] for a proof. Furthermore, it has its recursive form derived from the optimal price setting conditions and backward indexation scheme,

\[ D^y_t = (1 - \xi_t) \left( \frac{p^y_t / p_t^y}{\Pi^y_t / \Pi_{t-1}^y} \right)^{-\varepsilon} D^y_{t-1}, \]

see equation (B.52) in Appendix B.7 for details.

### 6.2 National accounting

We denote by \( \Sigma_t \) the nominal domestic value added (GDP). Based on the flows of goods and input factors, we may decompose GDP on both the expenditure and income sides. Namely, GDP by its expenditure components is

\[ \Sigma_t = S_t \tilde{P}_t^x x_t + P_t^c c_t + P_t^j j_t + P_t^g g_t - Q_t^N n_t, \]

whereas by its income components it is,

\[ \Sigma_t = P_t^x y_t + (P_t^N - Q_t^N) n_t + (S_t \tilde{P}_t^x - Q_t^x) x_t + \tau_t^c \cdot P_t^c c_t = (1 + \tau_t^x) \cdot W_t \ell_t + P_t^K k_t + (P_t^N - Q_t^N) n_t + (S_t \tilde{P}_t^x - Q_t^x) x_t + \tau_t^c \cdot P_t^c c_t \]

where \((1 + \tau_t^x) \cdot W_t \ell_t\) is labor income, \(P_t^K k_t\) is capital income, \((P_t^N - Q_t^N) n_t\) and \((S_t \tilde{P}_t^x - Q_t^x) x_t\) are profits, and \(\tau_t^c \cdot P_t^c c_t\) are indirect tax revenues. Note that

\[ (P_t^N - Q_t^N) y_t = \frac{1}{\varepsilon} \cdot P_t^N y_t \]

in steady state or in any flexible price equilibrium; analogous relationships also hold for the import and export sectors.
7. Calibration and Model Properties

7.1 Benchmark Calibration

We separate two groups of parameters: (i) parameters that determine the properties of the model along the non-stochastic balanced-growth path (termed as steady state parameters), namely $\alpha$, $\beta$, $\gamma$, $\delta$, $e$, $\omega_X$, $\omega_C$, $a_t$, $\sigma_G$, $\tau_Y$, $\tau_S$, $\tau_C$, and $\pi$, and the steady state characteristics of Rest-of-World’s variables, $N^*/\bar{a}$, $P^*\hat{a}$, (ii) parameters that only affect the transient and stochastic properties (termed as transient parameters), i.e. $\chi$, $\phi$, $\eta$, $\theta$, $\xi_H$, $\xi_T$, $\xi_S$, $\xi_C$, $\rho_H$, $\rho_G$, $\psi$, $\kappa$, $\rho_M$, together with parameters describing the properties of the stochastic processes.

We calibrate the steady state parameters as follows. First we set $\alpha$, $\delta$, $\beta$, and $\pi$ so that the implied real rate of growth is 2 %PA, the real rate of interest is 3 %PA, the government consumption share of GDP is 23 %, ultimate inflation is 3 %PA, and the rate of capacity depreciation is 6 %PA. Next, we assign $e = 11$, which leaves a reasonable 10 % for the monopolistic markups, and, at the same time, generates a profit share of GDP close to the recent observations presented in Figure 9. The fiscal policy parameters, $\sigma_G$, $\tau_Y$, $\tau_S$, $\tau_C$, are chosen on the basis of the recently observed fiscal ratios, too. Subsequently, we impose the restriction that World’s steady state is fully symmetric, which in particular requires a zero net asset position, $\bar{m}_0 = 0$, and $P^* = P^{x*} = Q^N / S$. Using the latter condition we may uniquely identify $\gamma$, $\omega_X$, $\omega_C$, and $N^*/a$ so that we can replicate the following desired steady state characteristics: the investment share of GDP 25 %, the import (and hence also export) share of GDP 70 %, the import intensity of households’ consumption 20 %, and labor effort normalized to 1. Last, we set $P^*$ to obtain Rest-of-World’s inflation 2 %PA. A summary of the calibration of the steady state parameters is provided in Table 1.

