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Some Empirical Evidence

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Rare Shocks vs. Non-linearities: What Drives Extreme Events in the Economy? Some Empirical Evidence

Michal Franta*

Abstract

A small-scale vector autoregression (VAR) is used to shed some light on the roles of extreme shocks and non-linearities during stress events observed in the economy. The model focuses on the link between credit/financial markets and the real economy and is estimated on US quarterly data for the period 1984–2013. Extreme shocks are accounted for by assuming t-distributed reduced-form shocks. Non-linearity is allowed by the possibility of regime switch in the shock propagation mechanism. Strong evidence for fat tails in error distributions is found. Moreover, the results suggest that accounting for extreme shocks rather than explicit modeling of non-linearity contributes to the explanatory power of the model. Finally, it is shown that the accuracy of density forecasts improves if non-linearities and shock distributions with fat tails are considered.

Abstrakt

K vysvětlení role extrémních šoků a nelinearit během napjatých období pozorovaných v ekonomice je použita vektorová autoregrese malého měřítka. Model se soustředí na vztah mezi úvěrovým/finančním trhem a reálnou ekonomikou a je odhadnut na čtvrtletních datech pro Spojené státy za období 1984–2013. Extrémní šoky jsou zohledněny předpokladem šoků v redukované formě následujících t-rozdělení. Nelinearita je modelována možností změny režimu v mechanismu přenosu šoků. Je ukázáno, že distribuce chyb vykazují silné konce (fat tails). Výsledky dále naznačují, že specifikace zohledňující extrémní šoky vysvětlují pozorovaná data lépe než specifikace s nelineárními vztahy. Nakonec je také ukázáno, že přesnost predikcí hustot se zvětší, jestliže bereme v úvahu nelinearity a distribuce šoků se silnými konci.

JEL Codes: C11, C32, E44.

Keywords: Bayesian VAR, density forecasting, fat tails, non-linearity.

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Nontechnical Summary

Extreme economic events – those such as the Great Recession starting in 2008 – are an integral part of economic reality. However, a detailed explanation of the mechanism of such events is often available only after they have happened. A more general understanding of such events could aid the formulation of measures that are not based solely on the experience of the latest crisis.

An important question can be posed regarding the dynamics of stress/extreme events. Is an extreme event a consequence of an extreme shock, or can the shock itself be of moderate size but the interactions between economic variables become unusual? A shock is an event which is by definition unexpected and independent of the current state of the economy. Therefore, it is difficult to set any pre-emptive measure. The focus should rather be placed on being prepared for the ex-post reaction to an extreme event. On the other hand, if the substantive element of the stress dynamics lies in the interactions between variables, regulation can be targeted at affecting such interactions.

This paper provides some empirical evidence on the roles of rare shocks and unusual interactions between economic variables. Rare shocks are modeled by error distributions with fat tails, i.e., distributions that ascribe a higher probability to extreme events by comparison with normally distributed errors. Unusual interactions are represented by non-linear models that are able to account for the situation where a change in one variable leads to a more than proportional change in another. More precisely, a threshold vector autoregression model with t-distributed shocks is employed.

The estimation results suggest that accounting for fat tails in error distributions is important for modeling stress events appropriately. Strong evidence of fat tails is found, and the finding is robust to the measure of financial/credit market conditions used. Modeling non-linearity, regardless of whether it arises in the form of changes in the dynamic relationships between endogenous variables or in changes in shock volatility, does not improve the explanatory power of the model very much. The importance of accounting for non-linearity is suggested by an examination of the out-of-sample fit. It turns out that allowing for regime changes and shock distributions with fat tails improves the density forecasting accuracy. Such modeling features result in superior forecasts of the tails of density forecasts. Moreover, the improvement in the accuracy of density forecasts is not solely related to the recent Great Recession.

1. Introduction

One of the responses of economic research to the Great Recession has consisted in a thorough examination of the shock distributions assumed in macroeconomic models. Attention has shifted towards non-Gaussian error structures, especially those exhibiting fat tails. For example, the Student's t-distribution is often considered because it ascribes higher probability to extreme events. Within the family of DSGE models, such investigation includes the studies by Chib and Ramamurthy (2014) and Cúrdia et al. (2014). Chiu et al. (2014) examine t-distributed shocks in vector autoregressions. These studies suggest that assuming error distributions with fat tails can provide a superior modeling tool for explaining observed data and for forecasting.

The research interest in fat-tailed error distributions can be understood from the evidence provided by Figure 1. The figure shows the absolute values of the normalized reduced-form errors estimated by a Bayesian VAR with output, inflation, the short-term interest rate and credit market conditions measured under the assumption of normally distributed errors. The exact specification of the model is discussed below. The occurrence of extreme values of shocks does not correspond to the normal distribution. For all variables the observed ratio of shocks that exceed three standard deviations is far greater than the 99.7 rule suggests.[1]

Another stream of research revived by the Great Recession focuses on potential non-linearities in economic relationships – the prominent example being the interaction between financial markets and the real economy. The non-linearity reflects mutually reinforcing feedback effects between the two sectors and can be represented by a change in the dynamic relationships between the variables and in the contemporaneous impacts of shocks. The non-linear nature of the interaction between the real economy and financial markets is examined, for instance, using the concept of the financial accelerator (Bernanke et al., 1996). In addition, starting with McCallum (1991) in a univariate setting and Balke (2000) in multivariate setting, an empirical investigation has also been carried out employing non-linear time-series models.[2]

The literature dealing with non-linearities draws on normally distributed errors. On the other hand, the above-mentioned studies examining fat-tailed error distributions are based on linear or linearized models. They allow for non-linear behavior only partially, by assuming stochastic volatility of shocks. However, non-linearity can also arise from a change in the dynamics of the model reflected by changes in the shock propagation mechanism.

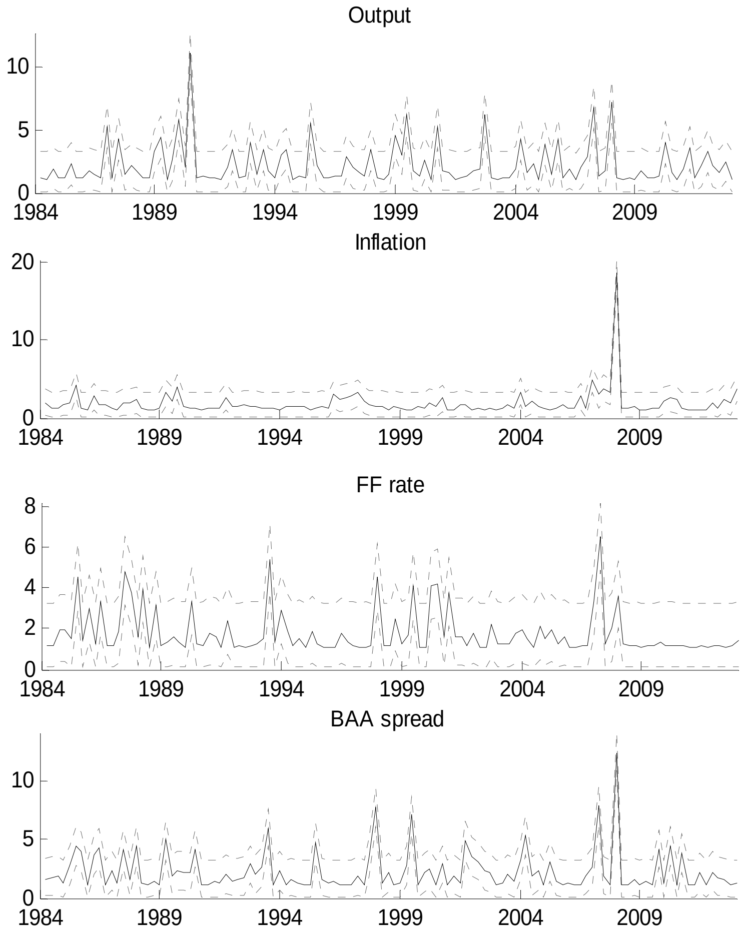
Importantly, models assuming fat-tailed error distributions can behave similarly to models accounting for non-linearities. So, the presence of extreme shocks in Figure 1 could be a consequence of a change in the volatility of shocks, a change in the shock propagation mechanism, or of non-normally distributed shocks per se. Neglecting non-linearity in the case where the data-generating process exhibits such a feature results in a superior fit of models with t-distributed errors in comparison with the same model estimated under Gaussianity. The statement can be reversed – ignoring fat tails of error distributions can falsely suggest the presence of non-

[1] The 99.7 rule states that 99.7% of normally distributed observations lie within the band constituted by three standard deviations around the mean. In Figure 1, the percentage share of observations outside the range for the four variables lies between 8.6 and 22.4.

[2] Balke (2000) draws on threshold VAR. Some recent papers – such as Serwa (2012) and Hubrich and Tetlow (2014) – employ Markov-switching VAR.

linear relationships or stochastic volatility. Identifying the source of extreme events in the economy can help us understand the dynamics of extreme events, with consequences for the right policy response.

Figure 1: Reduced-form Shocks (Absolute Value) in Standard Deviation Units from a Linear Model Estimated under Gaussianity



To understand the roles of non-linearities in economic relationships and fat-tailed error distributions during stress events, this paper examines the two factors empirically. The aim is to extend the existing literature and account for all possible sources of non-linear behavior. So, the contribution of this study is to provide a simple enough framework that allows both the detection of all forms of non-linearity and the implementation of error distributions with fat tails. The modeling framework is built on a small-scale threshold vector autoregression (TVAR). The various specifications are estimated assuming either normally distributed or t-distributed reduced-form shocks. The model is estimated on quarterly US data covering the period 1984Q1–2013Q4, including series describing conditions on financial or credit markets. The relatively simple structure of the model allows for estimation of all model parameters and generation of density forecasts that account completely for the parameters' uncertainty.

The estimation results suggest that accounting for fat tails in error distributions is important for modeling stress events appropriately. Strong evidence of fat tails is found, and the finding is robust to the measure of financial/credit market conditions used. Modeling non-linearity, regardless of whether it arises in the form of changes in dynamic relationships between endogenous variables or in changes in shock volatility, does not improve the explanatory power of the model very much. The robustness with respect to the measure of credit/financial market conditions used is examined in order to respond to concerns that fat tails are a consequence of modeling credit/financial market conditions by one variable only and thus of omitting important channels of interaction between the credit/financial market and the real economy.

The importance of accounting for non-linearity is suggested by an out-of-sample forecasting exercise. It turns out that allowing for regime changes and using a shock distribution with fat tails improve the density forecasting accuracy. A comparison of the forecasting performance of means and whole densities then suggests that the improvement in density forecasting accuracy is due to superior forecasts of the tails of density forecasts. Moreover, it turns out that the higher accuracy of density forecasts is not solely related to the recent Great Recession.

Another contribution of the paper is that it provides some evidence on the issue of how data frequency affects the possibility of detecting non-linear behavior of macroeconomic time series. Ng and Wright (2013) suggest that quarterly frequency can be too low to answer the question on the roles of extreme shocks and non-linear propagation of shocks. To address the issue of data frequency, the model is also estimated on monthly data. Turning to higher frequency data does not change the qualitative picture, i.e., accounting for extreme shocks is an important step towards appropriate modeling of stress events.

The rest of the paper is organized as follows. Section 2 presents the model, the estimation procedure, and the data set. In Section 3 the results and selected estimation issues are discussed. Finally, Section 4 concludes. Details of the estimation procedure, model comparison, data and convergence of the Gibbs sampler are presented in Appendixes A, B, C, and D, respectively. Detailed results relating to the out-of-sample forecasting performance exercises are presented in Appendixes E, F, and G.

2. Model, Estimation, and Data

Several recent papers have proposed ways of dealing with fat-tailed distributions in macroeconomic models. Cúrdia et al. (2014) work with a linearized DSGE model with shocks generated from a Student's *t*-distribution. Importantly, they allow for changes in the volatility of shocks in the form of log volatilities following a random walk. Similarly, Chiu et al. (2014) employ a VAR model with *t*-distributed structural shocks with time-varying variance.

In this paper, a threshold VAR model close to Balke (2000) is used. The use of the TVAR model allows us to consider a general form of non-linearity related to the shock propagation mechanism.[3] Moreover, by allowing for regime switches in the error covariance matrix, it allows

[3] The importance of modeling changes in dynamic relationships is strengthened by the fact that interest rates are currently close to the zero lower bound.

us to detect possible changes related to the volatility of shocks. In this regard, it resembles the approaches in Cúrdia et al. (2014) and Chiu et al. (2014).

An alternative modeling approach could be based on models that are more flexible in terms of the number of regime changes. For example, time-varying parameter VAR as introduced in Primiceri (2005) represents a model which changes regime every period (the model parameters follow a random walk or geometric random walk). The modeling potential of a more flexible approach is examined in this paper by adding more regimes to the TVAR framework. Another approach that could be used to investigate our research questions involves smooth transition models. Working with quarterly/monthly data suggests rather abrupt changes in the investigated relationships, so models with an immediate regime change are preferred. Moreover, ignoring smooth transitions does not prevent detection of regime change in the TVAR model, hence a more parsimonious model is employed. Finally, the Markov-switching approach could be considered. Here, we prefer to work with an explicit threshold variable so that we can interpret the regimes.

The set of endogenous variables includes a variable representing financial /credit markets. While having a single variable to represent the financial market is a strong simplification, it can still provide some guidance on the interaction between the real economy and financial markets. The situation would probably be more difficult within the family of DSGE models, which still do not provide a consensual approach to incorporating financial markets.[4] In addition, complex models usually need to be linearized, which prevents the investigation of non-linearities. More discussion on whether one variable suffices to capture credit/financial markets can be found in Section 2.3.

2.1 Model

Let's consider the following threshold vector autoregression with R regimes:

$$y_t = \sum_{i=1}^{R} x_t B_i I\left[r_{i-1} < y_{t-d}^{TR} \le r_i \right] + u_t , \tag{1}$$

where $y_t = \left[y_t^1, \ldots, y_t^M \right]$ is a row vector of M endogenous variables, $x_{t,p} = \left[1, y_{t-1}, \ldots, y_{t-p} \right]$ is a row vector of length $1 + Mp$, and B_i are $(1+Mp) \times M$ matrices of coefficients, $i = 1, \ldots, R$. The function $I[\cdot]$ indicates whether the lagged value of the threshold variable, y_{t-d}^{TR}, belongs in the interval given by r_{i-1} and r_i. The delay parameter $d \in \{1, 2, \ldots, d_0\}$ suggests that the threshold variable reflects the regime with a lag.
The reduced-form residuals, u_t, are independent and identically distributed either normally with zero mean and an $M \times M$ symmetric positive semi-definite covariance matrix Σ_i dependent on the regime:

$$u_t \sim N\left(0, \sum_{i=1}^{R} I\left[r_{i-1} < y_{t-d}^{TR} \le r_i \right] \Sigma_i \right), \tag{2}$$

[4] Cúrdia et al. (2014) draw on Smets and Wouters' DSGE model, which does not incorporate a financial sector.

or as a multivariate *t*-distribution with zero mean, an $M \times M$ symmetric positive semi-definite scale matrix Σ_i dependent on the regime, and *n* degrees of freedom:

$$u_t \sim MT\left(0, \sum_{i=1}^{R} I\left[r_{i-1} < y_{t-d}^{TR} \le r_i\right]\Sigma_i, n\right). \tag{3}$$

It can be shown (see Chahad and Ferroni, 2014, Appendix A) that a random variable distributed as a multivariate *t*-distribution can be viewed as a normally distributed variable with stochastic variance driven by a gamma distributed random variable. For the reduced-form residuals in (3) it holds that if

$$\omega_t \sim \Gamma\left(\frac{n}{2}, \frac{2}{n}\right) \tag{4}$$

and

$$u_t \sim MN\left(0, \omega_t^{-1} \sum_{i=1}^{R} I\left[r_{i-1} < y_{t-d}^{TR} \le r_i\right]\Sigma_i\right), \tag{5}$$

then the likelihood of (1) and (3) equals the likelihood of (1), (4), and (5). Note that volatility changes driven by the gamma distribution are of a different kind to the stochastic volatility usually employed in the literature, because there is no persistence in the process ω_t. Persistent changes in shock volatility are accounted for by changes in the matrix Σ_i. Furthermore, the time-varying error covariance matrix in (5) reflects both changes in the impact of a shock on the endogenous variables and changes in the volatility of shocks. The two sources of changes are not distinguished, as identification of structural shocks is not necessary.

The assumption of one degree of freedom parameter for all shocks may be viewed as too strong. Note that Cúrdia et al. (2014) model shock distributions separately and therefore they can discuss the fat tails of a specific shock. The approach of this paper follows Chahad and Ferroni (2014), with shocks modeled jointly as multivariate *t*-distributed vectors. Moreover, the reduced-form residuals in Figure 1 suggest fat tails for shocks to all variables.

2.2 Bayesian Inference

For inference, the Bayesian approach is taken. The joint posterior distribution is intractable and so MCMC methods are employed to draw from conditional distributions and estimate the marginal distributions of subsets of parameters. The crucial step is to extend the Gibbs sampler used to estimate the threshold vector autoregressions for the error distribution exhibiting fat tails. The TVAR model with normally distributed errors is discussed in Chen and Lee (1995) and Koop and Potter (2003), while the linear VAR with multivariate t-distributed shocks is examined in Chahad and Ferroni (2014). The Gibbs sampler combined with a Metropolis step and an adaptive rejection sampling algorithm are used in the estimation of the model. Adaptive rejection sampling as introduced in Gilks and Wild (1992) is employed. Overall, 150,000 iterations of the Gibbs sampler are carried out, with the first 50,000 used as a "burn-in" as they are presumably not from the target distribution. From the rest of the iterations, every tenth one is used for inference in order to avoid autocorrelation of draws. For details see Appendix A.2. Convergence diagnostics are presented in Appendix D.

For the threshold BVAR model (1) and (2) the independent Normal-inverse Wishart prior is assumed:

$$\beta_i \sim N\left(\beta^{PR}, V^{PR}\right) \text{ and } \Sigma_i \sim iW\left(\Sigma^{PR}, T^{TR}\right) \qquad i = 1,...,R, \tag{6}$$

where β_i is a vector created by stacking the columns of B_i. The prior on AR parameters B_i is specified as a Minnesota-style prior (Litterman, 1979). The prior distribution on the error covariance matrix Σ_i follows an inverse Wishart distribution. The scale matrix Σ^{PR} is an OLS estimate of the variance of a linear VAR estimated on a training sample. The estimate is multiplied by the length of the training sample to retain the interpretation of the scale matrix as the sum of squared residuals. The number of degrees of freedom is also set to the length of the training sample. The prior distribution for threshold value r_i and delay parameter d is Beta and multinomial, respectively, on the admissible parameter values. The effect of the priors on the results is discussed in Section 3.1.

The threshold BVAR model with fat tails (1), (4), and (5) follows the same prior for AR parameters B_i, the error covariance matrix Σ_i, the threshold value r_i, and the delay parameter d. The prior on the degrees of freedom of the multivariate *t* distributed shocks n (or equivalently the parameter of the gamma distribution assumed for the stochastic volatility component) is such that

$$pr(n/2) = \Gamma(20,2). \tag{7}$$

So, the prior allocates substantial prior weight to normal distribution of shocks. Finally, note that the chosen priors imply no difference in the dynamic relationships and the volatility of shocks across regimes. For details on the specification of the priors see Appendix A.1.

Density forecasts are simulated such that within each round of the Gibbs sampler an iterated forecast for up to seven quarters is computed based on the current draw of the parameter vector. More precisely, for a given draw of AR parameters, a draw from the error distribution with parameters obtained in the Gibbs iteration is taken to generate the forecast one quarter ahead. The next draw from the same error distribution yields the forecast two quarters ahead, and so on. A total of 50,000 iterations from the Gibbs sampler after another 50,000 as a "burn-in" are used to simulate the density forecasts.

2.3 Data

The models are estimated on US quarterly data covering the period 1984–2013 (see Appendix C for graphs of the time series). In order to reduce the number of possible regimes, we restrict our data set to the beginning of the Great Moderation, as it is usually viewed as a regime change associated with a decrease in the observed volatility of macroeconomic variables. However, as a robustness check, an extended sample starting in 1964 is also used for the estimation.

The real economy is represented by output (seasonally adjusted quarter-on-quarter change in real GDP), inflation (seasonally adjusted quarter-on-quarter change in CPI), and the short-term interest rate (the federal funds rate). The model is complemented with a variable representing the credit conditions. The set-up of the empirical exercise is very close to the analysis presented in Balke (2000). So, in the benchmark estimation, the credit conditions are represented by the BAA spread, i.e., the spread between the BAA-rated corporate bond yield and the 10-year Treasury constant maturity rate.

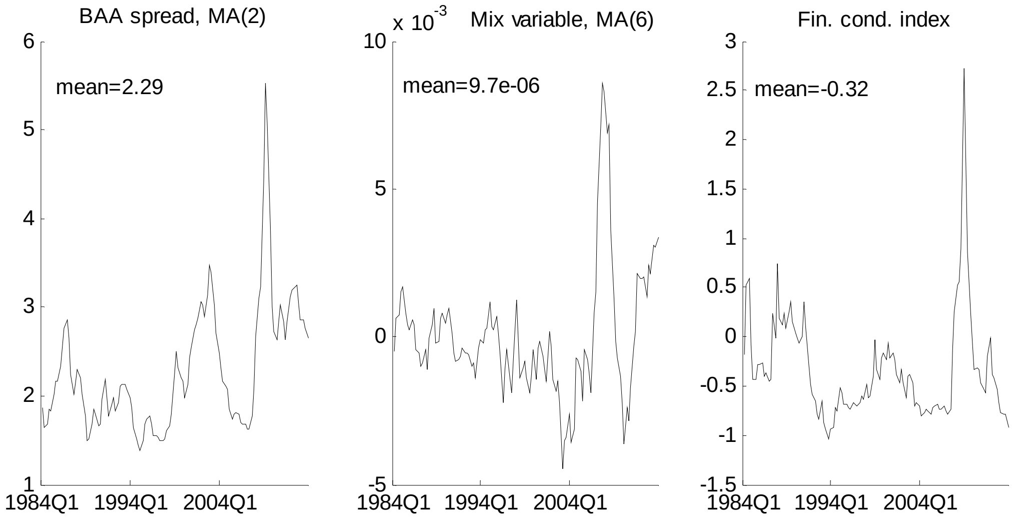
As a robustness check, the first difference of the mix of bank loans and commercial paper in total firm external finance (the Mix variable) is employed. The Mix variable is the ratio of the total amount of loans in the liabilities of non-financial corporate and non-corporate firms to the sum of the total amount of loans plus the amount of commercial paper issued by non-financial corporate firms. Such a measure is also considered in Balke (2000) and Ferraresi et al. (2013). To broaden our analysis from the credit markets to the financial markets in general, a financial conditions index (FCI) is used. The FCI is a real-time indicator extracted from a broad set of time series describing the money, debt, and equity markets and the leverage of financial intermediaries (Brave and Butters, 2012).[5]

Our efforts to demonstrate the robustness of the results with respect to the measure of the credit/financial market variable stem from the fact that we are forced to use a small-scale model and at the same time to capture the very complex relationship between the financial markets and the macroeconomy. Even the small-scale model used is heavily parameterized and the problem of overfitting arises. Therefore, only one variable representing the credit/financial markets can be used at a time. In this regard, the FCI includes general information from the financial markets, as it is basically a factor extracted from a broad set of financial variables.