The first-pass interim calibration of the transient parameters so far is somewhat less directly linked to the observed properties of the Czech economy, and it will be the subject of further research directed at advanced validation techniques based on the so-called minimal econometric interpretation of DSGE models, as advocated by Geweke [1999]. With our calibration we have attempted to broadly replicate the aggregate business-cycle features of the existing Quarterly Projection Model used at the Czech National Bank whenever there has been a meaningful mapping between the two. On top of this we have followed these guidelines: (i) nominal wage inflation set stickier than price inflation, while maintaining the degree of stickiness identical for the three types of prices ($P^Y$, $P^N$, $P^S$), (ii) the sacrifice ratio close to 1, (iii) a relatively low elasticity of substitution between imports and domestically produced goods (complements rather than substitutes), between 0 and 1, and a higher e.o.s. among national exports, above 1, (iv) the average forecast horizon of the monetary policy-maker of 4 quarters, together with policy rate smoothing sufficient to generate an initial increase in the nominal interest rate at the beginning of an unanticipated disinflation. The full transient parameterization can be found in Table 2.

7.2 Shock Simulations

We demonstrate the basic transient properties of the model by explaining the mechanics of three shock simulations: a disinflation shock, a labor participation shock, and a consumption demand shock. Our choice of these particular shocks was influenced by two factors: first, each of the three has, to some extent, historical relevance to the recent development of the Czech economy. Second, they are representatives of three distinct categories of shocks that may hit the model economy. Namely, the disinflation shock is a permanent monetary shock that induces changes in the steady state of nominal, but not real, variables; the labor participation shock, on
the other hand, permanently affects the real sector, whereas the consumption demand shock is solely a temporary one. The responses of the key macroeconomic variables to these 1% shocks are computed starting from the ultimate steady state of the model, where the deterministic part of the technological processes has converged their asymptotes,\footnote{The shock responses of an economy in transition, i.e. out of these asymptotes, may obviously differ, and this fact will be the subject of our further sensitivity analyses.} and are reproduced in Figures 10 to 12 in Appendix F. We plot the percentage deviations from the respective ultimate steady states; in the case of the permanent shocks we also indicate the new (after-shock) steady state by horizontal lines. The timescale is in quarters, and the shocks hit the economy at time 0. Note, however, that the graphs start a period before the shock to make them easier to read. Finally, all shocks are introduced as one-period, serially uncorrelated innovations.

**Disinflation shock (Figure 10)**—The disinflation shock is introduced as a permanent decline in the inflation target. We assume that the change in the target is sudden and not pre-announced,\footnote{This might seem too unrealistic, but this model experiment has developed to a standard in the monetary policy literature.} but, on the other hand, enjoys full credibility in that the new target is perfectly observed and believed. Note first that the real variables remain intact in the long run, whereas inflation, exchange rate depreciation and the nominal interest rate are all down by 1% permanently. To better understand the first-order forces behind the fall in consumer inflation, which takes less than 2 years, we log-linearize the pricing equations, producing a simple relationship called the Phillips curve in the New Keynesian literature,

\[
\Delta \pi_t = \frac{\beta}{\alpha} E_t \Delta \pi_{t+1} + \frac{(\alpha - \beta \xi)(1 - \xi)}{\alpha \xi} \hat{q}_t,
\]

where \(\alpha, \beta,\) and \(\xi\) have the same meaning as in the model, while \(\hat{q}_t\) denote percentage deviations of the real marginal cost (RMC) from its flexible-price (or steady-state) level for the respective producer.\footnote{See, inter alia, Christiano et al. [2005] for details on the derivation of the Phillips curve for a stationary economy.}