Following Balke (2000) the threshold variables are smoothed versions of the corresponding credit market conditions measures – we use the two-quarter moving average for the BAA spread and the six-quarter moving average for the Mix variable, and the FCI is not smoothed. Smoothing is employed to avoid frequent changes in regimes and regimes lasting only a quarter. The threshold variables are presented in Figure 2.

[5] Data sources: CPI, GDP, and the federal funds rate were downloaded from the IMF IFS database. The BAA spread, the components of the Mix variable, and the FCI were taken from FRED – the Federal Reserve Bank of St. Louis Database at http://research.stlouisfed.org/fred2/.

Figure 2: Threshold Variables and Their Means



Ng and Wright (2013) touch upon the possible influence of the data frequency on the possibility of detecting non-linear behavior. They mention Stock and Watson (2012), who use a dynamic factor model estimated on quarterly data and find no support for non-linear behavior or structural breaks. Extreme financial and uncertainty shocks seem to be the drivers of extreme events in 2008 within the framework of Stock and Watson (2012). On the other hand, working with a six-variable VAR estimated on monthly data, Sims (2012) suggests some role for non-linearity in the shock propagation mechanism. To contribute to the discussion, we also estimate the benchmark model on monthly data (1984M1–2014M9). Industrial production (seasonally adjusted annualized month-on-month change) is employed as an indicator of the performance of the real economy. The rest of the endogenous variables – inflation (seasonally adjusted annualized month-on-month change), the federal funds rate, and the BAA spread – remain the same. Note that the previous literature dealing with t-distributed errors in VAR models employed monthly data (Chiu et al., 2014, and Chahad and Ferroni, 2014). Cúrdia et al. (2014) estimate Smets and Wouters' DSGE model on quarterly data. The graphs of monthly data are presented in Appendix C.

3. Results

First, the type of shock distributions and the presence of regime switches are examined in terms of data fit. Then, the fat-tailedness is discussed and changes in the results with respect to data frequency are examined. Robustness issues are discussed in Subsection 3.1. Finally, the out-of-sample forecasting performance of the models is examined in Subsection 3.2.

The comparison of the specifications is based on the deviance information criterion (DIC), which takes into account data the fit along with the number of model parameters. The DIC is a generalization of the Akaike information criterion. Its definition and the computation procedure are discussed in Appendix B. Note that a model with a lower DIC value is preferable. Based on the criterion, the number of lags is chosen to be equal to four.[6]

Table 1 shows the DIC values for the model of quarterly data employing in turn the three measures of the credit/financial market conditions and different assumptions about the error distribution.

Table 1: Deviance Information Criterion (quarterly data)

	BAA spread Number of regimes		Mix variable Number of regimes		FCI Number of regimes	
Shocks:	1	2	1	2	1	2
Normal	666.26	1258.36	-154.30	-190.61	744.37	701.13
t-dist.	476.23	519.17	-337.05	-288.22	530.95	639.90

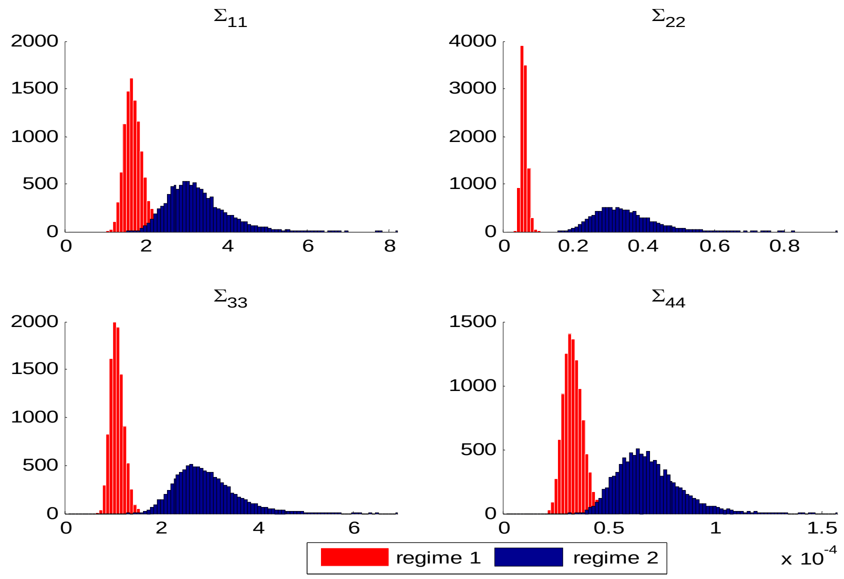
The table demonstrates that specifications with *t*-distributed errors are preferable regardless of whether or not a regime switch is assumed. So, within the family of linear models the usual assumption of normality is not favored by the data, in line with the above-mentioned literature. In addition, neither allowing for a regime switch in the shock propagation mechanism nor allowing for a change in the volatility of shocks leads to a preference for normally distributed errors. An important role of non-linearities is detected when the out-of-sample fit of the model specifications is discussed below.

For the model specifications with the BAA spread, allowing for non-linearity worsens the model fit for both types of error distributions. The situation is different if the Mix variable and the FCI are employed. When normal errors are assumed, allowing for threshold behavior provides a superior data fit. This is not the case for *t*-distributed errors. Thus, restricting attention to normal errors can result in a preference for a non-linear model, but the preferable approach is to allow for rare and extreme shocks in a linear model.

The preference for *t*-distributed shocks over all specifications provides robust evidence of fat tails of the error distributions. The specification with the Mix variable and the FCI suggests some role of non-linearities at least in the model with normal errors. Here, the limits of the methodology may have been reached. As already mentioned, having just one variable to represent the financial market may simply be insufficient to capture all the feedback effects between the real economy and the credit/financial markets. On the other hand, the small-scale model still seems to be an appropriate approach to avoid overfitting.

[6] The choice of the number of lags is carried out for the benchmark specification with one regime and normally distributed errors. The DIC suggests four lags. This number of lags is used in all the other specifications.

Figure 3: Posterior Distribution of the Diagonal Elements of Σ in Two Regimes



Note: The specification with the Mix variable and normally distributed errors is used.

The superior data fit of the model with normally distributed errors and regime switch for the specification with the Mix variable and the FCI is driven mainly by the possibility of volatility change. Figure 3 shows the posterior distributions of the diagonal elements of the error covariance matrix Σ. It turns out that the volatility of shocks is higher in the second regime than in the first. However, the posterior distribution of the AR parameters also exhibits a change between the two regimes, though not a significant one.

Table 2: Deviance Information Criterion (quarterly data, subsample 1984Q1–2008Q2)

	BAA spread		Mix variable		FCI	
	Number of regimes		Number of regimes		Number of regimes	
Shocks:	1	2	1	2	1	2
Normal	311.99	414.67	-311.60	-259.90	402.75	544.23
t-dist.	301.53	397.79	-319.23	-256.64	396.86	521.64

An important aspect of the above discussion is how much the results are driven by the period the data set covers. Table 2 reports the DIC for the same set of specifications estimated on the sub-set excluding the Great Recession (1984Q1–2008Q2). The preference for shock distributions with fat tails relative to normally distributed shocks is weakened. Still, the main conclusion is that the specification with one regime and *t*-distributed shocks is superior. However, the linear model with normal errors fits the data better than models allowing for regime switch regardless of the assumption about the error distribution. The Great Recession therefore results in a greater need for fat tails in shock distributions.

Next, focusing on the Great Moderation, the importance of accounting for changes in volatility may be weakened. As another robustness check, the benchmark specification with the BAA spread is estimated on the data sample starting in 1964Q1 for both types of error distribution and with one or two regimes. Table 3 shows that extending the estimation sample to include a period of high volatility of shocks does not alter the conclusions about the importance of fat-tailed shock distributions. The inclusion of a period with a significant volatility change does not alter the result that the model that allows for change in volatility is not preferred according to the DIC.

Table 3: DIC, Extended Sample (1964Q1–2013Q4)

		Number of regimes	
Shocks:		1	2
	Normal	1406.00	1769.97
	t-dist.	1298.43	1329.80

Note: The BAA spread is used as a measure of credit market conditions.

So, the data favor a fat-tailed distribution of shocks. The natural question follows: How fat are the fat tails? Looking at the posterior distribution of the degrees of freedom parameter n, strong evidence is found for fat-tailedness. The evidence is weaker for the specification with the BAA spread and one regime. Table 4 reports the mean and 90% confidence band of the posterior distribution.

Table 4: Posterior Mean of Degrees of Freedom

Number of regimes:		BAA spread	Mix variable	FCI
	1	20.98	5.29	5.10
		(5.01, 99.34)	(5.00, 5.49)	(5.00, 5.27)
	2	10.17	5.12	5.11
		(5.01, 41.14)	(5.00, 5.34)	(5.00, 5.30)

Note: 90% confidence bands reported in brackets.

Cúrdia et al. (2014) find that allowing for stochastic volatility leads to a dramatic increase in the degrees of freedom for monetary policy shocks. In the present framework, structural shocks are not identified. However, structural shocks are a linear combination of reduced-form shocks and thus one can discuss the propensity to produce fat-tail events with the estimated degrees of freedom for reduced-form shocks. Importantly, for the BAA spread and the Mix variable the estimated means of the degrees of freedom decrease if some non-linearity is allowed.[7] It seems that accounting for changes in volatility does not reduce the need for fat tails.

Responding to the above discussion on the data sampling frequency and the possibility of detecting non-linear behavior, we estimate the model on monthly data (we restrict our attention to the specification with the BAA spread). The DIC estimates suggest that the linear model with *t*-distributed errors is superior if quarterly data are used for the estimation (see Table 5).

[7] Confidence bands suggest that the difference in degrees of freedom between the models with one and two regimes is not considerable.

Furthermore, it turns out that the performance of specifications with fat-tailed shock distributions in terms of data fit is closer to that of specifications with normally distributed errors. The possibility of regime switches does not add explanatory power to the model.

Table 5: Deviance Information Criterion (monthly data)

		Number of regimes	
Shocks:		1	2
	Normal	1298.40	1322.11
	t-dist.	1222.71	1320.28

Note: The BAA spread is used as a measure of credit market conditions.

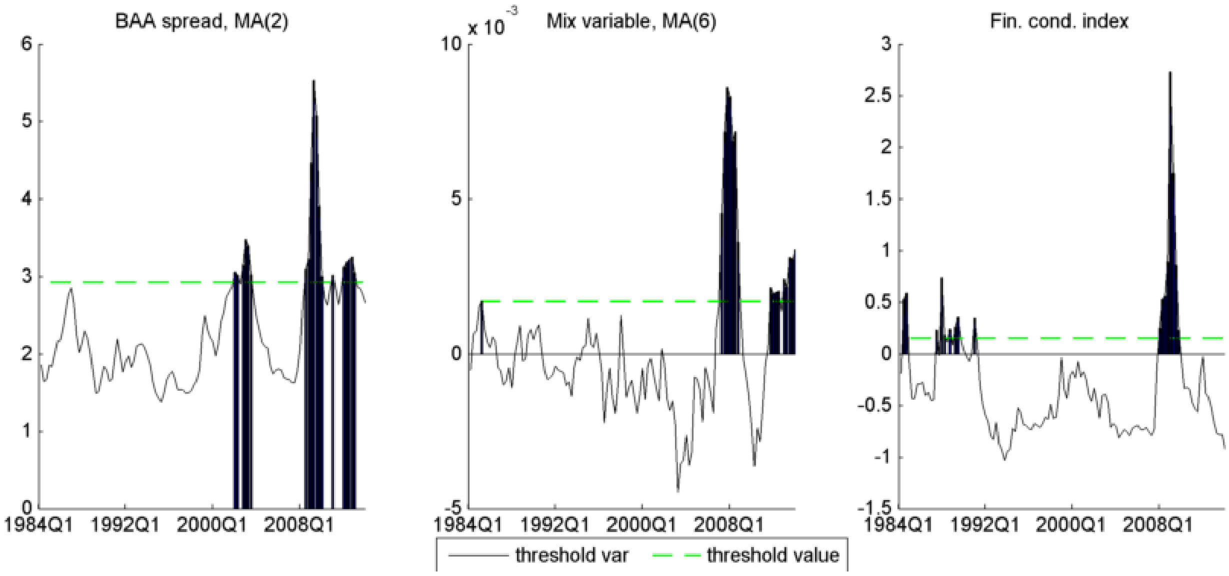
Table 6: Threshold Value in Models with Two Regimes

	Threshold variable function of:		
Shocks:	BAA spread	Mix variable	FCI
Normal	2.93	0.0017	0.15
	(2.87, 3.07)	(0.0015, 0.0018)	(0.09, 0.19)
t-dist.	2.92	0.0017	0.09
	(2.87, 3.05)	(0.0014, 0.0018)	(-0.01, 0.18)

Note: 90% confidence bands in brackets.

Regime switches can be characterized by threshold values. Table 6 shows that the mean of the posterior distribution of the threshold is not significantly affected by the assumption about the error distribution. The estimated regimes differ according to the threshold variable employed – see Figure 4 for regimes according to the specification with normally distributed errors. The specification with the BAA spread yields “periods of stress” related to the Great Recession and its aftermath. Next, the period relating to the financial and accounting scandals of Enron and Arthur Andersen and their aftermath is also indicated as a stress event. The BAA spread relates to the external finance premium of firms and also to possible flight-to-quality dynamics. The Mix variable is linked strictly to the supply of loans. The “extreme” episodes defined by the threshold for the Mix variable in a sense follow the episodes identified by the BAA spread. A broader measure of the conditions on the financial market, the FCI index, implies that in addition to the Great Recession, the “stress events” include, for example, Black Monday in 1987Q3.

Figure 4: Threshold Variables, Estimated Thresholds, and Implied Regimes



3.1 Robustness Issues

There are several robustness issues that are worth discussing. First, the assumption of two regimes imposes the condition that volatility and the shock propagation mechanism can change only once and at the same time. Therefore, for the specification with the BAA spread, the model is estimated assuming three regimes. The results are presented in Table 7. It turns out that adding a regime worsens the data fit. Moreover, the specification with three regimes and *t*-distributed shocks is still preferable to specifications with normally distributed errors.

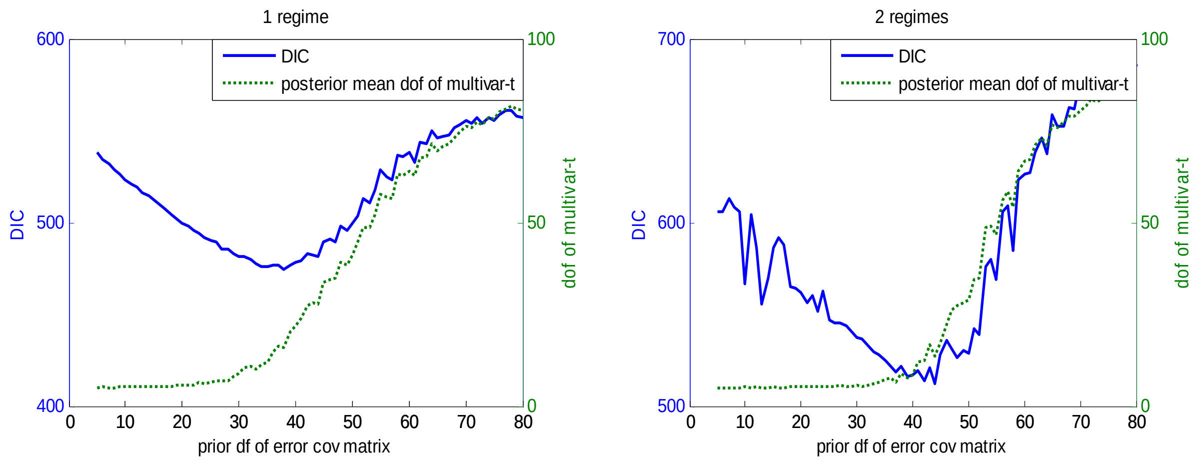
Table 7: Deviance Information Criterion, Three Regimes

		Number of regimes		
		1	2	3
Shocks	Normal	666.26	1258.36	1319.29
	t	476.23	519.17	597.00

Note: The BAA spread is used as a measure of credit market conditions.

Despite the computational burden, which reduces the time feasibility of the estimation procedure, adding a regime can provide some guidance on whether more flexible models can answer the research questions. Allowing for more regimes, however, does not add explanatory power to the model, so models with more regimes (e.g. TVP-VAR) seem not to be useful.

Figure 5: The Prior Degrees of Freedom for the Error Covariance Matrix and the Corresponding DIC and Posterior Mean of the Degrees of Freedom of the Multivariate t-Distribution

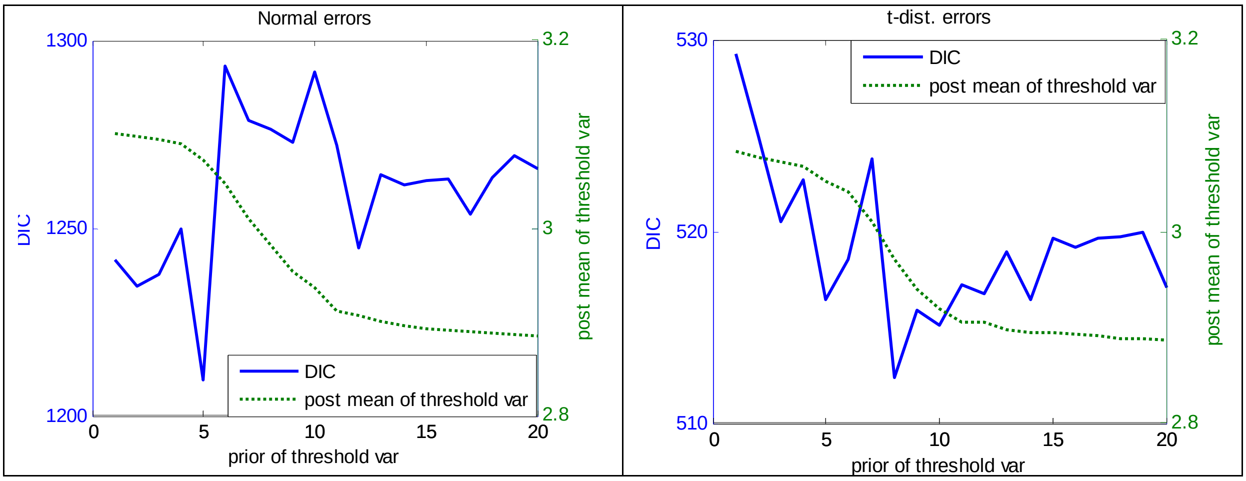


Note: The specification with the BAA spread is used.

The next robustness check concerns the influence of the prior on the error covariance matrix Σ_i on the estimated measure of fat-tailedness represented by degrees of freedom n. Figure 5 reports the DIC (left-hand scale) and the posterior mean of the degrees of freedom (right-hand scale) from the specification with the BAA spread, one or two regimes, and multivariate *t*-distributed errors.[8] The graphs show that the minimum DIC value is attained around 40. The preferred value of the parameter according to the DIC is very close to the one set for the prior – a training sample size equal to 40. More importantly, the posterior mean of the degrees of freedom covers values from those suggesting fat tails (less than 10) to those representing a multivariate t-distribution very close to the multivariate normal distribution (more than 50). The shift toward multivariate normal happens with higher prior degrees of freedom, representing a tighter prior on the error covariance matrix Σ_i. However, regardless of the size of the prior, the specification with one regime is always preferable according to the DIC.

[8] The model is estimated for 7,500 draws, with the first 2,500 discarded.

Figure 6: The Prior on the Beta Distribution for the Threshold Value and the Corresponding DIC and the Posterior Mean of the Threshold Value



The final robustness issue relates to the prior on the threshold value *r*. The underlying point of this exercise is to examine how much the results are driven by a low number of observations in a regime. A lower number of observations in a regime leads to fewer price estimates and to estimates that are driven more by the priors. Moreover, a low number of observations in a regime means overfitting. Tightening the prior on the threshold in the direction of the center of the interval ensures enough observations in each regime. So, Figure 6 shows the relationship between the shape parameter of the beta prior on the threshold value (1 represents a uniform distribution, 20 represents a very tight prior) and the DIC and the posterior mean of the threshold. The number of observations in a low-number-of-observations regime is 12 to 20. Not surprisingly, the posterior mean is closer to the mean of the threshold variable (2.29) if the prior is tighter. Next, according to the DIC a higher shape parameter on the beta distribution is preferred in the specification with *t*-distributed errors. The opposite applies if normal errors are assumed. However, the relative DIC values and thus the resulting preference for the model with t-distributed errors remain unchanged.

3.2 Out-of-sample Forecasting Performance

The forecasting performance of the model is assessed along two dimensions. First, a standard forecasting performance exercise based on the root-mean-square errors (RMSE) is carried out. Second, the accuracy of the density forecasts is discussed. The motivation for the density forecasting performance examination draws on the idea that during extreme events, non-linearity and fat tails of shock distributions can play a significant role. Therefore, the accuracy added by models with regime switch and *t*-distributed errors should be reflected in more precise estimation of the tails of the density forecasts. Focusing on the forecasting performance of means or medians can hide the value added of forecasting based on non-linear models with non-normal errors.

The evaluation period is 2002Q4–2013Q4. So, for each quarter from that period the model is estimated on the data set starting with 1984Q1 and ending with that quarter. Iterated density forecasts for up to seven quarters ahead are constructed and compared with the ex-post observed data. So, one-period-ahead density forecasts are compared with 45 ex-post observed values, two-period-ahead density forecasts with 44 ex-post observed values, and so on.