The main driving forces for inflation are therefore associated with fluctuations in the real marginal cost.\footnote{Recall that the real marginal cost may fluctuate in our model only because prices are sticky.} As headline inflation consists of contributions by import inflation and domestic intermediate inflation, we need to examine the RMC relevant for both types of producers. First, domestic intermediate firms produce with two input factors, so their RMC will then be determined by a combination of two real factor prices, the real wage, \(W/PY\), and the real rental price of capital (or productive capacity, in general), \(P^k/PY\). The supply of productive capacity is, however, predetermined from \(t-1\) and hence fixed at each date\footnote{See Item (12h) in the market clearing in Section 6}, meaning that the real rental price will exactly correlate with the output of the domestic intermediate sector. We observe that the level of domestic output drops enough to offset the increase in the real wage,\footnote{The weights of the real wage and real output in the RMC depend upon the calibration of the production function. In our case, with \(\gamma = 0.58\), it is approximately 3:2 in favor of the real wage.} so that the domestic RMCs are low and may bring inflation down. Second, on the import side, the RMC equals (in log terms) the difference between the price at which importers sell the goods domestically, and that at which they make purchases abroad. This is, in turn, mainly affected by fluctuations in nominal exchange rates: the initially observed rapid appreciation transforms gradually into a time-distributed slowdown of import price inflation, which again supports the disinflation process.
A fall in domestic economic activity is a typical feature of disinflation in monetary policy models. It also defines an overall measure of the flexibility or sluggishness of a monetary economy called the sacrifice ratio: how much output needs to be sacrificed to bring inflation down by 1%, both measured in annualized terms. The sacrifice ratio for our current calibration is 0.9, which is usually considered to be the lower bound for developed economies and the upper bound for emerging markets. The fall occurs because demand for domestically produced goods deteriorates temporarily: the consumption spending of households is dampened by high real interest rates, while price-elastic export demand suffers from an increase in the country’s terms of trade. The real interest rate is temporarily above the long run for two reasons. First, since the monetary authority has committed to lower inflation, it needs to tighten the monetary policy stance by raising nominal rates. But this effect is rather negligible in our particular calibration; it is, in fact, inflation expectations that cause the tough monetary policy conditions. Note that a low consumption profile is also accompanied by a reduction in households’ labor supply and the mentioned rise in the real wage. The former is consistent with our consumption-leisure choice, featuring highly elastic labor supply and habit formation. The latter is given by the calibration of the relative stickiness in nominal wages and prices: whenever wages are more sluggish they can only be followers during disinflation, leaving real wage rates high.

At this point, real investment demand may seem to behave oddly by expanding over time in response to tight monetary policy, with the exception of the first two quarters. Indeed, we can find a completely reverse reaction in other monetary policy models, such as Binette et al. [2004] or Cespedes et al. [2003]. But this phenomenon clearly relates to our particular assumptions about the final production structure and the time-to-build constraint. Investment goods have a zero content of domestic intermediates, being produced fully from imports. But import prices fall faster on average between the 2nd and 4th year after the shock than the domestic rental price of capital, making the future expected profitability of investment projects initiated today sufficiently high.

Finally, the behavior of the nominal exchange rate seems to be rather influential in many respects (low import inflation, deterioration of export demand, stimulation of investment demand) even though the exchange rate is largely disconnected from the real economy through our international currency pricing scheme. Recall that what matters for the exchange rate to be initially pro-inflationary or counter-inflationary is not the absolute reaction, but its magnitude relative to the new, after-shock, steady-state rate of depreciation/appreciation. In our case, the nominal exchange rate falls evidently faster and undershoots the horizontal line. To explain the initial jump, we need to analyze the fundamentals in the international financial market. When expanded forward, the UIP determines the current level of the nominal exchange rate as a result of two forces: the infinite sum of present and all future interest rate differentials (the higher the differential the more the appreciation at the beginning), and the terminal condition, i.e. the expected nominal exchange rate in the very distant future. The former channel is on the appreciation side: because disinflation takes time, the policy rate remains above its new long-run level for some period of time. But the financial market exploits the investment opportunity instantaneously, and the exchange rate appreciates. On the other hand, the terminal condition pushes toward depreciation: since relative prices, including those between Home and Rest-of-World, are determined fundamentally and hence intact by monetary policy, the nominal exchange rate needs to offset a certain build-up in domestic prices when inflation is still above the new target. What prevails depends on the other features of the model, in particular on the reactiveness of monetary policy and the stickiness of prices.

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22 See Devereux and Engel [2002].
Labor participation shock (Figure 11)—We first describe the effect of a 1% shift in intratemporal preferences on the real side of the economy in the long run. The starting point is the labor market reaction: the infinitely elastic labor supply, as is the case in our model, moves *ceteris paribus* by 1% up, letting the quantity of labor employed be determined by the demand side, see Figure 1. This results in a cut in steady-state labor with a magnitude close to 1%. But such a one-to-one reaction cannot be simply attributed to our unit-elasticity labor demand curve. More specifically, households’ long-run consumption declines on impact by 0.4% with the immediate consequence of the labor supply shift being in fact less than the size of the shock. On the other hand, the real production wage, which matters for labor demand, is affected by an increase in the relative price of domestic intermediate goods (not plotted in Figure 11); this latter effect indeed offsets roughly half of the reported decline in consumption. Moreover, there is also a relative fall in the price of investment and capital against other domestic prices, which makes the input factor substitution even more pronounced. As is evident from two real exchange rate measures reported in Figure 1 (the relative price of import and consumption goods, and the terms of trade), due to the changed households’ preferences domestic goods are now costlier relative to Rest-of-World.