Tables E1–E3 in Appendix E report the RMSE for the models with three variables representing credit/financial markets. No clear-cut conclusion can be made, as for different endogenous variables and different forecasting horizons the preferred model specification differs as well. However, it turns out that accounting for non-linearity in the form of a regime change does not add to the forecasting performance of the mean forecasts. For a majority of the variables and forecasting horizons, the specification with one regime is preferred in terms of the RMSE. In addition, for the specification with the FCI, only models with t-distributed errors provide superior median forecasting performance. For specifications with the FCI, allowing for fat tails leads to coefficient estimates that are more useful for forecasting the central tendency of the endogenous variables. Finally, comparing the RMSE of the models with the AR(1) model for each variable suggests superior forecasting performance for variants of the TVAR model (Table E4).

The accuracy of the density forecasts is measured using the Kullback-Leibler Information Criterion (KLIC) introduced in Vuong (1989) as a way of measuring the distance between two densities. Minimization of the KLIC can be reformulated as maximization of the expected logarithmic score:

$$E\left[\log f_{t+h,t}\left(x_{t+h}\right)\right],$$

which is estimated by the average logarithmic score:

$$\frac{1}{N}\sum_{t\in A}\ln f_{t+h,t}\left(\overline{x}_{t+h}\right),$$

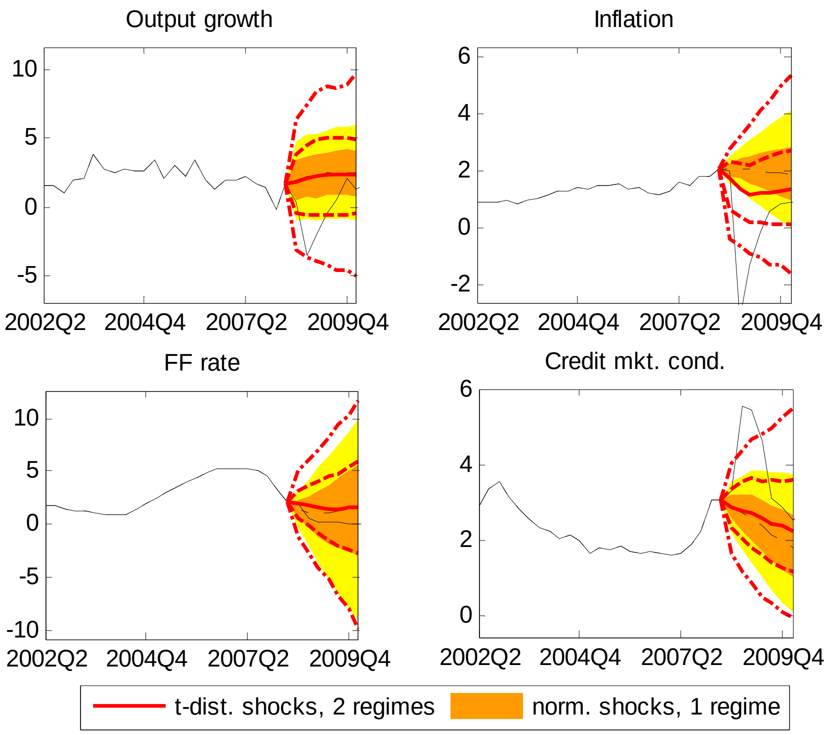
where $\overline{x}_t$ denotes an ex-post realization of a variable and $f_{t+h,t}$ is the simulated posterior density of that variable computed at time *t* at forecasting horizon *h*.

Tables F1–F3 in Appendix F show the average logarithmic scores. They suggest that accounting for non-linearity and shock distributions with fat tails improves density forecasts. This is especially striking when compared with the RMSE. For the model specifications with the BAA spread and the Mix variable, the linear model with normally distributed errors is superior in only two cases (out of the 28 cases, as seven forecasting horizons for four variables are considered). If the RMSE is considered, the linear model with normally distributed errors is preferred in approximately one half of cases. So, it can be concluded that fat tails and regime switches improve the forecasts of density tails.

Importantly, the results relating to forecasting performance are not driven by the Great Recession. Restricting the evaluation period to the sub-period 2002Q4–2008Q2 shifts the preference even more in the direction of non-linear models and models with t-distributed shocks. The average logarithmic scores do not suggest superiority of the linear model with normal errors in any case (see Tables G1–G3 in Appendix G).

The importance of non-linearities and fat tails in density forecasting is illustrated in Figure 7. The figure shows the density forecasts as if they were estimated using the data up to 2008Q2. It shows the median and centered 68% and 95% of the density forecasts. In addition, ex-post observed data are included. The Great Recession represents an ex-ante impossible event from the point of view of the linear model with normally distributed errors (shades of yellow). On the other hand, based on the model with two regimes and *t*-distributed shocks, the Great Recession represents a low probability event (red curves). For example, the ex-post observed output growth is contained by the centered 95% of the output growth density forecast.

Figure 7: Density Forecasts From Two Model Specifications Estimated on 1984Q1–2008Q2



Note: The BAA spread is used as a measure of credit market conditions. The red curves indicate the median and the centered 68% and 95% of the density forecasts of the model with *t*-distributed errors and two regimes. For the model with normal shocks and one regime, the median is denoted by the black dot-dash line and the centered 68% and 95% of the density forecasts are indicated by dark and light yellow. Observed data are denoted by a solid black line.

Density forecasts for the Federal Funds Rate presented in Figure 7 do not account for the zero lower bound because the influence of the bound is presumably similar regardless the model specification. However, the way how to deal with the zero lower bound in the estimation of density forecasts based on vector autoregressions and possible effects related to ignoring the bound are discussed in Franta et al. (2014).

4. Conclusions

This paper contributes to our understanding of the dynamics of extreme events in the economy. It attempts to provide some empirical evidence on the roles of rare and extreme shocks and non-linear behavior of economic variables. The empirical evidence draws on US macroeconomic variables that are able to capture feedback effects between the real economy and financial markets. Such feedback effects can strengthen an initial shock regardless of whether the shock originates in the financial market or in the real economy. Another possibility is that the shock itself is of an extreme size.

The model used to answer the question posed is simple enough to be completely estimated, yet flexible enough to detect non-linear behavior either in the dynamic relationships between the variables or in changes in the error covariance matrix. Based on the estimated parameters and measures of model fit, it turns out that the important finding of the model is that it is preferable to work with distributions that allow for fat tails. Modeling non-linearities does not necessarily help explain the data better. The importance of accounting for non-linearity is suggested by an examination of the out-of-sample fit. It turns out that allowing for regime changes and shock distributions with fat tails improves the density forecasting accuracy. Such modeling features result in superior forecasts of the tails of density forecasts. Moreover, the improvement in the accuracy of density forecasts is not related solely to the recent Great Recession.

Appropriate modeling of stress events is crucial for the design of financial sector regulation. If stress events affecting financial markets are a product of a rare and extreme shock, it is difficult to pre-empt the shock and regulation should be directed at dealing with the consequences of such shocks. On the other hand, if a stress event is mainly due to non-linearities arising during the stress period, the design of the system can be improved to avoid feedback effects leading to non-linearities. The findings presented in this paper provide support for a regulatory approach that prepares the system for unavoidable extreme shocks. Such regulation should create buffers that can be used when an extreme shock hits the macroeconomy.

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Appendix A: Bayesian Estimation

A.1 Priors

For AR parameters B_i and the covariance/scale matrix Σ_i the independent Normal-inverse Wishart prior is assumed:

$$\beta_i \sim N\left(\beta^{PR}, V^{PR}\right) \text{ and } \Sigma_i \sim iW\left(\Sigma^{PR}, T^{TR}\right) \qquad i = 1,\dots,R, \tag{A1}$$

where β_i is a vector created by stacking the columns of B_i. $\beta_i^{PR} = 0_{(1+pM)M\times 1}$ and V_i^{PR} are set such that the diagonal element equals ϕ_0 / l^2 for the coefficient on the lags of the LHS variable at lag *l*. The prior variance is set to $\phi_0\phi_1\sigma_{i,m}^2 / (l^2\sigma_{i,n}^2)$ for the coefficients on the lags of variables different from the LHS variable $(m \neq n)$ and $\phi_0\phi_2$ for the coefficients on the intercepts. $\sigma_{i,m}^2$ is the standard error of an AR(1) process for a particular variable *m* estimated on the whole sample. The hyperparameters are set to $\phi_0 = 0.2$, $\phi_1 = 0.5$, and $\phi_2 = 10^5$. The specification of the prior variance of the AR parameters and the values of the hyperparameter are taken from Canova (2007).

The prior on the error covariance/scale matrix Σ_i follows an inverse Wishart distribution with the scale matrix proportional to the OLS estimate of the error covariance matrix from the linear VAR on the training sample (1973Q1–1983Q1). The estimate is multiplied by the number of observations available for estimation $T^{TR} = 40$. The number of degrees of freedom is also set to T^{TR}. The motivation for such choice of degrees of freedom draws on the form of the conditional posterior of the error covariance matrix conditional on B_i:

$$E[\Sigma_i \mid B_i] = \frac{T^{TR}}{T^{TR}+T}\frac{\Sigma^{PR}}{T^{TR}} + \frac{T}{T^{TR}+T}\Sigma_i^*, \tag{A2}$$

where Σ_i^* is the maximum likelihood estimate of the error covariance matrix conditional on B_i. The posterior is a weighted average of the prior and the maximum likelihood estimate. The weights are chosen to reflect the sample size. The priors on the AR parameters and the scale matrix are independent. Hence, the prior on the AR parameters allows for different shrinking of the coefficients at lags of various dependent variables.

The prior for threshold values r_i, $i = 1,\dots,R-1$ is considered to follow a beta distribution:

$$r_i \sim \mathrm{B}(\beta_1, \beta_2) \qquad i = 1,\dots,R, \tag{A3}$$

defined on the interval $\left[r_0 = r_{q=0.1}, r_R = r_{q=0.9}\right]$, where r_q denotes the quantile of the threshold variable. The parameters of the beta distribution equal 10, which represents a tight prior in the sense of avoiding a posterior close to the edges of the interval. The model specifications estimated are heavily parameterized and this tight prior is intended to avoid overfitting, i.e., to ensure enough observations in regimes adjacent to the edge of the interval for the threshold value.

The prior for the delay parameter follows a multinomial distribution with probability of a particular delay equal to $1/d_0$. The prior of the degrees of freedom parameter is such that

$$pr(n/2) = \Gamma(20,2). \tag{A4}$$

So, the prior allocates substantial prior weight to normal distribution of shocks. Finally, note that the chosen priors imply no difference in the dynamic relationship and volatility across regimes.

A.2 Gibbs Sampler

In this section the Gibbs sampler for the model with multivariate *t*-distributed errors is described. The case of normally distributed errors is analogous; the only difference lies in skipping the parts that take draws of stochastic volatility ω_t and degrees of freedom *n*.

As noted in the introduction, the reduced-form errors u_t distributed according to the multivariate *t*-distribution can be viewed as multivariate normally distributed with zero mean and variance $\Sigma_i \otimes \Omega_i^{-1}$ in a regime *i*. The likelihood of the observed data is then

$$\begin{aligned} & L\left(y^T \mid x^T; B_i, \Sigma_i, \Omega_i, r_i, d\right) \propto \\ & \propto \prod_{i=1}^{R} \left[|\Sigma_i|^{T_i/2} |\Omega_i|^{M/2} \right] \exp\left\{ -\frac{1}{2} tr \left[\sum_{i=1}^{R} \left(y^{T_i} - x^{T_i} B_i \right)' \Sigma_i^{-1} \left(y^{T_i} - x^{T_i} B_i \right) \right] \right\}, \end{aligned} \tag{A5}$$

where T_i refers to the size of each regime and Ω_i is a diagonal matrix consisting of all ω_t in a particular regime *i*.