Because of the presence of habit, time to build, and nominal rigidities, the transition to the new steady state is time-distributed: it takes approximately 3 years until the variables approach a balanced-growth path again, and obviously longer in the case of productive capacity and investment. The inflation development is rather ambiguous in this experiment. The domestic component works as pro-inflationary but is, from the beginning, overriden by import prices. The domestic inflation profile suffers from an initially high and slowly adjusting level of intermediate production, even though the real production wage softens, to some extent, the unfavorable real marginal cost position of producers. On the other hand, import prices are driven by an initial rapid nominal appreciation. The mechanics of this are again rather complex: the depreciation effect of the sum of present and future negative interest rate differentials is dominated by the expected terminal condition, i.e. a low (or appreciated) nominal exchange rate in the distant future. The forces that position the terminal condition are, not surprisingly, closely related to the relative stickiness of wages and prices. Namely, the long-run increase in the real wage occurs consequently predominantly through a decline (i.e. lower cumulative inflation) in nominal prices. Even though domestic prices need to rise in real terms relative to Rest-of-World, the magnitude left to the ultimate nominal exchange rate adjustment is thus relatively low and results in the appreciation impact of the terminal condition.

Habit shock (Figure 12)—Within a DSGE model, a habit shock has the potential to mimic what is usually called or understood as an aggregate demand shock, namely a period of time in which inflation is boosted by domestic demand. First of all, without habit formation it would be impossible to generate an aggregate demand shock of the kind in an optimizing sticky-price model. Whenever there are inflationary pressures in the economy, monetary policy starts to react, pushing the real interest rate up. But according to the standard Euler equation for consumption, this would lead to an immediate reduction in households’ expenditures, and a high inflation profile would be accompanied by a negative cyclical gap in aggregate demand converging to the steady state from below. This pattern changes with habit.

A sudden increase in consumption raises demand for domestically produced goods, accelerating the real marginal cost of intermediate producers. Even though mitigated by the real wage

\[^{23}\text{Recall from the previous description that the real production wage is closer to the new steady-state level at the beginning, as compared to the reported real consumption wage.}\]
component of the RMC (recall the logic of the optimal sticky pricing introduced in the paragraph on disinflation) this transforms into higher domestic inflation. Monetary policy reacts, but now the high interest rate affects consumption demand slowly due to the presence of habit. The external sector works as pro-inflationary, too. Nominal depreciation gradually increases export demand because of the low terms of trade, and, at the same time, builds up import inflation, which peaks even higher than the domestic part of inflation. Note that (i) the relatively low elasticity of substitution imposed in our calibration on the production of final goods prevents imports from being replaced and pushed out of the market, and (ii) an increase in consumption and a small drop in the real wage appear simultaneously, because the habit shock immediately affects the marginal utility of consumption, thus breaking the regularities of the consumption-leisure choice. This is indeed a feature that makes the habit shock distinct from another way of generating aggregate demand disturbances, namely, an intertemporal preference shock. The latter has been used by McCallum and Nelson [1999], for example, and is also often encountered in the monetary policy and sticky price business cycle literature. With an intertemporal disturbance, the intratemporal choice would be left unaffected, with both the marginal utility of consumption and leisure being disturbed proportionally, and there would have to be an identical rise in consumption and the real wage in our model.

As explained also in the disinflation shock, the magnitude and sign of the initial reaction in the nominal exchange rate is rather ambiguous in general. In our current setting, the interest rate differential effect over the lifelong horizon is outweighed by long-run cumulation of the domestic price level, and the exchange rate accordingly depreciates. We are, nevertheless, able to induce nominal appreciation at the beginning of the shock by simply adjusting the monetary policy parameters appropriately so that the policy reaction is sharper and cumulative inflation lower. Finally, there are fewer investment projects undertaken in reaction to an increase in their acquisition costs caused both by interest rates and import prices. In this regard, an interesting modification of the habit shock simulation would be a shock hitting concurrently both the consumption and investment decisions of households.

8. Concluding Remarks

In this paper we describe the technical development stage of a long-run ongoing project aimed at implementing a new core projection model at the Czech National Bank, meant as a replacement...
The CNB’s New Projection Model

for the currently used QPM. The modeling team has recently focused on two other necessary aspects of the project, namely empirical validation of the model, and more importantly, its implementation within the FPAS. We therefore conclude with several comments relating to these issues.

Our empirical validation strategy is based on the now commonly accepted consensus that DSGE models are not meant to explain the full predictive p.d.f. for observed data, but rather only certain dimensions of it. This has recently led to advances in the area of estimation methods tailored specifically to these aspects of the system identification problem: recent examples include the Bayesian techniques applied, for example, in the influential paper by Smets and Wouters [2003]. We follow the so-called minimal econometric interpretation of models, introduced by DeJong et al. [1996] and termed by Geweke [1999]; specifically, we define the population properties of the observed data that need to be matched by their model counterparts in terms of the cross-spectrum functions over a certain subrange of desired frequencies. As a straightforward consequence, this invalidates, to some extent, the traditional methods based on the likelihood function, which, in this framework, becomes irrelevant. The technical background for this approach is found in Diebold et al. [1998] and Berkowitz [1996].

Furthermore, the implementation of the model requires us to establish a direct link between the model variables and their observed counterparts. The need for such a direct link is twofold. First, the model needs to be simulated from a particular initial condition derived from the current position of the economy. Second, the model predictions should be interpreted in terms of, and mapped to, the commonly understood and published macroeconomic indicators. Even though there is no decomposition into unobserved trends and cycles imposed by the model logic, as is the case in the QPM, the model presented herein still contains unobserved stochastic processes, such as technological development, productive capacity, etc. These, or their suitable transformation, are theoretically stationary in the long run, but are introduced with the purpose of generating economic transition: they are assumed to be substantially deviated from their ultimate steady state and to approach it gradually. Identification of their current position with respect to the ultimate steady state, together with projection of their likely future developments, are then among the key macroeconomic issues that underlie the production of a realistic prediction and a clear policy message with the model.

Our experience based on the first steps we have undertaken so far in the area of identifying the unobserved variables and producing the model prediction clearly indicates that formal statistical or econometric methods are largely uninformative. This is an obvious consequence of the fact, that no, or very little, information about the expected long run towards which the model economy converges is contained in the Czech historical data. In this regard, we have to carry out a number of structural economic analyses, e.g. on the convergence of relative prices or sectoral productivities.

\[24\] Such as the capital-to-output or capital-to-technology ratios.
References


A. Flows of Goods and Value Added

\[ Q^N \cdot N \rightarrow P^N \cdot N \]
\[ 0 \rightarrow P^Y \cdot Y \]
\[ P^J \cdot J \]
\[ Q^X \cdot X \rightarrow P^X \cdot X \]
\[ Q^C \cdot C \rightarrow P^C \cdot C \]
\[ P^G \cdot G \]

Rest of World

Rest of World
B. The Optimization Problems

B.1 Representative Consumption and Asset Holdings

The first-order conditions necessary to maximize (3.3) s.t. (3.1) and (3.5) taking net labor income as given, w.r.t. \( c_t, b_t, z_t, \) and \( K_t \):

\[
\Psi_{1,t} = \lambda_t p_t^C, \quad (B.28)
\]

\[
1 / \lambda_t = \beta / \lambda_t \cdot E_t \lambda_{t+1}, \quad (B.29)
\]

\[
1 / \lambda_t \cdot E_t \sum_{k=0}^T \beta^k \lambda_{t+k} p_{t+k}^f = (T + 1) \beta^T / \lambda_t E_t \lambda_2, \quad (B.30)
\]

\[
\mu_t = \beta \lambda_{t+1} / \lambda_t \cdot E_t \left( \mu_{t+1} + \lambda_{t+1} \lambda_{t+1} \right), \quad (B.31)
\]

where \( \lambda_t \) is the Lagrange multiplier associated with the nominal budget constraint, and \( \mu_t \) is the ratio of \( \lambda_t \) and the multiplier associated with capacity accumulation.