The posterior distribution of the stochastic component of the reduced-form error volatility, ω_t, conditional on the AR parameters, the scale matrix, and the degrees of freedom parameter, follows a gamma distribution – see a similar formula in Chahad and Ferroni (2014):

$$\omega_t \mid y^T, x^T; B_i, \Sigma_i, r_i, d, n \sim \Gamma\left(\frac{M+n}{2}, \frac{2}{\psi_t + n} \right), \tag{A6}$$

where

$$\psi_t = \left(y_t - \sum_{i=1}^{R} x_t B_i I\left[r_{i-1} < y_{t-d}^{TR} \le r_i \right] \right)' \left(\sum_{i=1}^{R} I\left[r_{i-1} < y_{t-d}^{TR} \le r_i \right] \Sigma_i \right)^{-1} \left(y_t - \sum_{i=1}^{R} x_t B_i I\left[r_{i-1} < y_{t-d}^{TR} \le r_i \right] \right).$$

Next, the posterior distribution of the AR parameters and the scale matrix for each regime (conditional on the rest of the parameter set) follow the same family of distributions as the prior distributions of the parameter concerned. The conditional posterior distributions of the AR parameters are multivariate normal with mean

$$\left[\left(V_i^{PR}\right)^{-1}+\Sigma_i^{-1}\otimes X_i'\Omega_i X_i\right]^{-1}\left[\left(V_i^{PR}\right)^{-1}\beta_i^{PR}+\left(\Sigma_i^{-1}\otimes X_i'\Omega_i X_i\right)\beta_i^{OLS}\right], \quad \text{(A7)}$$

and variance

$$\left[\left(V_i^{PR}\right)^{-1}+\Sigma_i^{-1}\otimes X_i'\Omega_i X_i\right]^{-1}. \quad \text{(A8)}$$

The conditional posterior of the scale matrix for regime *i* follows the inverse Wishart distribution with the following scale matrix and degrees of freedom

$$\left(Y_i-X_i\beta_i\right)'\Omega_i\left(Y_i-X_i\beta_i\right)+\Sigma^{PR} \text{ and } T_i+T^{TR}. \quad \text{(A9)}$$

The conditional posterior distribution of the degrees of freedom parameter cannot be expressed analytically and thus the adaptive rejection sampling algorithm introduced in Gilks and Wild (1992) is employed. The point of the algorithm is to approximate a log-concave function by piecewise linear upper bounds in the log-linear space and then take a random draw from that approximation. The draw is accepted with a probability related to the difference between the original function and its approximation. If the draw is rejected, the approximation of the function is refined. The log-concave conditional probability density function for the degrees of freedom parameter is as follows:

$$p\left(n\,|\,y^T,x^T;B_i,\Sigma_i,\Omega_i,r_i,d\right)\propto\frac{|\Omega|^{-\frac{M+n+1}{2}}}{\left[\Gamma\left(\frac{n}{2}\right)\left(\frac{2}{n}\right)^{n/2}\right]^T}\exp\left\{-\frac{n}{2}Tr\left(\Omega\right)\right\}\left(n/2\right)^2\exp\left\{-n/4\right\}. \quad \text{(A10)}$$

The interval of the possible values of parameter *n* is restricted to $[5,100]$. The lower bound ensures finite variance of the reduced-form shocks and the maximum value of 100 makes the difference between the normality and non-normality assumption on the residuals indistinguishable.

Draws from the conditional posterior distribution of the threshold are constructed as in Koop and Potter (2003). A Metropolis algorithm is used – a random draw of a proposed value for the threshold is drawn from a uniform distribution over the domain for the threshold values and the log of the conditional posterior probability of this value is compared with the log of the conditional posterior probability of the original value. The proposed value is accepted with a probability given by the difference between the two logs (with a maximum equal to unity). The draws of the proposed values are taken such that each regime contains at least 11 observations. This is to ensure that in each regime the posterior is driven by the data and not solely by the prior. The posterior probability is computed as the product of the prior and the observed data likelihood (conditional on the model parameters and the latent stochastic volatility state). The number of tries to find a new draw of the threshold is set such that the acceptance ratio is in the interval $[0.2, 0.4]$.

The conditional posterior of the delay parameter d follows a multinomial distribution with probability

$$p\left(d \mid y^T, x^T; B_i, \Sigma_i, \Omega_i, r_i, n\right) = \frac{L\left(B_i, \Sigma_i, \Omega_i, r_i, n, d \mid y^T\right)}{\sum_{\delta=1}^{d_0} L\left(B_i, \Sigma_i, \Omega_i, r_i, n, \delta \mid y^T\right)}. \tag{A11}$$

To summarize, the Gibbs sampler involves the following steps:

(0) The sampler is initialized by random draws from prior distributions for parameters d, r_i, and n. AR parameters B_i and error covariance matrices Σ_i are initialized by the relevant OLS estimates based on the whole data set.

(1) Given the data and the values of all the other parameters $[B_i, \Sigma_i, r_i, n, d]$ a random draw of $\omega^T = (\omega_1, ..., \omega_T)$ is taken from the Gamma distribution stated in (A6).

(2) Given the data, the values of all the other parameters $[\Sigma_i, r_i, n, d]$, and the unobserved stochastic volatility component Ω_i, a random draw of B_i for $i = 1, ..., R$ is taken from the Normal distribution with mean (A7) and variance (A8).

(3) Given the data, the values of all the other parameters $[B_i, r_i, n, d]$, and the unobserved stochastic volatility component Ω_i, a random draw of Σ_i is taken from the inverse Wishart distribution with the scale matrix and degrees of freedom stated in (A9).

(4) Given the data, the values of all the other parameters $[B_i, \Sigma_i, r_i, d]$, and the unobserved stochastic volatility component Ω_i, a random draw of the degrees of freedom parameter n is obtained using an adaptive rejection sampling algorithm applied on probability density function (A10).

(5) Given the data, the values of all the other parameters $[B_i, \Sigma_i, n, d]$, and the unobserved stochastic volatility component Ω_i, a random draw of the threshold value is obtained using independent Metropolis Hastings sampling with a uniform jumping distribution defined on a range for the threshold values. The maximum number of tries for accepting a proposed value is adjusted to get the desired acceptance ratio.

(6) Given the data, the values of all the other parameters $\left[B_i, \Sigma_i, r_i, n\right]$, and the unobserved stochastic volatility component Ω_i, a random draw of the delay parameter d is taken from the multinomial distribution with probabilities defined in (A11) for $d = 1, \ldots, d_0$.

(7) Steps (1) to (6) are repeated 150,000 times, the first 50,000 draws of the parameter set are discarded and the rest are used for inference.

Appendix B: Model Selection

Here, the procedure for models with *t*-distributed errors is described, because in the case of normal errors the procedure is straightforward. We exploit the fact that the likelihood of the observed data of the model is the same regardless of whether the reduced-form residuals are viewed as distributed normally with stochastic volatility or distributed according to the multivariate *t*-distribution. The computation of the likelihood of a model with stochastic errors can be a difficult task because the system contains a latent state (ω_t). Therefore, we view the model as a threshold vector autoregression with *t*-distributed errors and compute the likelihood of the observed data according to the following formula:

$$L\left(y^T \mid x^T; B_i, \Sigma_i, r_i, n, d\right) = \left[\left(n\pi\right)^{-M/2} \frac{\Gamma\left(\left(n+M\right)/2\right)}{\Gamma\left(n/2\right)}\right]^T \prod_{i=1}^{R} \left|\Sigma_i\right|^{-T_i/2} * \\ * \prod_{t=1}^{T} \left[1 + \frac{1}{n}\left(y_t - B_i x_t\right)' \Sigma_i^{-1} \left(y_t - B_i x_t\right)\right]^{\frac{-(M+n)}{2}}, \tag{B1}$$

which can be viewed as the integrated likelihood with respect to the latent state Ω_i. The AR coefficients and the scale matrix in the last term relate to the regime for period *t*.

The model selection is based on the deviance information criterion (DIC) introduced in Spiegelhalter et al. (2002):

$$DIC = \overline{D} + p_D, \tag{B2}$$

where $\overline{D}$ measures the goodness of fit and can be approximated by

$$\frac{1}{G} \sum_{g \in G} \left[-2 \ln L\left(y^T \mid x^T; B_i^g, \Sigma_i^g, r_i^g, d^g\right)\right]. \tag{B3}$$

The parameter *G* denotes the number of Gibbs iterations. The approximation can be computed within the Gibbs sampler. For each iteration, the likelihood is evaluated at the draw of the parameter set.

The parameter p_D represents the model complexity. It is a measure of the number of effective parameters in the model. It can be approximated by

$$\frac{1}{G} \sum_{g \in G} \left[-2 \ln L\left(y^T \mid x^T; B_i^g, \Sigma_i^g, r_i^g, d^g\right)\right] - \left[2 \ln L\left(y^T \mid x^T; \overline{B}_i^g, \overline{\Sigma}_i^g, \overline{r}_i^g, \overline{d}^g\right)\right], \tag{B4}$$

where $\overline{\mathrm{P}}$ for a parameter P denotes the mean computed using Gibbs draws of the parameter.

A possible limitation of the DIC is that it tends to select over-fitted models. However, this seems not to be the case in the presented setting, as more heavily parametrized models are not preferred by the DIC. The conclusion about the preference for a linear model with t-distributed errors is not affected by the choice of model selection criterion.[9]

[9] A criterion that responds to the over-fitting problem with the DIC is introduced in Aldo (2007).

Appendix C: Data

Figure C1: Quarterly Data, 1984Q1–2013Q4

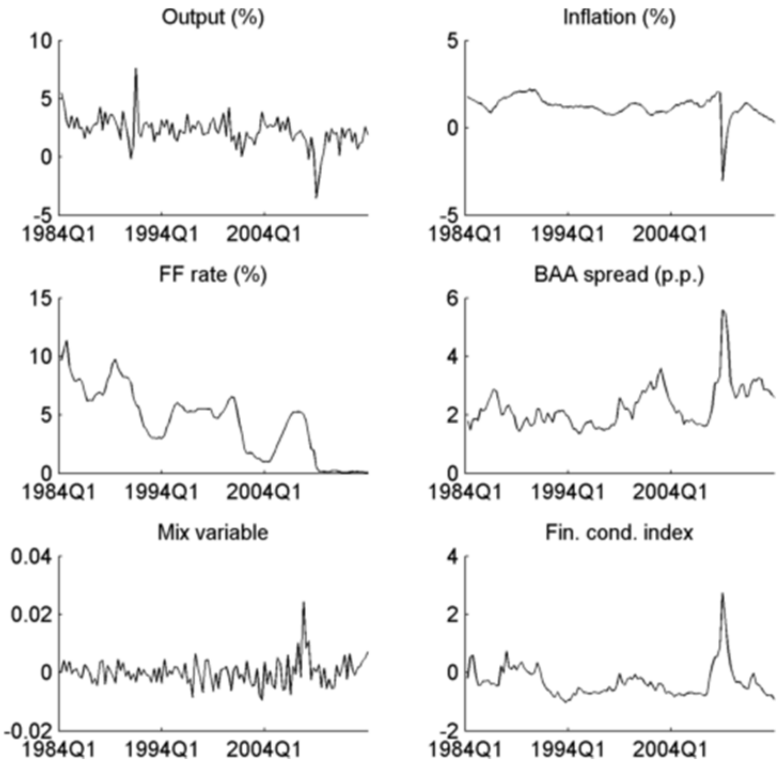
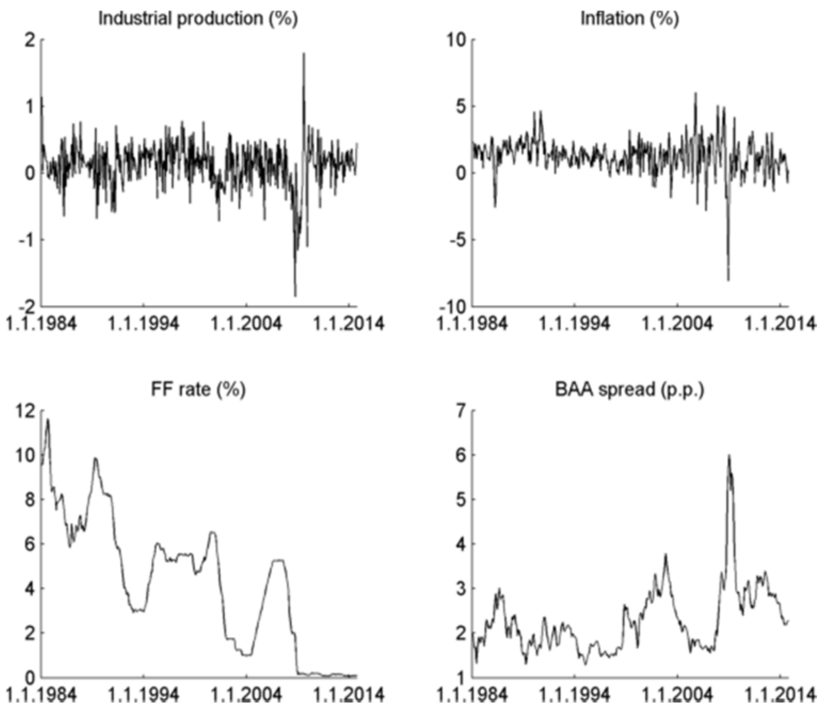


Figure C2: Monthly Data, 1984M1–2014M9

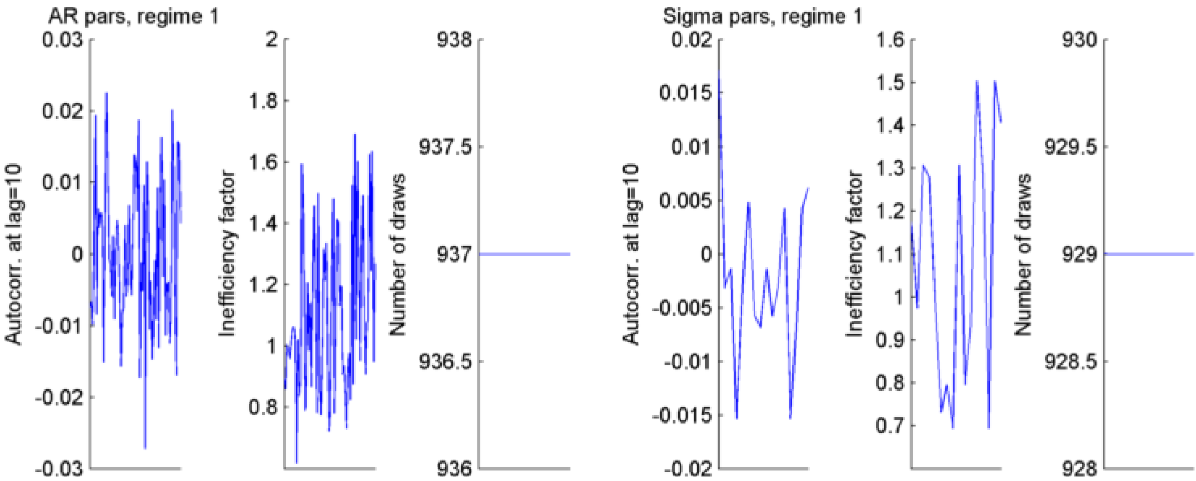


Appendix D: Convergence Diagnostics

Convergence diagnostics for the specification with the BAA spread, *t*-distributed errors, and two regimes is presented; other specifications give similar results in this respect and are available upon request.

Three measures are discussed: autocorrelation of the chain of draws from the Gibbs sampler (at a lag equal to 10), the inefficiency factor, and the measure of the number of draws to get a stationary distribution from the Gibbs sampler (Raftery and Lewis, 1992). The convergence diagnostics for the AR parameters and the elements of Σ are presented in Figure D1. Results are reported for the first regime only.

Figure D1: Convergence Diagnostics for the AR Parameters and the Elements of the Error Covariance Matrix, First Regime



Note: The x-axis contains all the AR parameters and elements of the error covariance matrix.

All three statistics suggest convergence of the sampler. For the threshold value, the delay parameter, and degrees of freedom, the convergence diagnostics provide a similar conclusion (Table 1).

Table D1: Convergence Diagnostics for Selected Parameters

Parameter:	Threshold value	Delay parameter	Degrees of freedom
Statistics:			
Autocorr. at lag=10	0.0432	0.0188	0.0135
Inefficiency factor	0.4283	1.2421	0.3599
Number of draws	1043	8888	960

Appendix E: RMSE

Table E1: RMSE, Models with BAA Spread

1 regime, normal errors					2 regimes, normal errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	1.23	0.89	0.37	0.47	1	1.32	0.76	**0.33**	0.52
2	1.39	1.12	0.69	**0.74**	2	1.44	0.85	**0.60**	0.76
3	1.49	1.14	0.97	**0.87**	3	1.59	0.88	**0.89**	0.89
4	1.52	1.06	1.27	**0.97**	4	1.56	0.96	**1.23**	1.04
5	1.56	**0.98**	**1.55**	**1.05**	5	1.57	1.03	1.57	1.17
6	1.55	**0.98**	**1.81**	**1.11**	6	1.60	1.02	1.92	1.26
7	**1.58**	**0.97**	**2.06**	**1.14**	7	1.64	1.04	2.28	1.31
1 regime, t-distributed errors					2 regimes, t-distributed errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	**1.14**	0.92	0.36	**0.46**	1	1.27	**0.73**	0.33	0.52
2	**1.28**	1.22	0.68	0.74	2	1.39	**0.81**	0.61	0.75
3	**1.41**	1.31	1.00	0.91	3	1.54	**0.84**	0.91	0.90
4	**1.46**	1.33	1.32	1.03	4	1.54	**0.95**	1.26	1.05
5	**1.53**	1.28	1.61	1.10	5	1.56	**0.98**	1.62	1.18
6	**1.54**	1.27	1.88	1.16	6	1.55	1.02	1.99	1.28
7	1.59	1.25	2.13	1.19	7	1.60	1.02	2.37	1.35

Note: The lowest value for a particular variable and horizon is in bold.

Table E2: RMSE, Models with Mix Variable

1 regime, normal errors					2 regimes, normal errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	1.22	0.81	**0.35**	**0.0064**	1	1.14	**0.80**	0.62	0.0066
2	1.29	1.02	**0.65**	**0.0061**	2	1.23	0.94	0.96	0.0065
3	1.35	1.07	**0.94**	0.0063	3	1.28	1.05	1.32	0.0072
4	1.37	**1.06**	**1.26**	0.0061	4	1.34	1.13	1.63	0.0074
5	1.48	**1.02**	**1.55**	0.0062	5	1.52	1.19	1.95	0.0077
6	1.49	**1.03**	**1.82**	0.0063	6	1.62	1.19	2.18	0.0083
7	1.56	**1.01**	**2.06**	0.0064	7	1.76	1.21	2.42	0.0086
1 regime, t-distributed errors					2 regimes, t-distributed errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	**1.10**	0.92	0.36	0.0066	1	1.18	0.83	0.54	0.0064
2	1.18	1.21	0.67	0.0063	2	**1.15**	**0.94**	0.89	0.0067
3	1.25	1.31	0.98	**0.0062**	3	**1.22**	**1.03**	1.17	0.0071
4	1.31	1.34	1.30	**0.0061**	4	**1.26**	1.09	1.47	0.0070
5	1.44	1.31	1.60	**0.0061**	5	**1.44**	1.10	1.77	0.0072
6	**1.47**	1.30	1.87	**0.0062**	6	1.50	1.11	2.03	0.0075
7	**1.53**	1.28	2.12	**0.0063**	7	1.59	1.09	2.27	0.0081

Note: The lowest value for a particular variable and horizon is in bold.

Table E3: RMSE, Models with FCI

	1 regime, normal errors					2 regimes, normal errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	1.17	1.03	0.36	0.40	1	1.31	0.99	0.49	0.46
2	1.43	1.51	0.71	0.57	2	1.68	1.39	0.94	0.68
3	1.55	1.85	1.06	0.67	3	1.93	1.72	1.37	0.84
4	1.59	2.06	1.45	0.74	4	2.04	2.04	1.76	0.98
5	1.57	2.19	1.80	0.79	5	2.20	2.38	2.09	1.14
6	1.63	2.27	2.13	0.85	6	2.45	2.89	2.41	1.31
7	1.67	2.35	2.43	0.92	7	2.78	3.67	2.76	1.53
	1 regime, t-distributed errors					2 regimes, t-distributed errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	**1.10**	**0.87**	**0.34**	**0.36**	1	1.36	0.90	0.57	0.41
2	**1.31**	1.14	**0.66**	**0.52**	2	1.52	**1.05**	1.06	0.56
3	**1.43**	1.21	**0.97**	**0.61**	3	1.69	**1.12**	1.51	0.67
4	**1.48**	1.23	**1.31**	**0.69**	4	1.66	**1.15**	1.88	0.76
5	**1.51**	1.18	**1.63**	**0.75**	5	1.74	**1.16**	2.19	0.85
6	**1.53**	**1.17**	**1.91**	**0.80**	6	1.81	1.21	2.46	0.92
7	**1.58**	**1.15**	**2.17**	**0.83**	7	1.86	1.27	2.72	0.99

Note: The lowest value for a particular variable and horizon is in bold.