B.2 Staggered Wage Setting

The first-order condition necessary to maximize s.t. with respect to \( w_{t|t} \) and \( L_{t|t} \):

\[
\frac{w_{t|t}}{W_t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{E_t \sum_{t=0}^\infty (\xi_t \beta)^k \ell_{t+k} \lambda_{t+1} (\lambda_t / \lambda_{t+k})^{-\varepsilon}}{\sum_{t=0}^\infty (\xi_t \beta)^k \ell_{t+k} \lambda_{t+1} (1 - \tau_t^w) W_{t+k} / P_{t+k}^C (\lambda_t / \lambda_{t+k})^{-\varepsilon}} \cdot (B.32)
\]

\[
L_{t|t} = \left( w_{t|t} / W_t \right)^{-\varepsilon} \ell_t. \quad (B.33)
\]

B.3 Employment Intermediation

The cost-minimization first-order condition in the employment intermediation sector implies the following sector-wide demand for individual labor services:

\[
\ell_t(s) = \left[ w_t(s) / W_t \right]^{-\varepsilon} \ell_t, \quad (B.34)
\]

\[
W_t = \int_0^1 w_t(s)^{-\varepsilon} ds. \quad (B.35)
\]

B.4 Foreign Exchange Dealers

The first-order condition necessary to maximize (3.11) w.r.t. \( \tilde{b}_t \):

\[
a_t^\delta E_t \frac{\beta \lambda_{t+1}}{\lambda_t} \cdot \frac{S_{t+1} \tilde{b}_t}{S} = 1 + \xi_a a_t \tilde{b}_t. \quad (B.36)
\]

B.5 Final Goods Producers

The cost-minimizing sector-wide demand curves for intermediate goods in the export sector:

\[
n_t^x = (P_t^N / Q_t^X)^{-\xi} X_t, \quad (B.37)
\]

\[
y_t^x = (P_t^Y / Q_t^X)^{-\xi} X_t. \quad (B.38)
\]
and in the consumption sector:
\[ n_t^C = \left( P_t^N/Q_t^C \right)^{-\xi} C_t, \]  
(B.39)  
\[ y_t^C = \left( P_t^Y/Q_t^C \right)^{-\xi} C_t. \]  
(B.40)  
where \( P_t^N \) and \( P_t^Y \) are the cost-minimizing price indexes,
\[ P_t^N = \left[ \int_0^1 p_t^N(s) \, ds \right]^{1/\varepsilon}, \]  
(B.41)  
\[ P_t^Y = \left[ \int_0^1 p_t^Y(s) \, ds \right]^{1/\varepsilon}, \]  
(B.42)  
common to all final goods producers.

### B.6 Staggered Price Setting

The first-order condition necessary to maximize s.t. with respect to \( p_{t|t}^Y \) and \( Y_{t|t} \):
\[
\frac{p_{t|t}^Y}{P_t^Y} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{E_t \sum_0^\infty (\xi_t)^k p_{t|t+k}^Y y_{t+k} Q_{t+k}^Y / P_{t+k}^Y \left( \Pi_t^Y / \Pi_{t+k}^Y \right)^{1-\varepsilon}}{E_t \sum_0^\infty (\xi_t)^k p_{t|t+k}^Y y_{t+k} \left( \Pi_t^Y / \Pi_{t+k}^Y \right)^{1-\varepsilon}},
\]  
(B.43)  
\[ Y_{t|t} = \left( p_{t|t}^Y / P_t^Y \right)^{-\varepsilon} y_t. \]  
(B.44)  
Analogously for the import sector,
\[
\frac{p_{t|t}^N}{P_t^N} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{E_t \sum_0^\infty (\xi_t)^k p_{t|t+k}^N n_{t+k} Q_{t+k}^N / P_{t+k}^N \left( \Pi_t^N / \Pi_{t+k}^N \right)^{1-\varepsilon}}{E_t \sum_0^\infty (\xi_t)^k p_{t|t+k}^N n_{t+k} \left( \Pi_t^N / \Pi_{t+k}^N \right)^{1-\varepsilon}},
\]  
(B.45)  
\[ N_{t|t} = \left( p_{t|t}^N / P_t^N \right)^{-\varepsilon} n_t, \]  
(B.46)  
and the export sector,
\[
\frac{\tilde{P}_{t|t}^X}{P_t^X} = \frac{E_t \sum_0^\infty (\xi_t)^k p_{t|t+k}^X x_{t+k} Q_{t+k}^X / P_{t+k}^X \left( \tilde{\Pi}_t^X / \tilde{\Pi}_{t+k}^X \right)^{1-\varepsilon}}{E_t \sum_0^\infty (\xi_t)^k p_{t|t+k}^X x_{t+k} \left( \tilde{\Pi}_t^X / \tilde{\Pi}_{t+k}^X \right)^{1-\varepsilon}},
\]  
(B.47)  
\[ X_{t|t} = \left( \tilde{P}_{t|t}^X / P_t^X \right)^{-\varepsilon} x_t. \]  
(B.48)  
where \( P_t^X = \tilde{P}_t^X S_t \) is the export price index converted to Home currency units.

The laws of motion for the sector-wide gross rates of inflation are
\[ \Pi_t^Y = \left[ \xi_t \left( \Pi_{t-1}^Y \right)^{1-\varepsilon} + \left( 1 - \xi_t \right) \left( p_{t|t}^Y / P_t^Y \cdot \Pi_t^Y \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}, \]  
(B.49)  
\[ \Pi_t^N = \left[ \xi_t \left( \Pi_{t-1}^N \right)^{1-\varepsilon} + \left( 1 - \xi_t \right) \left( p_{t|t}^N / P_t^N \cdot \Pi_t^N \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}, \]  
(B.50)  
\[ \Pi_t^X = \left[ \xi_t \left( \Pi_{t-1}^X \right)^{1-\varepsilon} + \left( 1 - \xi_t \right) \left( p_{t|t}^X / P_t^X \cdot \Pi_t^X \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}. \]  
(B.51)
B.7 Aggregation

The Home intermediate goods price dispersion:

$$D^y_t = \int_0^1 \left[ \frac{p^y_t(s)}{P^y_t} \right]^{-\epsilon} ds = (1 - \xi^y_t) \sum_{k=0}^{\infty} (\xi^y_t)^k \left( \frac{P^y_{t-k}}{p^y_t} \right)^{-\epsilon} =$$

$$= \xi^y_t \sum_{k=0}^{\infty} (\xi^y_t)^k \left( \frac{\Pi^y_{t-k}}{\Pi^y_t} \cdot \frac{p^y_{t-k}}{p^y_t} \right)^{-\epsilon} =$$

$$= (1 - \xi^y_t) \left( \frac{\Pi^y_{t}}{\Pi^y_{t-1}} \right)^{-\epsilon} + \xi^y_t \left( \frac{\Pi^y_{t}}{\Pi^y_{t}} \right)^{-\epsilon} D^y_{t-1}. \quad (B.52)$$

Similarly,

$$D^N_t = (1 - \xi^N_t) \left( \frac{P^N_{t}}{P^N_{t}} \right)^{-\epsilon} + \xi^N_t \left( \frac{\Pi^N_{t}}{\Pi^N_{t-1}} \right)^{-\epsilon} D^N_{t-1}, \quad (B.53)$$

$$D^X_t = (1 - \xi^X_t) \left( \frac{P^X_{t}}{P^X_{t}} \right)^{-\epsilon} + \xi^X_t \left( \frac{\Pi^X_{t}}{\Pi^X_{t-1}} \right)^{-\epsilon} D^X_{t-1}, \quad (B.54)$$

$$D^W_t = (1 - \xi^W_t) \left( \frac{W^W_{t}}{W^W_{t}} \right)^{-\epsilon} + \xi^W_t \left( \frac{\Omega^W_{t}}{\Omega^W_{t-1}} \right)^{-\epsilon} D^W_{t-1}. \quad (B.55)$$

C. Basic Notation

- \(a_t\): Sector-specific stochastic processes
- \(b_t\): Nominal net asset position (bonds)
- \(C_t, c_t\): Private consumption goods
- \(D_t\): Price dispersion
- \(e_t\): Preference or technology shocks
- \(F_t, f_t\): Real government transfers
- \(G_t, g_t\): Government consumption goods
- \(h_t\): Habit in consumption
- \(i_t\): Gross nominal interest rate
- \(J_t, j_t\): Investment goods
- \(K_t, k_t\): Production capacity
- \(L_t, \ell_t\): Labor
- \(N_t, n_t\): Import intermediate goods
- \(P_t, p_t\): Nominal price
- \(Q_t\): Nominal marginal cost
- \(r_t\): Real interest rate
- \(s_t\): Nominal exchange rate
- \(\tau_t\): Tax rate
- \(v_t\): Insurance against the wage-setting risk
- \(W_t, w_t\): Nominal wage
- \(X_t, x_t\): Export goods
- \(Y_t, y_t\): Domestic intermediate goods
- \(z_t\): Investment projects
- \(\Pi_t\): Gross rate of price inflation
- \(\Sigma_t\): Nominal domestic value added (gross domestic product)
- \(\Phi_t\): Nominal labor income
- \(\Psi_t\): Nominal instantaneous net cash-flows
- \(\Omega_t\): Gross rate of nominal wage inflation
D. Calibration and Properties of the Model