Table E4: RMSE, AR(1) Model

	Output gr.	Inflation	FF rate	BAA	Mix var.	FCI
1	1.27	0.77	0.42	0.49	**0.0063**	0.41
2	1.44	0.95	0.76	0.80	0.0061	0.63
3	1.51	1.00	1.09	1.03	0.0063	0.78
4	1.54	1.02	1.42	1.17	0.0062	0.89
5	1.55	1.01	1.72	1.27	0.0062	0.98
6	1.57	1.01	1.99	1.33	0.0062	1.05
7	1.60	1.00	2.24	1.38	0.0063	1.10

Appendix F: Average Logarithmic Score

Table F1: Average Logarithmic Score, Models with BAA Spread

1 regime, normal errors					2 regimes, normal errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-1.06	**-0.84**	-0.82	-0.74	1	-1.00	-0.90	-0.83	-0.66
2	-1.12	**-0.97**	-0.87	-0.73	2	-1.01	-1.01	-0.90	-0.67
3	-1.20	-1.13	-0.92	-0.68	3	-1.08	**-1.02**	-0.95	-0.65
4	-1.24	-1.20	-0.96	-0.62	4	-1.12	**-1.03**	-1.00	-0.63
5	-1.29	-1.21	-1.01	-0.58	5	-1.17	**-1.10**	-1.04	-0.61
6	-1.31	-1.24	-1.05	-0.54	6	-1.20	**-1.16**	-1.08	-0.59
7	-1.33	-1.25	-1.08	-0.50	7	-1.22	-1.22	-1.11	-0.56
1 regime, t-distributed errors					2 regimes, t-distributed errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-1.00	-1.04	**-0.81**	-0.67	1	**-0.93**	-0.95	-0.82	**-0.64**
2	-1.07	-1.23	**-0.86**	-0.66	2	**-0.95**	-1.03	-0.88	**-0.63**
3	-1.13	-1.41	**-0.90**	-0.61	3	**-1.02**	-1.06	-0.93	**-0.60**
4	-1.17	-1.49	**-0.94**	**-0.56**	4	**-1.06**	-1.06	-0.97	-0.57
5	-1.22	-1.50	**-0.97**	**-0.51**	5	**-1.09**	-1.10	-1.00	-0.55
6	-1.23	-1.48	**-1.00**	**-0.47**	6	**-1.10**	-1.16	-1.04	-0.53
7	-1.24	-1.47	**-1.04**	**-0.43**	7	**-1.13**	**-1.18**	-1.07	-0.50

Note: Bold indicates the highest average logarithmic score within the TVAR models.

Table F2: Average Logarithmic Score, Models with Mix Variable

1 regime, normal errors					2 regimes, normal errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.99	**-0.83**	-0.80	-0.73	1	**-0.86**	-0.88	-0.82	-0.72
2	-1.02	**-0.92**	-0.84	-0.72	2	-0.84	-0.94	-0.86	-0.72
3	-1.06	-1.06	-0.88	-0.71	3	**-0.85**	**-0.91**	-0.90	-0.71
4	-1.07	-1.11	-0.92	-0.69	4	**-0.84**	**-0.92**	-0.93	-0.69
5	-1.16	-1.12	-0.95	-0.63	5	**-0.88**	**-0.89**	-0.96	-0.65
6	-1.17	-1.10	-0.98	-0.66	6	**-0.94**	**-0.95**	-0.99	-0.66
7	-1.23	-1.13	-1.00	-0.63	7	-0.94	**-0.94**	-1.01	-0.63
1 regime, t-distributed errors					2 regimes, t-distributed errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.92	-0.96	**-0.79**	-0.69	1	-0.87	-0.88	-0.80	**-0.68**
2	-0.94	-1.12	-0.83	**-0.68**	2	**-0.84**	-0.94	**-0.83**	-0.68
3	-0.99	-1.26	-0.85	**-0.67**	3	-0.88	-0.97	**-0.85**	-0.69
4	-1.00	-1.30	-0.88	**-0.66**	4	-0.87	-1.00	**-0.86**	-0.67
5	-1.07	-1.29	-0.91	**-0.61**	5	-0.93	-1.02	**-0.88**	-0.64
6	-1.08	-1.29	-0.93	**-0.63**	6	-0.94	-1.04	**-0.90**	-0.63
7	-1.12	-1.32	-0.95	**-0.61**	7	**-0.94**	-1.03	**-0.92**	-0.64

Note: Bold indicates the highest average logarithmic score within the TVAR models.

Table F3: Average Logarithmic Score, Models with FCI

1 regime, normal errors					2 regimes, normal errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-1.04	**-0.81**	**-0.80**	-0.74	1	-0.95	-0.90	-0.81	-0.73
2	-1.08	**-0.92**	-0.86	-0.71	2	-0.94	-1.00	-0.87	-0.73
3	-1.14	-1.07	-0.91	-0.69	3	-0.96	**-1.00**	-0.93	-0.73
4	-1.15	-1.13	-0.96	-0.66	4	**-0.97**	**-0.98**	-0.99	-0.72
5	-1.21	-1.15	-1.01	-0.64	5	-1.01	**-0.97**	-1.05	-0.70
6	-1.24	-1.17	-1.04	**-0.62**	6	-1.05	**-0.99**	-1.10	-0.69
7	-1.26	-1.19	-1.07	**-0.61**	7	-1.08	**-1.01**	-1.14	-0.68
1 regime, t-distributed errors					2 regimes, t-distributed errors				
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.98	-1.03	-0.81	**-0.66**	1	**-0.92**	-0.94	-0.83	-0.69
2	-1.03	-1.18	**-0.85**	**-0.65**	2	**-0.92**	-1.03	-0.89	-0.68
3	-1.09	-1.33	**-0.89**	**-0.64**	3	**-0.95**	-1.05	-0.95	-0.67
4	-1.10	-1.38	**-0.93**	**-0.64**	4	-0.98	-1.05	-1.00	-0.66
5	-1.13	-1.40	**-0.96**	**-0.63**	5	**-1.01**	-1.05	-1.06	-0.65
6	-1.14	-1.36	**-0.98**	-0.63	6	**-1.04**	-1.07	-1.11	-0.64
7	-1.15	-1.36	**-1.00**	-0.62	7	**-1.06**	-1.06	-1.16	-0.63

Note: Bold indicates the highest average logarithmic score within the TVAR models.

Appendix G: Average Logarithmic Score, 2002Q4–2008Q2

Table G1: Average Logarithmic Score, Models with BAA Spread, 2002Q4–2008Q2

	1 regime, normal errors					2 regimes, normal errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.90	-0.67	-0.84	-0.70	1	-0.94	**-0.63**	-0.81	-0.80
2	-0.88	-0.62	-0.91	-0.74	2	-0.88	**-0.57**	-0.86	-0.89
3	-0.89	-0.62	-0.90	-0.76	3	-0.85	**-0.56**	-0.83	-0.93
4	-0.89	-0.59	-0.87	-0.76	4	-0.83	**-0.55**	-0.79	-0.95
5	-0.90	-0.57	-0.82	-0.74	5	-0.81	**-0.53**	-0.75	-0.95
6	-0.92	-0.54	-0.80	-0.72	6	-0.84	**-0.51**	-0.71	-0.95
7	-0.94	-0.53	-0.77	-0.71	7	-0.84	**-0.50**	-0.68	-0.95
	1 regime, t-distributed errors					2 regimes, t-distributed errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.82	-0.68	-0.83	**-0.66**	1	**-0.81**	-0.64	**-0.79**	-0.78
2	-0.81	-0.65	-0.86	**-0.68**	2	**-0.80**	-0.61	**-0.82**	-0.84
3	-0.82	-0.64	-0.85	**-0.68**	3	**-0.80**	-0.59	**-0.80**	-0.87
4	-0.81	-0.62	-0.83	**-0.67**	4	**-0.80**	-0.58	**-0.77**	-0.86
5	-0.82	-0.60	-0.80	**-0.65**	5	**-0.80**	-0.56	**-0.74**	-0.85
6	-0.84	-0.58	-0.78	**-0.64**	6	**-0.80**	-0.54	**-0.71**	-0.86
7	-0.85	-0.57	-0.77	**-0.62**	7	**-0.81**	-0.53	**-0.68**	-0.85

Note: Bold indicates the highest average logarithmic score within the TVAR models.

Table G2: Average Logarithmic Score, Models with Mix Variable, 2002Q4–2008Q2

	1 regime, normal errors					2 regimes, normal errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.84	-0.66	-0.81	-0.81	1	**-0.80**	-0.65	-0.82	-0.78
2	-0.81	-0.61	-0.85	-0.79	2	-0.80	**-0.61**	-0.83	-0.73
3	-0.86	-0.59	-0.83	-0.75	3	-0.83	-0.59	-0.80	-0.65
4	-0.85	-0.56	-0.82	-0.69	4	-0.80	**-0.55**	-0.76	-0.59
5	-0.89	-0.54	-0.78	-0.64	5	-0.81	**-0.54**	-0.75	-0.57
6	-0.90	-0.53	-0.76	-0.65	6	-0.82	**-0.53**	-0.69	-0.57
7	-0.91	-0.53	-0.73	-0.63	7	-0.81	**-0.52**	-0.67	**-0.54**
	1 regime, t-distributed errors					2 regimes, t-distributed errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.81	-0.67	-0.80	-0.77	1	-0.81	**-0.63**	**-0.76**	**-0.70**
2	**-0.79**	-0.63	-0.83	-0.74	2	-0.81	-0.62	**-0.77**	**-0.66**
3	-0.81	-0.61	-0.82	-0.73	3	**-0.80**	**-0.59**	**-0.74**	**-0.62**
4	-0.80	-0.59	-0.79	-0.67	4	**-0.79**	-0.56	**-0.69**	**-0.58**
5	-0.82	-0.58	-0.77	-0.63	5	**-0.78**	-0.55	**-0.68**	**-0.56**
6	-0.83	-0.56	-0.75	-0.65	6	**-0.79**	-0.55	**-0.66**	**-0.57**
7	-0.85	-0.57	-0.74	-0.63	7	**-0.78**	-0.54	**-0.64**	-0.56

Note: Bold indicates the highest average logarithmic score within the TVAR models.

Table G3: Average Logarithmic Score, Models with FCI, 2002Q4–2008Q2

	1 regime, normal errors					2 regimes, normal errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.89	-0.65	-0.82	-0.66	1	-0.83	**-0.63**	-0.83	-0.65
2	-0.90	-0.60	-0.89	-0.66	2	-0.83	**-0.58**	-0.88	-0.64
3	-0.85	-0.58	-0.89	-0.67	3	-0.83	**-0.56**	-0.87	-0.64
4	-0.83	-0.55	-0.87	-0.66	4	-0.86	**-0.54**	-0.89	-0.63
5	-0.86	-0.54	-0.83	-0.66	5	-0.91	**-0.52**	-0.89	-0.62
6	-0.87	-0.52	-0.80	-0.66	6	-0.93	**-0.51**	-0.90	-0.61
7	-0.89	-0.52	-0.77	-0.65	7	-0.94	**-0.50**	-0.91	**-0.60**

	1 regime, t-distributed errors					2 regimes, t-distributed errors			
	Output gr.	Inflation	FF rate	Cr. cond.		Output gr.	Inflation	FF rate	Cr. cond.
1	-0.81	-0.66	**-0.81**	-0.65	1	**-0.79**	-0.65	-0.82	**-0.64**
2	-0.81	-0.62	**-0.86**	-0.65	2	**-0.80**	-0.61	-0.86	**-0.64**
3	**-0.80**	-0.61	**-0.85**	-0.66	3	-0.82	-0.60	-0.86	**-0.63**
4	**-0.79**	-0.59	**-0.84**	-0.65	4	-0.87	-0.58	-0.89	**-0.62**
5	**-0.81**	-0.57	**-0.81**	-0.65	5	-0.90	-0.57	-0.91	**-0.62**
6	**-0.82**	-0.56	**-0.79**	-0.65	6	-0.93	-0.56	-0.94	**-0.61**
7	**-0.83**	-0.56	**-0.77**	-0.65	7	-0.94	-0.56	-0.96	-0.60

Note: Bold indicates the highest average logarithmic score within the TVAR models.

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