Table 1: Steady-state parameters

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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>( \alpha )</td>
<td>Real growth (gross rate)</td>
<td>1.02(^{\frac{3}{2}})</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>1.01(^{\frac{1}{4}})</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Labor intensity of production function</td>
<td>0.58</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Depreciation of productive capacity (1 - (1 - 0.06)^{\frac{3}{2}})</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Elasticity of substitution in monopolistic markets</td>
<td>11</td>
</tr>
<tr>
<td>( \omega_x )</td>
<td>Import intensity of exports</td>
<td>0.59</td>
</tr>
<tr>
<td>( \omega_c )</td>
<td>Import intensity of consumption</td>
<td>0.22</td>
</tr>
<tr>
<td>( a_L )</td>
<td>Consumption-leisure preference</td>
<td>0.42</td>
</tr>
<tr>
<td>( \sigma_G )</td>
<td>Government consumption share of GDP</td>
<td>0.23</td>
</tr>
<tr>
<td>( \tau_w )</td>
<td>Labor income tax rate</td>
<td>0.25</td>
</tr>
<tr>
<td>( \tau_s )</td>
<td>Health and social insurance rate</td>
<td>0.20</td>
</tr>
<tr>
<td>( \tau_c )</td>
<td>Consumption sales (indirect) tax rate</td>
<td>0.25</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Price inflation (gross rate)</td>
<td>1.03(^{\frac{1}{4}})</td>
</tr>
<tr>
<td>( N^*/\pi )</td>
<td>Rest-of-World’s import demand index</td>
<td>3.36</td>
</tr>
<tr>
<td>( \bar{\pi}^* )</td>
<td>Rest-of-World’s price inflation (International currency, gross rate)</td>
<td>1.02(^{\frac{2}{3}})</td>
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### Table 2: Transient parameters

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<thead>
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<th>Symbol</th>
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<td>$\chi$</td>
<td>Habit preference parameter</td>
<td>0.80</td>
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<tr>
<td>$\phi$</td>
<td>Portfolio adjustment costs</td>
<td>0.10</td>
</tr>
<tr>
<td>$\eta$</td>
<td>E.o.s. between old and new capital</td>
<td>0.50</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>E.o.s. between imports and domestic intermediates</td>
<td>0.50</td>
</tr>
<tr>
<td>$\theta$</td>
<td>E.o.s. between pairs of national exports</td>
<td>1.50</td>
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<tr>
<td>$\xi_w$</td>
<td>Fraction of non-optimizing households</td>
<td>0.90</td>
</tr>
<tr>
<td>$\xi_f$</td>
<td>Fraction of non-optimizing intermediate firms</td>
<td>0.80</td>
</tr>
<tr>
<td>$\xi_N$</td>
<td>Fraction of non-optimizing importers</td>
<td>0.80</td>
</tr>
<tr>
<td>$\xi_X$</td>
<td>Fraction of non-optimizing exporters</td>
<td>0.80</td>
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<tr>
<td>$\rho_H$</td>
<td>Autoregression in habit</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>Autoregression in government consumption</td>
<td>0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Reactiveness of monetary policy</td>
<td>5.00</td>
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<tr>
<td>$\kappa$</td>
<td>Discount factor of monetary policy</td>
<td>0.80</td>
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<tr>
<td>$\rho_M$</td>
<td>Policy rate smoothing</td>
<td>0.50</td>
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E. Stylized facts

Figure 2: Shifts in relative prices (deflators of demand components)

Figure 3: Consumption expenditure shares of GDP
Figure 4: Investment expenditure shares of GDP

Figure 5: Labor participation
Figure 6: Imports and GDP

Figure 7: Exports and GDP
Figure 8: Costly disinflation and sticky wages

Figure 9: Income composition of GDP; Composition of tax revenues (2004)
F. Shock Simulations

Figure 10: Disinflation shock
Figure 11: Labor participation shock
Figure 12: Consumption habit shock
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